Analysis

A Collection of Notes and Problems

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Chapter 1

The Real Numbers

1.1 Introductory Proofs

Theorem 1.1.1. There is no rational number whose square is 2

Proof. A rational number can be expressed as p/q where both p and q are integers. We can prove this using contradiction. If there does exist a rational number whose square is 2, then

$$\left(\frac{p}{q}\right)^2 = 2$$

We can rearrange this to find that

$$p^2 = 2q^2$$

. Based on this, we know that p^2 is even. We must now show that since p^2 is even p must also be even. Here we can say that since $p^2 = p * p$ and p^2 is divisible by 2, p must also be divisible by two, since p is an integer. We can now introduce another variable a to say $2a = p$ so

$$\frac{4a^2}{q^2} = 2$$

rearranging we find

$$\frac{q^2}{a^2} = 2$$

now using the previous argument we know that q is also even. This now tells us that p/q are further divisible and thus do not define a rational number.