Analysis Chapter 2 Test

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1. State the definition for convergence of a sequence

2. Consider the sequence (a_n), where $a_n = 1/\sqrt{n}$.
Show that the limit of the sequence is

$$\lim \left(\frac{1}{\sqrt{n}} \right) = 0$$

3. Show

$$\lim \left(\frac{n+1}{n} \right) = 1$$

4. Prove the Algebraic Limit Theorem. Let $\lim a_n = a$, and $\lim b_n = b$. Then,

(i) $\lim (ca_n) = ca,$ for all $c \in \mathbf{R}$
(ii) $\lim (a_n + b_n) = a + b$
(iii) $\lim (a_n b_n) = ab$
(iv) $\lim (a_n / b_n) = a/b,$ provided $b \neq 0$

5. (Cesaro Means) Show that if (x_n) is a convergent sequence, then the sequence given by the averages

$$y_n = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

also converges to the same limit.
Give an example to show that it is possible for the sequence (y_n) of averages to converge even if (x_n) does not.

6. Prove the Monotone Convergence Theorem.
If a sequence if monotone and bounded, then it converges.

7. Prove that

$$\sum_{n=1}^{\infty} \frac{1}{n^2}$$

converges

8. Prove the Harmonic Series does not converge

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

9. For the sequence

$$(a_n) = \left(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \cdots \right)$$

state two subsequences, and two non-obvious examples of invalid subsequences.

10. Prove
Subsequences of a convergent sequence converge to the same limit as the original sequence.

11. Prove the Bolzano-Weierstrass Theorem. *Every bounded sequence contains a convergent subsequence.*