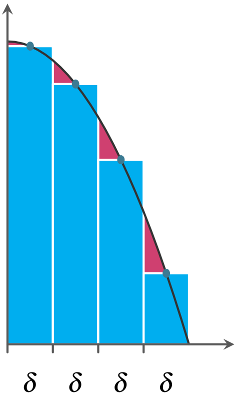
Survey

Fill it out!!

https://forms.gle/XL79oruU8BHrQcaeA

Calculus



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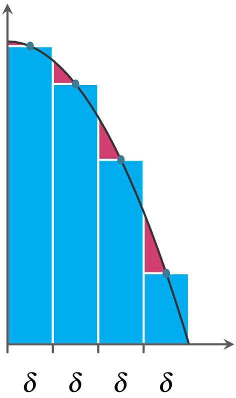

Riemann Sum/Integral: $\int_a^b f(x)dx = lim_{\delta \rightarrow 0} \sum_i \delta f(a_i)$

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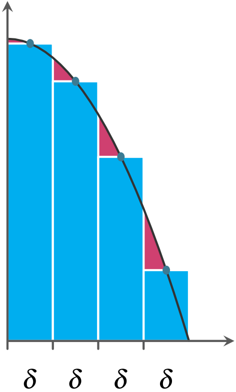

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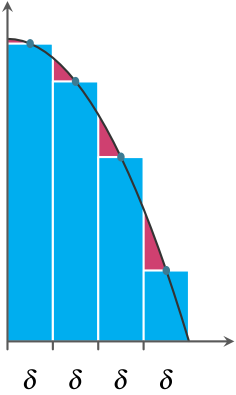

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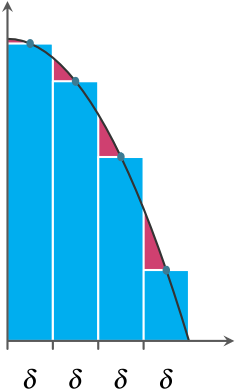

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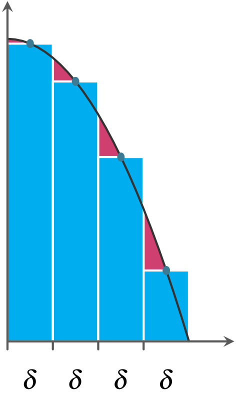

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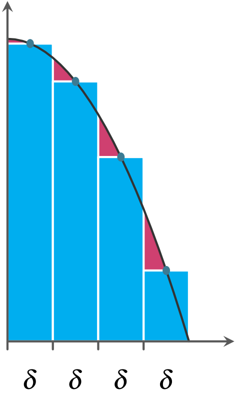

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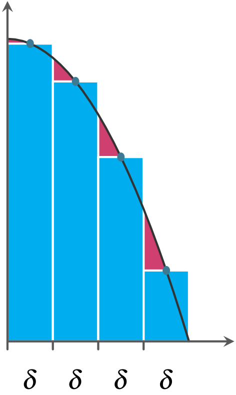

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Thus $F'(x) = f(x)$.

CS70: Continuous Probability.

$\boxed{\text{Continuous Probability 1}}$

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Continuous Probability 1

1. Examples
2. Events
3. Continuous Random Variables

Uniformly at Random in $[0, 1]$.

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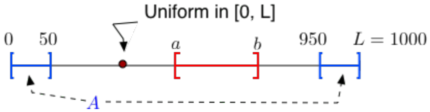

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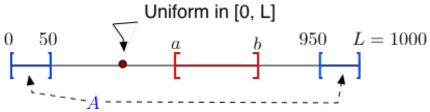

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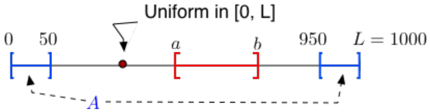

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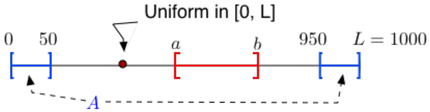

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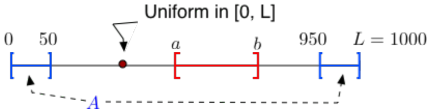

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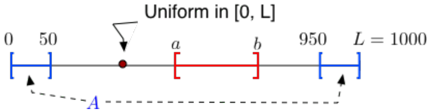

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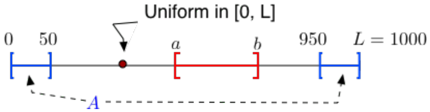

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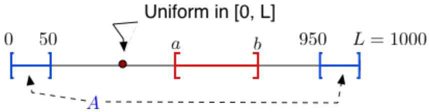

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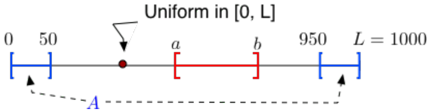

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Makes sense: $b - a$ is the fraction of $[0,1]$ that $[a,b]$ covers.

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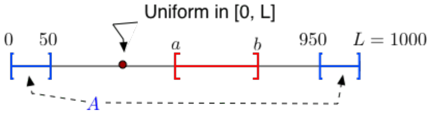

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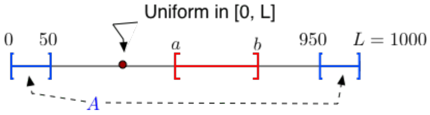

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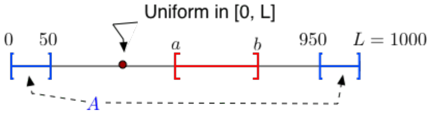

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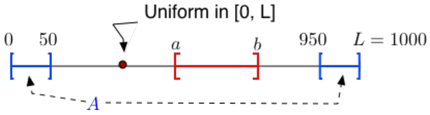

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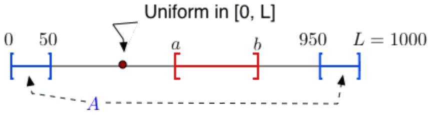

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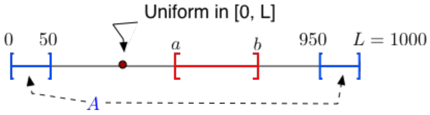

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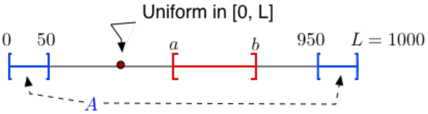

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Continuous space:

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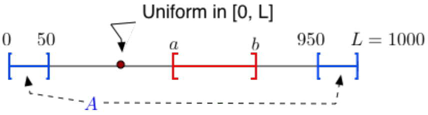

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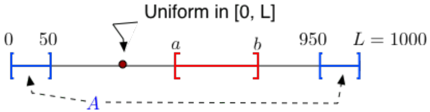

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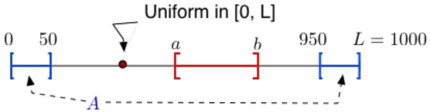

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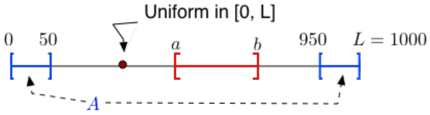

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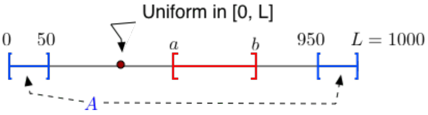

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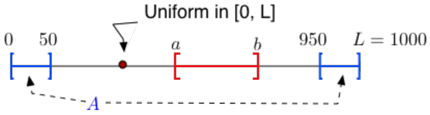
Event A = interval, or union of intervals.

Uniformly at Random in $[0, 1]$.



Note: A **radical** change in approach.

Finite prob. space: $\Omega = \{1, 2, \ldots, N\}$, with $Pr[\omega] = p_\omega$.

$\implies Pr[A] = \sum_{\omega \in A} p_\omega$ for $A \subset \Omega$.

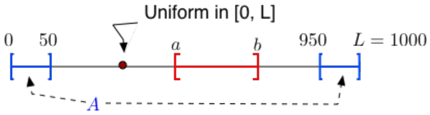
Continuous space: e.g., $\Omega = [0, 1]$,

$Pr[\omega]$ is typically 0.

Instead, start with $Pr[A]$ for some events A.

Event A = interval, or union of intervals.

Uniformly at Random in $[0, 1]$.



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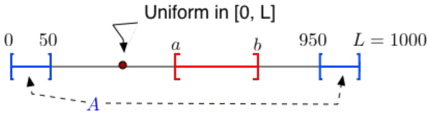

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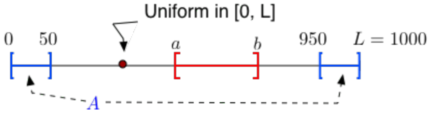

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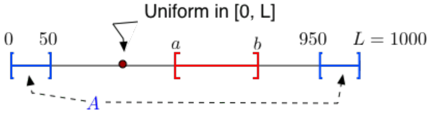
Uniformly at Random in $[0, 1]$.



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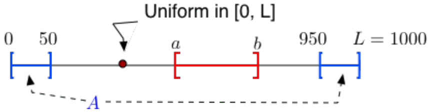

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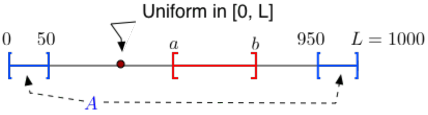
Uniformly at Random in $[0,1]$.



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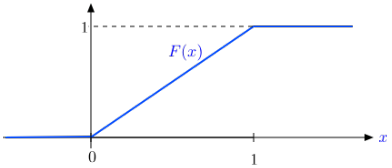

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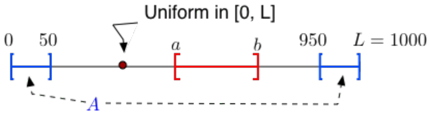

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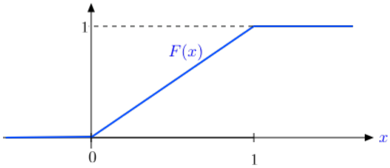

Then we have $Pr[X \in (a,b]] = Pr[X \leq b] - Pr[X \leq a]$

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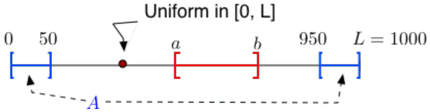

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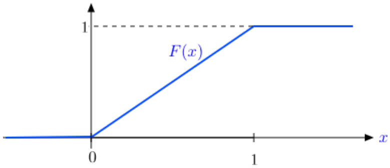

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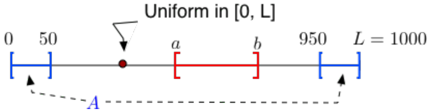

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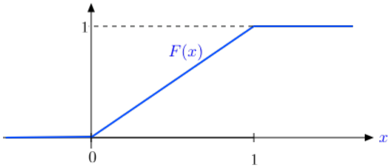

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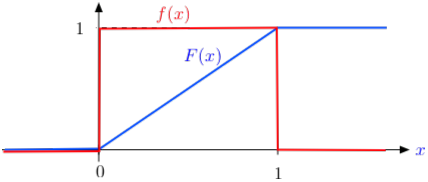
Thus, $F(\cdot)$ specifies the probability of all the events!

Uniformly at Random in $[0,1]$.



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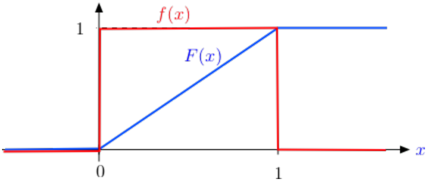

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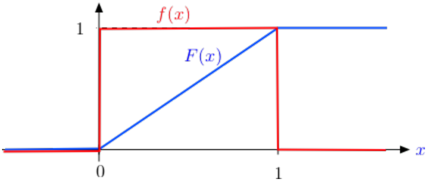

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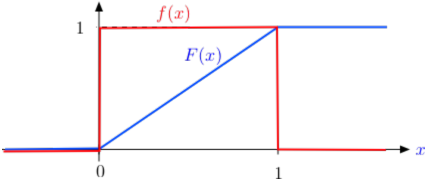

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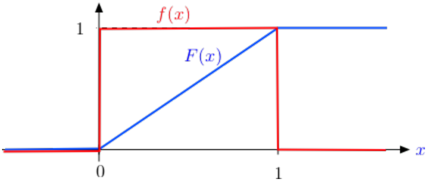

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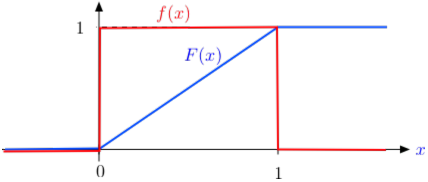

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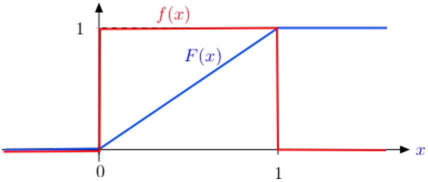

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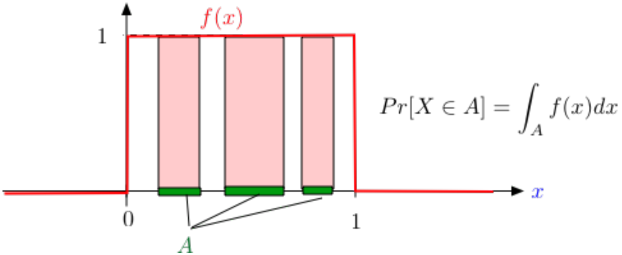
$$Pr[X \in A] = \int_A f(x)dx.$$

Uniformly at Random in $[0, 1]$.



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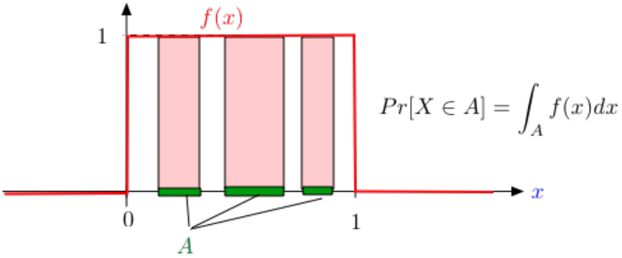
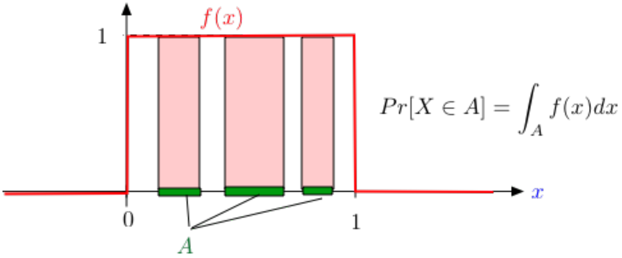


Think of $f(x)$ as describing how
one unit of probability is spread over $[0, 1]$:

Uniformly at Random in $[0, 1]$.



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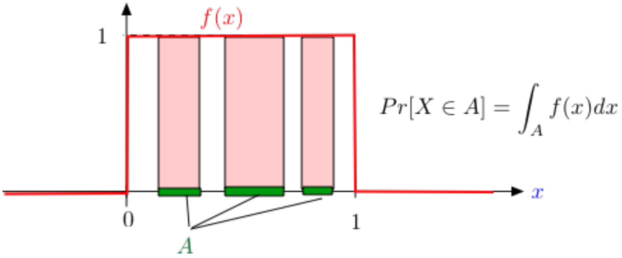

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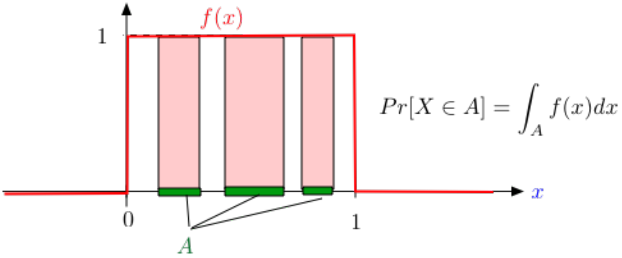
Then $Pr[X \in A]$ is the probability mass over A.

Uniformly at Random in $[0, 1]$.



Think of $f(x)$ as describing how
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Observe:

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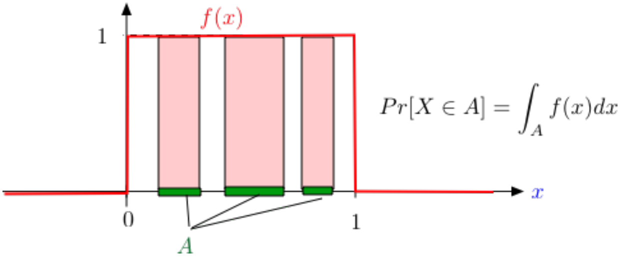

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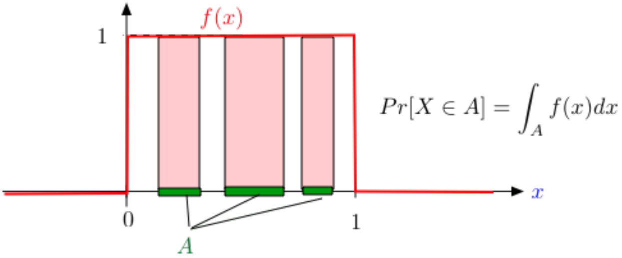
Observe:

- This makes the probability automatically additive.

Uniformly at Random in $[0, 1]$.



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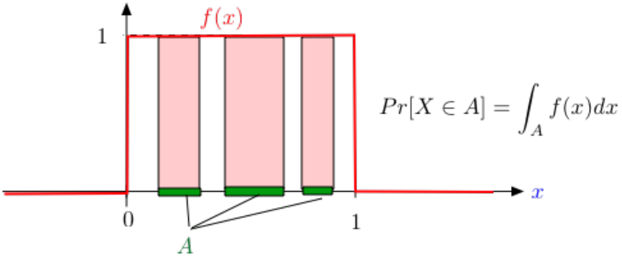
Uniformly at Random in $[0, 1]$.



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Then $Pr[X \in A]$ is the probability mass over A.

Observe:

- This makes the probability automatically additive.
- We need $f(x) \geq 0$ and $\int_{-\infty}^{\infty} f(x)dx = 1$.

Uniformly at Random in $[0, 1]$.

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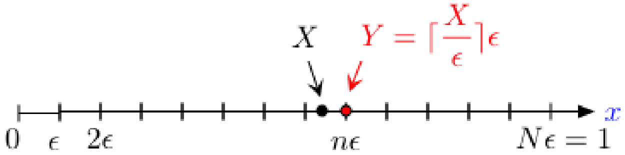

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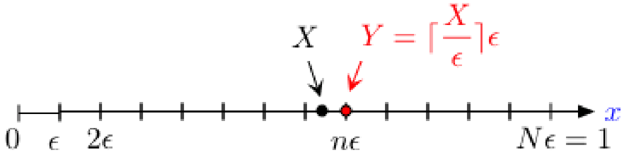
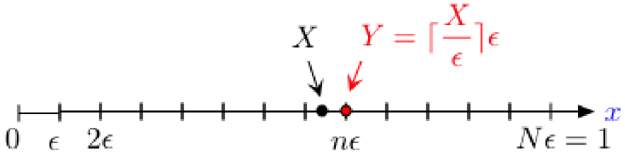


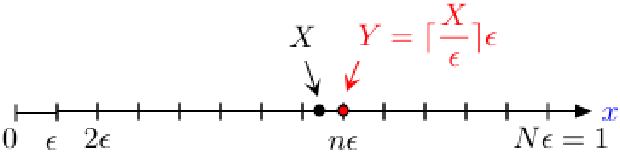
Discrete Approximation:

Uniformly at Random in $[0, 1]$.



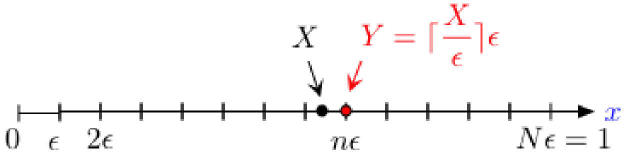
Discrete Approximation: Fix $N \gg 1$

Uniformly at Random in $[0, 1]$.



Discrete Approximation: Fix $N \gg 1$ and let $\varepsilon = 1/N$.

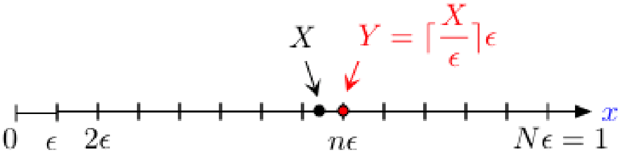
Uniformly at Random in $[0,1]$.



Discrete Approximation: Fix $N \gg 1$ and let $\varepsilon = 1/N$.

Define $Y = n\varepsilon$ if $(n-1)\varepsilon < X \leq n\varepsilon$ for $n = 1, \ldots, N$.

Uniformly at Random in $[0, 1]$.



Discrete Approximation: Fix $N \gg 1$ and let $\varepsilon = 1/N$.

Define $Y = n\varepsilon$ if $(n-1)\varepsilon < X \leq n\varepsilon$ for $n = 1, \ldots, N$.

Then $|X - Y| \leq \varepsilon$

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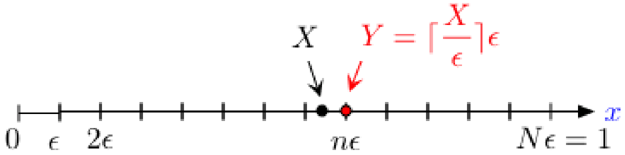

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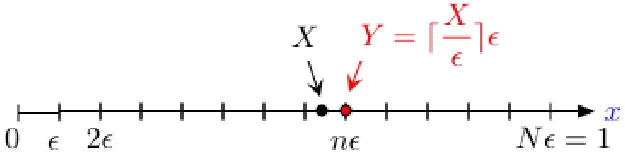

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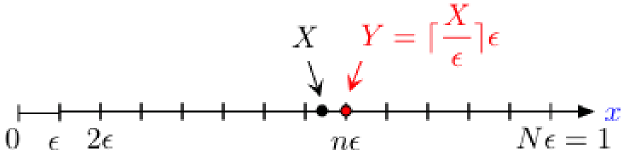

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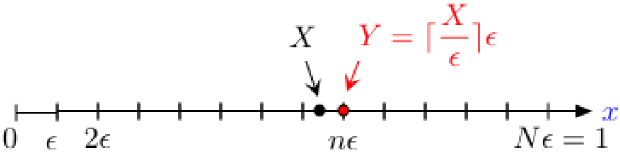
Also, $Pr[Y = n\varepsilon] = \frac{1}{N}$ for $n = 1, \ldots, N$.

Uniformly at Random in $[0,1]$.



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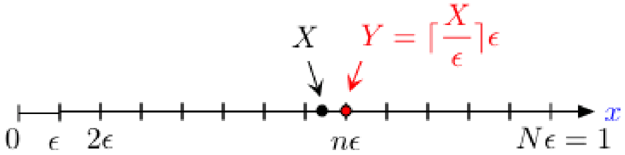
Also, $Pr[Y = n\varepsilon] = \frac{1}{N}$ for $n = 1, \ldots, N$.

Thus, X is ‘almost discrete.’

Uniformly at Random in $[0,1]$.



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Calculus view: $Pr[Y = n\varepsilon]$ is area of rectangle in Riemann sum.

Nonuniformly at Random in $[0, 1]$.

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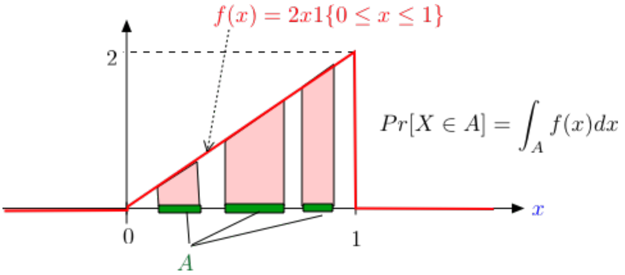

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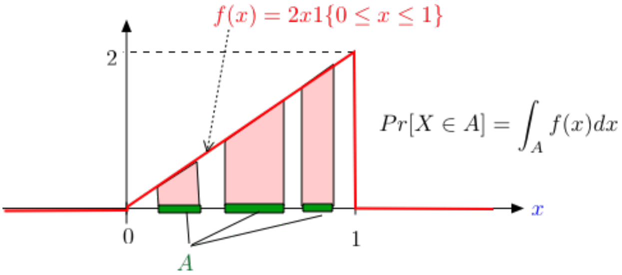
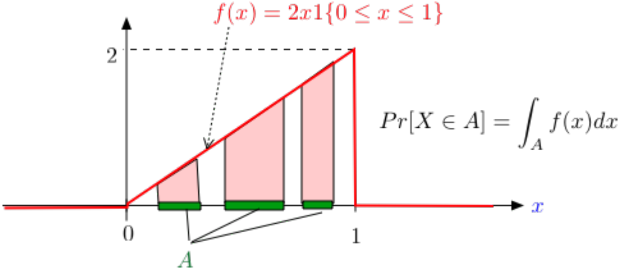


This figure shows a different choice of $f(x) \geq 0$ with $\int_{-\infty}^{\infty} f(x)dx = 1$.

Nonuniformly at Random in $[0, 1]$.



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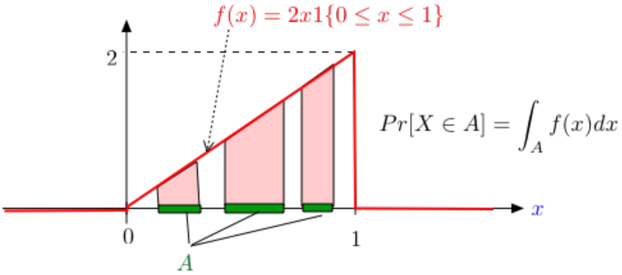
It defines another way of choosing X at random in $[0, 1]$.

Nonuniformly at Random in $[0, 1]$.



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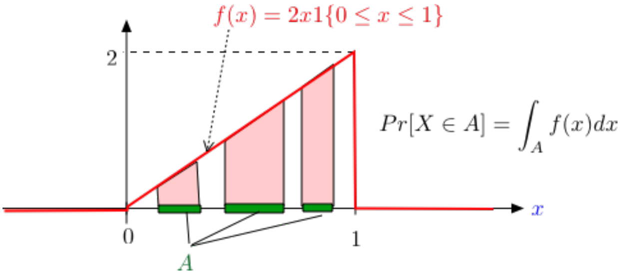

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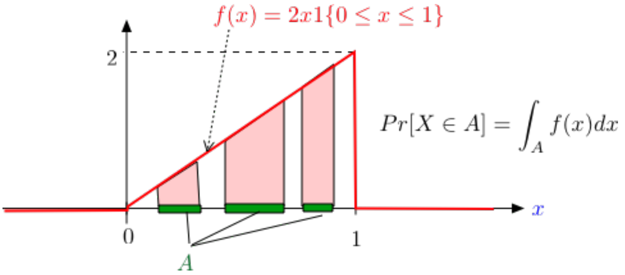

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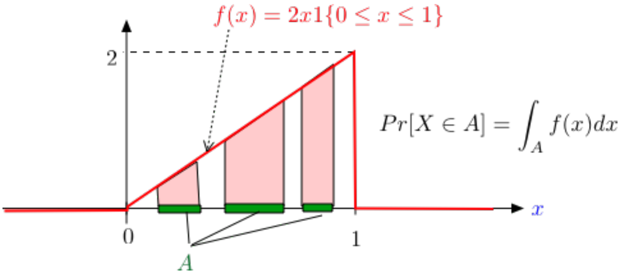

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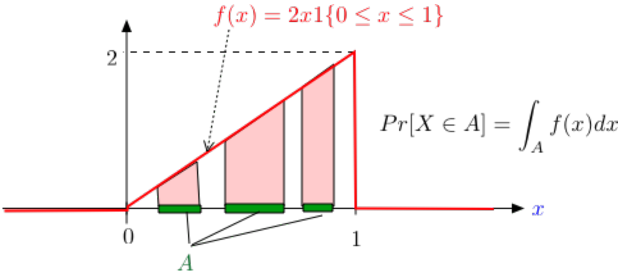

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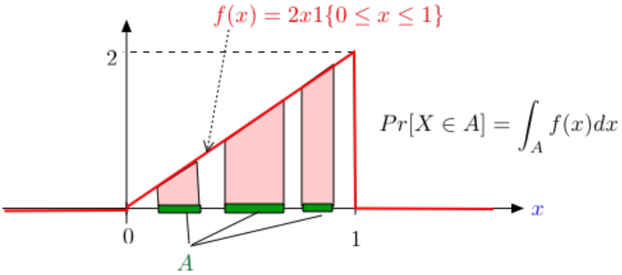

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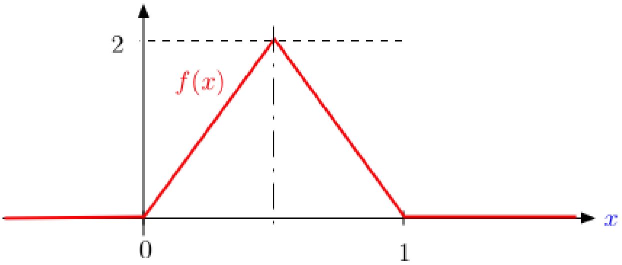

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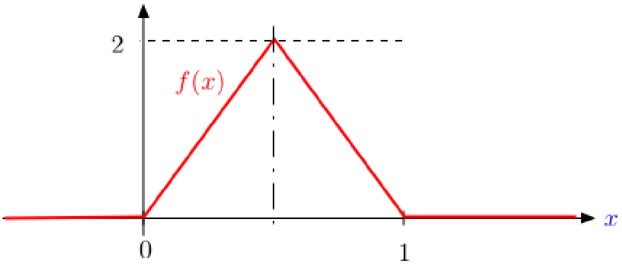

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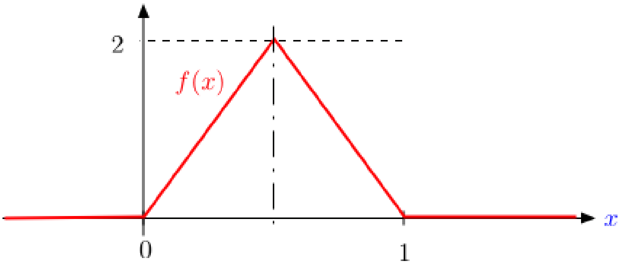

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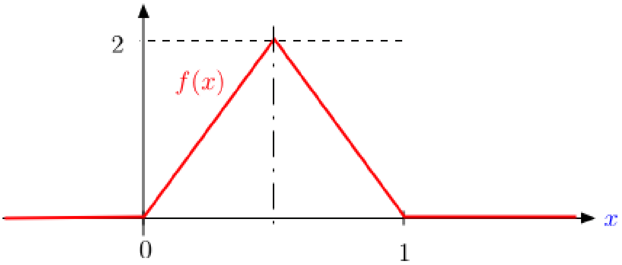

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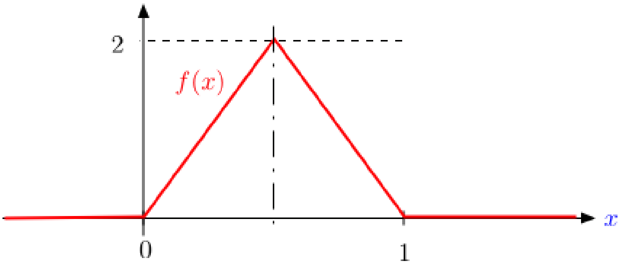

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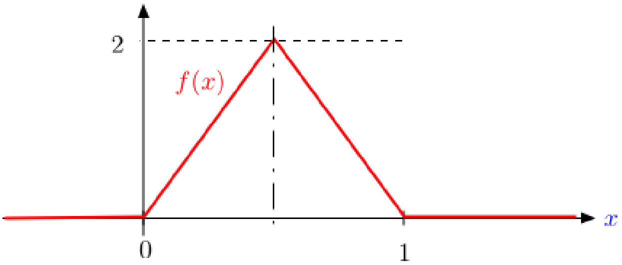

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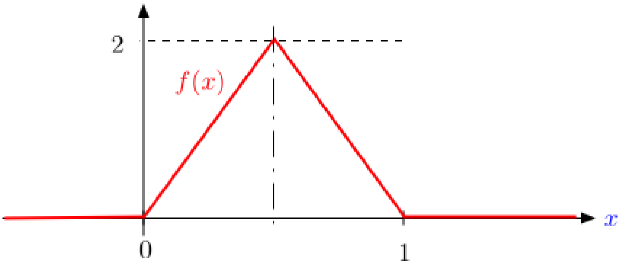

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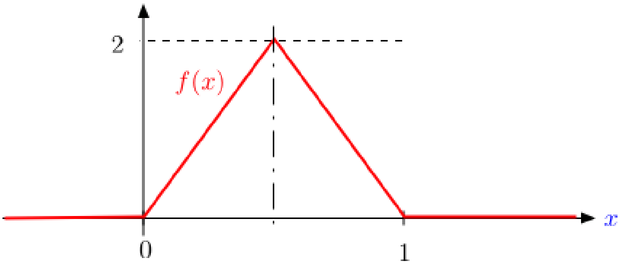

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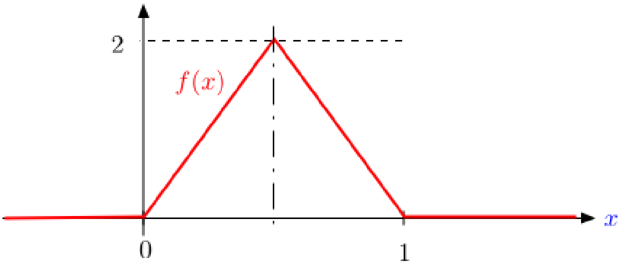

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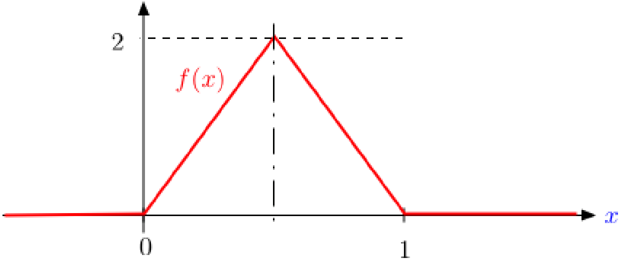

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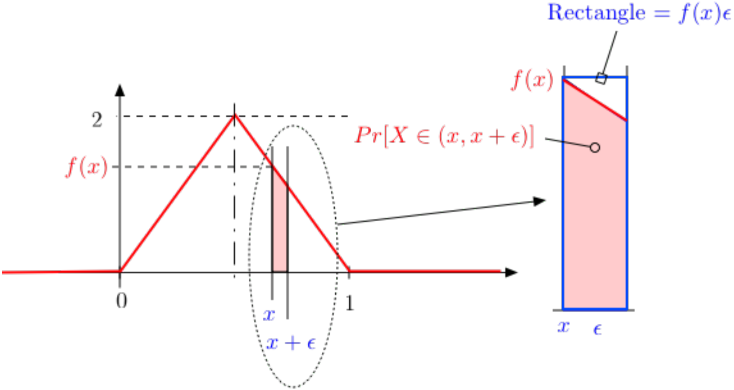

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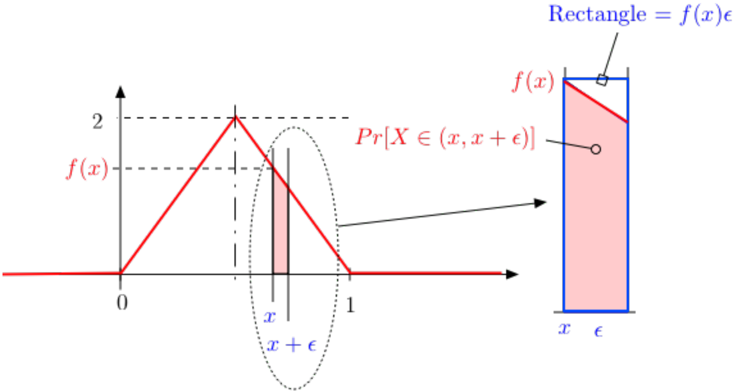

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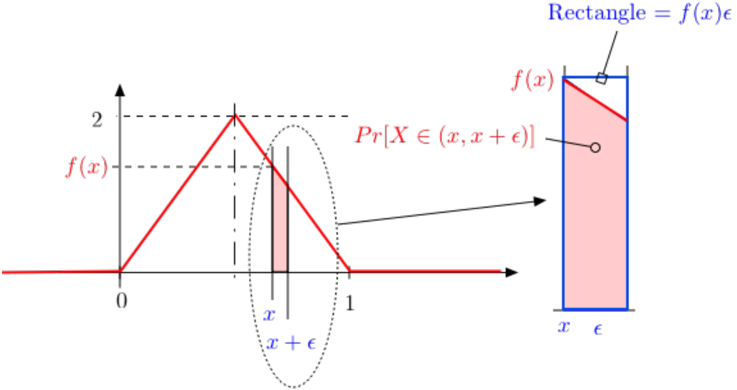

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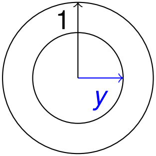

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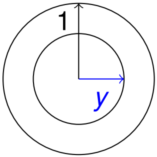

Random Variable: Y distance from center.
Probability within y of center:

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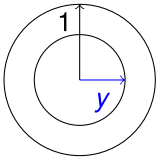

Random Variable: Y distance from center.
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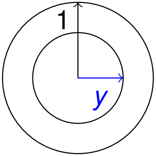

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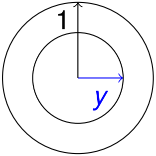

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Hence,

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Calculation of event with dartboard..

Probability between .5 and .6 of center?

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Use whichever is convenient.

Target

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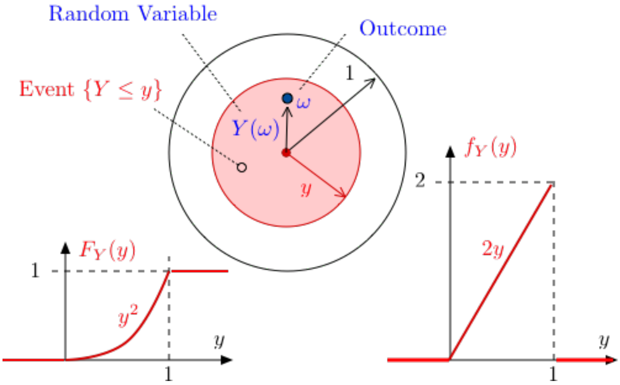
Random Variable
Outcome
Event $\{Y \leq y\}$
1
ω
$Y(\omega)$
y
$f_Y(y)$
2
$2y$
y
1
$F_Y(y)$
1
y^2
y
1

$U[a, b]$

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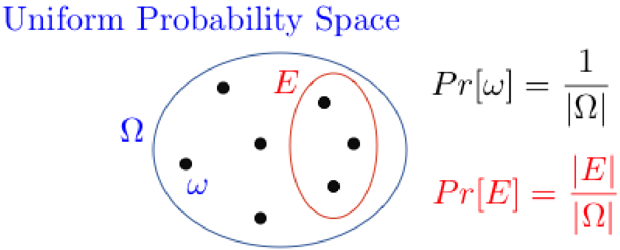
Uniform Probability Space
Ω
ω
E
$Pr[\omega] = \frac{1}{|\Omega|}$
$Pr[E] = \frac{|E|}{|\Omega|}$

Expo(λ)

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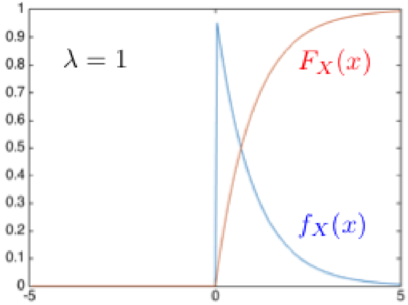

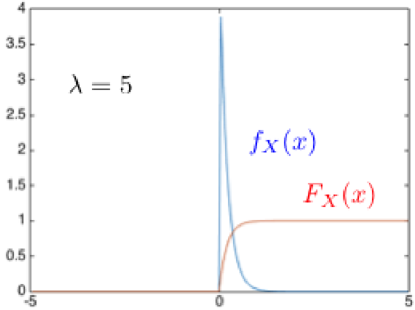

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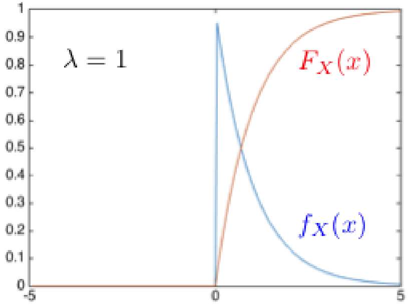

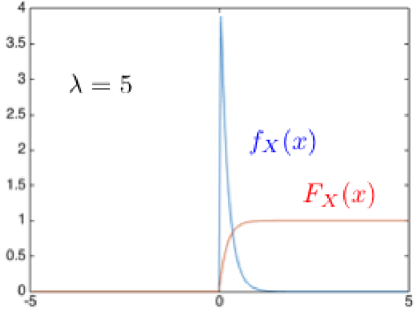

Note that $Pr[X > t] = e^{-\lambda t}$ for $t > 0$.

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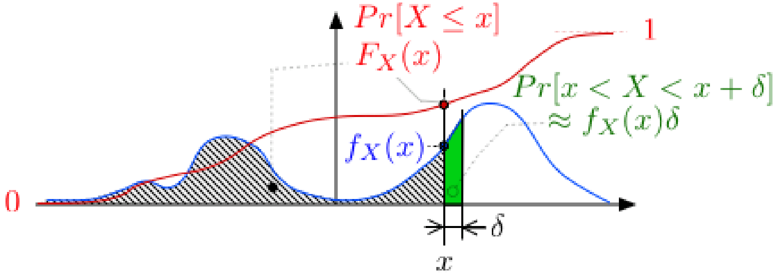
Recall that $Pr[X \in (x, x+\delta)] \approx f_X(x)\delta$.
X "takes" value $n\delta$, for $n \in Z$, with $Pr[X = n\delta] = f_X(n\delta)\delta$

A Picture



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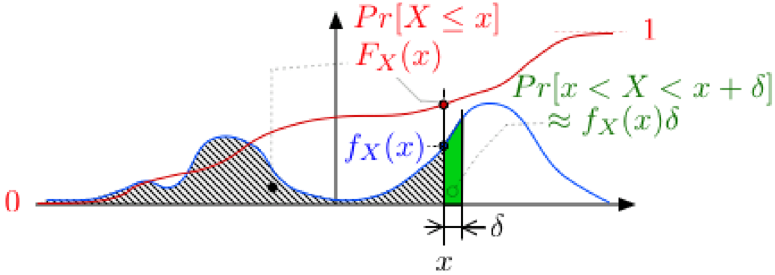

The pdf $f_X(x)$ is a nonnegative function that integrates to 1.

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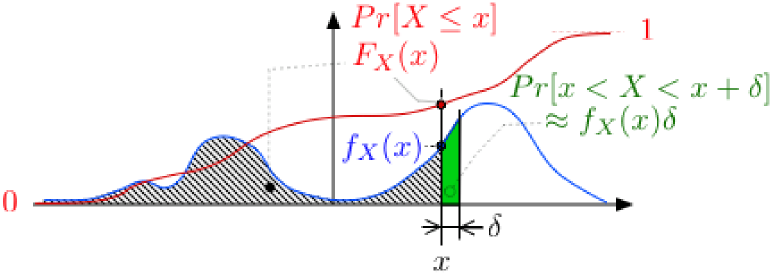

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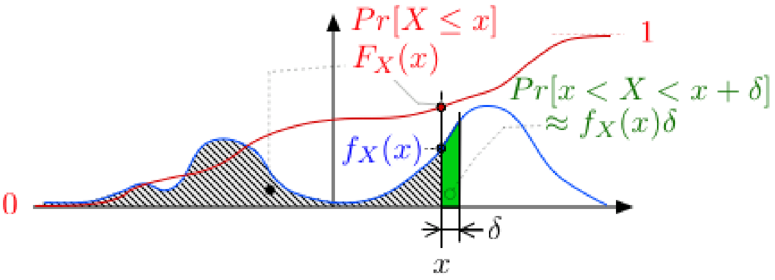

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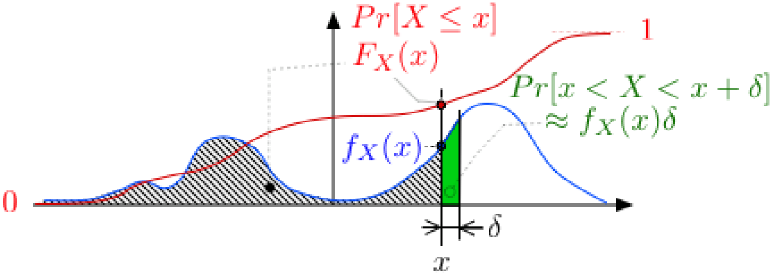

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Interpretation. Think of (X,Y) as being discrete on a grid with mesh size ε and $Pr[X = m\varepsilon, Y = n\varepsilon] = f_{X,Y}(m\varepsilon, n\varepsilon)\varepsilon^2$.

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where $|A|$ is the area of A.

Interpretation. Think of (X,Y) as being discrete on a grid with mesh size ε and $Pr[X = m\varepsilon, Y = n\varepsilon] = f_{X,Y}(m\varepsilon, n\varepsilon)\varepsilon^2$.

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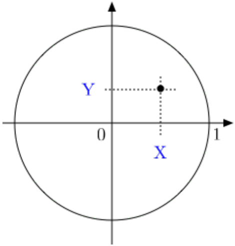

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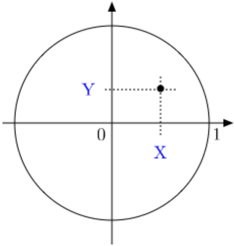

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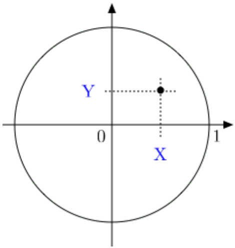

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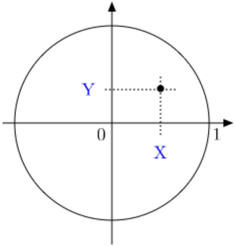

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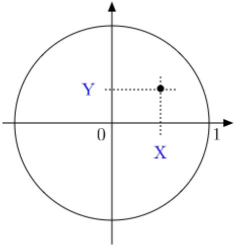

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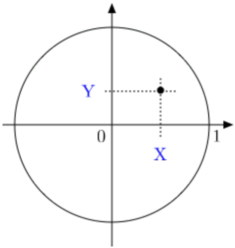

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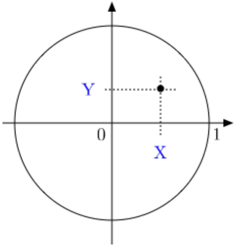

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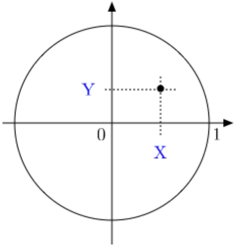

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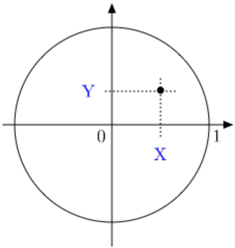

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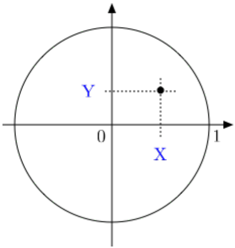

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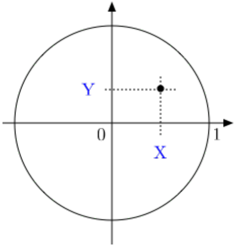

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Thus, $f_{X,Y}(x,y) = f_X(x)f_Y(y)$.

$\square$

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Proof: As in the discrete case.

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Corollary: For independent random variables, $f_{X|Y}(x,y) = f_X(x)$.

Independent Random Variables?

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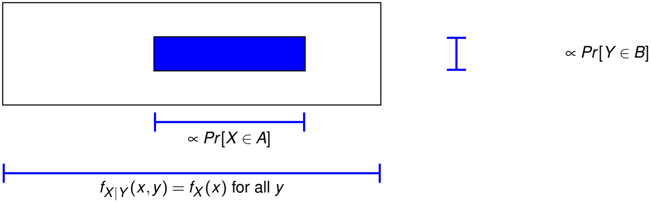

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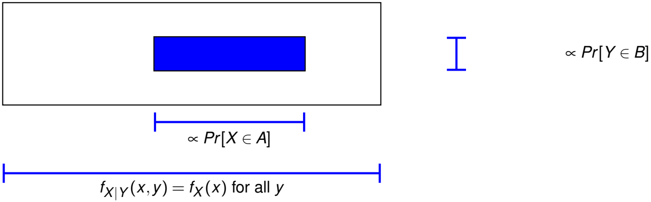

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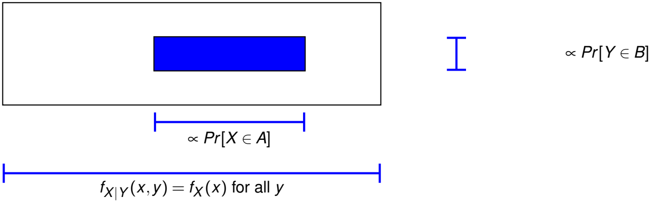

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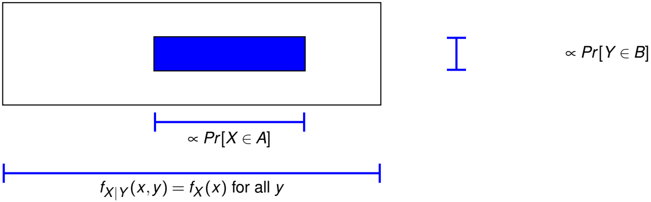

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Independent!

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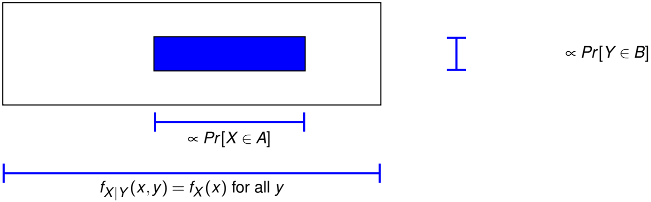
Uniform on a rectangle? Independent?



Also: $Pr[X \in A, Y \in B] \propto$ Area of rectangle $\propto Pr[X \in A] \times Pr[Y \in B]$.

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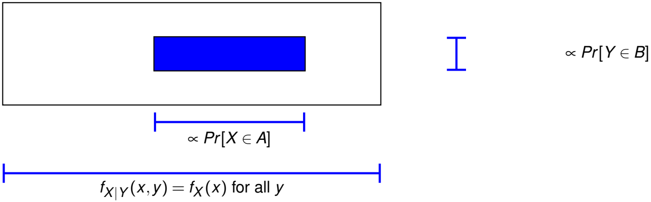

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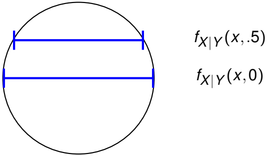

Not independent!

Summary

$\boxed{\text{Continuous Probability 1}}$

1. pdf:

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5. Target: $f_X(x) = 2x1\{0 \leq x \leq 1\}; F_X(x) = x^2$ for $0 \leq x \leq 1$.
6. Joint pdf: $Pr[X \in (x, x+\delta), Y = (y, y+\delta)) = f_{X,Y}(x,y)\delta^2$.
   6.1 Conditional Distribution: $f_{X|Y}(x,y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$.
   6.2 Independence: $f_{X|Y}(x,y) = f_X(x)$

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- Continuous RVs are essentially the same as discrete RVs
- Think that $X \approx x$ with probability $f_X(x)\varepsilon$
- Sums become integrals, ...
- The exponential distribution is magical: memoryless.