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Models Of Solar Wind

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Abstract

Solar wind is a complex phenomenon of gas being released from the Sun. Since at least 1957, models have attempted to replicate the mechanism by which solar wind accelerates outwards from the Sun's corona to beyond Earth's orbit. The goal of this project is to investigate and compare these early models. We will see how previous researchers arrived at each model from the equations of motion and magnetohydrodynamic equations, as well as looking at the assumptions used and the drawbacks to each model that result from these simplifications. Comparing the models we will see how each improves upon the last, and develop a greater understanding of how more recent models of the solar wind were developed.

1 Introduction

In this project we will be investigating solar wind. Solar wind originates from the Sun's outer atmosphere, known as the Sun's corona. The corona is an aura of plasma that surrounds the Sun and extends for millions of miles, most visible during solar eclipses. The corona reaches temperatures of over 1 million kelvin. The solar wind released from the corona is a continuous stream of plasma that escapes the gravity of the Sun and flows outward into solar space, towards the Earth and other interstellar objects. It is made up of charged particles, including electrons, alpha particles and protons. The term 'solar wind' was first used by Parker, when he recognized that the same phenomenon was responsible for both Chapman's model of heat flow from the Sun, and for Biermann's hypothesis suggesting that comet tails blow away from the sun because of gas streaming outward. The solar wind is non-uniform, but always points outward from the Sun. Owing to the high temperature of the corona, the Sun's gravity is insufficient to retain charged particles in the corona, resulting in streams of charged particles that are accelerated to reach speeds of up to 800 km/s. The details of how particles get accelerated to such high speeds is not fully understood, since the thermal velocity of a hydrogen ion is only 260 km/sec (Parker, 1958) – substantially less than the 500 km/sec required for particles to escape the Sun's gravitational pull. This suggests that there is another other mechanism for the acceleration of particles that creates solar wind, which we shall explore.

The idea of particles flowing out of the sun has been suggested since at least 1859, when Carrington and Richard Hodgson both independently observed a "sudden localized increase in brightness from the solar disc" (noa, 2019), which would later be known as a solar flare. Several days later a geomagnetic storm was seen, followed by a "failure of telegraph communications all over Europe and North Africa" (Obridko and Vaisberg, 2017), leading Carrington to believe that these two events were connected, and laying the foundations for the idea that mass is ejected from the Sun in a constant stream. In 1956 Biermann claimed that there was a stream of plasma from the Sun that was dense enough to affect comet tails. Parker later used the materials from Biermann's hypothesis to calculate that it was possible that such a flow existed, and that the Sun's atmosphere was not static but highly dynamic. Parker's paper was denied publication by several scientific journals for almost a year, and although it was published in 1958 it was not widely popular until further papers and data followed in later years.

Much of the current research into solar wind comes via the Parker Solar Probe (Garner, 2017). The probe was launched on 12$^{\text{th}}$ August 2018, and will use Venus' gravity during seven flybys to gradually bring it about 3.8 million miles away from the Sun's surface – more then seven times closer to the Sun then any other spacecraft. During its flight the aim is to expand our knowledge of the origin of solar wind, using *in situ* measurements and imaging. As of 11$^{\text{th}}$ December 2019, the probe had completed three orbits. Using the initial data, Tim Horbury examined more closely the magnetic 'switchbacks' in which the solar magnetic field bends back on itself in a whip-like motion (Tran, 2019). The origin of these switchbacks is not certain, but they are thought to be "signatures of the process that heats the Sun's outer atmosphere". Bale and Badman (2019) used the initial data to compare with the Potential Field Source Surface (PFESS) model, which is an approximate description of the solar coronal magnetic field, based on observed photoelectric fields. From this comparison, Bale and Badman (2019) suggest that low-latitude coronal holes are a key source of slow solar wind.

As the solar wind flows from the Sun it decreases the mass of the Sun by about 1.3 – 1.9 million tonnes a year. However, given the large mass of the Sun, this equates to only 0.01% of its mass that has been lost through this process. The solar wind itself has both fast and slow solar winds, reaching speeds of 750 km/s and 400 km/s respectively. The way in which the Sun's magnetic field interacts with the plasma means that the slow solar wind is most prevalent at the equatorial region of the Sun, while the fast solar wind is located closer to the poles. Owing to the constant flow of the plasma and the rotation of the Sun, the magnetic field is twisted into a spiral and this creates a complex structure. The difference in speed of the winds is shown in Figure 1;

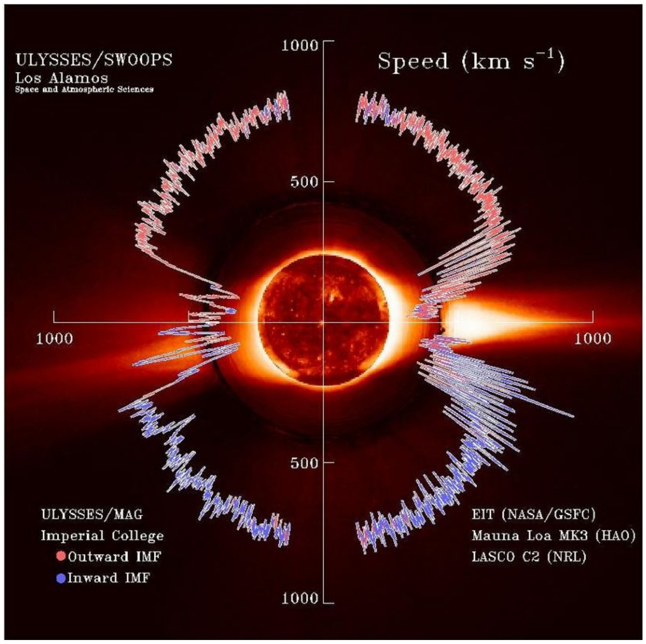

Figure 1: Solar wind from the Sun

We will first discuss Parker's model for the solar wind in our Universe, before addressing some of its limitations. In particular, we will address Parker's assumption that gravity and pressure gradient are the dominant forces in the equation of motion for solar wind. We will then move on to consider models that include magnetic fields, such as Webers and Davis' model developed in 1967 (Weber and Davis, 1967), and consider the influence of magnetic fields on solar wind and its acceleration. We will also consider Parker's own review of his original paper (Parker, 1965), following the collection of data during the space race.

2 Parker's basic model (1958)

2.1 Static equilibrium

Parker's model neglected any solar magnetic fields, choosing to solve a hydrodynamic problem rather then a hydromagnetic one. First, there is no hydrostatic equilibrium solution with vanishing pressure at infinity. We will state that the total gas pressure for gas that is fully ionized is 2NkT, where N is the number of particles, T is the temperature and k is Boltzmann's constant, which is 1.38×10^{-23}J/K. We must account for both electrons and protons in the model, under the assumption that there is an equal number of electrons and protons and that they have the same average temperature. The gas is fully ionized as a result of the high corona temperature. Assuming static equilibrium we have the following hydrostatic equation

$$\rho \mathbf{g} - \bigtriangledown P. \tag{1}$$

We here assume that the gravity is acting radially inwards towards the Sun, such that $\mathbf{g} = -\left(GM_\odot/r^2\right)\mathbf{e}_r$, which leads to the barometric relation of

$$0 = \frac{d}{dr}(2\text{NkT}) + \frac{GM_\odot MN}{r^2}. \tag{2}$$

For which $M_\odot$ is the mass of the mass of the Sun, G is the gravitational constant 6.67×10^{-11} and M is the mass of the hydrogen. Far away from the Sun there is no local heat source, hence we get the steady-state heat-flow equation $\bigtriangledown \cdot [k(T) \bigtriangledown T] = 0$, requiring T(r) to fall off at distance $r^{\frac{1}{n+1}}$. Suppose there is no heating after r=a, where a is the corona, we get that

$$T(r) = T_0 \left(\frac{a}{r}\right)^{1/(n+1)}.$$

We then integrate equation (2) to get

$$N(r) = N_0 \left(\frac{r}{a}\right)^{1/(n+1)} \exp\left(\left[\frac{\lambda(n+1)}{n}\right]\left[\left(\frac{a}{r}\right)^{n/(n+1)} - 1\right]\right), \tag{3}$$

$$\lambda = \frac{GM_\odot MN}{2kT_0 a}.$$

Where λ is a dimensionless parameter. If $n > 0$ we get that N(r) will become infinite as r becomes larger. As we move further away from the Sun's corona the density will tend to infinity, which cannot be. If $n < 0$, N(r) will vanish at infinity, and at $n = 0$ we get that

$$\left[\frac{n+1}{n}\right]\left[1 - \left(\frac{a}{r}\right)^{n/n+1}\right] \longrightarrow \ln\left(\frac{a}{r}\right),$$

giving us

$$N(r) = N_0 \left(\frac{a}{r}\right)^{\lambda-1}.$$

We also get that the pressure $p(r) = 2NkT$ varies for $n \neq 0$ with

$$p(r) = p_0 \times \exp\left(\left[\frac{\lambda(n+1)}{n}\right]\left[\left(\frac{a}{r}\right)^{n/(n+1)} - 1\right]\right)$$

and for n=0 we get

$$p(r) = p_0 \left(\frac{a}{r}\right)^{\lambda}.$$

As r approcess infinity we have

$$p(\infty) = p_0 \times \exp\left[\frac{-\lambda(n+1)}{n}\right].$$

With $a = 10^6$ km, $T_0 = 1.5 \times 10^6$ K, $M_\odot = 2 \times 10^{33}$ gm and $n = 2.5$ for ionised hydrogen we get that $\lambda = 5.35$, which gives us $p(\infty) = 0.55 \times 10^{-3} p_0$. If we take standard coronal conditions for temperature and density we get that $p_0 \approx 1.3 \times 10^{-2}$ dyn/cm^2, making $p(\infty) = 0.6 \times 10^{-5}$ dyn/cm^2 for $n = 2.5$.

We see that it is not possible for there to be a general pressure to balance $p(\infty)$ with the expected n out for large distances, meaning it is not possible for the solar corona to have complete hydrostatic equilibrium at large distances.

2.2 Uniform expansion

Now looking at the stationary expansion of the solar corona assuming T(r) is given, we see that it satisfies the Cauchy momentum equation

$$NMv\frac{dv}{dr} = -\frac{d}{dr}(2NkT) - GNMM_{\odot}\frac{1}{r^2}. \tag{4}$$

We have the equation of mass continuity being

$$\frac{d}{dr}\left(r^2Nv\right) = 0,$$

Assuming the corona is spherical symmetric we get

$$r^2N(r)v(r) = C = N_0v_0a^2,$$

$$N(r)v(r) = N_0v_0\left(\frac{a}{r}\right)^2. \tag{5}$$

Introducing the dimensionless variables $\lambda = GMM_{\odot}/2akT_0$,$\tau = T(r)/T_0$,$\psi = \frac{1}{2}Mv^2/kT_0$ and $\xi = r/a$, and using the above equation to eliminate N, we have

$$\frac{d\psi}{d\xi}\left(1 - \frac{\tau}{\psi}\right) = -2\xi^2\frac{d}{d\xi}\left(\frac{\tau}{\xi^2}\right) - \frac{2\lambda}{\xi^2}. \tag{6}$$

We assume that T_0 is the uniform temperature from $r = a$ to some radius b, and beyond b the heating from the corona vanishes. Hence making T negligible beyond b. Taking $\tau = 1$ for $r < b$, then integrating equitation (6) yields

$$\psi - \ln\psi = 4\ln\xi + \frac{2\lambda}{\xi} - 2\lambda + \psi_0 - \ln\psi_0. \tag{7}$$

The constant of integration ψ_0 is such that $\psi_0 = \psi$ at $r = a$. Using the approach used in Boyd and Sanderson (2003) we have

$$v_c^2 = \frac{2kT_0}{M} = \frac{2kT_0N}{MN} = \frac{P}{\rho}, \tag{8}$$

This leads to

$$\left(\frac{v}{v_c}\right)^2 - \ln\left(\frac{v}{v_c}\right)^2 = 4\ln\frac{r}{a} + \frac{2\lambda a}{r} - 2\lambda + \psi_0 - \ln\psi_0. \tag{9}$$

If we look at the solution of the curve at which $\lambda = 2$ we get the contours of the equation to be,
At $\psi_0 - \log\psi_0 = 1$ we have the line that intersects the point (1,1). This figure shows the solution curves for Parker's solar wind model, with a critical point at $(1, 1)$ that translates to when $v = v_c$ and $r = a$. To the right and left of the critical point shows solutions that are double-valued, and hence physically unacceptable. Above the critical point are the solutions in which the solar wind is entirely supersonic, and below which they are entirely subsonic; neither correspond to what has been observed. As the solar wind is subsonic at the Sun and supersonic near the Earth, the only acceptable solution is that of the curve that goes through the critical point, where it transitions from subsonic to supersonic and has a positive gradient.

Then solving this equation near the Earth such that $r = 214r_{\odot}$ and $a = 20r_{\odot}$ with $r_{\odot}$ being the solar radius, leads to the numerical solutions for v/v_c being

$$[0.008913395121, 3.452329350, -0.008913395121, -3.452329350]$$

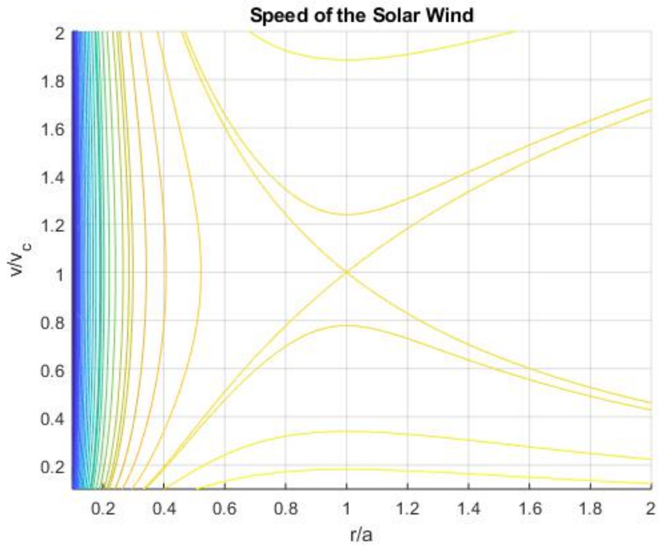

Figure 2: Parker's solution for the solar wind model

T_0 in terms of 10^6 K	Velocity at $r = 10R_0$ km	Velocity at $r = 214R_0$ km
1	128	443
1.5	157	543
2	181	627
2.5	203	701
3	222	768
4	257	887

With 3.452329350 being the only appropriate solution, we determine a speed of around 543kms^{-1} near the Earth's orbit with an assumed temprature of $T_0 = 1.5 \times 10^6$ K. The table above shows the velocity of the gas according to the model, for various values of T_0 at the Earth's orbit and at the Sun's corona. The terminal velocity of $T_0 = 1.5 \times 10^6$ K is sufficient to push the gas out of the solar gravitational field, which has an escape velocity of 500kms^{-1}. However, this model predicts the density near the Earth to be two orders of magnitude too high. The model could therefore be improved by considering magnetic fields, which Parker's model did not include. We should also consider that the solar wind is not uniformly distributed from the Sun, and instead look at an open region of the Sun's field referred to as "coronal holes".

2.3 Coronal holes

Instead of a spherical uniform expansion from the Sun, we now consider a local spherical expansion centered at an active region of the corona. The region is confined to a narrow cone with a cross-sectional area of A(r), which varies with the area becoming larger as distance from the active region becomes greater.We denote s to be the distance from the origin of the spherical expansion to the centre of the Sun. This give us the continuity equation of

$$N(r)v(r) = N_0 v_0 \left(\frac{a-s}{r-s}\right)^2, \tag{10}$$

where s is the distance from the centre of the Sun to the origin of the spherical expansion of the gas. This modifies equation (7) to become

$$\psi - \log\psi = 4\log\left(\frac{\xi - \frac{s}{a}}{1 - \frac{s}{a}}\right) + \frac{2\lambda}{\xi} - 2\lambda + \psi_0 - \log\psi_0. \tag{11}$$

With the same gravitational force as before, we get that ψ must equal unity where ξ is equal to

$$\xi = \left(\frac{\lambda}{4}\right)\left[1 + \left(1 - \frac{8s}{\lambda a}\right)\right]^{\frac{1}{2}}.$$

We note that $s/a < \lambda/8$, otherwise there is no solution to equation with values such that $\psi < 1$ at $\xi = 1$ and going to $\psi > 1$ as $\xi \to \inf$. The figure below plots the solution of equation , with one line where we take $s/a = 0.4$ and $\lambda = 0.5$ and we plot this, and the other line for which we take $s/a = 0$ and λ being the same. We demonstrate an increase in the value of ψ with the only change being the distance of the origin of the spherical expansion to the centre of the Sun. Decreasing s increases the cross-sectional area A(r) as it leaves the corona, whereas increasing s decreases the cross-sectional area, in a sense narrowing the cone. The decrease in cross-section results in higher values for ψ, but the only part of ψ that can change is v, hence decreasing the cross-sectional area increases the speed of the solar wind without adding any additional temperature to the system, and allows corona holes to be a source of increased wind speeds,

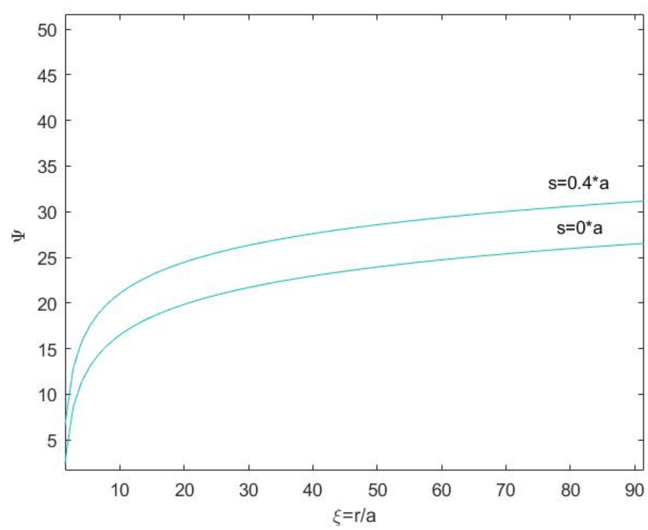

Figure 3: Solar wind speeds with a narrow cone

2.4 Cross-sectional Area

To show that decreasing the cross-sectional area of the narrow cone of expansion at the corona affects the velocity of the solar wind, we consider the velocity $\mathbf{v} = v_r \mathbf{e}_r$ and A(r) that varies with radius (Leer and Holzer, 1979). We will consider the conservation of mass,

$$N v_r A(r) = constant = q \tag{12}$$

and the conservation of momentum being equation , while using the expressions of ρ and P as stated in equation (8) to get

$$NMv_r \frac{dv_r}{dr} = -\frac{dP}{dr} - NM\frac{GM_\odot}{r^2}. \tag{13}$$

Considering separately one-fluid models appropriate to a polytropic equation of state we get that our pressure P can be represented as

$$P = P_0 \left(\frac{N}{N_0}\right)^\alpha. \tag{14}$$

For an isothermal case such as ours, we will have that $\alpha = 1$. Combining the equations (12)-(14) we get

$$v_r \frac{dv}{dr} = -\frac{1}{NM}\frac{d}{dr}\left(P_0\left(\frac{N}{N_0}\right)\right) - \frac{GM_\odot}{r^2}$$

$$\Rightarrow v_r \frac{dv}{dr} = -\frac{1}{NM}\frac{P_0}{N_0}\frac{d}{dr}\left(\frac{q}{v_r A}\right) - \frac{GM_\odot}{r^2}.$$

We get that

$$\frac{d}{dr}\left(\frac{q}{v_r A}\right) = -q\left(\frac{1}{v_r^2 A}\frac{dv_r}{r} + \frac{1}{v_r A^2}\frac{dA}{r}\right),$$

Which finally leads to

$$\frac{1}{v_r}\frac{dv_r}{dr}\left(v_r^2 - \frac{2kT}{M}\right) = \frac{2kT}{M}\frac{1}{A}\frac{dA}{dr} - \frac{GM_\odot}{r^2}. \tag{15}$$

We see that there is a critical point at $v_r = 2kt/M$ where the right hand side of the equation will become zero. This leads to a critical point radius of

$$r_c = \frac{GM_\odot MA}{2kTr}\frac{1}{\frac{dA}{dr}} = \frac{GM_\odot M}{4\beta_c kT}.$$

With $\beta = (r/2A)(dA/dr)$, where the subscript denotes the evaluation at the critical point. If we then integrate equation (15) from the solar surface ($r = r_0$) to the critical point $r = r_c$ we obtain

$$\ln\left(\frac{v_0}{v_c}\right) = \ln\left(\frac{A_c}{v_0}\right) - \frac{1}{2} + 2\beta_c - \frac{GM_\odot}{2v_c^2},$$

Which leads the expression for v_0 to be

$$v_0 = v_c\frac{A_c}{A_0}\exp\left(-\frac{GM_\odot}{2v_c^2} - \frac{1}{2} + 2\beta_c\right).$$

We have that mass flux is $j_m = \rho v = MNv$. This gives the mass flux at the critical point to be

$$j_c = NMv_0\frac{A_0}{A_c}exp\left(\frac{GM_\odot}{2v_c^2} + \frac{1}{2} - 2\beta_c\right).$$

We can see from the above equation that there are several ways to increase the mass flux, including increasing the velocity of the plasma as it leaves the base of the corona, and decreasing the ratio between the area at the base of the corona and the area at the critical point. If we consider $A(r) = cr^{\alpha}$, where c and α are positive constants, we get that $B_c = \alpha$, leading to

$$r_c = \frac{GM_{\odot}M}{4\alpha kT} \approx \frac{1}{\alpha}2.7711 \times 10^{-14}\text{m}.$$

We also get that the mass flux is,

$$j_c = NMv_0 \left(\frac{r_0}{r_c}\right)^{\alpha} exp\left(\frac{GM_{\odot}}{2v_c^2} + \frac{1}{2} - 2\alpha\right)$$

As $r_0 < r_c$ we have that $r_0/r_c < 1$. If we decrease the value of α this will actually increase the total mass flux density at the critical point, and will therefore increase the value of r_c. Hence we find that decreasing the cross-sectional area $A(r)$ will increase the total mass flux, in turn increasing the velocity of the solar wind.

3 Weber and Davis Model

3.1 Magnetohydrodynamic Model

In the Weber and Davis Model, flow is taken to be magnetohydrodynamic, whereby the flow interacts with the magnetic field of the Sun and the magnetic field is assumed to only vary with latitude, and to have symmetry about the axis of rotation. We will mainly be focusing on the equatorial plane, also assuming that the flow has no viscosity, with infinite conductivity, and its pressure is scalar acting in all directions. We have that the solar wind has a velocity as follows

$$\mathbf{v} = u\mathbf{e}_r + v_{\phi}\mathbf{e}_{\phi} \tag{16}$$

and a magnetic field of

$$\mathbf{B} = B_r\mathbf{e}_r + B_{\phi}\mathbf{e}_{\phi}. \tag{17}$$

With the conservation of mass it is required that

$$\rho ur^2 = constant, \tag{18}$$

with ρ being the mass density. As the solar wind is such a good conductor we will have that $\mathbf{E} = -\mathbf{v} \times \mathbf{B}/c$. From Maxwells's equations we see that

$$c(\bigtriangledown \times \mathbf{E})_{\phi} = c(\bigtriangledown \times (-\mathbf{v} \times \mathbf{B}/c))_{\phi} = \frac{1}{r}\frac{d}{dr}\left[r(uB_{\phi} - v_{\phi}B_r)\right] = 0.$$

As the fluid itself is a perfect conductor, $\mathbf{v}$ must be parallel to the magnetic field in a frame rotating with the Sun. Hence, along with the equation above, we must get that

$$r(uB_{\phi} - v_{\phi}B_r) = constant = -\Omega r^2 B_r, \tag{19}$$

with Ω being the angular velocity of the roots of the lines of force in the Sun. Considering that Gauss's law from magnetism states that $\bigtriangledown \cdot \mathbf{B} = 0$, we also get that

$$\bigtriangledown \cdot \mathbf{B} = \frac{1}{r^2}\frac{d}{dr}(r^2 B_r) + \frac{1}{rsin(\theta)}\frac{d}{d\phi}B_{\phi} = \frac{1}{r^2}\frac{d}{dr}(r^2 B_r) = 0,$$

This implies that

$$r^2 B_r = const = r_0^2 B_0. \tag{20}$$

This model is symmetrical in the ϕ component, hence with the momentum term and the magnetic force term we get that the only term that matters is in the steady-state ϕ equation of motion are

$$\rho \frac{u}{r} \frac{d}{dr} (rv_\phi) = \frac{1}{c} (\mathbf{J} \times \mathbf{B})_\phi = \frac{1}{4\pi} [(\bigtriangledown \times \mathbf{B}) \times \mathbf{B}] = \frac{B_r}{4\pi r} \frac{d}{dr} (rB_\phi). \tag{21}$$

From equations (18) and (20) we find that

$$\frac{B_r}{4\pi\rho u} = \frac{B_r r^2}{4\pi\rho u r^2} = const,$$

This allows us to integrate the equation (21), which if first done by rearranging as,

$$\frac{d}{dr} (rv_\phi) = \frac{B_r r^2}{4\pi\rho u r^2} \frac{d}{dr} (rB_\phi).$$

$$rv_\phi - \frac{B_r}{4\pi\rho u} rB_\phi = const = L. \tag{22}$$

Where rv_ϕ represents the ordinary angular momentum per unit mass, and with the second term in the equation being the torque associated with the magnetic stresses. We introduce a new variable, M_A, known as the radial Alfvénic Mach number, which is given by

$$M_A^2 = \frac{4\pi\rho u^2}{B_r^2}.$$

Then from equation (19) we get that B_ϕ can be written as,

$$B_\phi = \frac{1}{u}(v_\phi B_r - \Omega r B_r)$$

Substituting this into equation (22) along with M_A^2 we get for the angular velocity

$$v_\phi = \Omega r \frac{M_A^2 L r^{-2} \Omega^{-1} - 1}{M_A^2 - 1}.$$

Near the surface of the Sun the radial Mach number M_A is much smaller then 1, but at 1 a.u. it is approximately 10. Thus, there is a point between the surface of the Sun and the Earth where $M_A = 1$, referred to as the Alfvénic critical point. At this point we will denote the radius as r_a and the radial velocity u_a. The denominator of the above equation will be equal to zero. To keep the expression for v_ϕ finite we require that L has the constant value of

$$L = \Omega r_a^2.$$

We also have from previous equations that M_a^2/ur^2. Evaluating this at the critical point we get that

$$M_A^2 = \frac{ur^2}{u_a r_a^2} = \frac{\rho_a}{\rho}. \tag{23}$$

This allows us to reduce our equation for the angular velocity to be,

$$v_\phi = \Omega r \left(\frac{\frac{ur^2}{u_a r_a^2} \Omega r_a^2 \Omega^- 1 - 1}{M_A^2 - 1} \right)$$

$$v_\phi = \Omega r \left(\frac{\frac{u}{u_a} - 1}{M_A^2 - 1} \right)$$

$$v_\phi = \frac{\Omega r}{u_a} \left(\frac{u_a - u}{1 - M_A^2} \right). \tag{24}$$

Then we can evaluate the angular component of the magnetic field, to be given as,

$$B_\phi = -B_r \frac{\Omega r}{u_a} \left(\frac{r_a^2 - r^2}{r_a^2 (1 - M_A^2)} \right). \tag{25}$$

Note that L is not determined by the value of B_ϕ or v_ϕ, it is determined by the Alfvénic critical point. For small values of r, the B_ϕ can therefore be considered fixed, along with v_ϕ. Looking at the asymptotic behaviour of these functions if we consider $r >> r_a$, we get that u, the radial velocity, is almost a constant, meaning that $M_A \propto r$ with both v_ϕ and B_ϕ vary as $1/r$. Considering $r << r_a$, with $u << u_a$, we can use the Taylor expansion of (24) and (25) to get,

$$B_\phi = -B_r \frac{\Omega r}{u_a} \left(1 + \frac{-1 + \frac{u}{u_a}}{r_a^2} r^2 + ... \right),$$

and

$$v_\phi = \Omega r \left(1 - \frac{u}{u_a} + \frac{u}{u_a} \left(\frac{r^2}{r_a^2} \right) - \left(\frac{u}{u_a} \right)^2 \left(\frac{r^2}{r_a^2} \right) + ... \right).$$

Simplifying this and ignoring values of some of the higher order terms, as they are relatively small, results in

$$B_\phi = -B_r \frac{r\Omega}{u_a} \left(1 - \frac{r^2}{r_a^2} \left(1 - \frac{u}{u_a} \right) + ... \right),$$

$$v_\phi = \Omega r \left(1 - \frac{u^2}{u_a^2} \left(1 - \frac{r}{r_a} \right) + ... \right).$$

At the surface of the Sun the torque caused by the magnetic field causes the angular-momentum to be mainly lost. As we go away from the Sun the radial velocity of the solar wind increases and the magnetic stress decreases. If we look at the radial momentum equation we have

$$\rho u \frac{du}{dr} = -\frac{d}{dr} P - \rho \frac{GM_\odot}{r^2} + \frac{1}{c} (\mathbf{J} \times \mathbf{B})_r + \rho \frac{v_\phi^2}{r}.$$

This equation is very similar to that of Parker's model with the addition of the azimuthal velocity and the addition of the magnetic field. We also have that as the gas is fully ionized hydrogen the effective particle mass is only half the hydrogen mass m, giving us the equation of state

$$P = \frac{2kT}{m} \rho$$

With T being the temperature of the gas, assuming the same temperature for ions and electrons, and k being Boltzmann's constant. Instead of determining the temperature from specified sources of energy and using the energy equation, we will use an approximation. The approximation, which uses the polytrope law, tends to be used when the sources are poorly known, such as the properties of the solar wind near the Sun. We will have γ in the approximation, referred to as the polytropix index. The approximation is given by

$$P = P_0 \left(\frac{\rho}{\rho_0} \right)^\gamma = P_a \left(\frac{\rho}{\rho_a} \right)^\gamma, \tag{26}$$

We write the magnetic force as

$$\frac{1}{c}\left(\mathbf{J}\times\mathbf{B}\right)_r = -\frac{1}{4\pi r}B_\phi\frac{d}{dr}(rB_\phi) \tag{27}$$

We use (26) and (27) and substitute into the equation of radial momentum to get;

$$\frac{d}{dr}\left(\frac{1}{2}u^2+\frac{\gamma}{\gamma-1}\frac{P_a}{\rho_a}\left(\frac{\rho}{\rho_a}\right)^{\gamma-1}-\frac{GM_\odot}{r}\right)=\frac{v_\phi^2}{r}-\frac{1}{8\phi\rho r^2}\frac{d}{dr}(rB_\phi)^2$$

If we set the right hand side equal to zero we get Parker's equation of motion for the solar wind. In our equation, however, the terms on the right hand side are present due to the inclusion of the magnetic force along with the radial velocity. Using our expressions for ρ,v_ϕ and B_ϕ from equations (23),(24) and (25), respectively, we get an expression for the change in radial velocity with respect to the radius, given by,

$$\begin{aligned}\frac{du}{dr}=\frac{u}{r}&\left[\left(\frac{2\gamma P_a}{\rho_a M_A^{2(\gamma-1)}}-\frac{GM_\odot}{r}\right)\left(M_A^2-1\right)^3+\Omega^2r^2\left(\frac{u}{u_a}-1\right)\left(\left(M_A^2+1\right)\frac{u}{u_a}-3M_A^2+1\right)\right]\\ \times&\left[\left(u^2-\frac{\gamma P_a}{\rho_a M_A^{2(\gamma-1)}}\right)(M_A^2-1)^3-\Omega^2r^2M_A^2\left(\frac{r_a^2}{r^2}-1\right)^2\right]^{-1}.\end{aligned} \tag{28}$$

The Alfvénic critical point $r=r_a$ is also a critical point of the radial equation. The radial equation above can be integrated to give us the total energy flux per square radian, which is a constant for our solution,

$$F=\rho ur^2\left[\frac{u^2}{2}+\frac{\gamma}{\gamma-1}\frac{P_a}{\rho_a}M_A^{-2(\gamma-1)}-\frac{GM_\odot}{r}+\frac{\Omega^2r_a^4}{2r^2}\left(1+\frac{(2M_A^2-1)(r^2-r_a^2)^2}{r_a^4(M_A^2-1)^2}\right)\right]. \tag{29}$$

For this equation the first term is our kinetic energy associated with the radial velocity. The second is the sum of the enthalpy and energy being transported by the thermal conductivity of the solar wind, which is a result of the approximation we get from using the polytropic law. Our third term is the gravitational energy, and the fourth term is the sum of the magnetic and rotational energy. We can simplify this equation using our equations (24) and (25) to obtain a cleaner form for our F

$$F_{rot+mag}=\rho ur^2\left(\frac{v_\phi^2}{2}-\frac{B_\phi B_r}{4\pi\rho}\frac{\Omega r}{u}\right).$$

We have the first term, which represents the kinetic energy flux associated with the angular velocity, and the second term is the energy transported out by the magnetic field, refered to as the Poynting energy flux. The total energy flux is given by equation (29), which is a constant.

3.2 Conclusion and numerical example of Weber and Davis Model

Now we can make some calculations from our equation to get the order of magnitude values for the angular momentum lost by the Sun and the angular velocity of the solar wind. We will need to determine values for the total mass, energy fluxes and the critical radius and velocity, r_a and u_a. We will also need to apply the boundary conditions at r_0 to find the radial magnetic filed, pressure and density at that point. Our final value that we will need to determine is the value for γ, which determines how much energy is supplied to the solar wind itself. We would normally set r_0 to be at the Sun's surface and determine the boundary conditions there, but this presents many challenges as it is much harder to find the boundary conditions. Instead we will set $r_0 = 1$a.u., and set the boundary conditions to be at Earth's orbit, because we know what the boundary conditions are at the orbit of the Earth compared to the Sun. The boundary conditions are the speed of the solar wind, density and temperature, along with the magnetic field strength. We will use the subscript E instead of 0 to distinguish between values set at the Earth orbit and those at the solar radius. The values for our parameters are given below,

$$\begin{aligned} u_E &= 400 \text{ km sec}^{-1}, \quad B_{rE} = 5\gamma, \\ \rho_E &= 11.7 \times 10^{-}24 \text{ gm cm}^{-3}, \quad T_E = 2 \times 10^5 \text{K}. \end{aligned} \tag{30}$$

We have specified $P_E/\rho_E = 4.7 \times 10^{18}$ ergs gm^{-1} and the constant $\rho ur^2 = 1.05 \times 10^{11}$ gm sec^{-1}sterad^{-1}, taking $\Omega = 3 \times 10^{-6}$ radians sec^{-1} and $GM_\odot = 1.33 \times 10^{26}$dynes cm^{2}gm$^-1$. Using these parameters for equations and we get

$$\frac{2kT_a}{m} = \frac{P_a}{\rho_a} = \frac{P_E}{\rho_E}\left(\frac{\rho_a}{\rho_E}\right)^{\gamma-1} = \frac{2kT_E}{m} M_{AE}^{-2(\gamma-1)} = \frac{2kT_E}{m}\left(\frac{B_{rE}^2}{4\pi\rho_E u_E^2}\right)^{\gamma-1}.$$

To use our equation (29) to get a solution we first need to find r_a,u_a,F and γ. However, for our solution to extend from very small values of r to vary large values of r we need it to pass through all three critical points of (r, u). Each critical point makes both the numerator and denominator of equation (28) equal to zero. We know that the pair (r_a, u_a) does this but we need to find the remaining pairs (r_c, u_c) and (r_f, u_f). This gives us four equations with eight unknowns. In addition to our equation (28) we also have the energy equation that must be satisfied at $r = r_a, r_f, r_{c,2}$ and r_E, giving us the four additional equations needed to determine the unknown values. However, if we try to specify γ, which is the power supply, the system of eight equations would be overdetermined unless we regarded u or ρ at $r_0 = r_E$ as unknown. As the eight equations are non-linear there might also be more than one solution to them.

Unfortunately I was unable to reproduce the following data from the paper myself. After taking (25) and using the the nondimensionalised terms stated before I was able to simplify the equation, but then was unable to find the values of the pairs of variables that would make the numerator and denominator equal to zero. Being unable to find these points I could not numerically compute the solutions of the equations to plot them, hence I take the values that are given in the paper to demonstrate how the model predicts the velocity of the solar wind, and to discuss how this model differs from the previous model by Parker.

The solution that uses the values assigned in equation (30) yields $\gamma = 1.221$, $r_a = 24.3r_\odot$, $u_a = 3.32 \times 10^7$ cm sec^{-1}, $r_c = 3r_\odot$, $u_c = 1.82 \times 10^7$cm sec^{-1}, $r_f = 24.6r_\odot$, $u_f = 3.33 \times 10^7$cm sec^{-1} and $F/\rho ur^2 = 9.02 \times 10^{14}$ ergs gm^{-1}sec with $r_\odot$ as the solar radius. We get u as a function of r. Along with the other properties of the solution we can get a direct numerical solution of equation . We get the following two graphs for angular velocity and angular momentum respectively,
The model by Weber and Davis makes it possible to understand the large-scale properties of the solar wind, along with its angular momentum flux in the wind itself. This model did have some difficulties. With the use of the polytropic approximation γ it is impossible to set physical conditions at both the Earth and at the Sun, and as the model uses the boundary conditions at the Earth it determines that the radial velocity at the Sun is still very high, the temperature is 2.7×10^6 K and the density is only 3×10^7 particles cm^{-3}. Values reasonably close to the values known to exist at the Sun can be achieved by slowly varying the value of γ between the Sun and the critical point, but the energy supply by any constant γ is still incorrect.

Although there is no co-rotation with the gas and the Sun, the gas being released does take away a substantial amount of angular momentum and torque due to the magnetic field, allowing it to also decelerate the Sun's angular motion and ultimately slowing it down. The Weber and Davis model provides a good understanding of the coupling between the magnetic field and the plasma motion, especially the effects on the motion of the plasma in the angular direction. In addition, we can gather information on the angular-momentum loss of the star due to their expanding coronas.

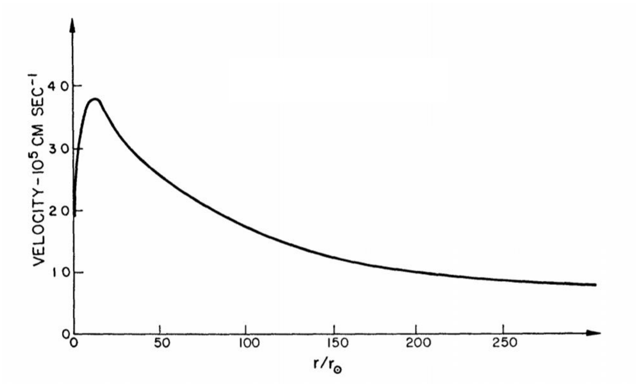

Figure 4: The angular velocity of the solar wind

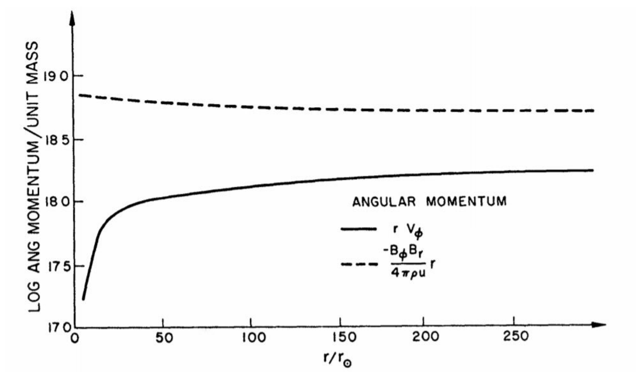

Figure 5: Angular momentum and magnetic torque

4 Two-fluids solar wind model

Before we go into a two-fluid model we must first discuss Alfvén waves and cyclotron heating. Alfvén waves, named after Hannes Alfvén, are low-frequency waves that require a magnetic field to exist, and constitute the traveling oscillation of the ions in that field. The mass of these ions provides the inertia, with the magnetic field tension providing the restoring force. Alfvén waves move in the direction of the magnetic field lines, and can help accelerate the plasma by pushing it along the magnetic field lines and away from the Sun (Guo et al., 2016).

The final model we will be looking at is the model from McKenzire and Marek (1995). Previous work on this model also includes Leer and Axford (1971) and Axford and Mckenzie (1993), and was later expanded on in Mckenzie et al. (1997). This is a two-fluid model of electron and protons, where the assumption is that the flow is radial everywhere and neglects the effects of viscosity. As shown in Scarf and Noble (1965), the effects of the viscosity are not important for the radial expansion of the solar wind. We will be looking at the acceleration of the high speed solar wind based on the conservation equations for steady plasma flow, which is a radially directed flux tube of a sectional area $A(r)$. The model is developed from dissipation length characterisation of wave heating of the coronal plasma close to the Sun, on which the high speed solar wind is based. Solutions of this model, along with correct particle and energy flux and a realistic magnetic field, yield the requirements on the density at the base of the corona, assuming a dissipation length at about $0.25-0.5$ solar radii. The features of the solution of the model are such that the acceleration is rapid, having a sonic point at about two solar radii, along with a maximum proton temperature of about $8-10\times10^6$ K. For the dissipation to be so efficient it would require that the Alfvén waves responsible must have a frequency in the range of 0.01 Hz – 10kHz.

4.1 Properties of solar wind

The properties for the high speed solar wind that we will be using in this model (McKenzire and Marek, 1995) are,

1. Asymptotic speed of $V_\infty \sim 750 - 800$ km sec^{-1} with small fluctuations in speed.
2. Particle flux of about 2×10^8 cm^{-2}sec.
3. Radial magnetic field strength of 2.8 nT at 1 a.u. with the field being unipolar in each stream.
4. The average temperatures of the protons and electrons at 1 a.u. are $T_p \sim 200,000$ K and $T_e \sim 100,000$ K, respectively.
5. For electrons, the distribution function contains a field-aligned beam that evidently originates in a region close to the sun with T_e not exceeding $\sim 10^6$ K
6. Proton temperature perpendicular to the magnetic field is greater than that parallel to the field, meaning $T_p^\perp > T_p^\parallel$.
7. The magnetic field fluctuation occurs with periods > 200 sec; this corresponds to Alfvén waves propagating away from the Sun with energy fluxes of a few percent of the total solar wind energy flux.
8. Minor species including helium have roughly the same temperature per atomic mass as the protons, and move faster by approximately the Alfvén speed, such that $T_i \sim AT_p$ and $V_i \sim V + V_A$.
9. The composition appears to be constant, with helium abundance of about 5%.
10. The high solar wind is dominant during periods of low solar activity and occupies the whole heliosphere at solar latitudes greater than about 20 degrees.
11. The wind is fully developed at about 0.3 a.u. in accordance with observations, and is also close to its terminal speed at about $20r_\odot$, but sometimes only $8r_\odot$.
12. The electron densities are low, at $n_0 \sim 2 \times 10^7 - 10^8$ cm^{-3}, in comparison with coronal streamers.
13. The electron temperature does not appear to exceed about 10^6 K.

We can deduce from the results gathered that pressure gradient associated with electrons and Alfvén waves is not the major factor in accelerating the wind, as they act too slowly and hence cannot lead to the rapid acceleration observed close to the Sun. We also know that because the protons are hotter than the electrons, the minor species are strongly favoured relative to protons, such that that the high frequency waves involved allow discrimination in terms of ion cycltron waves. At 0.3 a.u., the ion cylotron waves have periods less than the Ferraro-Plumpton period at the base of the corona. Thus, $\tau_0 = 2H/V_A \sim 30 - 100$ sec, with H being the scale height of the corona.

4.2 Equations for the model

There appears to be only one possibility to account for the high speed solar wind: it is the consequence of heating close to the Sun by the dissipation of waves with periods considerably less than that of τ_0. These waves preferentially heat the protons and minor species such as helium. Given that the magnetic field strength may be 5-10 gauess at the base of the coronal holes, the frequency range of the waves is about 0.01 Hz – 10 kHz. Adopting a simple approach to the heating and acceleration of the solar wind, we assume a single damping length L, giving the wave energy flux F_w to be

$$F_w = p_w(3V + 2V_A). \tag{31}$$

With $p_w = \rho < \delta v^2 > /2$ being the wave pressure, and ρ and V the fluid density and speed, respectively. $< \delta v^2 >$ is the mean square wave amplitude and $V_A = B/\sqrt{4\pi\rho}$, with B as the magnetic field strength. We also have Q being the dissipation function, assumed to have the form,

$$Q_\perp = Q_{\perp 0} \exp\left[\frac{-(s - s_0)}{L}\right]. \tag{32}$$

We have s, which is measured along the magnetic field, and $s_0 = r_0 = r_\odot$. The area factor A is such that the magnetic flux is conserved, meaning

$$BA = B(s_0)A(s_0) = constant = \Phi. \tag{33}$$

For solar minimum conditions the field is essentially an axisymmetric current sheet in the equatorial plane with the Sun's field being a dipole, thus for the polar field line for which s=r we have,

$$\frac{B(r)}{M} = \frac{2}{r^3} + \frac{1}{a(a+r)^2} \tag{34}$$

With M being the magnetic dipole moment of the Sun. We choose the parameters such that the open field lines emerge from the latitudes $\theta > 60$ degrees, thus having $a \approx 3.98 r_\odot$. This model does not allow for the region of the slow solar wind, that being $\theta < 20$ degrees, however it should be reasonably accurate close to the Sun at around $r < 20 r_\odot$. We can compensate for this omission by estimating the field strength at the base of the coronal hole, based on mapping only the radial components of the field at large distances above 30 degrees latitude into the north and south polar coronal holes, which are defined as $\theta > 60$. Thus, the average field strength in the coronal holes is taken as 5.5 gauss. We also have three integrals of the motion of the solar wind, the magnetic flux BA, mass flux J and the energy flux E, given by,

$$\rho V A = mn(r)V(r)A(r) = J, \tag{35}$$

$$J\left[\frac{V^2}{2} + \frac{5p}{2\rho} - \frac{GM_\odot}{r}\right] + A(F_w + F_c) = E. \tag{36}$$

With $p = p_i + p_e$ being the plasma pressure, $GM_\odot/r$ the solar gravitational potential and $F_c = -\kappa_e dT_e/dr$ as the heat conduction flux. Proton heat conduction can be neglected as κ_p is presumed to be controlled by the wave-particle interactions associated with the heating process, and is relatively small. However, electron heat conduction is controlled by Coulomb collisions and may be the rule beyond the heating region, thus we can assume,

$$T_e = T_{0e} = 10^6 \text{ K for } r < 3r_\odot \tag{37}$$

$$F_c = -\kappa_{0e} T_e^{5/2} \frac{d}{dt} T_e \text{ for } r > 3r_\odot \tag{38}$$

We neglect radiative cooling since it is not important in coronal holes. We can show that if p_w/p is small at the base of the corona and the wave dissipation and plasma heating processes have the same length scales, p_w can be neglected relative to p, which simplifies the solution. From observed quantities at large distances from the Sun, assuming that p tends to zero at infinity, we can deduce that $J/Am \sim 2 \times 10^8$ cm^{-2}sec^{-1} and $E \sim nmV_\infty^3/2 \sim 1.2$ ergscm^{-2}sec at 1 a.u., where m is the mean ion mass, being around 1.9×10^{-24} gm with 5% helium.

Finally, there is the plasma momentum and wave energy exchange equations, given as,

$$\rho V \frac{dV}{dr} = -\frac{dp}{dr} - \frac{dp_w}{dr} - \frac{GM_\odot \rho}{r^2} \tag{39}$$

$$\frac{1}{A}\frac{d}{dr}(AF_w) = V\frac{dp_w}{dr} - Q_\perp \tag{40}$$

We neglect the terms involving dp_w/dr and note that $AF_{w0}/J \sim V_\infty^2/2 + GM_\odot/r_\odot$,$AF_c = A_0F_{c0}$ and

$$F_{w0} \sim Q_{\perp 0}\int_{r_\odot}^{\infty}\left(\exp -\frac{r-r_\odot}{L}\right)\left(\frac{A(r)}{A(r_\odot)}\right) dr \tag{41}$$

Equation (39) can be rewritten because $p = nkT = nk(T_p^{\|} + T_e)$,

$$mnV\frac{dv}{dr} = -\frac{d}{dr}\left(nk(T_p^{\|} + T_e)\right) - nk(T_p^{\|} - T_p^{\perp})\frac{1}{A}\frac{dA}{dr} - \frac{dp_w}{dr} - \frac{GM_\odot \rho}{r^2}. \tag{42}$$

We also have the following energy equations for electrons and protons, given as,
Electron energy:

$$\frac{3}{2}\frac{dT_e}{dr} - \frac{T_e}{V}\frac{dv}{dr} = \frac{Q_e}{nVk} + \frac{m}{J}\frac{d}{dr}\left(A(r)K_e\frac{dT_e}{dr}\right). \tag{43}$$

Proton energy ($\|$):

$$\frac{dT_p^{\|}}{dr} = \frac{-2T_p^{\|}}{V}\frac{dV}{dr} + \frac{2\nu_p(T_p^{\perp} - T_p^{\|})}{V} + \frac{Q_{\|}}{nVk}. \tag{44}$$

Proton energy ($\perp$):

$$\frac{1}{A}\frac{d(AT_p^{\perp})}{dr} = -\frac{\nu_p(T_p^{\perp} - T_p^{\|})}{V} + \frac{Q_\perp}{nVk}. \tag{45}$$

4.3 Conclusion and results of the model

Equations (34)-(36) and (42)-(45), if neglecting the wave pressure term, represent the plasma equations and describe the motion of the plasma along the magnetic field. We can justify ignoring the wave pressure term because if the acceleration of the gas is rapid, the wave pressure gradient should be much smaller then the plasma pressure gradient. We assume that there is only cyclotron heating involved, such that $Q_{\|} = 0$. We can then solve these equations with the assumption that $T_p^{\perp} > T_p^{\|}$ for large distances from the Sun. For (40) with (31), $V_A = B/\sqrt{4\pi\rho}$ can be rewritten to be the form of,

$$\frac{1}{A}\frac{d}{dr}\left[A\frac{(V-V_A)^2}{V_A}2\rho_w\right] = \frac{-(V+V_A)}{V_A}Q_\perp. \tag{46}$$

This describes the evolution of the wave action. On choosing a value for L, a complete solution may be found that has both the correct magnetic field geometry and correct particle flux, with an asymptotic flow speed. Each solution is such that it corresponds to a particular value for n_0, which is the coronal base density and a mean ion temperature. For $2 \times 10^7 < n_0 < 10^8$ cm^{-3} we would need $0.25r_\odot < L < 0.5r_\odot$ and a $T_{max} \sim 9.5 - 12 \times 10^6$ K. We get that the transonic point occurs at about $2r_\odot$, with the Alfvén point for which $V = V_A$ at about $r \sim 8r_\odot$. The solution for the velocity is shown by the graph below, from McKenzire and Marek (1995),

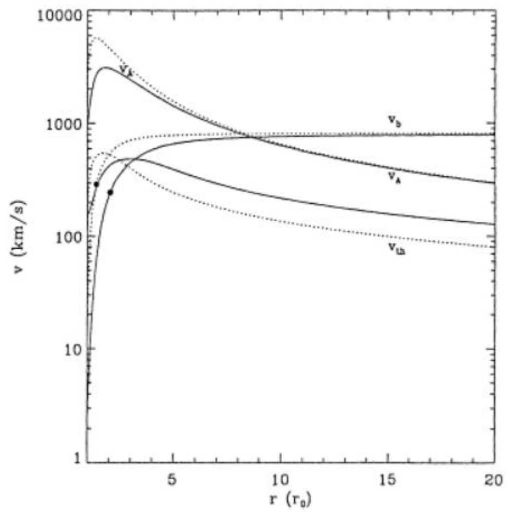

Figure 6: Variation of solar wind speed with $L = 0.25r_\odot$ (dotted line) and $L = 0.5r_\odot$

The graph shows the speeds for the solar wind, Alfvén speed and the thermal speed between $1 - 20r_\odot$. With these results we only have the single free parameter, that being the dissipation length L. The results from the model have some properties very close to the observed high speed solar wind, including the correct speed and particle flux, with a realistic magnetic field. We also get rapid acceleration near the Sun as we would expect to see with low coronal densities. This is a relatively accurate model for solar wind, which has improved upon models previously discussed by the inclusion of a two-fluid system with a magnetic field, allowing it to have both a correct speed and similar density. To further improve upon this model it would be necessary to include the source spectrum, along with the dissipation process for high frequency waves. We see further work done on this model in Mckenzie et al. (1997), which extended the results from above by including the effects of the wave pressure and anisotropic proton temperature.

5 Conclusion

We have seen how the basics of solar wind models derive from a combination of the momentum equations and magnetohydrodynamics equations, with assumptions to simplify and evaluate the models. We see that using some assumptions makes the models easier to solve, but in doing so we lose accuracy and are unable to reproduce all of the real-world observations. Parker's basic model from 1956, for example, was able to produce correct speeds near the Earth and Sun, but was unable to produce the correct densities. We show how the equations could be manipulated into a state that was solvable, but not necessarily realistic. To further build upon this work I would numerically solve the equations (28) and (29), such that I could reproduce the solutions and discuss in more detail what they show. I would consider more recent models and study how they were derived, and what their strengths and weaknesses are. From this I would hope to make my own models, which could feasibly be reproduced while also matching real-world observations as closely as possible. One area for development, for example, would be the inclusion of a more realistic magnetic field.

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