The natural numbers.

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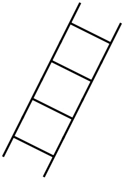
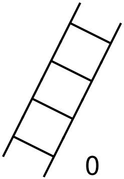


The natural numbers.

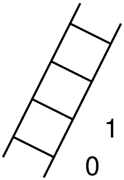
0,



0

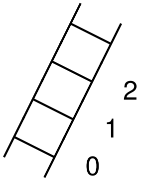
The natural numbers.

0, 1,



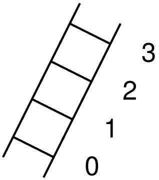
The natural numbers.

0, 1, 2,



The natural numbers.

0, 1, 2, 3,



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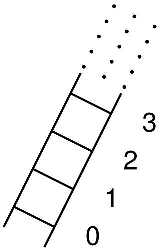

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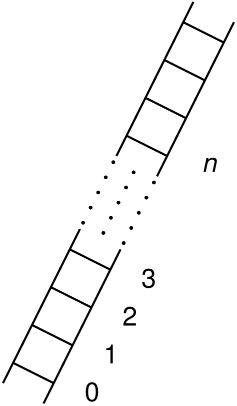

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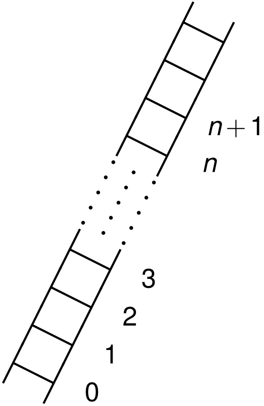

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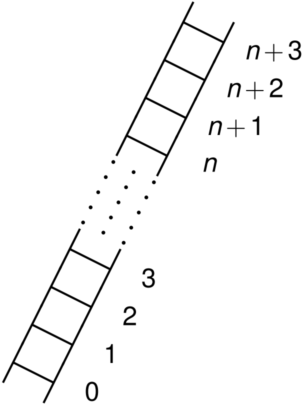

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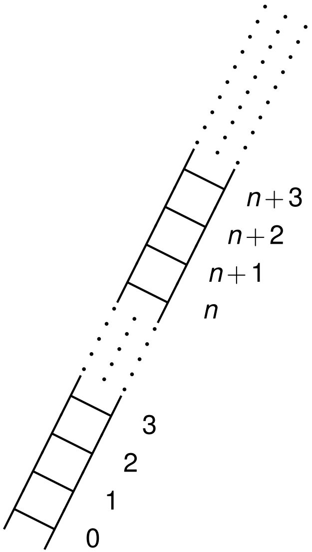

$0, 1, 2, 3,$

$\ldots, n, n+1, n+2, n+3, \ldots$

A formula.

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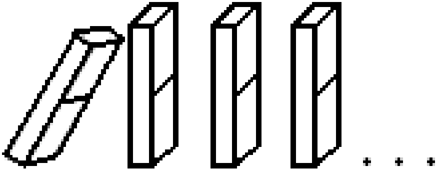
Notes visualization

Note's visualization: an infinite sequence of dominos.



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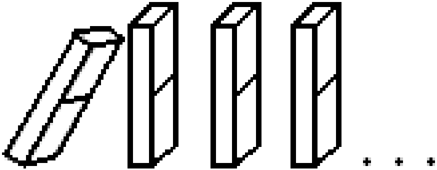

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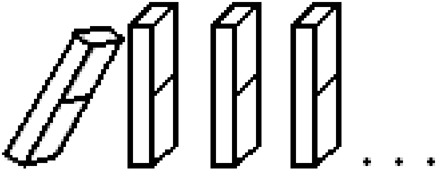

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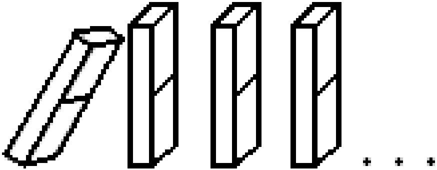

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Climb an infinite ladder?

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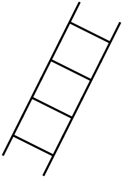

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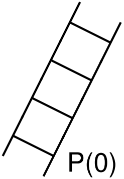

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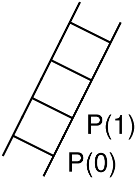

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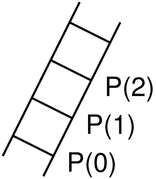

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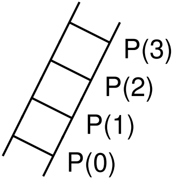

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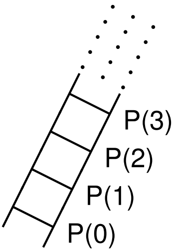

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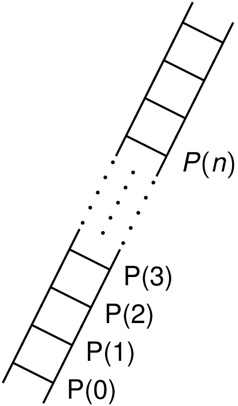

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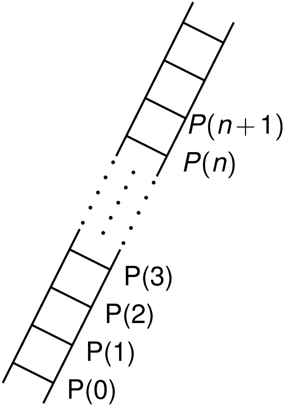

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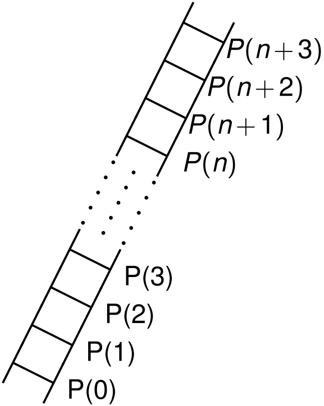

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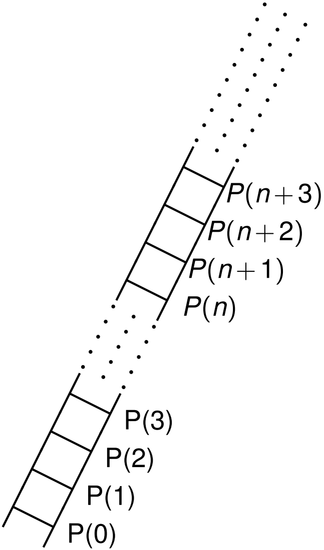

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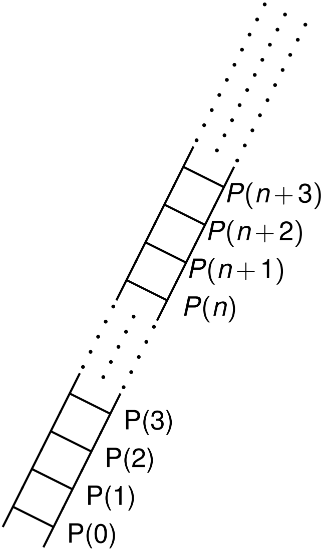

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Your favorite example of forever..

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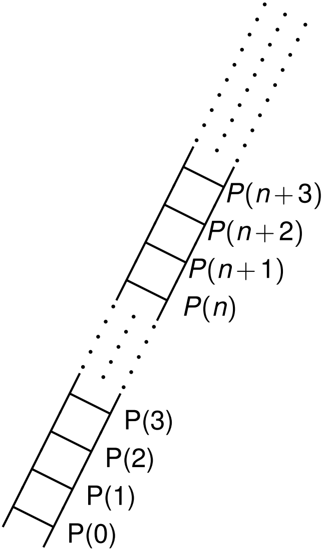

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Your favorite example of forever..or the natural numbers...

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The canonical way of proving statements of the form

$$(\forall k \in N)(P(k))$$

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$P(n)$ true for all natural numbers n!!!

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- $P(k) \implies P(k+1)$
  - Assume $P(k)$, "Induction Hypothesis"
  - Prove $P(k+1)$. "Induction Step."

$P(n)$ true for all natural numbers n!!!
Get to use $P(k)$ to prove $P(k+1)$!

Induction

The canonical way of proving statements of the form

$$(\forall k \in N)(P(k))$$

- For all natural numbers n, $1 + 2 \cdots n = \frac{n(n+1)}{2}$.
- For all $n \in N$, $n^3 - n$ is divisible by 3.
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Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=0}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$$\sum_{i=0}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{k(k+1)+2(k+1)}{2} = \frac{(k+1)(k+2)}{2}.$$

How about $k+2$. Same argument starting at $k+1$ works!

Induction Step. $P(k) \implies P(k+1)$.

Is this a proof? It shows that we can always move to the next step.

Need to start somewhere. $P(0)$ is $\sum_{i=0}^{0} i = \frac{(0)(0+1)}{2}$ **Base Case.**

Statement is true for $n = 0$ $P(0)$ is true

plus inductive step $\implies$ true for $n = 1$ $(P(0) \wedge (P(0) \implies P(1))) \implies P(1)$

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Predicate, $P(n)$, True for all natural numbers! **Proof by Induction.**

Poll: What did Gauss use in the proof?

(A) Every natural number has a next number.

(B) The recursive leap of faith.

(C) $2^k > k$.

(D) $\forall k \in \mathbb{N}, \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}$.

Another Induction Proof.

Theorem: For every $n \in N$, $n^3 - n$ is divisible by 3. $(3|(n^3 - n)$).

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$(k+1)^3 - (k+1)$

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Two color theorem: example.

Any map formed by dividing the plane into regions by drawing straight lines can be properly colored with two colors.

.

Proper coloring: for each line segment the regions on the two sides have different colors.1

Fact: Swapping red and blue gives another valid colors.

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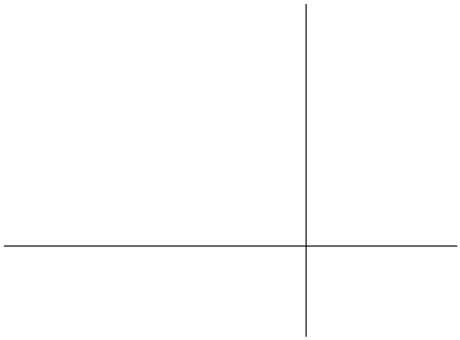

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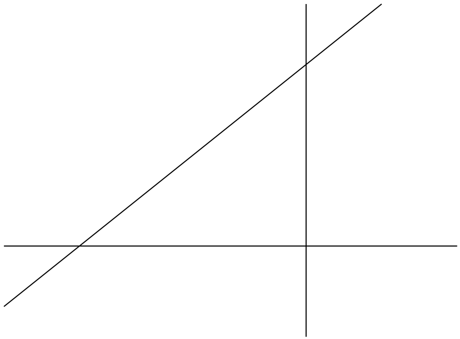
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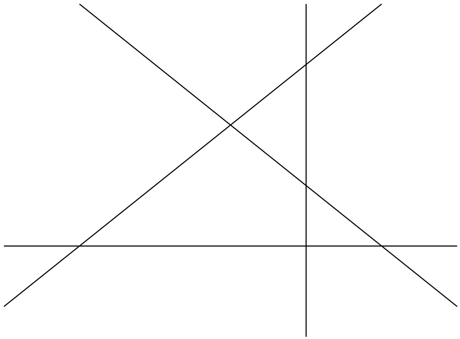

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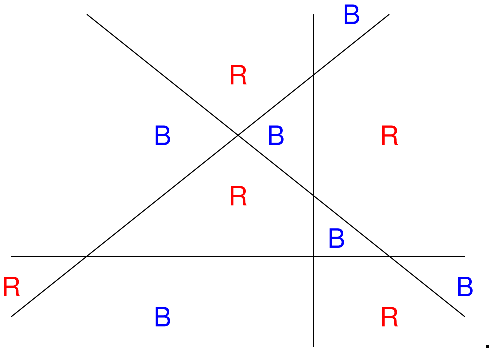

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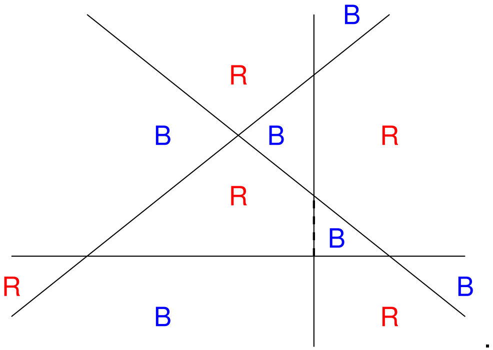

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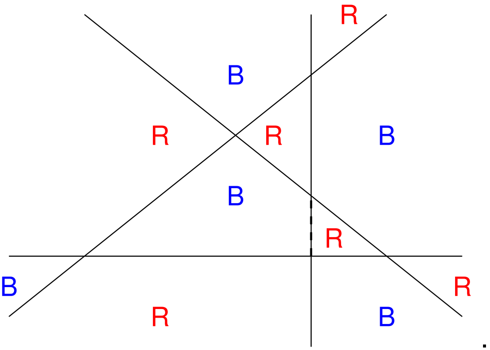

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Two color theorem: proof illustration.

Base Case.

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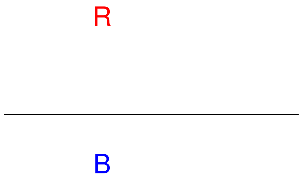

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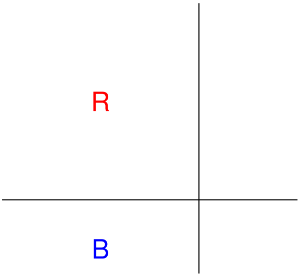
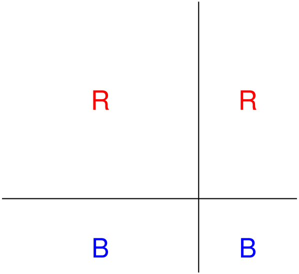


1. Add line.

Two color theorem: proof illustration.



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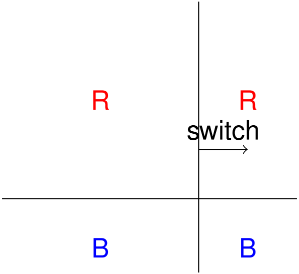
Two color theorem: proof illustration.



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(Fixes conflicts along new line, and makes no new ones along previous line.)

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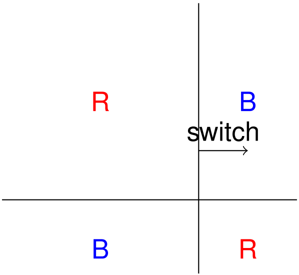

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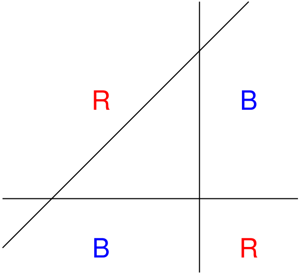

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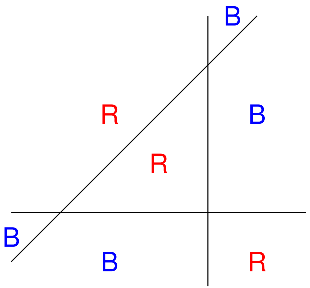

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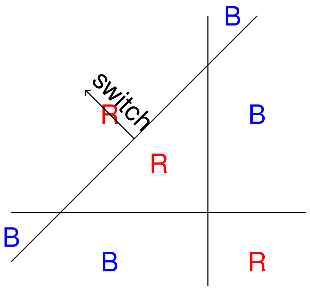

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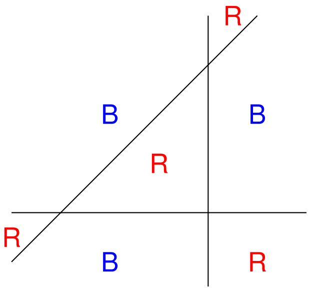

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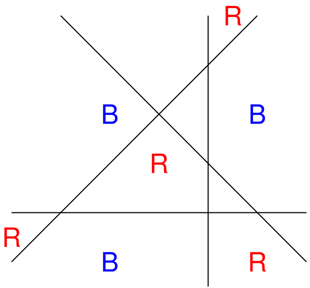

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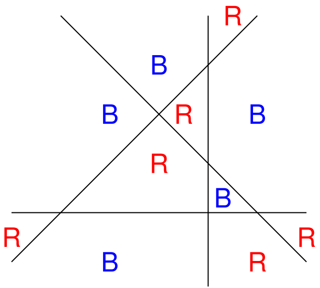

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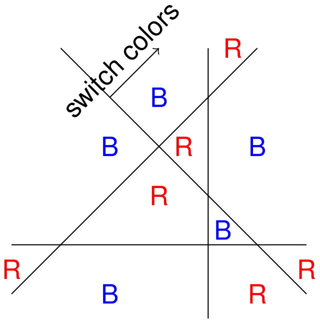

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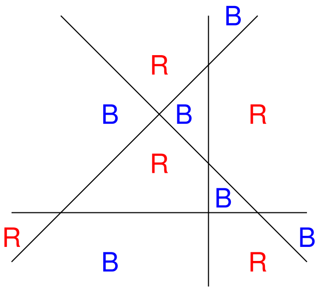

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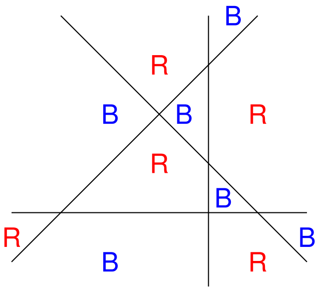

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Algorithm gives $P(k) \implies P(k+1)$.

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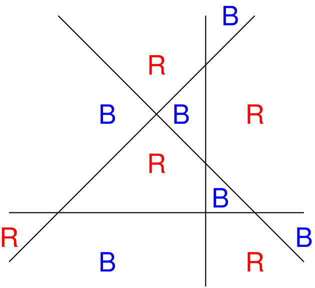

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Poll: what did we use in the proof.

(A) Switching a 2-coloring is a valid coloring.

(B) Definition of 2-coloring.

(C) Definition of adjacent.

(D) Definition of region.

(E) The four color theorem.

Strenthening Induction Hypothesis.

Theorem: The sum of the first n odd numbers is a perfect square.
Theorem: The sum of the first n odd numbers is n^2.

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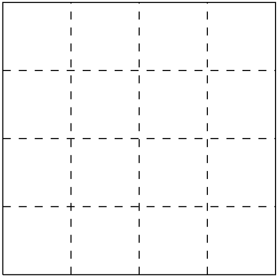
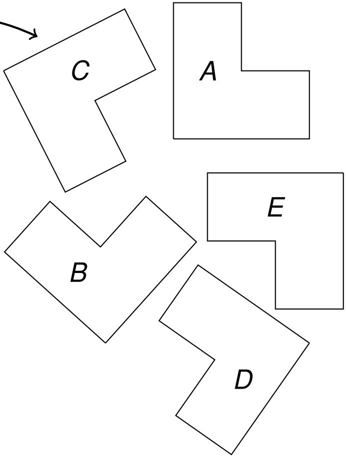
$\square$

Tiling Cory Hall Courtyard.

Use these L-tiles.

To Tile this 4×4 courtyard.





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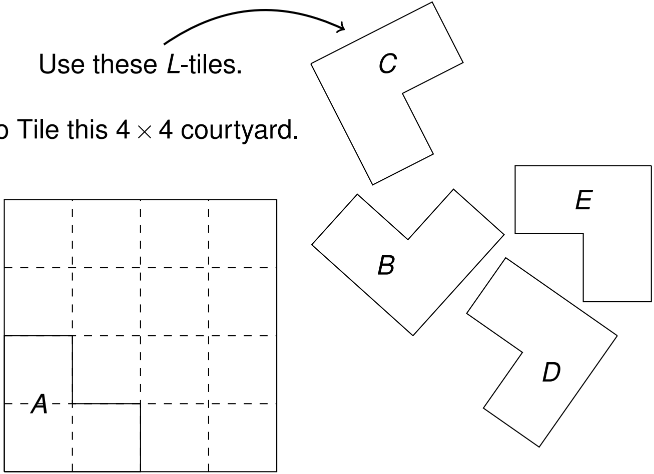

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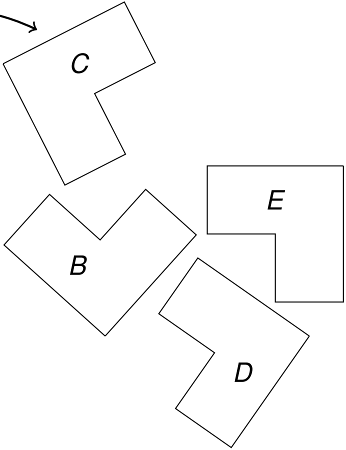

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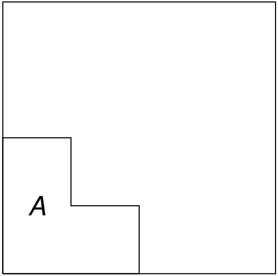

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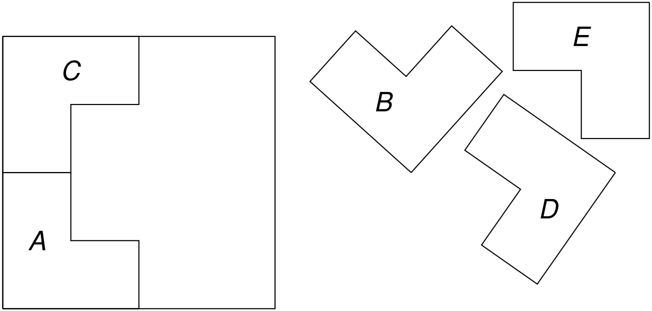

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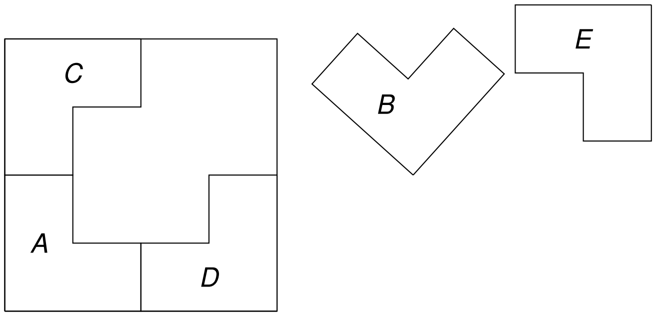

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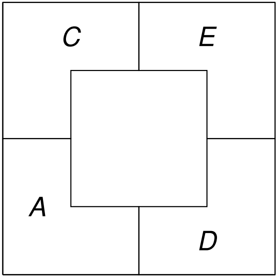

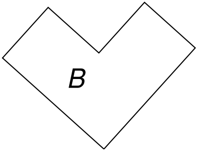

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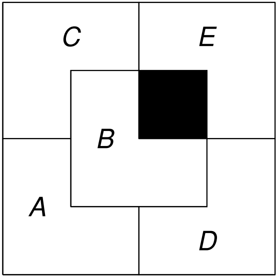

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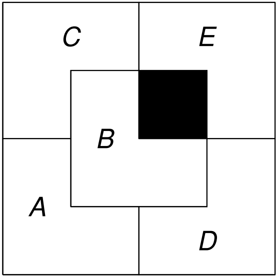

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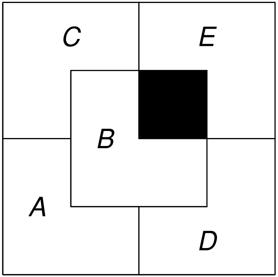


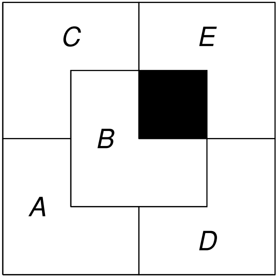
Alright!

Tiled 4×4 square with 2×2 L-tiles.

Tiling Cory Hall Courtyard.

Use these L-tiles.

To Tile this 4×4 courtyard.



Alright!

Tiled 4×4 square with 2×2 L-tiles.

with a center hole.

Tiling Cory Hall Courtyard.

Use these L-tiles.

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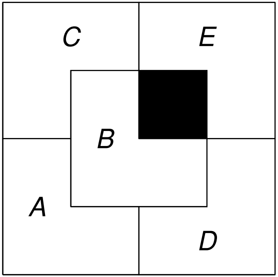

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Can we tile any $2^n \times 2^n$ with L-tiles (with a hole)

Tiling Cory Hall Courtyard.

Use these L-tiles.

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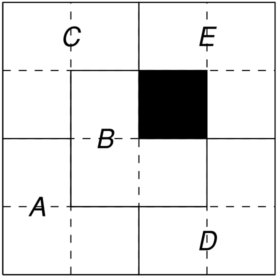

Alright!

Tiled 4×4 square with 2×2 L-tiles.

with a center hole.

Can we tile any $2^n \times 2^n$ with L-tiles (with a hole) for every n!

Hole have to be there? Maybe just one?

Theorem: Any tiling of $2^n \times 2^n$ square has to have one hole.

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Theorem: Any tiling of $2^n \times 2^n$ square has to have one hole.

Proof: The remainder of 2^{2n} divided by 3 is 1.

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Base case: true for $k = 0$.

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$$2^{2(k+1)}$$

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$$2^{2(k+1)} = 2^{2k} * 2^2$$

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$$
\begin{aligned}
2^{2(k+1)} &= 2^{2k} * 2^2 \\
&= 4 * 2^{2k} \\
&= 4 * (3a + 1) \\
&= 12a + 3 + 1
\end{aligned}
$$

Hole have to be there? Maybe just one?

Theorem: Any tiling of $2^n \times 2^n$ square has to have one hole.

Proof: The remainder of 2^{2n} divided by 3 is 1.

Base case: true for $k = 0$. $2^0 = 1$

Ind Hyp: $2^{2k} = 3a + 1$ for integer a.

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2^{2(k+1)} &= 2^{2k} * 2^2 \\
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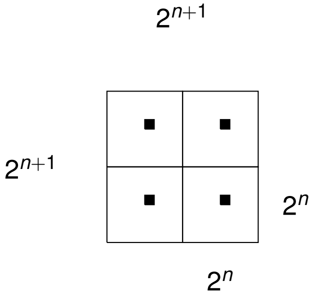

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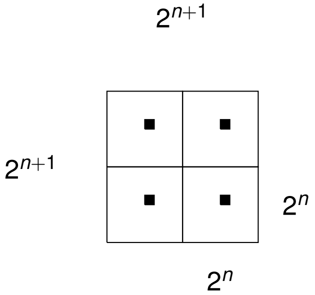

What to do now???

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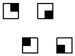

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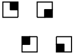

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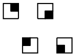

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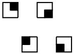

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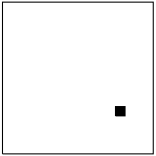

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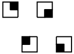
Flipping the orientation can leave hole anywhere. 

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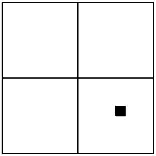

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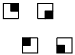

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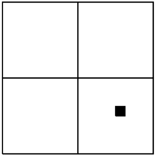

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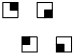

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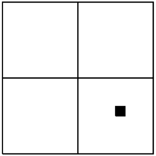

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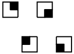
Flipping the orientation can leave hole anywhere. 

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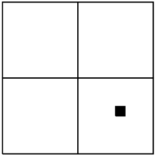

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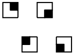

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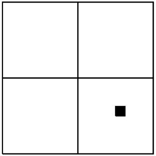

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(Contrapositive of Induction principle (assuming $P(0)$)

It assumes that there is a smallest m where $P(m)$ does not hold.

The **Well ordering principle** states that for any subset of the natural numbers there is a smallest element.

Examples: even numbers, odd numbers, primes, non-primes, etc..

Well Ordering Principle and Induction.

If $(\forall n)P(n)$ is not true, then $(\exists n)\neg P(n)$.

Consider smallest m, with $\neg P(m)$, $m \geq 0$

$P(m-1) \implies P(m)$ must be false (assuming $P(0)$ holds.)

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Note: can do with different definition of smallest.
For example. Use reduced form: a/b and order by $a+b$.

Well ordering principle.

Thm: All natural numbers are interesting.

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Thus, there is no smallest uninteresting natural number.

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But this is interesting.
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Thus: All natural numbers are interesting.

Tournaments have short cycles

Def: A **round robin tournament on** n **players**: every player p plays every other player q, and either $p \to q$ (p beats q) or $q \to p$ (q beats p.)

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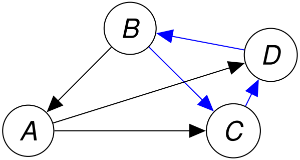

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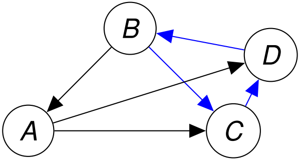

Theorem: Any tournament that has a cycle has a cycle of length 3.

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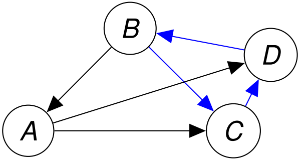

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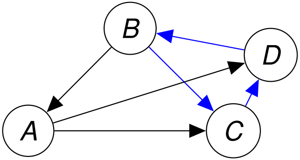

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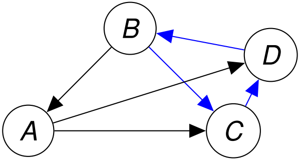

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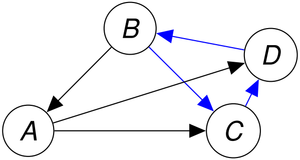

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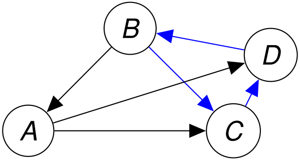

Theorem: Any tournament that has a cycle has a cycle of length 3.

Tournament has a cycle of length 3 if at all.

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Assume the the **smallest cycle** is of length k.

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Case 1: Of length 3.

Tournament has a cycle of length 3 if at all.

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Case 1: Of length 3. Done.

Tournament has a cycle of length 3 if at all.

Assume the the **smallest cycle** is of length k.

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Case 2: Of length larger than 3.

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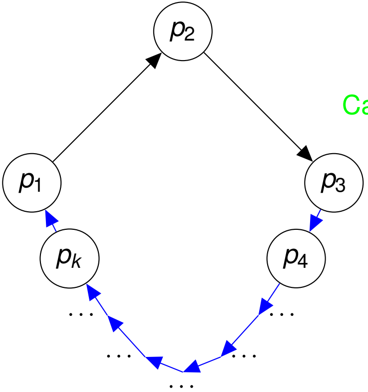

Case 1: "$p_3 \to p_1$" $\implies$ 3 cycle

Contradiction.

$\square$

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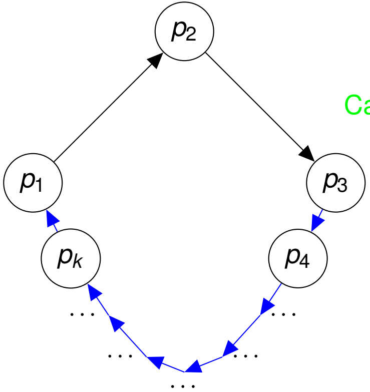

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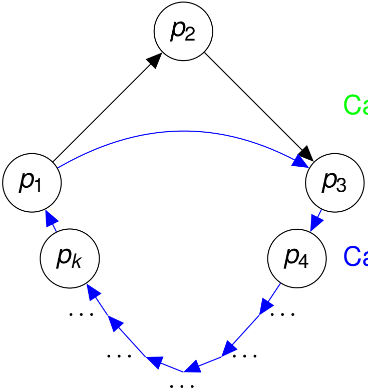

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Contradicts assumption of smallest k!

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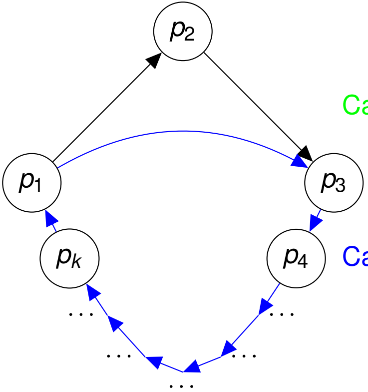

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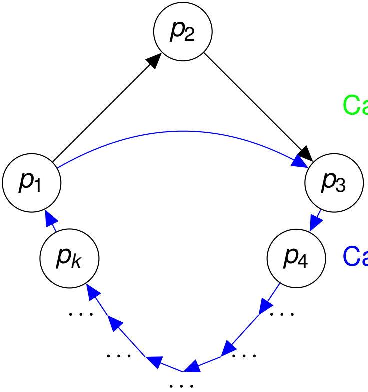

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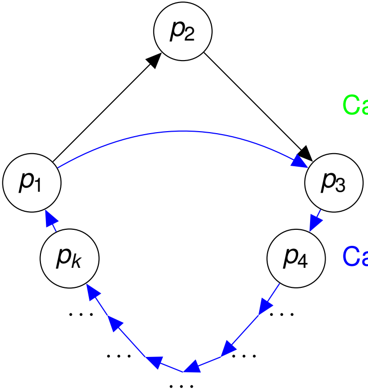

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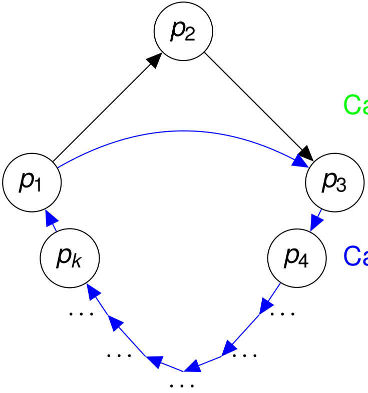

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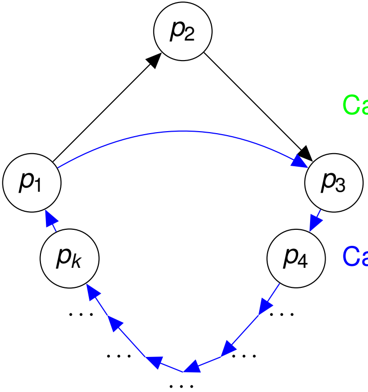

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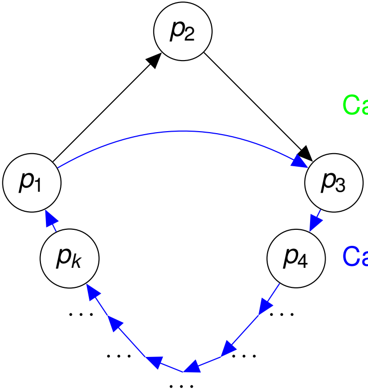

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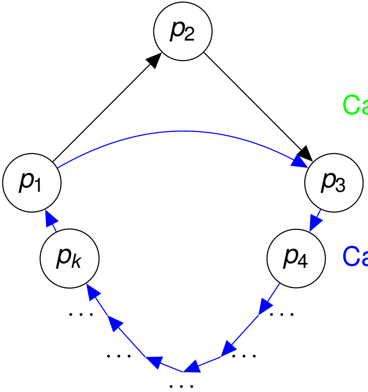

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Tournaments have long paths.

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$$2 \to 1 \to \cdots \to 7$$

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Tournament on $n+1$ people,

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Tournament on $n+1$ people,

Remove arbitrary person $\to$ yield tournament on $n-1$ people.

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Tournament on $n+1$ people,
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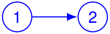

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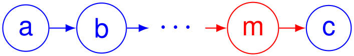

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Tournament on $n+1$ people,
Remove arbitrary person $\to$ yield tournament on $n-1$ people.

By induction hypothesis: There is a sequence $p_1, \ldots, p_n$
contains all the people where $p_i \to p_{i+1}$

$$a \to b \to \cdots \to m \to c$$

If p is big winner, put at beginning.

Tournaments have long paths.

Def: A **round robin tournament on** n **players**: all pairs p and q play, and either $p \to q$ (p beats q) or $q \to p$ (q beats q.)

Def: A **Hamiltonian path**: a sequence
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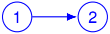

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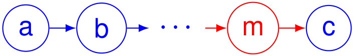

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Horses of the same color...

Theorem: All horses have the same color.

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As we will see, it is more subtle to catch errors in proofs of correct theorems!!

Sad Islanders...

Island with 100 possibly blue-eyed and green-eyed inhabitants.

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Visitor: "I see someone has green eyes."

Result: Poll.

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Result: Poll. On day 100,

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Why?

They know induction.

Thm: If there are n villagers with green eyes they do ritual on day n.

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Proof:
Base: $n = 1$. Person with green eyes does ritual on day 1.

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But they didn't do the ritual.

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Wait! Visitor added no information.

Common Knowledge.

Using knowledge about what other people's knowledge (your eye color) is.

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On day 1, everyone knows everyone sees more than zero.

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On day 99, everyone knows no one sees 98
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Another example:

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No one knows other people see that he has no clothes.

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Another example:
Emperor's new clothes!
No one knows other people see that he has no clothes.
Until kid points it out.

Summary: principle of induction.

Today: More induction.

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$(P(0)$

Summary: principle of induction.

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$(P(0) \wedge ((\forall k \in N)(P(k) \implies P(k+1))))$

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$$(P(0) \wedge ((\forall k \in N)(P(k) \implies P(k+1)))) \implies (\forall n \in N)(P(n))$$

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Ind. Step: Prove.

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Ind. Step: Prove. For all values, $n \geq n_0$, $P(n) \implies P(n+1)$.
Statement is proven!

Summary: principle of induction.

Today: More induction.

$(P(0) \wedge ((\forall k \in N)(P(k) \implies P(k+1)))) \implies (\forall n \in N)(P(n))$

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Strengthen theorem statement.
Sum of first n odds is n^2.

Summary: principle of induction.

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Also Today: strengthened induction hypothesis.

Strengthen theorem statement.
Sum of first n odds is n^2.
Hole anywhere.

Summary: principle of induction.

Today: More induction.

$$(P(0) \wedge ((\forall k \in N)(P(k) \implies P(k+1)))) \implies (\forall n \in N)(P(n))$$

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Not same as strong induction.

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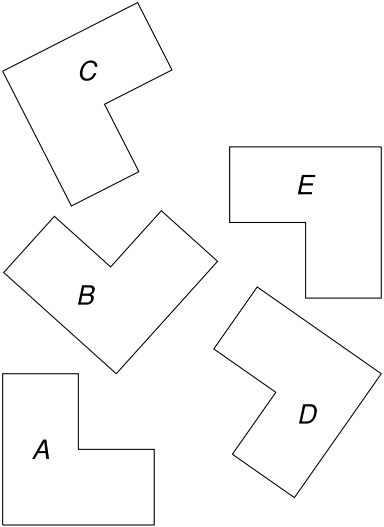
Induction $\equiv$ Recursion.

Tiling Cory Hall Courtyard.



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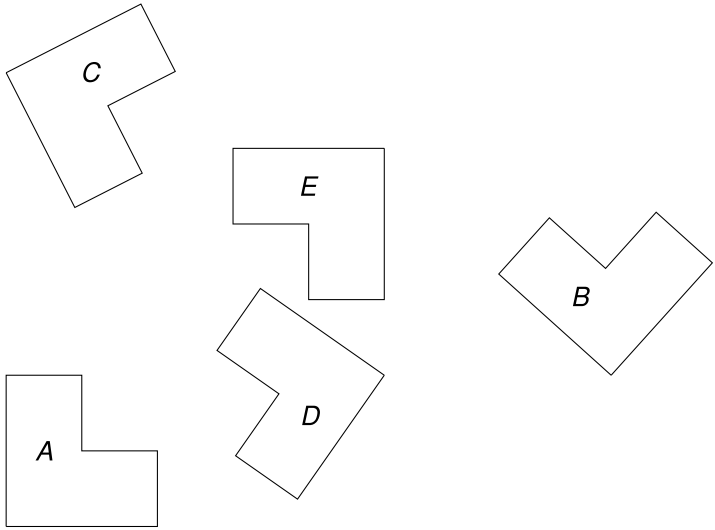

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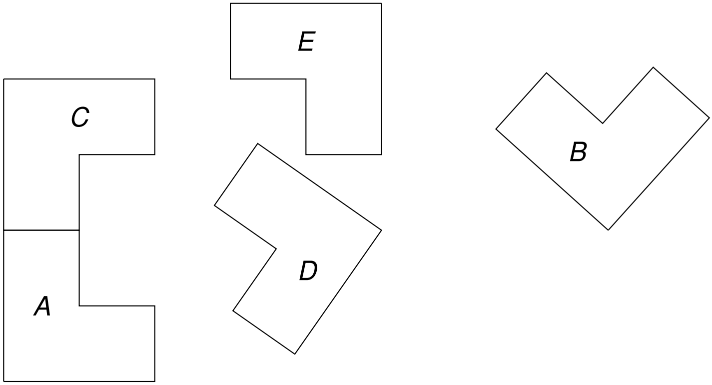

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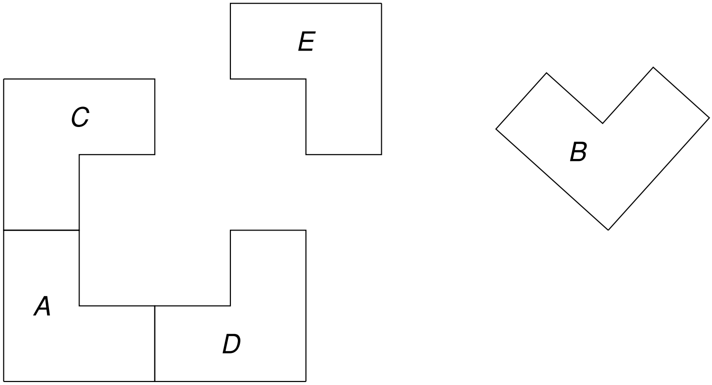

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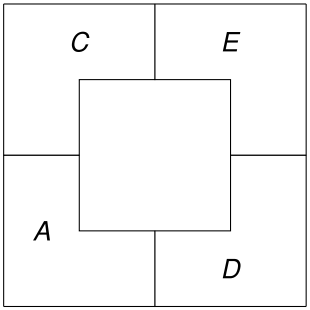

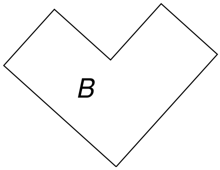

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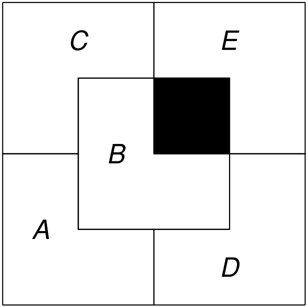

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