

$$\begin{aligned} f(z) \\ w = \\ u + \\ vi \\ f(x+iy) = w = u+iv \end{aligned}$$

$$\begin{aligned} f \\ u(x,y) \text{ and } v(x,y) \end{aligned}$$

$$f(z) = u(x,y) + i v(x,y)$$

$$f(re^{i\theta}) = u + i v$$

$$\begin{aligned} z = \\ re^{i\theta} \\ f(z) = u(r,\theta) + i v(r,\theta) \end{aligned}$$

$$\begin{aligned} f(z) = \\ x^2 - \\ y^2 + \end{aligned}$$

$$\begin{aligned} f(z) = \\ x^2 - \\ y^2 + \\ 2ixy \text{ This gives us the values for } \\ u(x,y) = \\ x^2 - \\ y^2 \end{aligned}$$

$$\begin{aligned} v(x,y) = \\ 2xy \text{ In polar coordinates this is given by } z = \\ re^{i\theta} \end{aligned}$$

$$\begin{aligned} f(z) = \\ z^2 = \\ r^2 e^{i2\theta} \text{ if we express this in terms of sine and cosine } r^2 e^{i2\theta} = \\ r^2 \cos 2\theta + \\ r^2 i \sin 2\theta \\ u(r,\theta) = \\ r^2 \cos 2\theta \\ v(r,\theta) = \\ r^2 \sin 2\theta \end{aligned}$$

$$\text{The limit } \lim_{z \rightarrow 0} f(z) \text{ does not exist because the limit does not agree from all directions. We can consider two directions from the real axis so the limit becomes } \lim_{x \rightarrow 0} x x =$$

$$1 \text{ From the imaginary axis it is } \lim_{y \rightarrow 0} y - y =$$

$$\begin{aligned} -1 \\ w = u + i v \end{aligned}$$

$$\begin{aligned} M \\ R \\ M \\ |f(z)| \leq M \end{aligned}$$

$$\begin{aligned} R \\ \sqrt{[u(x,y)]^2 + [v(x,y)]^2} \end{aligned}$$

$$\begin{aligned} f \\ M \end{aligned}$$

$$\begin{aligned} f \\ z_0 \\ \lim_{z \rightarrow z_0} f(z) \end{aligned}$$

$$\begin{aligned} f(z_0) \\ \lim_{z \rightarrow z_0} f(z) = \\ f(z_0) \end{aligned}$$

$$\begin{aligned} \epsilon \\ \delta \\ |f(z) - f(z_0)| < \epsilon \text{ whenever } |z - z_0| < \delta \end{aligned}$$

$$\begin{aligned} f(z) \\ z_0 \\ f(z) \neq \\ 0 \end{aligned}$$

$$\begin{aligned} f(z) \\ z_0 \\ |f(z_0)|/2 \\ \epsilon \\ \delta \end{aligned}$$

$$|f(z) - f(z_0)| < |f(z_0)|/2 \text{ whenever } |z - z_0| < \delta$$

$$\begin{aligned} z \\ |z - \\ z_0| < \\ \delta \end{aligned}$$

$$\begin{aligned} f(z) = \end{aligned}$$