70: Discrete Math and Probability Theory

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Programming + Microprocessors $\equiv$ Superpower!

What are your super powerful programs/processors doing?
Logic and Proofs!
Induction $\equiv$ Recursion.

What can computers do?
Work with discrete objects.
Discrete Math $\implies$ immense application.

Computers learn and interact with the world?
E.g. machine learning, data analysis, robotics, ...
Probability!

My hopes and dreams.

We teach you to think more clearly and more powerfully.

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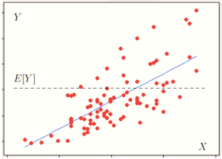
..And to deal clearly with uncertainty itself.

Probability Unit

- How can we predict unknown future events (e.g., gambling profit, next week rainfall, traffic congestion, …)?
  - Constructive Models: Model the overall system (including the sources of uncertainty).
    - For modeling uncertainty, we'll develop probabilistic models and techniques for analyzing them.
  - Deductive Models: Extract the "trend" from the previous outcomes (e.g., linear regression).



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Explains policies, has office hours, homework, midterm dates, etc.

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Questions

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Questions $\Longrightarrow$ piazza:

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Read it!!!!
Announcements, logistics, critical advice.

Wason's experiment:1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.

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Suppose we have four cards on a table:

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- Suppose you see that Alice went to Baltimore, Bob drove,
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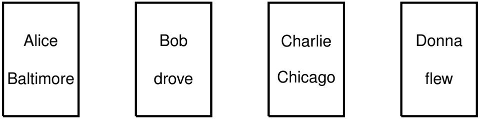

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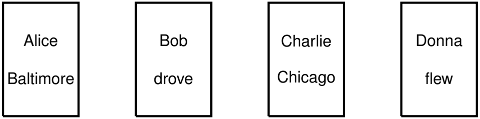

- Which cards must you flip to test the theory?

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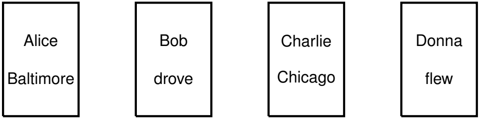

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Answer:

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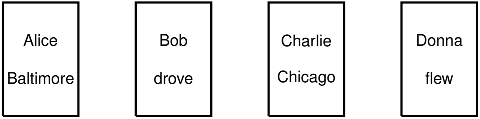

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Answer: Later.

CS70: Lecture 1. Outline.

Today: Note 1.

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The language of proofs!

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Today: Note 1.   Note 0 is background. Do read it.

The language of proofs!

1. Propositions.
2. Propositional Forms.
3. Implication.
4. Truth Tables
5. Quantifiers
6. More De Morgan's Laws

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational
2+2 = 4
2+2 = 3
826th digit of pi is 4
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Any even > 2 is sum of 2 primes
$4 + 5$
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Propositional Forms.

Put propositions together to make another...

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Put them together..

Propositions:

P_1 - Person 1 rides the bus.

Put them together..

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P_1 - Person 1 rides the bus.
P_2 - Person 2 rides the bus.

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Put them together..

Propositions:

P_1 - Person 1 rides the bus.

P_2 - Person 2 rides the bus.

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But we can't have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn't.

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Propositional Form:

$$\neg(((P_1 \vee P_2) \wedge (P_3 \vee P_4)) \vee ((P_2 \vee P_3) \wedge (P_4 \vee \neg P_5)))$$

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Can person 3 ride the bus?

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Can person 3 ride the bus?
Can person 3 and person 4 ride the bus together?

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But we can't have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn't.

Propositional Form:

$\neg(((P_1 \vee P_2) \wedge (P_3 \vee P_4)) \vee ((P_2 \vee P_3) \wedge (P_4 \vee \neg P_5)))$

Can person 3 ride the bus?
Can person 3 and person 4 ride the bus together?

This seems ...

Put them together..

Propositions:
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We can program!!!!

We need a way to keep track!

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True when both P and Q are True .

P	Q	$P \wedge Q$
T	T	T
T	F	
F	T	
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DeMorgan's Law's for Negation: distribute and flip!

$\neg(P \wedge Q)$

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F	F	T	T

DeMorgan's Law's for Negation: distribute and flip!

$$\neg(P \wedge Q) \quad \equiv \quad \neg P \vee \neg Q$$

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One use for truth tables: Logical Equivalence of propositional forms!

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Quick Questions

P	Q	$P \wedge Q$
T	T	T
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Quick Questions

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F	T	F
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P	Q	$P \vee Q$
T	T	T
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Is $(T \wedge Q) \equiv Q$?

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes?

P	Q	$P \vee Q$
T	T	T
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Quick Questions

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T	T	T
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Yes!

Quick Questions

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P	Q	$P \vee Q$
T	T	T
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F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
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Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$?

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
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Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$? F or False.

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
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Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$? F or False.

What is $(T \vee Q)$? T

What is $(F \vee Q)$? Q

Distributive?

$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)?$$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$,

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

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RHS: $(T \wedge Q) \vee (T \wedge R)$

Distributive?

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Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

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P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

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$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$,

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:
P is True .
LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.
RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.
P is False .
LHS: $F \wedge (Q \vee R) \equiv F$.
RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:
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$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Foil 1:

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

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LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Foil 1:

$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$?

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Foil 1:

$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$?

Foil 2:

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:
P is True .
LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.
RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.
P is False .
LHS: $F \wedge (Q \vee R) \equiv F$.
RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Foil 1:
$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$?

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$(A \wedge B) \vee (C \wedge D) \equiv (A \vee C) \wedge (A \vee D) \wedge (B \vee C) \wedge (B \vee D)$?

Implication.

$P \implies Q$ interpreted as

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If P, then Q.

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Examples:

Statement: If you stand in the rain, then you'll get wet.

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Examples:

Statement: If you stand in the rain, then you'll get wet.
P = "you stand in the rain"

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$P \implies Q$ interpreted as

If P, then Q.

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Examples:

Statement: If you stand in the rain, then you'll get wet.
P = "you stand in the rain"
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Statement: "Stand in the rain"

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Examples:

Statement: If you stand in the rain, then you'll get wet.
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Statement: "Stand in the rain"
Can conclude: "you'll get wet."

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Statement:
If a right triangle has sidelengths $a \leq b \leq c$, then $a^2 + b^2 = c^2$.

Implication.

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P = "a right triangle has sidelengths $a \leq b \leq c$",
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Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

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The statement "$P \implies Q$"

only is False if P is True and Q is False .

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The chemical plant pollutes river. Can we conclude fish die?

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Some Fun: use propositional formulas to describe implication?

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Some Fun: use propositional formulas to describe implication?
$((P \implies Q) \wedge P) \implies Q$.

Implication and English.

$P \implies Q$

Poll.

- If P, then Q.

Implication and English.

$P \implies Q$

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- If P, then Q.
- Q if P.

  Just reversing the order.

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Implication and English.

$P \implies Q$

Poll.

- If P, then Q.
- Q if P.

  Just reversing the order.
- P only if Q.

Implication and English.

$P \implies Q$

Poll.

- If P, then Q.
- Q if P.
  Just reversing the order.
- P only if Q.
  Remember if P is true then Q must be true.

Implication and English.

$P \implies Q$

Poll.

- If P, then Q.
- Q if P.

  Just reversing the order.
- P only if Q.

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  this suggests that P can only be true if Q is true.

Implication and English.

$P \implies Q$

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  since if Q is false P must have been false.

Implication and English.

$P \implies Q$

Poll.

- If P, then Q.
- Q if P.
  Just reversing the order.
- P only if Q.
  Remember if P is true then Q must be true.
  this suggests that P can only be true if Q is true.
  since if Q is false P must have been false.
- P is sufficient for Q.

Implication and English.

$P \implies Q$

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  Just reversing the order.
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Implication and English.

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- P is sufficient for Q.

  This means that proving P allows you
  to conclude that Q is true.

  Example: Showing $n > 4$ is sufficient for showing $n > 3$.

Implication and English.

$P \implies Q$

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- If P, then Q.
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  Just reversing the order.
- P only if Q.
  Remember if P is true then Q must be true.
  this suggests that P can only be true if Q is true.
  since if Q is false P must have been false.
- P is sufficient for Q.
  This means that proving P allows you
  to conclude that Q is true.
  Example: Showing $n > 4$ is sufficient for showing $n > 3$.
- Q is necessary for P.
  For P to be true it is necessary that Q is true.
  Or if Q is false then we know that P is false.

Implication and English.

$P \implies Q$

Poll.

- If P, then Q.
- Q if P.
  Just reversing the order.
- P only if Q.
  Remember if P is true then Q must be true.
  this suggests that P can only be true if Q is true.
  since if Q is false P must have been false.
- P is sufficient for Q.
  This means that proving P allows you
  to conclude that Q is true.
  Example: Showing $n > 4$ is sufficient for showing $n > 3$.
- Q is necessary for P.
  For P to be true it is necessary that Q is true.
  Or if Q is false then we know that P is false.
  Example: It is necessary that $n > 3$ for $n > 4$.

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	
F	T	
F	F	

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	
F	F	

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Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	$\neg P \vee Q$
T	T	
T	F	
F	T	
F	F	

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	T
F	F	T

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T	T	T
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Truth Table: implication.

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T	F	F
F	T	
F	F	

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	$\neg P \lor Q$
T	T	T
T	F	F
F	T	T
F	F	

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
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T	T	T
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F	T	T
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Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	$\neg P \vee Q$
T	T	T
T	F	F
F	T	T
F	F	T

$\neg P \vee Q \quad \equiv P \implies Q.$

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	$\neg P \lor Q$
T	T	T
T	F	F
F	T	T
F	F	T

$\neg P \lor Q \quad \equiv P \implies Q.$

These two propositional forms are logically equivalent!

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don't die, the plant does not pollute.

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don't die, the plant does not pollute.
    (contrapositive)

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
    - If the plant pollutes, fish die.
    - If the fish don't die, the plant does not pollute. (contrapositive)
    - If you stand in the rain, you get wet.

Contrapositive, Converse

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- **Definition:** If $P \implies Q$ and $Q \implies P$ is P if and only if Q or $P \iff Q$.
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Universe examples include..

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or $\exists k \in \mathbb{Z}, n = kd$.
$2|4$? True.
$4|2$? False.

Back to: Wason's experiment:1

Theory:

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Theory: "If a person travels to Chicago, he/she flies."

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Which cards do you need to flip to test the theory?

$Chicago(x)$ = "x went to Chicago." $Flew(x)$ = "x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, Chicago(x)$

Back to: Wason's experiment:1

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$Chicago(A)$ = False .

Back to: Wason's experiment:1

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No.

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No. $Chicago(A) \implies Flew(A)$ is true.
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$Flew(B)$ = False . Do we care about $Chicago(B)$?
Yes.

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So $Chicago(Bob)$ must be False .

Back to: Wason's experiment:1

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So $Chicago(Bob)$ must be False .

$Chicago(C)$ = True .

Back to: Wason's experiment:1

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So $Chicago(Bob)$ must be False .

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Yes.

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So $Chicago(Bob)$ must be False .

$Chicago(C)$ = True . Do we care about $Flew(C)$?
Yes. $Chicago(C) \implies Flew(C)$ means $Flew(C)$ must be true.

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$Chicago(C)$ = True . Do we care about $Flew(C)$?
Yes. $Chicago(C) \implies Flew(C)$ means $Flew(C)$ must be true.

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Yes. $Chicago(C) \implies Flew(C)$ means $Flew(C)$ must be true.

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Yes. $Chicago(C) \implies Flew(C)$ means $Flew(C)$ must be true.

$Flew(D)$ = True . Do we care about $Chicago(D)$?
No.

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$Flew(D)$ = True . Do we care about $Chicago(D)$?
No. $Chicago(D) \implies Flew(D)$ is true when $Flew(D)$ is true.

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$Chicago(C)$ = True . Do we care about $Flew(C)$?
Yes. $Chicago(C) \implies Flew(C)$ means $Flew(C)$ must be true.

$Flew(D)$ = True . Do we care about $Chicago(D)$?
No. $Chicago(D) \implies Flew(D)$ is true when $Flew(D)$ is true.

Only have to turn over cards for Bob and Charlie.

More for all quantifiers examples.

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- “doubling a number always makes it larger”

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$$(\forall x \in N)\ (2x > x)$$

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  $(\forall x \in N)\ (2x > x)$ **False** **Consider** $x = 0$

More for all quantifiers examples.

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  $$(\forall x \in N)\ (2x > x) \quad \textcolor{red}{\textbf{False}} \quad \textbf{Consider } x = 0$$

  Can fix statement...

More for all quantifiers examples.

- "doubling a number always makes it larger"

$$(\forall x \in N)\ (2x > x) \quad \textbf{False} \quad \textbf{Consider } x = 0$$

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- "Square of any natural number greater than 5 is greater than 25."

More for all quantifiers examples.

- "doubling a number always makes it larger"

  $(\forall x \in N)\ (2x > x)$ **False** **Consider** $x = 0$

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  $(\forall x \in N)$

More for all quantifiers examples.

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  $$(\forall x \in N)\ (2x > x) \quad \textbf{False} \quad \textbf{Consider } x = 0$$

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- "Square of any natural number greater than 5 is greater than 25."

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More for all quantifiers examples.

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  $(\forall x \in N)\ (2x > x)$ **False** **Consider** $x = 0$

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  $(\forall x \in N)\ (2x \geq x)$ **True**

- "Square of any natural number greater than 5 is greater than 25."

  $$(\forall x \in N)(x > 5 \implies x^2 > 25).$$

More for all quantifiers examples.

- "doubling a number always makes it larger"

  $(\forall x \in N)\ (2x > x)$ **False** **Consider** $x = 0$

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  Idea alert:

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  Idea alert: Restrict domain using implication.

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  Idea alert: Restrict domain using implication.

Later we may omit universe if clear from context.

Quantifiers..not commutative.

- In English: “there is a natural number that is the square of every natural number”.

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$$(\exists y \in N)\ (\forall x \in N)\ (y = x^2)$$

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Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

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English: there is an x in S where $P(x)$ does not hold.

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That is,

$$\neg(\forall x \in S)(P(x)) \iff \exists(x \in S)(\neg P(x)).$$

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What we do in this course! We consider claims.

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Claim: $(\forall x)\ P(x)$

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Claim: $(\forall x)\ P(x)$ "For all inputs x the program works."

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Claim: $(\forall x)\ P(x)$ “For all inputs x the program works.”
For False , find x, where $\neg P(x)$.

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What we do in this course! We consider claims.

Claim: $(\forall x)\ P(x)$ "For all inputs x the program works."
For False , find x, where $\neg P(x)$.
Counterexample.

Quantifiers....negation...DeMorgan again.

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Next Time: proofs!