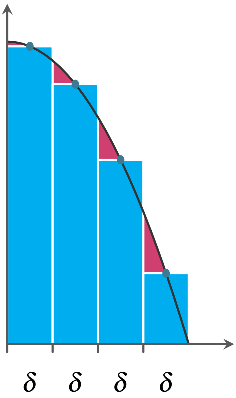
Survey

Fill it out!!
tinyurl.com/cs70-survey

Calculus



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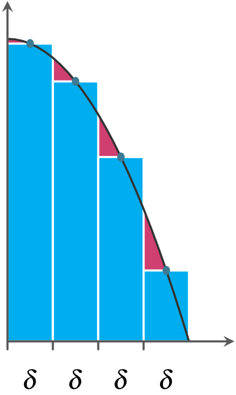

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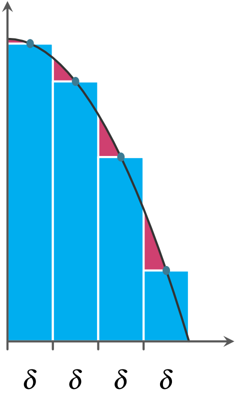

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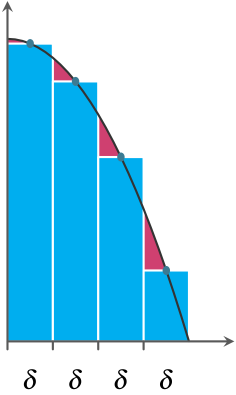

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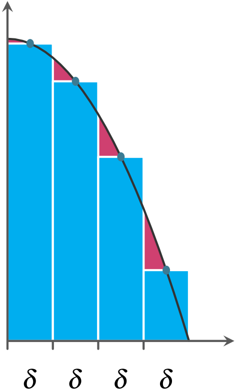

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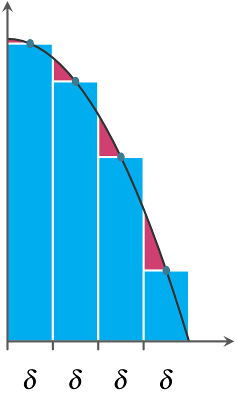

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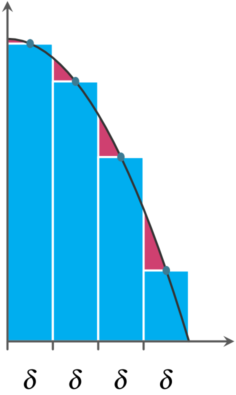

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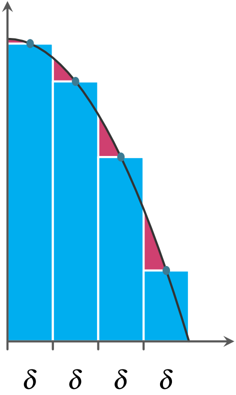

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Thus $F'(x) = f(x)$.

CS70: Continuous Probability.

$\boxed{\text{Continuous Probability 1}}$

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Continuous Probability 1

1. Examples
2. Events
3. Continuous Random Variables

Uniformly at Random in $[0, 1]$.

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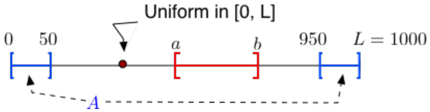

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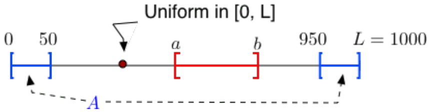

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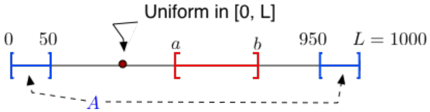

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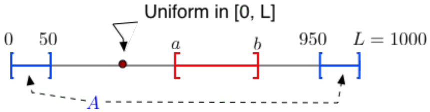

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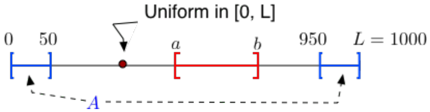

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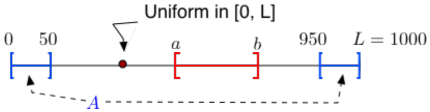

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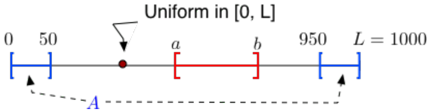

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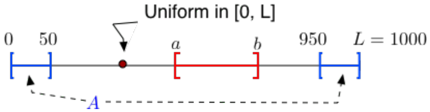

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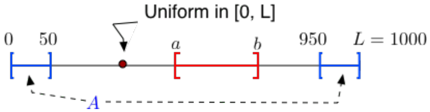

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Makes sense: $b - a$ is the fraction of $[0,1]$ that $[a,b]$ covers.

Poll

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Next lecture.

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More generally, events in this space are unions of intervals.

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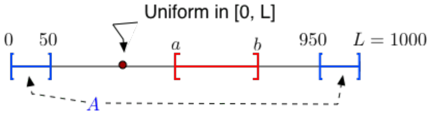

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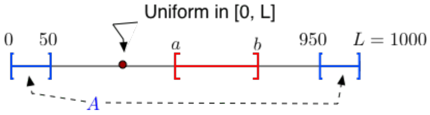
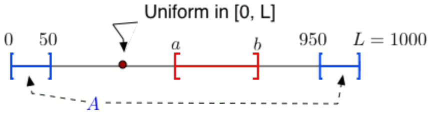


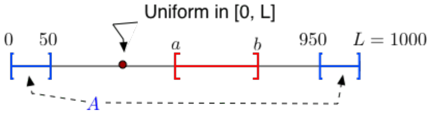
Note: A **radical** change in approach.

Uniformly at Random in $[0, 1]$.



Note: A **radical** change in approach.
Finite prob. space:

Uniformly at Random in $[0,1]$.



Note: A **radical** change in approach.
Finite prob. space: $\Omega = \{1, 2, \ldots, N\}$,

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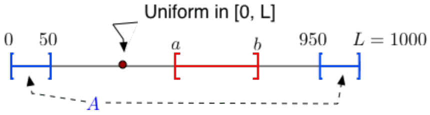

Note: A **radical** change in approach.
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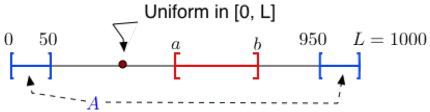

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Finite prob. space: $\Omega = \{1, 2, \ldots, N\}$, with $Pr[\omega] = p_\omega$.
$\implies Pr[A] = \sum_{\omega \in A} p_\omega$ for $A \subset \Omega$.

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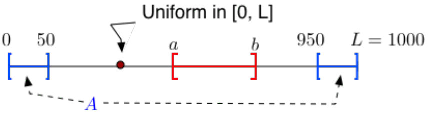


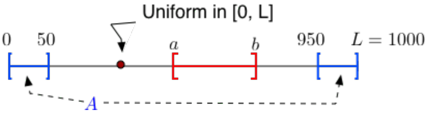
Note: A **radical** change in approach.

Finite prob. space: $\Omega = \{1, 2, \ldots, N\}$, with $Pr[\omega] = p_\omega$.

$\implies Pr[A] = \sum_{\omega \in A} p_\omega$ for $A \subset \Omega$.

Continuous space:

Uniformly at Random in $[0,1]$.



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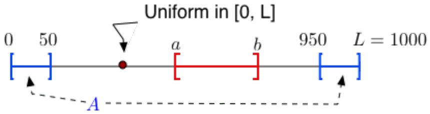

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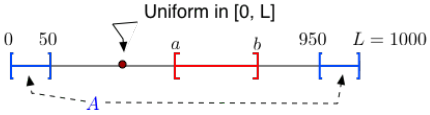
Continuous space: e.g., $\Omega = [0,1]$,
$Pr[\omega]$ is typically 0.

Uniformly at Random in $[0,1]$.



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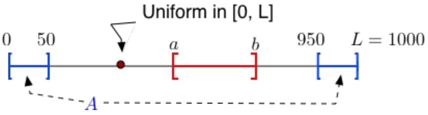
Instead, start with $Pr[A]$ for some events A.

Uniformly at Random in $[0,1]$.



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Continuous space: e.g., $\Omega = [0,1]$,
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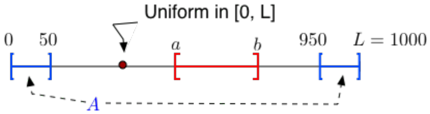
Instead, start with $Pr[A]$ for some events A.
Event A = interval,

Uniformly at Random in $[0, 1]$.



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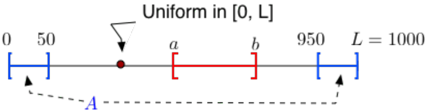
Event A = interval, or union of intervals.

Uniformly at Random in $[0,1]$.



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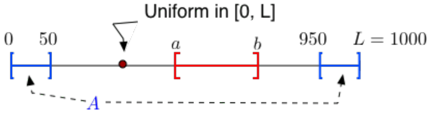
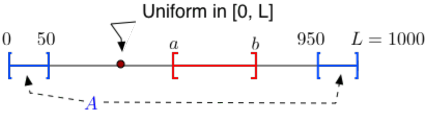
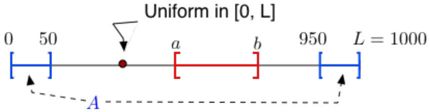


Uniformly at Random in $[0, 1]$.



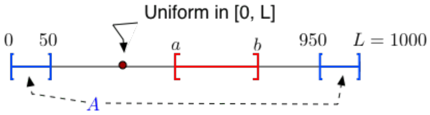
$Pr[X \leq x] = x$ for $x \in [0, 1]$.

Uniformly at Random in $[0, 1]$.



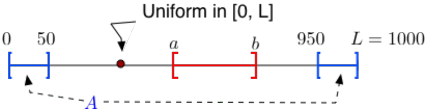
$Pr[X \leq x] = x$ for $x \in [0, 1]$. Also, $Pr[X \leq x] = 0$ for $x < 0$.

Uniformly at Random in $[0,1]$.



$Pr[X \leq x] = x$ for $x \in [0,1]$. Also, $Pr[X \leq x] = 0$ for $x < 0$.
$Pr[X \leq x] = 1$ for $.2x > 1$.

Uniformly at Random in $[0, 1]$.



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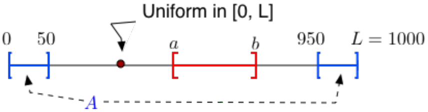
Define $F(x) = Pr[X \leq x]$.

Uniformly at Random in $[0,1]$.



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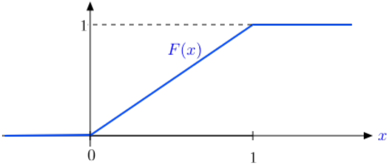

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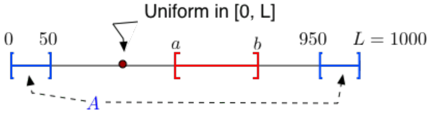

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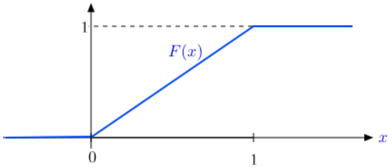
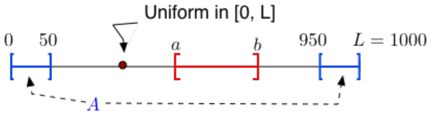


Then we have $Pr[X \in (a, b]] = Pr[X \leq b] - Pr[X \leq a]$

Uniformly at Random in $[0,1]$.



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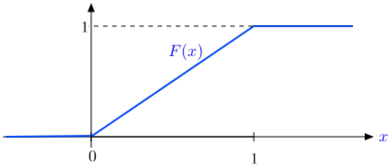

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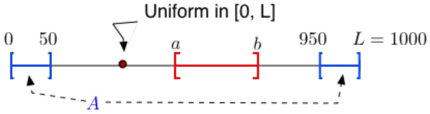
Uniformly at Random in $[0,1]$.



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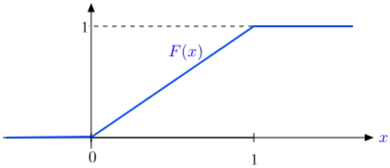
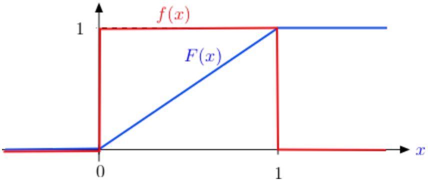


Then we have $Pr[X \in (a,b]] = Pr[X \leq b] - Pr[X \leq a] = F(b) - F(a)$.

Thus, $F(\cdot)$ specifies the probability of all the events!

Uniformly at Random in $[0,1]$.



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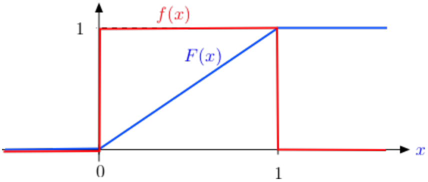

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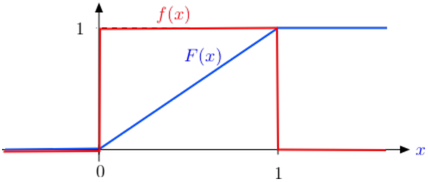
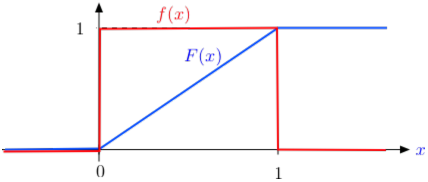


$$Pr[X \in (a, b]] = Pr[X \leq b] - Pr[X \leq a] = F(b) - F(a).$$

An alternative view is to define $f(x) = \frac{d}{dx} F(x) =$

Uniformly at Random in $[0,1]$.



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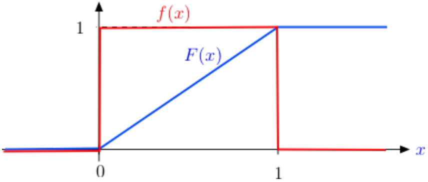
Uniformly at Random in $[0,1]$.



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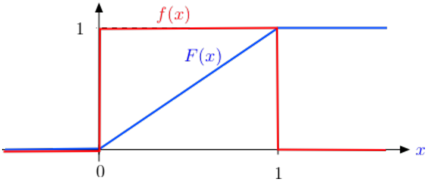

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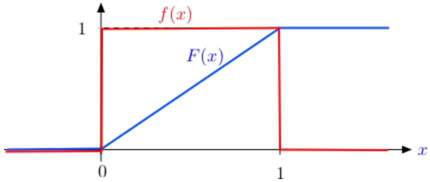
Thus, the probability of an event is the integral of $f(x)$ over the event:

Uniformly at Random in $[0,1]$.



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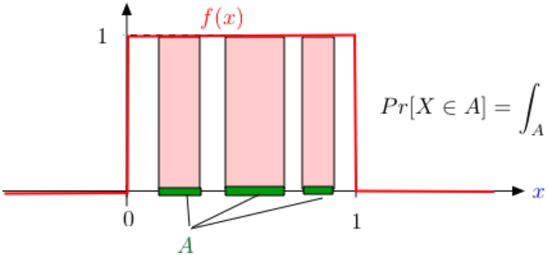
Thus, the probability of an event is the integral of $f(x)$ over the event:

$$Pr[X \in A] = \int_A f(x)dx.$$

Uniformly at Random in $[0, 1]$.



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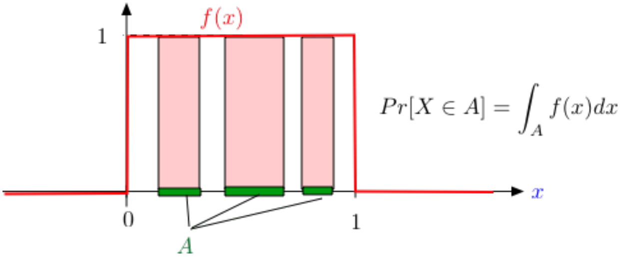
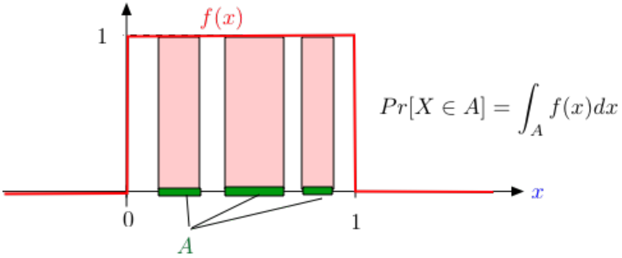
Uniformly at Random in $[0, 1]$.



Think of $f(x)$ as describing how
one unit of probability is spread over $[0, 1]$:

Uniformly at Random in $[0, 1]$.



Think of $f(x)$ as describing how
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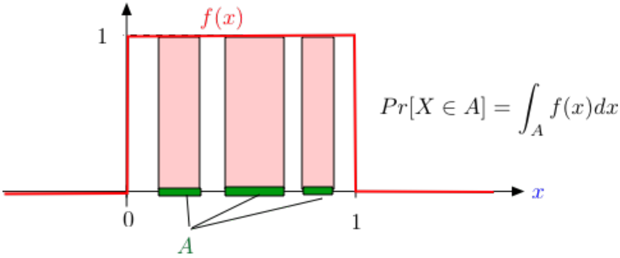
Uniformly at Random in $[0, 1]$.



Think of $f(x)$ as describing how
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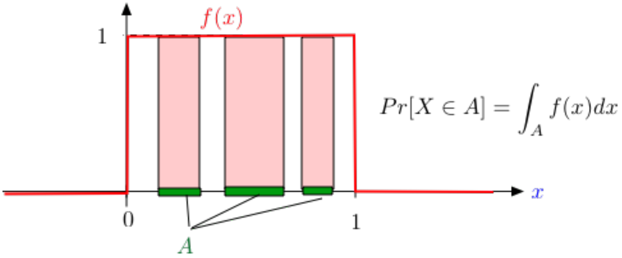
Then $Pr[X \in A]$ is the probability mass over A.

Uniformly at Random in $[0, 1]$.



Think of $f(x)$ as describing how
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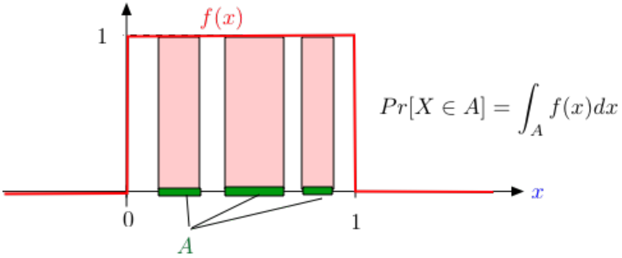
Then $Pr[X \in A]$ is the probability mass over A.

Observe:

Uniformly at Random in $[0, 1]$.



Think of $f(x)$ as describing how
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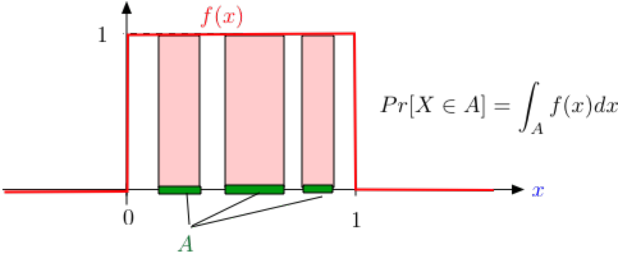
Then $Pr[X \in A]$ is the probability mass over A.

Observe:

- This makes the probability automatically additive.

Uniformly at Random in $[0, 1]$.



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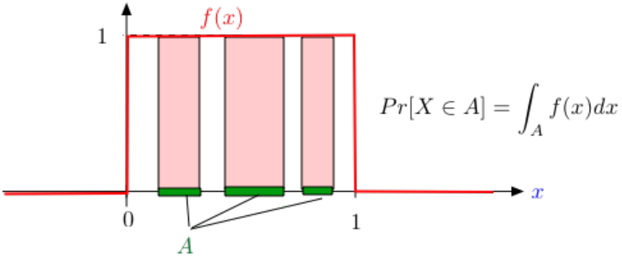
Uniformly at Random in $[0, 1]$.



Think of $f(x)$ as describing how
one unit of probability is spread over $[0, 1]$: uniformly!

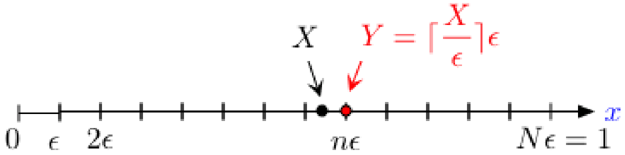
Then $Pr[X \in A]$ is the probability mass over A.

Observe:

- This makes the probability automatically additive.
- We need $f(x) \geq 0$ and $\int_{-\infty}^{\infty} f(x)dx = 1$.

Uniformly at Random in $[0, 1]$.

Uniformly at Random in $[0, 1]$.



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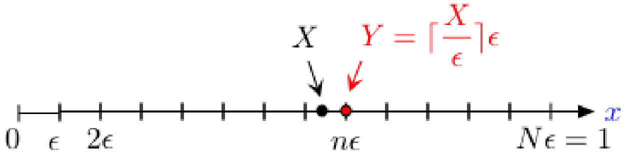
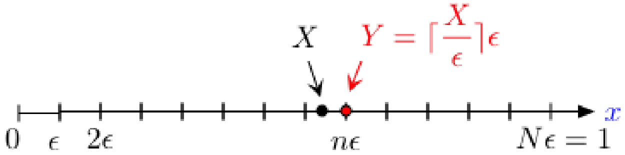


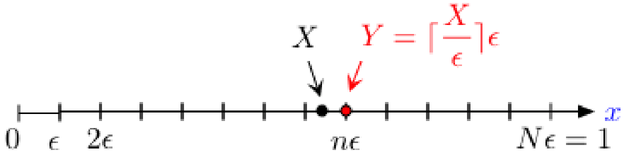
Discrete Approximation:

Uniformly at Random in $[0, 1]$.



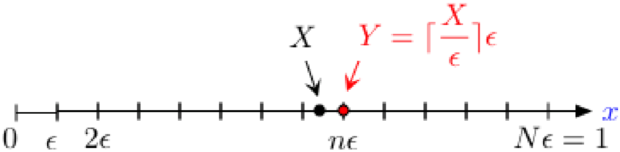
Discrete Approximation: Fix $N \gg 1$

Uniformly at Random in $[0, 1]$.



Discrete Approximation: Fix $N \gg 1$ and let $\varepsilon = 1/N$.

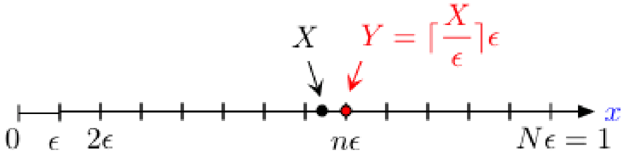
Uniformly at Random in $[0,1]$.



Discrete Approximation: Fix $N \gg 1$ and let $\varepsilon = 1/N$.

Define $Y = n\varepsilon$ if $(n-1)\varepsilon < X \leq n\varepsilon$ for $n = 1, \ldots, N$.

Uniformly at Random in $[0,1]$.



Discrete Approximation: Fix $N \gg 1$ and let $\varepsilon = 1/N$.

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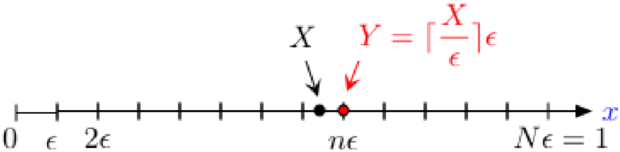
Then $|X - Y| \leq \varepsilon$

Uniformly at Random in $[0,1]$.



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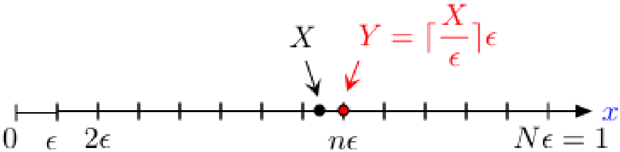
Define $Y = n\varepsilon$ if $(n-1)\varepsilon < X \leq n\varepsilon$ for $n = 1, \ldots, N$.

Then $|X - Y| \leq \varepsilon$ and Y is discrete:

Uniformly at Random in $[0,1]$.



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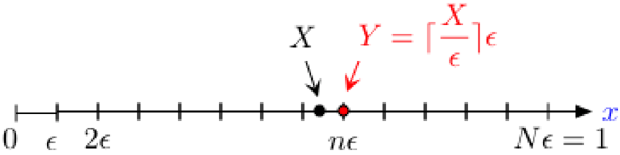

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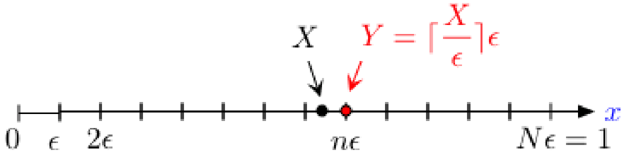

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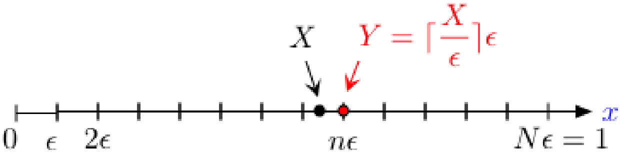

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Calculus view: $Pr[Y = n\varepsilon]$ is area of rectangle in Riemann sum.

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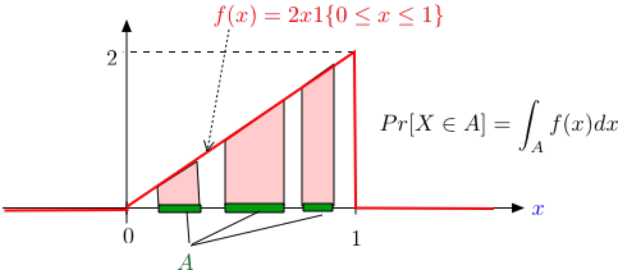

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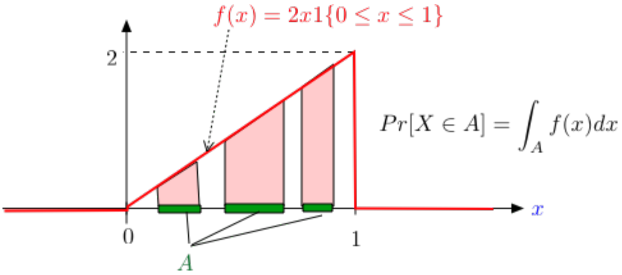

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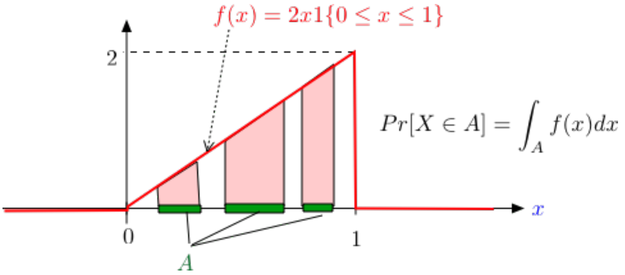

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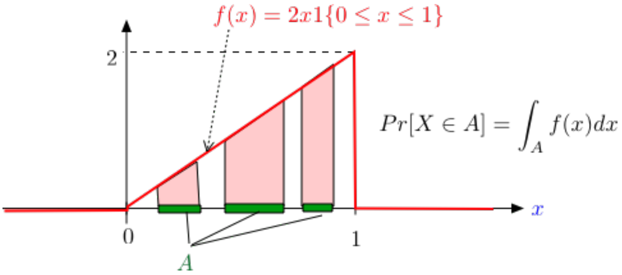

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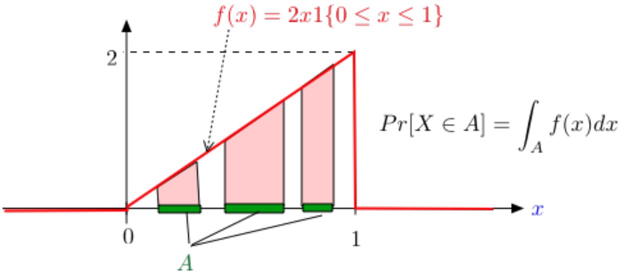

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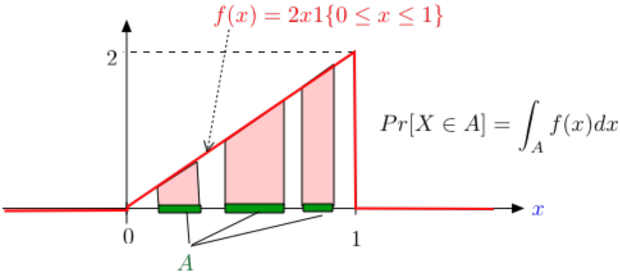

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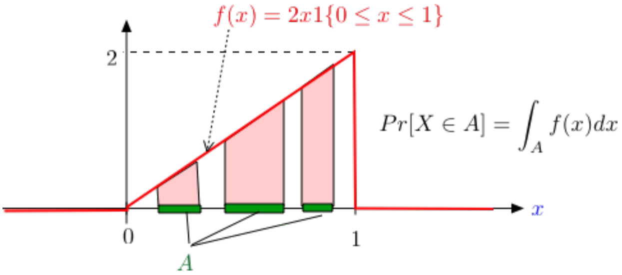

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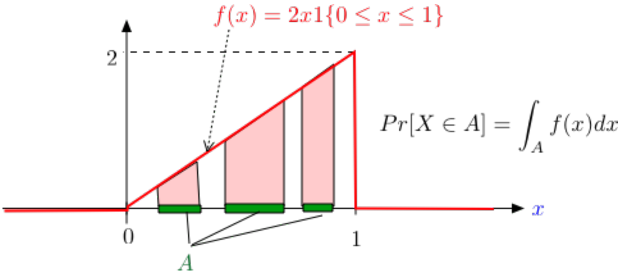

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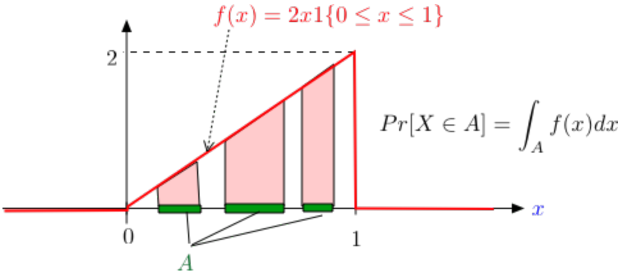

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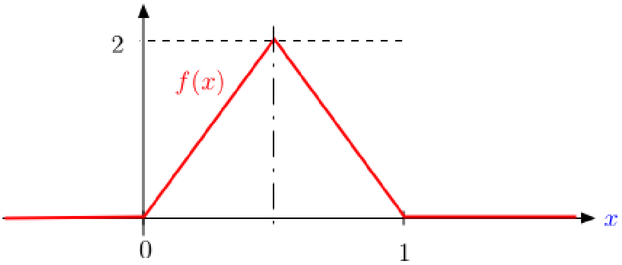

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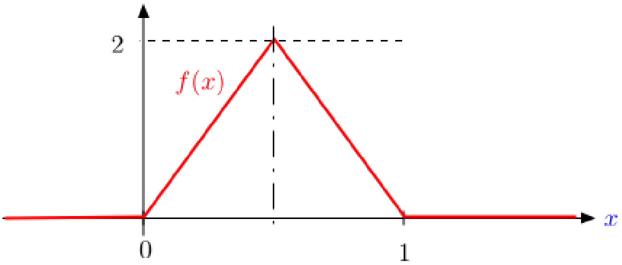

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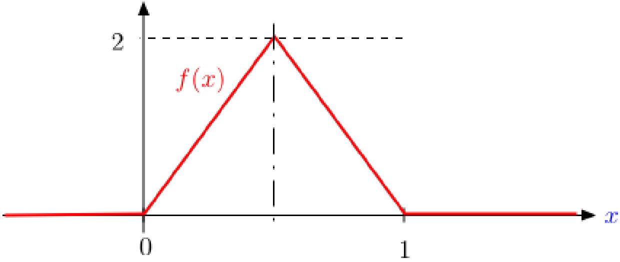

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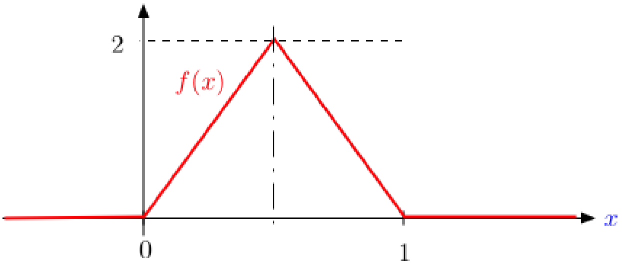

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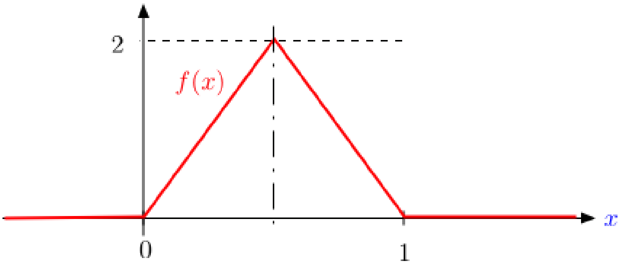

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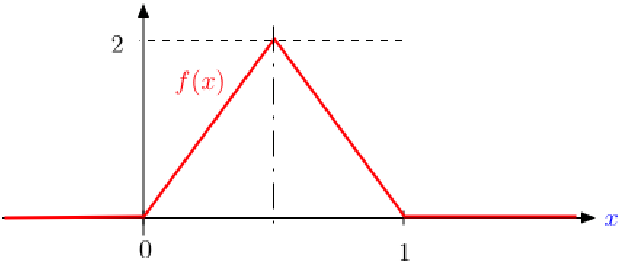

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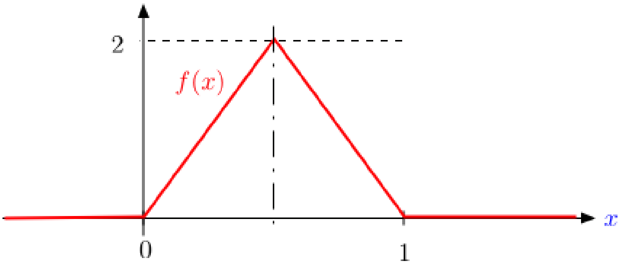

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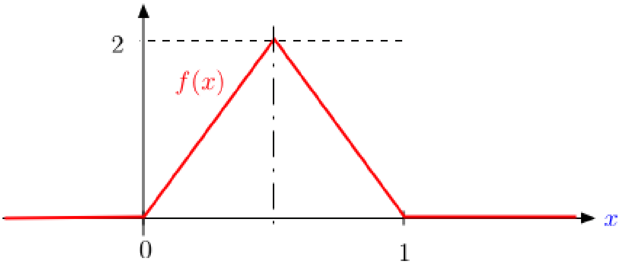

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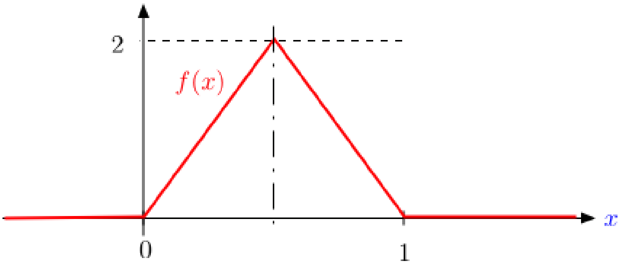

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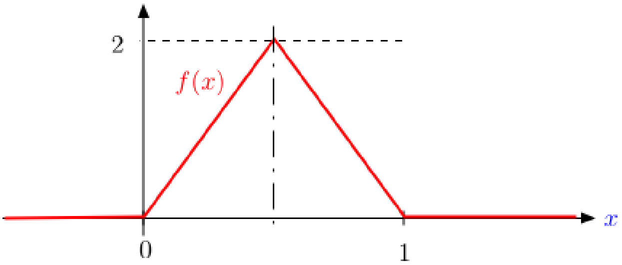

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When F and f correspond RV X: $F_X(x)$ and $f_X(x)$.

$Pr[X \in (x, x+\varepsilon)]$

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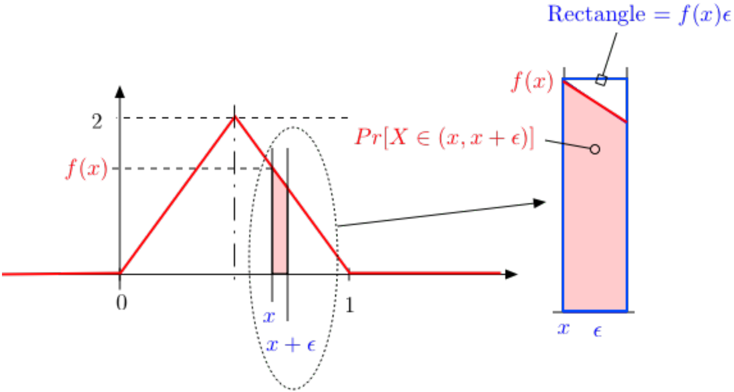

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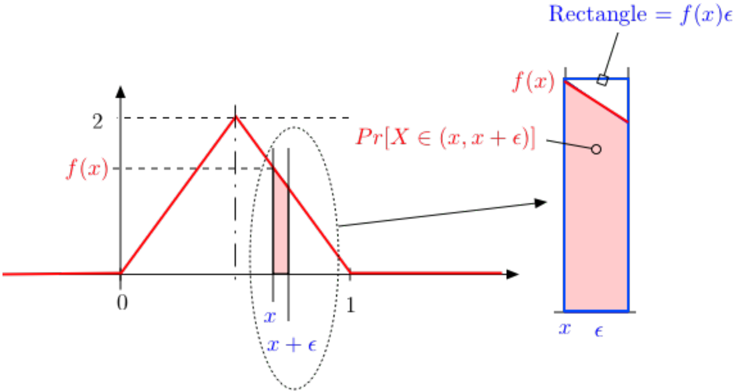

Thus, the pdf is the 'local probability by unit length.'

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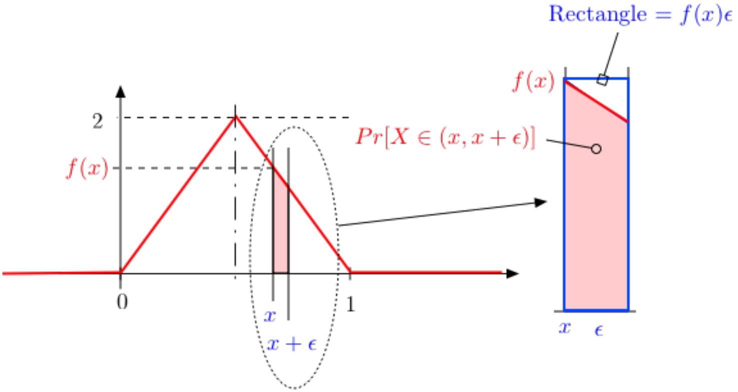

Thus, the pdf is the ‘local probability by unit length.’

It is the ‘probability density.’

Discrete Approximation

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Fix $\varepsilon \ll 1$

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Example: CDF, pre-poll

Example: hitting random location on gas tank.

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Example: hitting random location on gas tank.
Random location on circle.

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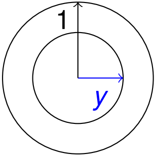

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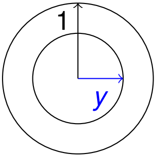

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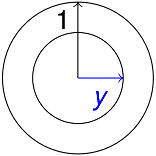

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(D) Next slide.

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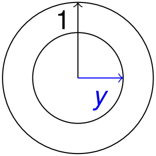

Random Variable: Y distance from center.

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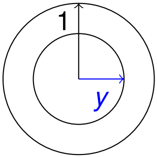

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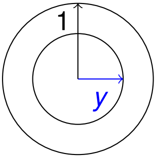

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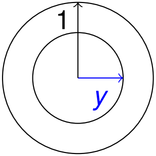

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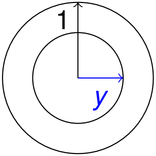

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Hence,

$$F_Y(y) = Pr[Y \leq y] = \begin{cases} 0 & \text{for } y < 0 \\ y^2 & \text{for } 0 \leq y \leq 1 \\ 1 & \text{for } y > 1 \end{cases}$$

Calculation of event with dartboard..

Probability between .5 and .6 of center?

Calculation of event with dartboard..

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PDF.

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The cumulative distribution function (cdf) and probability distribution function (pdf) give full information.

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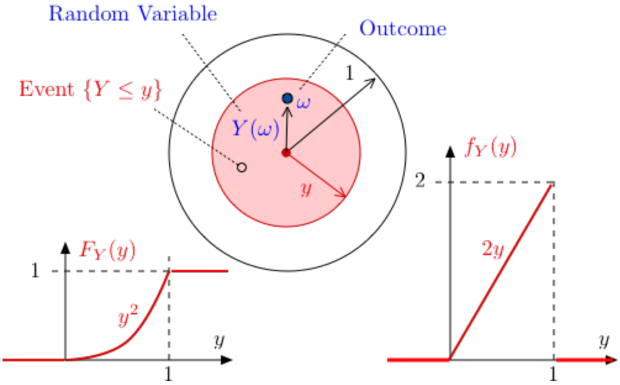
The cumulative distribution function (cdf) and probability distribution function (pdf) give full information.

Use whichever is convenient.

Target

Target



Random Variable
Outcome
1
Event $\{Y \leq y\}$
ω
$Y(\omega)$
y
$f_Y(y)$
2
$2y$
y
1
$F_Y(y)$
1
y^2
y
1

$U[a, b]$

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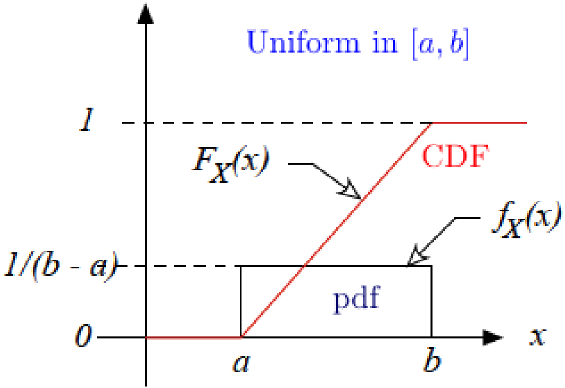

Exponential derivation:Poll.

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$Pr[Y > y]$ is defined as “Survival function.”

Expo(λ)

"Limit of geometric."

$Expo(\lambda)$

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From last slide: $S(t) = Pr[X > t] = e^{-\lambda t}$ for $t > 0$.
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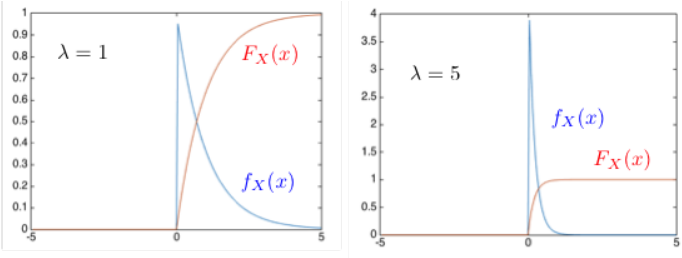

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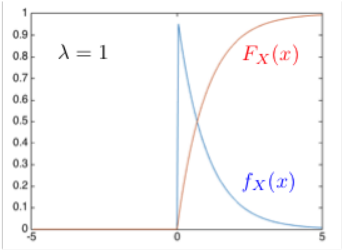

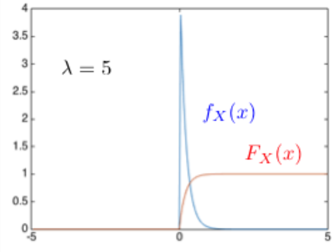

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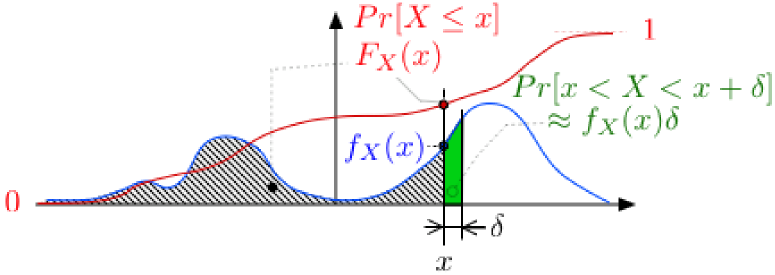
A Picture



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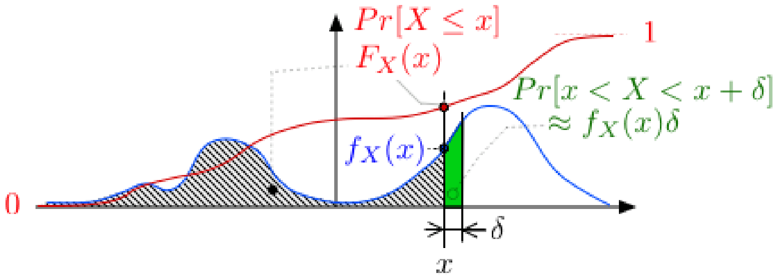

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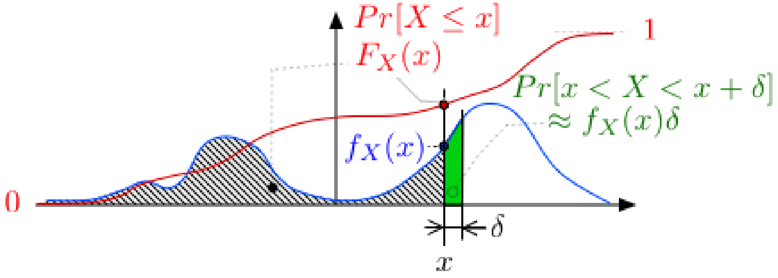

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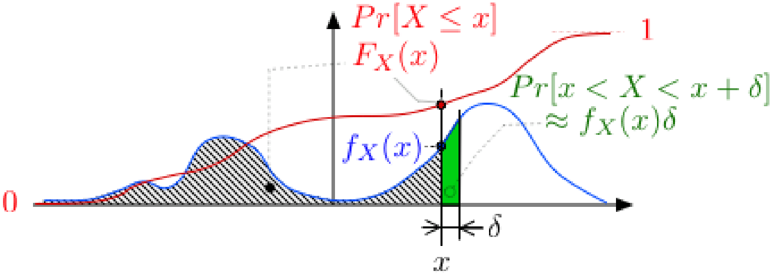

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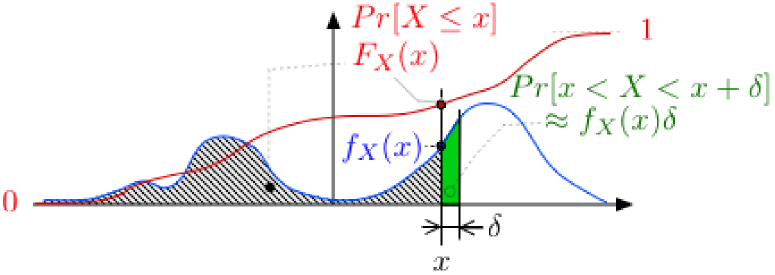

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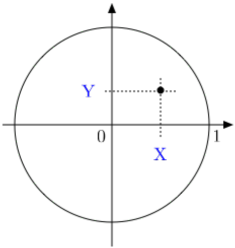

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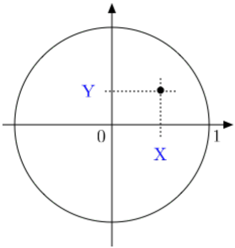

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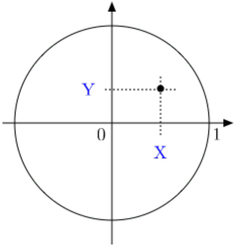

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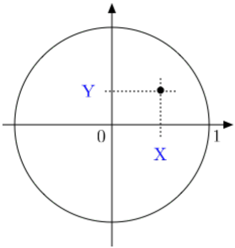

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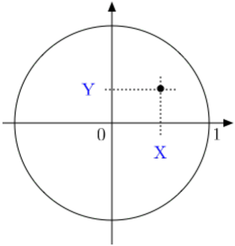

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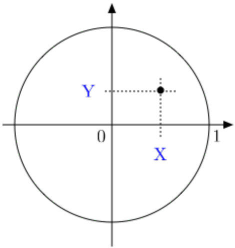

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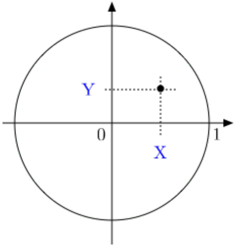

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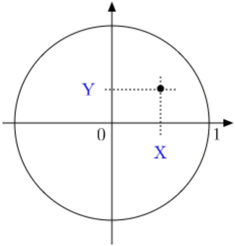

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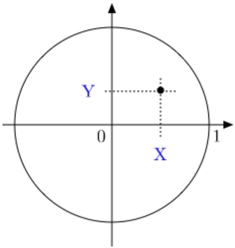

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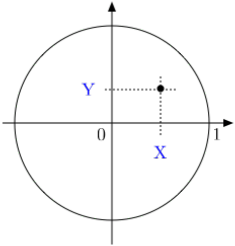

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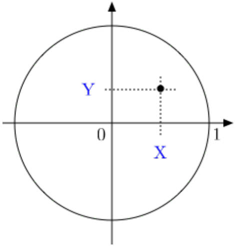

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Thus, $f_{X,Y}(x,y) = f_X(x)f_Y(y)$.

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Corollary: For independent random variables, $f_{X|Y}(x,y) = f_X(x)$.

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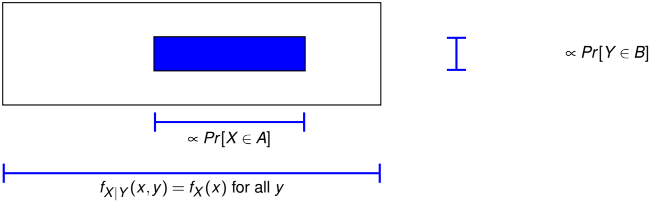

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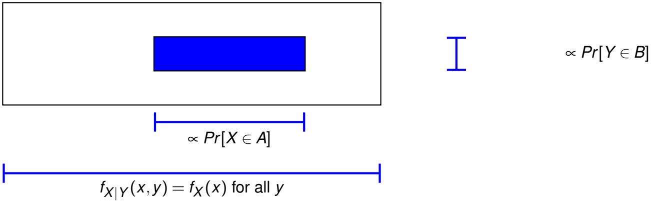

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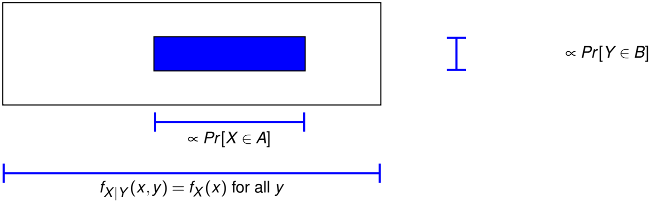

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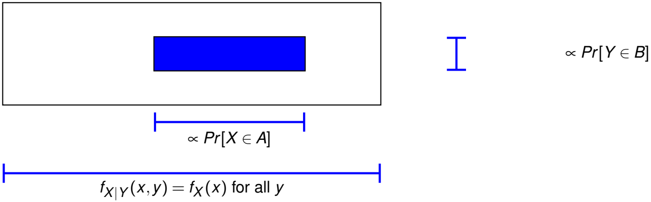

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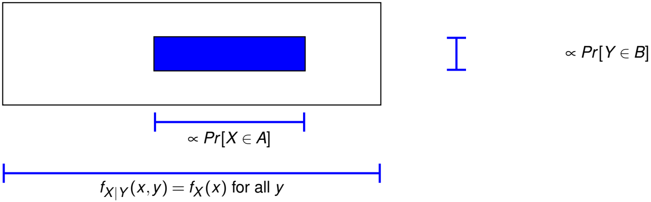

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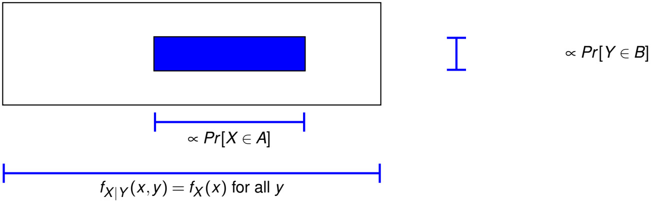

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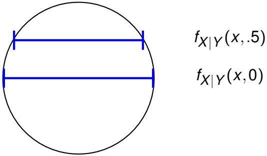

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