COMP7405 - Assignment 3

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1 Interface Introduction

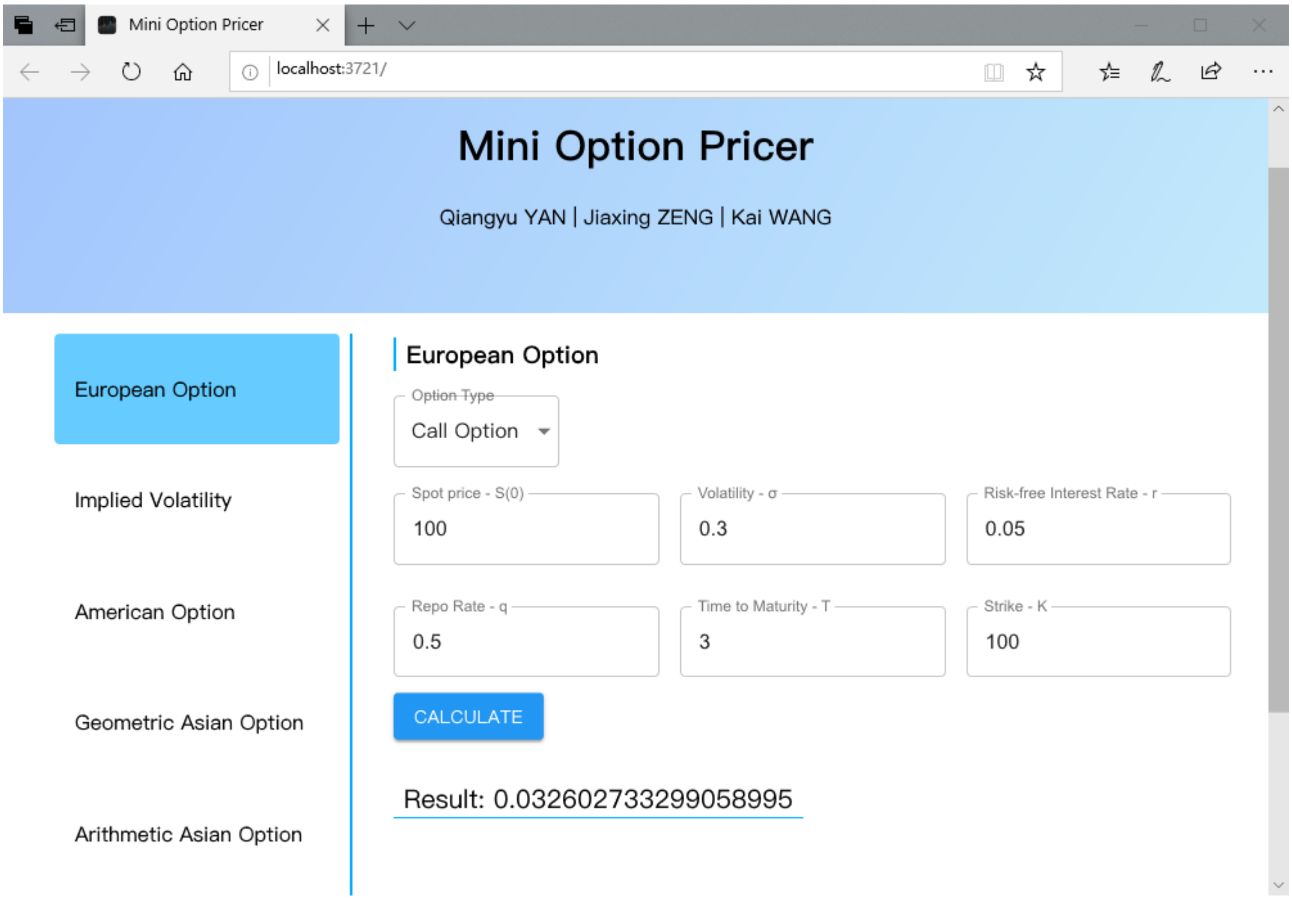


Figure 1: UI SnapShot

The User Interface is developed using *Flask* (A python web framework) and *HTML* & *CSS* & *JavaScript*. It's a webpage.

Look at the snapshot above. There is a menu on the left panel which can be click and navigate to different option pricer calculator. On the right panel, it provides several input fields, such as Option Type, Spot Price and so on.

After filling in these input fields, we can click on the button "CALCULATE" and the result will be displayed below.

2 Functionalities Explanation

```
.                                 # For more details, please refer to source codes.
|-- option_pricer
|    |-- binomial_tree.py         # American options
|    |-- black_scholes.py         # European options
|    |-- closed_form_formulas.py  # Geometric Asian/Basket options
|    |-- implied_volatility.py    # Implied volatility calculator
|    |-- MC.py                    # Monte Carlo Method (Arithmetic Asian/Basket option)
|    |-- server.py                # HTTP Server
|    |-- static                   # Web User Interface compiled file directory
|-- README.txt                    # Setup Guide
|-- requirement.txt               # Python Dependencies
|-- web                           # Web User Interface source code
```

Due to the page limitation, we do not list all the details. If you feel any doubts, please email to `qiangyuyan@gmail.com` or our school emails(`.@hku.hk`).

3 Test cases and analysis

Form previous knowledges (Black-Sholes formula), we know that in some situations(dividend) there maybe some variables that the option value may not be monotone respect to one of them, so all the analysis only based on the testing result, which means it may be slightly different under other conditions.

Table 1: European Option

No.	$S(0)$	K	T	q	σ	r	Call	Put
0	100	100	3	0.2	0.3	0.05	3.7385	34.9281
1+	**120**	100	3	0.2	0.3	0.05	7.4270	27.6404
1-	**80**	100	3	0.2	0.3	0.05	1.4346	43.6004
2+	100	**120**	3	0.2	0.3	0.05	2.0720	50.4758
2-	100	**80**	3	0.2	0.3	0.05	6.8517	20.8272
3+	100	100	**4**	0.2	0.3	0.05	2.9468	39.8870
3-	100	100	**2**	0.2	0.3	0.05	4.6054	28.0571
4+	100	100	3	**0.3**	0.3	0.05	0.9993	46.4131
4-	100	100	3	**0.1**	0.3	0.05	11.0827	23.0717
5+	100	100	3	0.2	**0.4**	0.05	7.1639	38.3535
5-	100	100	3	0.2	**0.2**	0.05	1.0750	32.2646
6+	100	100	3	0.2	0.3	**0.10**	5.6942	24.8948
6-	100	100	3	0.2	0.3	**0.03**	3.1088	39.6207

From Table 1, we can get: Based on Case 0, (1) Call Option value will increase(decrease) when $S(0)/\sigma/r$ increase(decrease), or $K/T/q$ decrease/increase; (2) Put Option value will increase(decrease) when $K/T/q/\sigma$ increase(decrease), or $S(0)/r$ decrease(increase).

Table 2: Implied Volatility

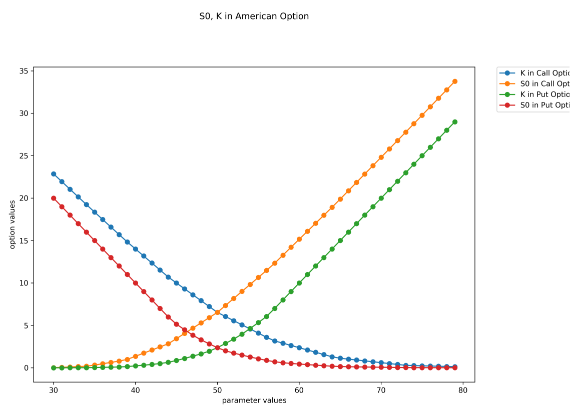
No.	Price	$S(0)$	K	T	q	r	σ_C	σ_P
0	25	100	100	1	0.2	0.05	0.9158	0.5044
1+	**27**	100	100	1	0.2	0.05	0.9801	0.5656
1-	**23**	100	100	1	0.2	0.05	0.8522	0.4429
2+	25	**120**	100	1	0.2	0.05	0.6165	0.7021
2-	25	**90**	100	1	0.2	0.05	1.0863	0.3346
3+	25	100	**110**	1	0.2	0.05	0.9904	0.2608
3-	25	100	**90**	1	0.2	0.05	0.8260	0.7054
4+	25	100	80	**2**	0.2	0.05	0.8541	0.1970
4-	25	100	80	**0.5**	0.2	0.05	1.0978	0.8143
5+	25	100	100	1	**0.3**	0.05	1.0775	0.3460
5-	25	100	100	1	**0.1**	0.05	0.7538	0.6205
6+	25	100	100	1	0.2	**0.07**	0.8994	0.5448
6-	25	100	100	1	0.2	**0.03**	0.9319	0.4620

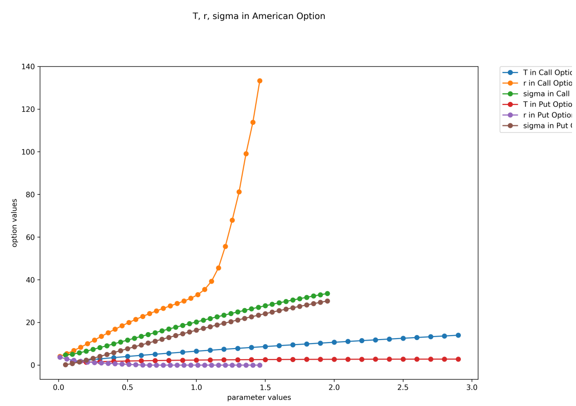
From Table 2, we can get: Based on Case 0, (1) Implied Volatility of Call Option will increase(decrease) when $OptionPrice/K/q$ increase(decrease), or $S(0)/T/r$ decrease/increase; (2) Implied Volatility of Put Option will increase(decrease) when $OptionPrice/S(0)/r$ increase(decrease), or $K/T/q$ decrease(increase).

Table 3: American Option

No.	$S(0)$	K	T	σ	r	N	Call	Put
0	100	100	3	0.3	0.05	5	27.6113	15.2564
1+	**120**	100	3	0.3	0.05	5	41.9701	9.1557
1-	**80**	100	3	0.3	0.05	5	13.2526	23.8372
2+	100	**120**	3	0.3	0.05	5	18.7749	26.2878
2-	100	**80**	3	0.3	0.05	5	36.4478	6.1963
3+	100	100	**4**	0.3	0.05	5	32.4964	16.4948
3-	100	100	**2**	0.3	0.05	5	21.9150	13.3923
4+	100	100	3	**0.4**	0.05	5	33.8292	21.6133
4-	100	100	3	**0.2**	0.05	5	21.3850	8.7914
5+	100	100	3	0.3	**0.10**	5	34.0587	10.9048
5-	100	100	3	0.3	**0.03**	5	25.1147	17.3811
6+	100	100	3	0.3	0.05	**7**	27.3803	11.0762
6-	100	100	3	0.3	0.05	**3**	28.1473	15.2788

From Table 3, we can get: Based on Case 0, (1) Call Option value will increase(decrease) when $S(0)/T/\sigma/r/N$ increase(decrease), or K decrease/increase; (2) Put Option value will increase(decrease) when $K/T/\sigma$ increase(decrease), or $S(0)/r/N$ decrease(increase).





For more information, we crate the chart to represent the relationship. Based on $K = 50, r = 0.1, \sigma = 0.2, S(0) = 50$, and the step of binomial tree is 10, we test K from 30 to 80, $S(0)$ from 30 to 80, T from 0.1 to 3, r from 0.01 to 1.5, σ from 0,05 to 2.

Table 4: Geometric Asian Option

Test No.	$S(0)$	K	T	σ	r	n	Call	Put
0	100	100	3	0.3	0.05	50	13.2591	8.4827
1+	**120**	100	3	0.3	0.05	50	26.6181	3.6722
1-	**80**	100	3	0.3	0.05	50	4.3379	17.7309
2+	100	**120**	3	0.3	0.05	50	6.5450	18.9828
2-	100	**80**	3	0.3	0.05	50	24.3475	2.3569
3+	100	100	**4**	0.3	0.05	50	15.1536	9.0403
3-	100	100	**2**	0.3	0.05	50	10.8693	7.5519
4+	100	100	3	**0.4**	0.05	50	15.7598	12.5588
4-	100	100	3	**0.2**	0.05	50	10.5384	4.6197
5+	100	100	3	0.3	**0.10**	50	15.6126	5.2850
5-	100	100	3	0.3	**0.03**	50	12.3055	10.1408
6+	100	100	3	0.3	0.05	**100**	13.1388	8.4311
6-	100	100	3	0.3	0.05	**30**	13.4200	8.5513

From Table 4, we can get: Based on Case 0, (1) Call Option value will increase(decrease) when $S(0)/T/\sigma/r$ increase(decrease), or K decrease/increase; (2) Put Option value will increase(decrease) when $K/T/\sigma$ increase(decrease), or $S(0)/r$ decrease(increase); (3) n usually won't affect the option value a lot.

Table 5: Arithmetic Option

No.	$S(0)$	K	T	σ	r	n	$m(k)$	$Call_0$	Put_0	$Call_1$	Put_1
0	100	100	3	0.3	0.05	50	100	[14.5328, 14.8193]	[7.7782, 7.9165]	[14.7196, 14.7412]	[7.8024, 7.8112]
0*	100	100	3	0.3	0.05	50	100	[14.6382, 14.9257]	[7.6845, 7.8224]	[14.7232, 14.7450]	[7.7949, 7.8038]
1+	**120**	100	3	0.3	0.05	50	100	[28.4026, 28.8034]	[3.1498, 3.2390]	[28.6764, 28.7044]	[3.1634, 3.1719]
1-	**80**	100	3	0.3	0.05	50	100	[5.0763, 5.2405]	[16.8179, 17.0013]	[5.1948, 5.2107]	[16.8699, 16.8796]
2+	100	**120**	3	0.3	0.05	50	100	[7.5411, 7,7620]	[17.9305, 18.1435]	[7.6950, 7.7152]	[17.9870, 17.9983]
2-	100	**80**	3	0.3	0.05	50	100	[25.8763, 26.2178]	[1.9273, 2.0356]	[26.1147, 26.1284]	[1.9846, 1.9913]
3+	100	100	**4**	0.3	0.05	50	100	[16.9239, 17.2637]	[8.1950, 8.3406]	[17.1441, 17.1746]	[8.2193, 8.2300]
3-	100	100	**2**	0.3	0.05	50	100	[11.6636, 11.8892]	[7.0222, 7.1473]	[11.8116, 11.8251]	[7.0453, 7.0518]
4+	100	100	3	**0.4**	0.05	50	100	[17.9294, 18.3309]	[11.2506, 11.4307]	[18.1931, 18.2340]	[11.2855, 11.3010]
4-	100	100	3	**0.2**	0.05	50	100	[11.1576, 11.3450]	[4.3411, 4.4312]	[11.2811, 11.2906]	[4.3569, 4.3607]
5+	100	100	3	0.3	**0.10**	50	100	[17.2140, 17.5077]	[4.8335, 4.9367]	[17.4057, 17.4299]	[4.8492, 4.8557]
5-	100	100	3	0.3	**0.03**	50	100	[13.4598, 13.7419]	[9.3022, 9.4564]	[13.6463, 13.6669]	[9.3326, 9.3425]
6+	100	100	3	0.3	0.05	**100**	100	[14.5575, 14.8447]	[7.6720, 7.8094]	[14.5923, 15.6135]	[7.7473, 7.7561]
6-	100	100	3	0.3	0.05	**30**	100	[14.5566, 14.8441]	[7.8557, 7.9952]	[14.8895, 14.9119]	[7.8707, 7.8797]

Default random seed is 10. Case 0^* is using 5 as random seed. $m(k)$ means in thousands, 100k-100,000. $Call_0$ and Put_0 are the result without control, $Call_1$ and Put_1 are the result with control variate.

From Table 5, we can get: Based on Case 0, (1) Call Option value will increase(decrease) when $S(0)/T/\sigma/r$ increase(decrease), or K decrease/increase; (2) Put Option value will increase(decrease) when $K/T/\sigma$ increase(decrease), or $S(0)/r$ decrease(increase); (3) n usually won't affect the option value a lot.

Table 6: Geometric Asian Basket

No.	$S_1(0)$	$S_2(0)$	K	T	σ_1	σ_2	r	ρ	Call	Put
0	100	100	100	3	0.3	0.3	0.05	0.5	22.1021	11.4916
1+	**120**	**120**	100	3	0.3	0.3	0.05	0.5	36.6832	6.7364
1-	**80**	**80**	100	3	0.3	0.3	0.05	0.5	10.5840	19.3097
2+	100	100	**120**	3	0.3	0.3	0.05	0.5	14.6855	21.2891
2-	100	100	**80**	3	0.3	0.3	0.05	0.5	32.5363	4.7116
3+	100	100	100	**4**	0.3	0.3	0.05	0.5	25.8367	12.1100
3-	100	100	100	**2**	0.3	0.3	0.05	0.5	17.6665	10.3751
4+	100	100	100	3	**0.5**	**0.5**	0.05	0.5	28.4494	23.4691
4–	100	100	100	3	**0.1**	**0.1**	0.05	0.5	14.7662	1.2113
4-	100	100	100	3	**0.1**	0.3	0.05	0.5	17.9247	6.5864
5+	100	100	100	3	0.3	0.3	**0.10**	0.5	29.0230	6.4235
5-	100	100	100	3	0.3	0.3	**0.03**	0.5	19.5131	14.2249
6+	100	100	100	3	0.3	0.3	0.05	**0.9**	25.8788	12.6224
6-	100	100	100	3	0.3	0.3	0.05	**0.3**	20.1535	10.8394

From Table 6, we can see: Based on Case 0, (1)Call Option value will increase(decrease) when $S_1(0)\&S_2(0)/T/\sigma_1\&\sigma_2/r/\rho$ increase(decrease), or K decrease(increase); (2)Put Option value will increase(decrease) when $K/T/\sigma_1\&\sigma_2/\rho$ increases(decreases), or $S_1(0)\&S_2(0)/r$ decreases(increases).

Table 7: Arithmetic Asian Basket

No.	$S_1(0)$	$S_2(0)$	K	T	σ_1	σ_2	r	ρ	$m(k)$	$Call_0$	Put_0	$Call_1$	Put_1
0	100	100	100	3	0.3	0.3	0.05	0.5	100	[24.1454, 24.6262]	[10.5387, 10.7284]	[24.4580, 24.5200]	[10.5670, 10.5913]
0*	100	100	100	3	0.3	0.3	0.05	0.5	100	[24.4490, 24.9341]	[10.4730, 10.6621]	[24.4576, 24.5201]	[10.5659, 10.5902]
1+	**120**	**120**	100	3	0.3	0.3	0.05	0.5	100	[39.5611, 40.1950]	[6.0868, 6.2355]	[39.9785, 40.0551]	[6.1012, 6.1212]
1-	**80**	**80**	100	3	0.3	0.3	0.05	0.5	100	[11.7723, 12.0930]	[18.0303, 18.2597]	[11.9840, 12.0300]	[18.0785, 18.1061]
2+	100	100	**120**	3	0.3	0.3	0.05	0.5	100	[16.2783, 16.6952]	[19.8146, 20.0828]	[16.5535, 16.6121]	[19.8698, 19.9024]
2-	100	100	**80**	3	0.3	0.3	0.05	0.5	100	[34.9971, 35.5335]	[4.2431, 4.3546]	[35.3517, 35.4157]	[4.2524, 4.2676]
3+	100	100	100	**4**	0.3	0.3	0.05	0.5	100	[28.7083, 29.2935]	[10.9846, 11.1824]	[29.0876, 29.1743]	[11.0139, 11.0426]
3-	100	100	100	**2**	0.3	0.3	0.05	0.5	100	[18.9245, 19.2925]	[9.6463, 9.8201]	[19.1646, 19.2037]	[9.6725, 9.6910]
4+	100	100	100	3	**0.5**	**0.5**	0.05	0.5	100	[34.2764, 35.2139]	[21.0047, 21,2980]	[34.8770, 35.0872]	[21.0632, 21.1196]
4–	100	100	100	3	**0.1**	**0.1**	0.05	0.5	100	[14.9704, 15.1371]	[1.1592, 1.2010]	[15.0839, 15.0903]	[1.1596, 1.1619]
4-	100	100	100	3	**0.1**	0.3	0.05	0.5	100	[19.1806, 19.5245]	[5.4966, 5.6117]	[19.4154, 19.4533]	[5.5160, 5.5331]
5+	100	100	100	3	0.3	0.3	**0.10**	0.5	100	[31.3679, 31.8890]	[5.8200, 5.9544]	[31.7094, 31.7730]	[5.8338, 5.8517]
5-	100	100	100	3	0.3	0.3	**0.03**	0.5	100	[21.4166, 21.8780]	[13.1101, 13.3245]	[21.7161, 21.7771]	[13.1454, 13.1724]
6+	100	100	100	3	0.3	0.3	0.05	**0.9**	100	[25.9909, 26.5407]	[12.3960, 12.6073]	[26.3479, 26.3605]	[12.4286, 12.4340]
6-	100	100	100	3	0.3	0.3	0.05	**0.3**	100	[23.1861, 23.6333]	[9.5634, 9.7409]	[23.4685, 23.5544]	[9.5887, 9.6203]

Default random seed is 10. Case 0^* is using 5 as random seed. $m(k)$ means in thousands, 100k-100,000.
$Call_0$ and Put_0 are the result without control, $Call_1$ and Put_1 are the result with control variate.

From Table 7, we can see: Based on Case 0, (1)Call Option value will increase(decrease) when $S_1(0)\&S_2(0)/T/\sigma_1\&\sigma_2/r/\rho$ increase(decrease), or K decrease(increase); (2)Put Option value will increase(decrease) when $K/T/\sigma_1\&\sigma_2/\rho$ increases(decreases), or $S_1(0)\&S_2(0)/r$ decreases(increases).

Summary:

(1) Based on our cases, usually `Stock Price(`$S(0)$`)/Volatility(`σ`)/ Interest rate(`r`)` and `Call Option value` are positively correlated; `Strike Price(`K`)/Volatility(`σ`)/Maturity Time(`T`)` and `Put Option value` are positively correlated;

(2) Based on our cases, usually `Call Option value` and `Strike Price(`K`)` are negatively correlated; `Put Option value` and `Stock Price(`$S(0)$`)/Interest rate(`r`)` are negatively correlated.

(3) Based on our cases, `Maturity Time(`T`)` usually is positively correlated with `Call Option Value`, but sometimes(as shown in Table 1) it will be negatively correlated with `Call Option Value`.

4 Contributions

Task	YAN Qiangyu	ZENG Jiaxing	WANG Kai
European Option		√	
Implied Volatility		√	
American Option			√
Geometric Asian Option			√
Arithmetic Asian Option	√		
Geometric basket Option			√
Arithmetic basket Option	√		
UI Design		√	
Test	√	√	√
Report	√	√	