Models of the Solar Wind

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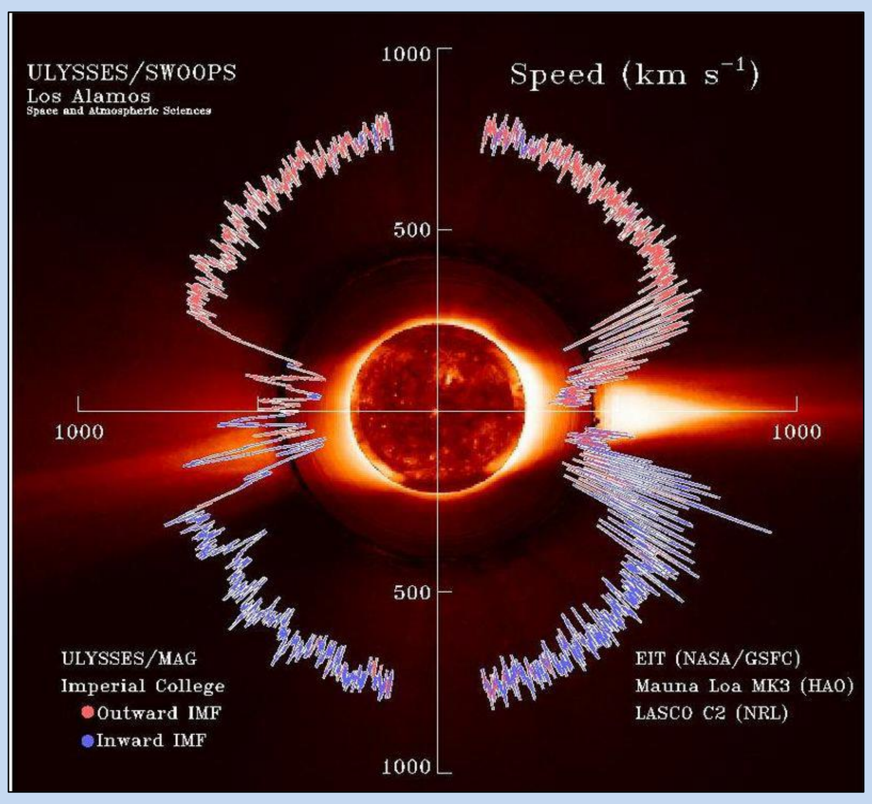
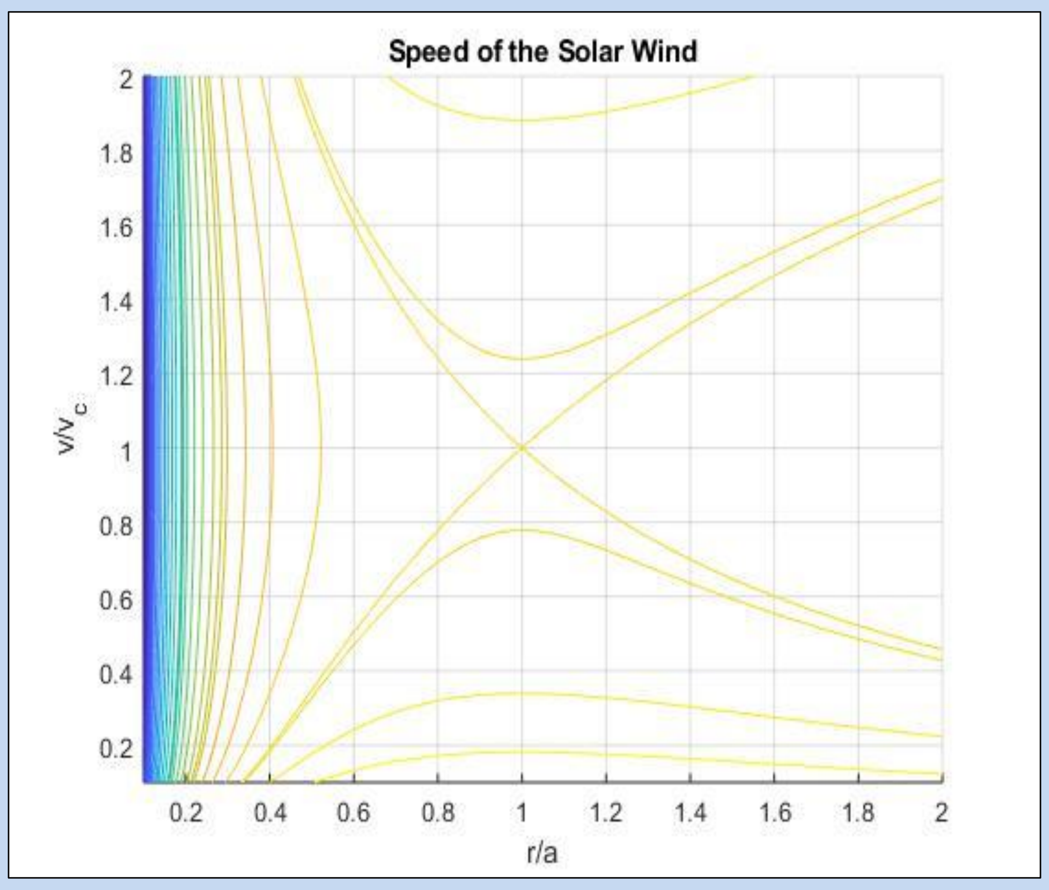
Overview

Solar wind originates from the outer atmosphere of the Sun, known as the corona. The corona reaches temperatures of over 1 million kelvin, but this only partially explains the supersonic speeds achieved by the plasma. In this project I will compare several models of solar wind to investigate how it is accelerated and how the solar wind moves through the solar system.

I will consider two separate models for the solar wind, created by Parker (1958) and Weber and Davis (1967) . These two models use different assumptions to model the solar wind, with Parker's basic model being the simpler of the two. Parker's model determined that the thermal velocity could only be about 250 km/sec, which is less then the required 500 km/sec for particles to escape the gravitational pull of the Sun. From this we conclude that there must be other mechanisms influencing the acceleration of the wind. This is where we consider the second model, which incorporates the magnetic field of the Sun and its role in explaining how the wind reaches its escape velocity. We also look at how coronal holes can help increase the speed without increasing the temperature to unrealistic amounts.





Parker's basic solar wind model

The assumptions made in this model are: (a) solar wind is a hydrodynamic flow and hence does not interact with the magnetic field of the Sun; and (b) that the heating of the corona stops after r=a. We are also looking at the plasma as a whole instead of dividing it into protons and electrons, which would accelerate to different speeds.

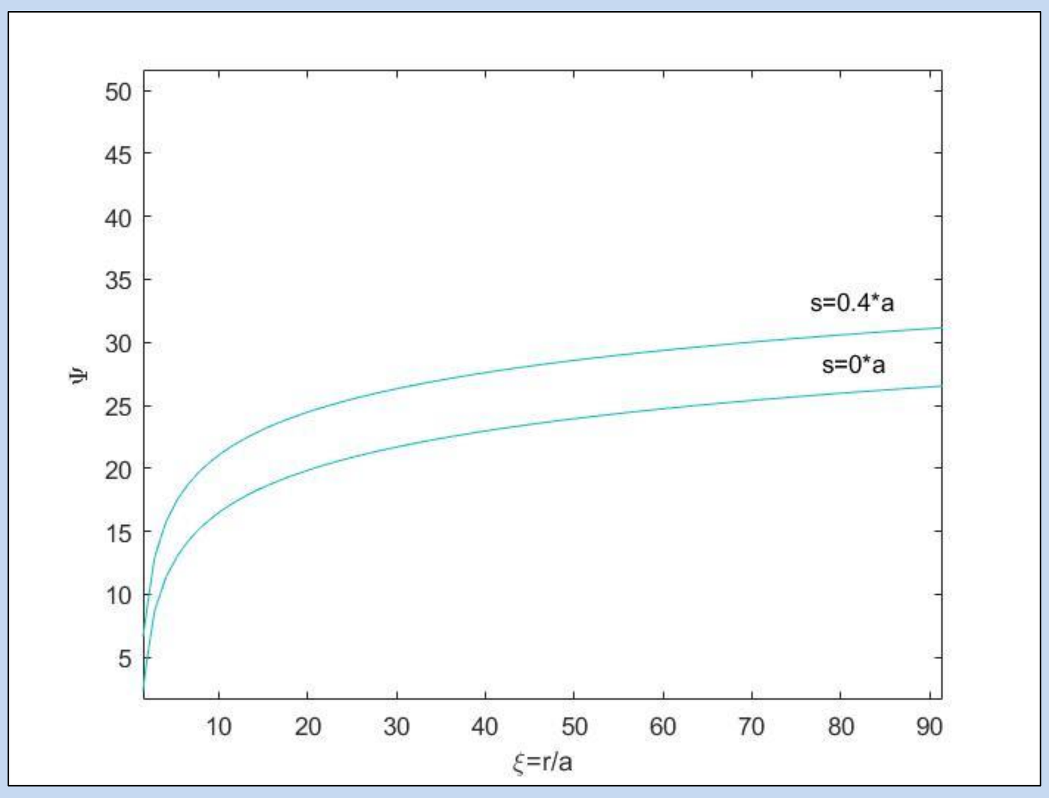
The equation for this model is as follows,

$$\left(\frac{v}{v_c}\right)^2 - \log\left(\frac{v}{v_c}\right)^2 = 4\log\left(\frac{r}{a}\right) + \frac{2\lambda a}{r} - 2\lambda + \psi_0 - \log(\psi_0) \qquad (1)$$

ψ_0 is the constant of integration and $\psi_0 = \psi$ at r=a. Where $\psi = \frac{Mv^2}{2kT_0}$, we also have v_c which is the critical point from subsonic speeds to supersonic. This equation comes from solving Cauchy momentum equation,

$$\rho\left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\nabla\right)\boldsymbol{v} = \boldsymbol{J}\times\boldsymbol{B} - \nabla p \qquad (2)$$

With $\rho = NM$ and $p = 2NkT$ and N being the number of particles, along with the Boltzmann's constant k and temperature T. The graph shows the solutions of equation (1) given λ=2. The only viable solutions are those that start from a small v/v_c and tend to become bigger as r/a increases. The line which goes through the point (1,1) shows a velocity that goes from subsonic to supersonic at the point a (where the heating stops).



Coronal holes

Instead of a spherical expansion from the Sun we can look at a localised point known as a coronal hole, which is an active region of the corona. We consider a narrow cone with a cross-sectional area of $A(r)$. We also have an s variable, which denotes the distance from the origin of spherical expansion to the Sun's centre. We get a continuity equation of,

$$N(r)v(r) = N_0 v_0 \left(\frac{a-s}{r-s}\right)^2 \qquad (3)$$

Using this we get,

$$\left(\frac{v}{v_c}\right)^2 - \log\left(\frac{v}{v_c}\right)^2 = 4\log\left(\frac{r/a + s/a}{1 + s/a}\right) + \frac{2\lambda a}{r} - 2\lambda + \psi_0 - \log(\psi_0) \qquad (4)$$

This leads to the following graph, where we have set $\lambda = 0.5$ and we have varied the values for s/a. We can see that increasing the s value means the cross-sectional area is decreased, resulting in a higher wind speed without adding any additional energy into the system.

Davis and Weber's Model

In Davis and Weber's model they considered a similar model to Parker, but also accounted for the effect of the magnetic field and how it could accelerate the solar wind. The flow is assumed to have no viscosity and infinite conductivity. This leads to a radial momentum equation derived from equation (2), as before, to be,

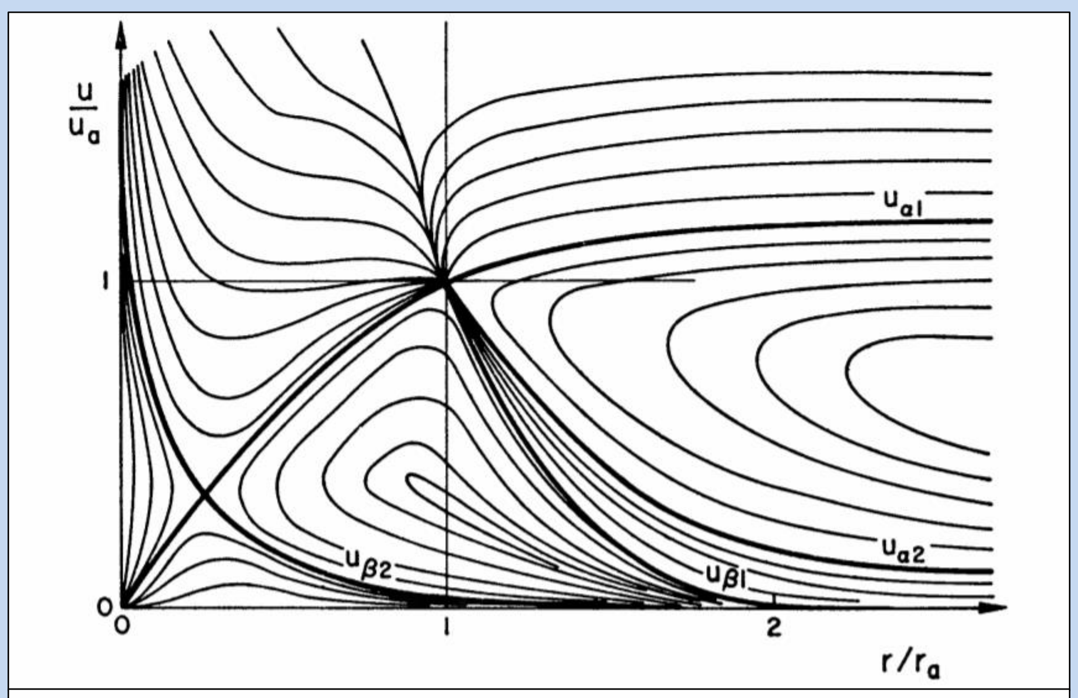
$$\rho u\frac{du}{dr} = -\frac{d}{dr}p - \rho\frac{GM_{\odot}}{r^2} + \frac{1}{c}(\boldsymbol{J}\times\boldsymbol{B})_r + \rho\frac{v_{\phi}^2}{r} \qquad (5)$$

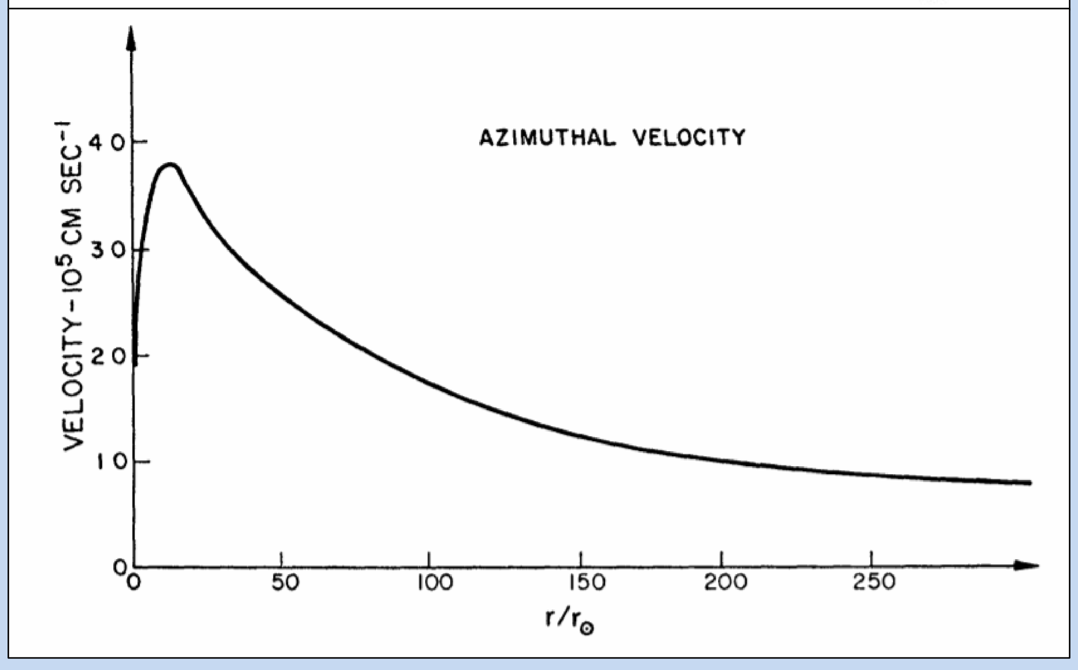
We can write the magnetic force as, $\frac{1}{c}(\boldsymbol{J}\times\boldsymbol{B})_r = \frac{-1}{4\pi r}B_{\phi}\frac{d}{dr}(rB_{\phi})$, we can then substitute some other forms of our variables to get an ODE for u, which is our radial velocity. The family of solutions for this ODE are shown in the graph on the right. In this graph at (1,1) is where the solution is at the Alfvénic critical point (u_a, r_a) and is where the radial Mach number $M_A = (ur^2)/(u_a r_a^2)$is also equal to 1. We see this plot is very similar to Parker's solution though showing more complex structure.

After the determination of u as a function of r given some assumptions for the variables we can use this to solve for the azimuthal velocity of the solar wind equation,

$$v_{\phi} = \frac{\Omega r}{u_a}\frac{u_{a-u}}{1-M_A^2} \qquad (6)$$

The azimuthal velocity plot matches the velocity of the Sun at the surface increasing it its apex point at around 11.5 the solar radius, then slowing down as r tends to infinity. We can use the equation azimuthal velocity to also find the angular momentum of the particles using rv_{ϕ}.





How do the solutions of the two models compare?

We see from the plots that Parker's model and the model proposed by Davis and Weber yield very similar solutions for the radial velocity of the solar wind, with both passing through the transonic critical point. These model also suffer from similar problems. Namely, it is impossible to reproduce observed physical conditions. Parker's model estimates density near the Earth to be several orders of magnitude too high, while the Davis and Weber model uses boundary conditions at the Earth so, as a consequence, the radial wind velocity near the sun is still very high. The Davis and Weber model does, however, have the advantage of also showing the angular momentum of the solar wind, and demonstrates how the gas takes away a substantial amount of angular momentum from the Sun, and therefore how the rotation of the Sun as a whole is decreased by the wind. The next stage of my project compares these two models to a more recent model by McKenzie, Banaszkiewicz and Axford (1995).

References:
- Weber, E. and Davis, L. (1967). The angular momentum of the solar wind. 148(148):217-227.
- Parker, E. (1958). Dynamics of the interplanetary gas and magnetic fields.
- McKenzire, J. and Marek, B. (1995). Acceleration of the high speed solar wind.