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CS70: Lecture 2. Outline.

Today: Proofs!!!

1. By Example.
2. Direct. (Prove $P \implies Q$.)
3. by Contraposition (Prove $P \implies Q$)
4. by Contradiction (Prove P.)
5. by Cases

If time: discuss induction.

Quick Background and Notation.

Integers closed under addition.

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A natural number $p > 1$, is **prime** if it is divisible only by 1 and itself.

Divides.

$a|b$ means

(A) a divides b.

(B) There exists $k \in \mathbb{N}$, with $a = kb$.

(C) There exists $k \in \mathbb{N}$, with $k = ka$.

(D) b divides a.

Direct Proof.

Theorem: For any $a,b,c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

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Therefore Q.

Another direct proof.

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Examples:
$n = 121$ Alt Sum: $1 - 2 + 1 = 0$.

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$n = 605$ Alt Sum: $6 - 0 + 5 = 11$

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Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

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Add $99a + 11b$ to both sides.

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Assume: Alt. sum: $a - b + c = 11k$ for some integer k.

Add $99a + 11b$ to both sides.

$$100a + 10b + c = 11k + 99a + 11b$$

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

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Direct proof of $P \implies Q$:
Assumed P: $11|a - b + c$. Proved Q: $11|n$.

The Converse

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n^2 is even, $n^2 = 2k$, ...$\sqrt{2k}$ even?

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Did we prove?

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- Proof assumed no primes *in between* p_k and q.

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Theorem: $x^5 - x + 1 = 0$ has no solution in the rationals.

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Reduced form $\frac{a}{b}$: a and b can't both be even!

Proof by cases.

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The fourth case is the only one possible, so the lemma follows. $\square$

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Question: Which case holds? Don't know!!!

Be careful.

Theorem: $3 = 4$

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Proof: Assume $3 = 4$.

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Don't assume what you want to prove!

Be really careful!

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Theorem: $1 = 2$

Proof: For $x = y$, we have

$$\begin{aligned}(x^2 - xy) &= x^2 - y^2 \\ x(x - y) &= (x + y)(x - y)\end{aligned}$$

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(A) Assumed what you were proving.

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(C) $x - y$ is zero.

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$P \implies Q$ does not mean $Q \implies P$.

Summary: Note 2.

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To Prove: $P \implies Q$. Assume P. Prove Q.

By Contraposition:
To Prove: $P \implies Q$ Assume $\neg Q$. Prove $\neg P$.

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To Prove: P Assume $\neg P$. Prove False .

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CS70: Note 3. Induction!

Poll. What's the biggest number?

(A) 100

(B) 101

(C) n+1

(D) infinity.

(E) This is about the "recursive leap of faith."

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1. The natural numbers.
2. 5 year old Gauss.
3. ..and Induction.
4. Simple Proof.

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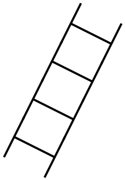

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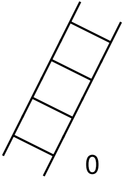
0,



0

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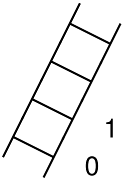
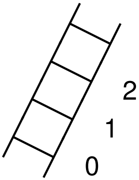
0, 1,



The natural numbers.

0, 1, 2,



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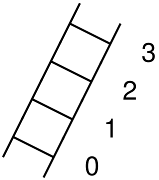

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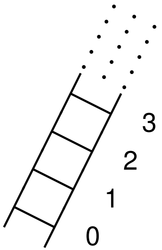

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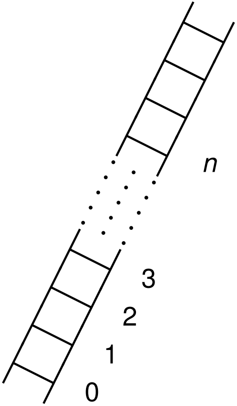

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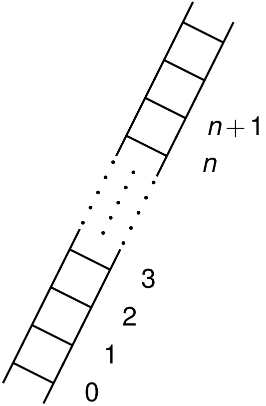

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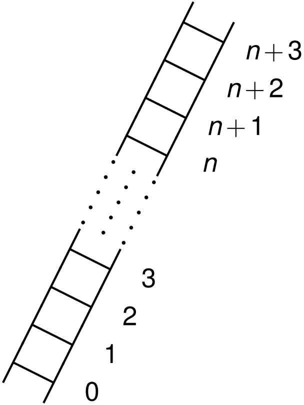

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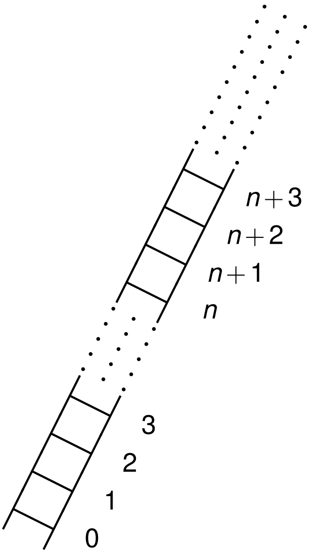

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A formula.

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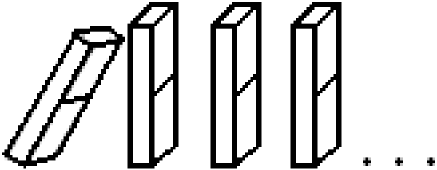
Notes visualization

Note's visualization: an infinite sequence of dominos.



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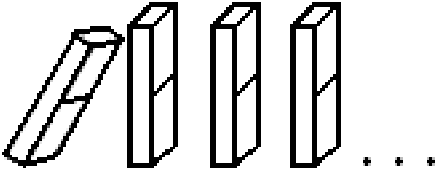

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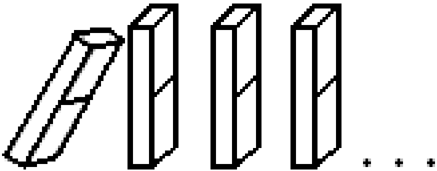

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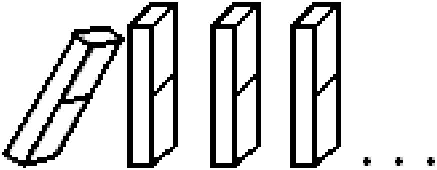

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Climb an infinite ladder?

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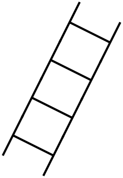

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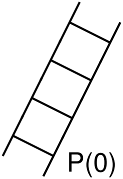

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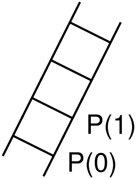

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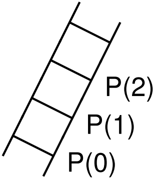

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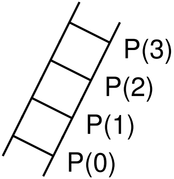

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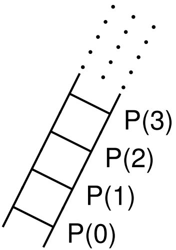

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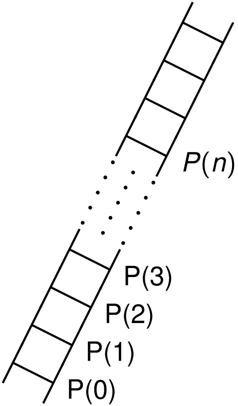

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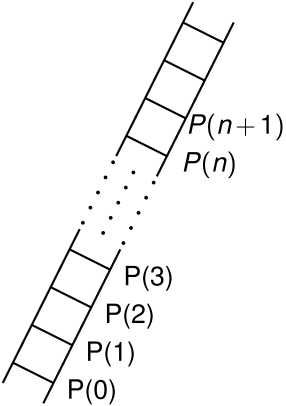

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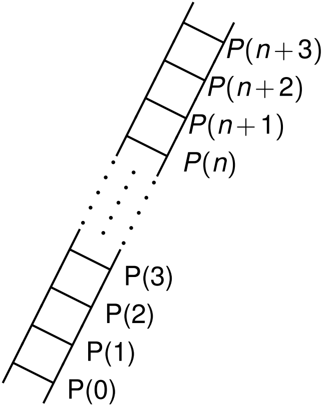

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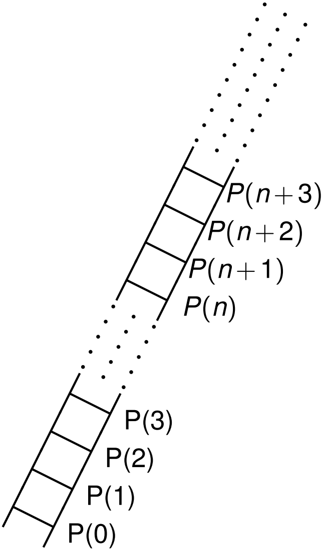

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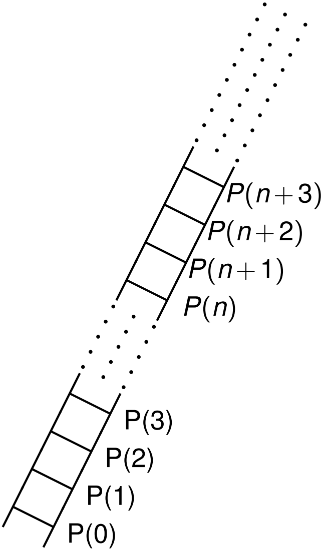

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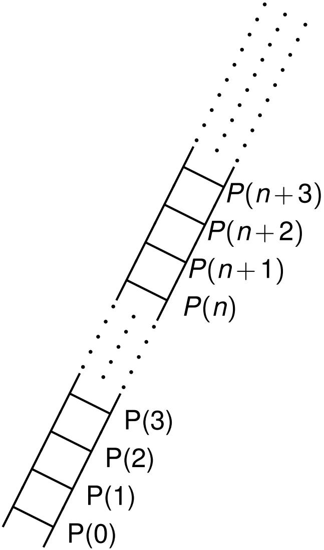
Your favorite example of forever..

Climb an infinite ladder?



$$P(0)$$

$$\forall k, P(k) \implies P(k+1)$$

$$P(0) \implies P(1) \implies P(2) \implies P(3) \ldots$$

$$(\forall n \in N) P(n)$$

Your favorite example of forever..or the natural numbers...

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$

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Is predicate, $P(n)$ true for $n = k + 1$?

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$\sum_{i=1}^{k+1} i$

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$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1)$

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Need to start somewhere. $P(0)$ is $\sum_{i=0}^{0} i = 0 = \frac{(0)(0+1)}{2}$ **Base Case.**

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true for $n = k \implies$ true for $n = k+1$ $(P(k) \wedge (P(k) \implies P(k+1))) \implies P(k+1)$

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Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}.$$

How about $k+2$. Same argument starting at $k+1$ works!

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The canonical way of proving statements of the form

$$(\forall k \in N)(P(k))$$

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Next Time.

More induction!

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“See you” on Tuesday!