$f(z)$
$w = u + vi$
$f(x+iy) = w = u+iv$

x
y
$u(x,y) and v(x,y)$

$f(z) = u(x,y)+iv(x,y)$

$f(re^{i\theta}) = u+iv$

$z = re^{i\theta}$
$f(z) = u(r,\theta)+iv(r,\theta)$

$f(z) = z^2$
z

$f(z) = x^2 - y^2 + 2ixy This gives us the values for$
$u(x,y) = x^2 - y^2$
$v(x,y) = 2xy In polar coordinates this is given by z = re^{i\theta}$
$f(z) = z^2 = r^2 e^{i2\theta} if we express this in terms of sine and cosine r^2 e^{i2\theta} = r^2 \cos 2\theta + r^2 i \sin 2\theta$
$u(r,\theta) = r^2 \cos 2\theta$
$v(r,\theta) = r^2 \sin 2\theta$
$\bar{z} the limit \lim_{z\to 0} f(z) does not exist because the limit does not agree from all directions. We can consider two directions from the real ax$
$so the limit becomes \lim_{x\to 0} xx = 1 From the imaginary axis it is \lim_{y\to 0} y-y = -1$
$\bar{w}$
$w = u+iv$

u
v
R
M
$|f(z)| \leq M$

z
R
$\sqrt{[u(x,y)]^2 + [v(x,y)]^2}$

f
M
f
z_0
$\lim_{z\to z_0} f(z)$
$f(z_0)$
$\lim_{z\to z_0} f(z) = f(z_0)$
ϵ
δ
$|f(z)-f(z_0)| < \epsilon whenever |z-z_0| < \delta$

$f(z)$
z_0
$f(z) \neq 0$
$f(z)$
z_0
$|f(z_0)/2$
ϵ
δ
$|f(z)-f(z_0) < |f(z_0)|2 whenever |z-z_0| < \delta$

z
$|z - z_0| < \delta$
$f(z) =$