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Poll

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Everything below is true. Mark if you know it and perhaps why it is true.

(A) Two points determine a line: $mx + b$
(B) A root of $P(x)$, is a where $P(a) = 0$.
(C) A degree d polynomial has at most d roots.
(D) Arithmetic modulo a prime p is a "field".

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(A) If a polynomial has a root at a, $P(x) = Q(x)(x - a)$.
(B) A line has at most one root, if not always zero.
(C) System: $y_1 = mx_1 + b, y_2 = mx_2 + b$ has unique solution (m, b.)
(D) Degree of a polyomial $P(x)^2$ is $2d$ if $P(x)$ is degree d.

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Arithmetic $\pmod p \implies$ work with $O(\log p)$ bit numbers.

Proof sketches.

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Two polynomials: $P(x), Q(x)$, $P(x) - Q(x)$ has too many roots.

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Proof works for reals, rationals, and complex numbers.

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Arithmetic modulo a prime m is a **finite field** denoted by F_m or $GF(m)$.

Intuitively, a field is a set with operations corresponding to addition, multiplication, and division.

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Modular Arithmetic Fact: Exactly one polynomial degree $\leq d$ over $GF(p)$, $P(x)$, that hits $d+1$ points.

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3 kids hand out 3 points. Any two know the line.

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(Almost) the same as what is missing: one $P(i)$.

Runtime.

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Runtime: polynomial in k, n, and $\log p$.

1. Evaluate degree $k-1$ polynomial n times using $\log p$-bit numbers.
2. Reconstruct secret by solving system of k equations using $\log p$-bit arithmetic.

A bit more counting.

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Infinite number for reals, rationals, complex numbers!

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n people, k is enough.

(A) The modulus needs to be at least $n+1$.
(B) The modulus needs to be at least k.
(C) Use degree k polynomial, hand out n points.
(D) Use degree n polynomial, hand out k points.
(E) Use degree $k-1$ polynomial, hand out n points.
(F) The modulus needs to be at least 2^s, where s is value of secret.
(G) The modulus needs to be at least 2^s, where s is size of secret.

Secret Sharing.

n people, k is enough.

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(A), (B), (E), (F)

Erasure Codes.

Satellite

GPS device

Erasure Codes.

Satellite

3 packet message.

GPS device

Erasure Codes.

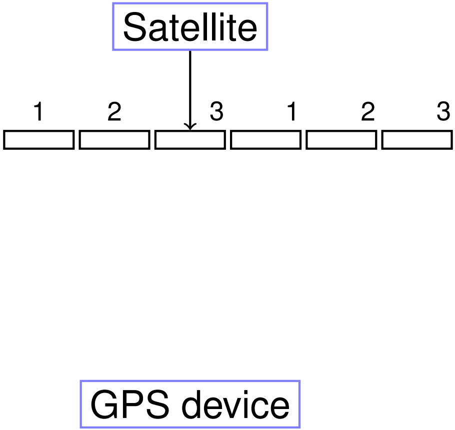
Satellite

3 packet message.

Lose 3 out 6 packets.

GPS device

Erasure Codes.



3 packet message. So send 6!

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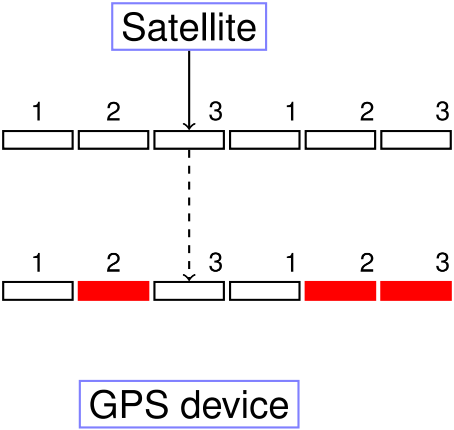
Erasure Codes.



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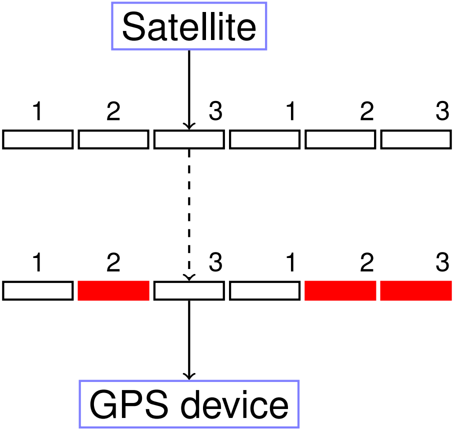

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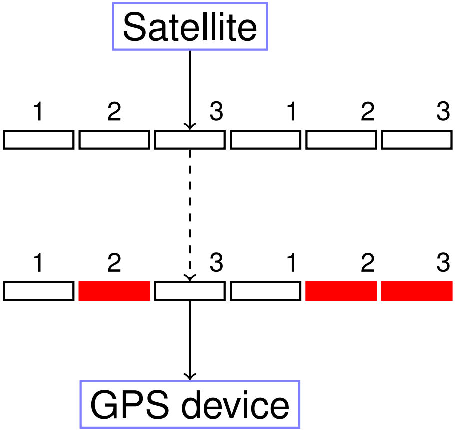

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Lose 3 out 6 packets.

Gets packets 1,1,and 3.

Solution Idea.

n packet message, channel that loses k packets.

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Must send $n+k$ packets!

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Any n point values allow reconstruction of degree $n-1$ polynomial.

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Alright!!!!!!

Use polynomials.

The Scheme

Problem: Want to send a message with n packets.

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A degree $n-1$ polynomial determined by any n points!

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2. $P(x) = m_{n-1}x^{n-1} + \cdots m_0 \pmod p$.
3. Send $P(1), \ldots, P(n+k)$.

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GPS device

Erasure Codes.

Satellite

n packet message.

GPS device

Erasure Codes.

Satellite

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Lose k packets.

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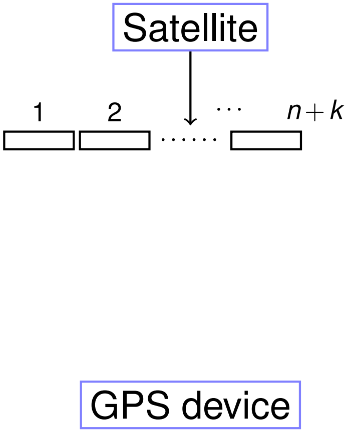
Erasure Codes.



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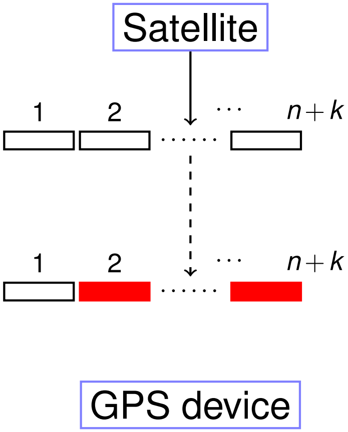

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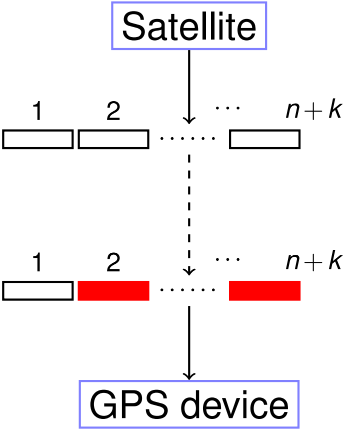

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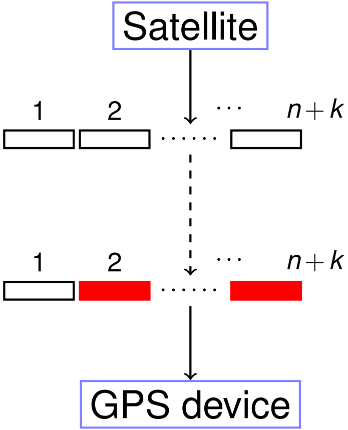

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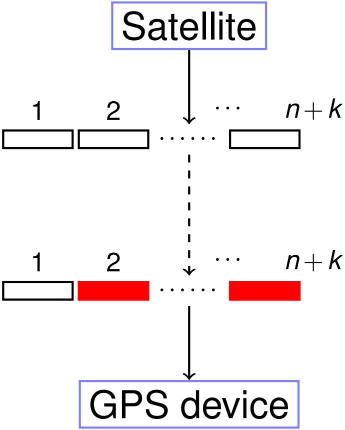

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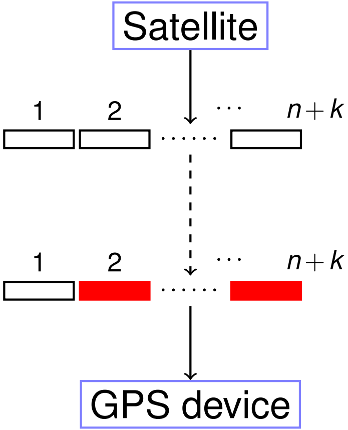

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Size: Can choose a prime between 2^{b-1} and 2^b.
(Lose at most 1 bit per packet.)

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Send message of 1,4, and 4.

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Example

Make polynomial with $P(1) = 1$, $P(2) = 4$, $P(3) = 4$.

Modulo 7 to accommodate at least 6 packets.

Linear equations:

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Packets: $(1,1),(2,4),(3,4),(4,7),(5,2),(6,0)$

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Notice that packets contain “x-values”.

Bad reception!

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Channeling Sahai

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You want to encode a secret consisting of 1,4,4.

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Send n packets b-bit packets, with k errors.

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Modulus should be larger than $n + k$ and also larger than 2^b.

Polynomials.

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- ..give Secret Sharing.

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Additional Challenge: Finding which packets are corrupt.

Error Correction

Satellite

GPS device

Error Correction

Satellite

3 packet message.

GPS device

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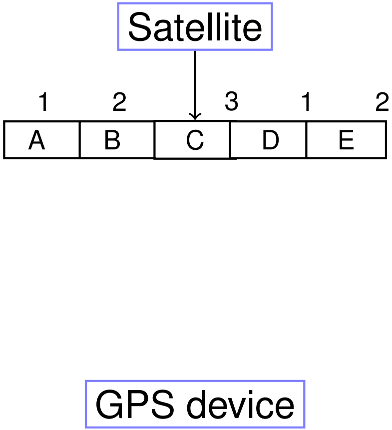
3 packet message.

Corrupts 1 packets.

GPS device

Error Correction



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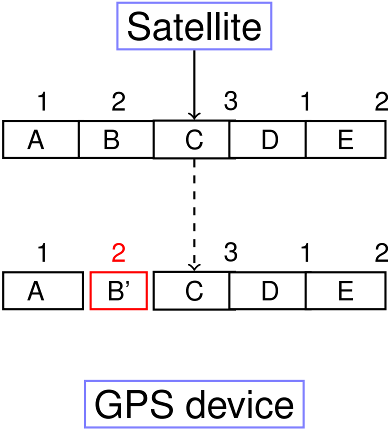

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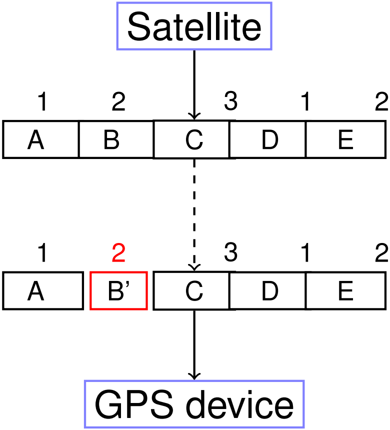

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The Scheme.

Problem: Communicate n packets $m_1, \ldots, m_n$ on noisy channel that corrupts $\leq k$ packets.

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Properties: proof.

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(1) $P(i) = R(i)$ for at least $n+k$ points i,
(2) $P(x)$ is unique degree $n-1$ polynomial
that contains $\geq n+k$ received points.

Proof:
(1) Sure. Only k corruptions.

Properties: proof.

$P(x)$: degree $n-1$ polynomial.
Send $\quad P(1),\ldots,P(n+2k)$
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Total points contained by both: $2n+2k$.

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Receive $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$.

$P(i) = R(i)$ for $n + k = 3 + 1 = 4$ points.

Slow solution.

Brute Force:
For each subset of $n+k$ points

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Reconstructs $P(x)$ and only $P(x)$!!

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Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

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All equations..

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p_2 + p_1 + p_0 &\equiv 3 \pmod 7 \\
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Assume point 2 is wrong

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Runtime: $\binom{n+2k}{k}$ possibilitities.

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p_{n-1}(m)^{n-1} + \cdots p_0 &\equiv R(m) \pmod{p}
\end{aligned}
$$

Error!! .... Where???
Could be anywhere!!! ...so try everywhere.
Runtime: $\binom{n+2k}{k}$ possibilitities.

Something like $(n/k)^k$ ...Exponential in k!.

In general..

$P(x) = p_{n-1}x^{n-1} + \cdots p_0$ and receive $R(1), \ldots R(m = n+2k)$.

$$
\begin{aligned}
p_{n-1} + \cdots p_0 &\equiv R(1) \pmod{p} \\
p_{n-1}2^{n-1} + \cdots p_0 &\equiv R(2) \pmod{p} \\
&\vdots \\
p_{n-1}i^{n-1} + \cdots p_0 &\equiv R(i) \pmod{p} \\
&\vdots \\
p_{n-1}(m)^{n-1} + \cdots p_0 &\equiv R(m) \pmod{p}
\end{aligned}
$$

Error!! .... Where???
Could be anywhere!!! ...so try everywhere.
Runtime: $\binom{n+2k}{k}$ possibilitities.

Something like $(n/k)^k$ ...Exponential in k!.

How do we find where the bad packets are efficiently?!?!?!

Ditty...

Oh where, Oh where

Ditty...

Oh where, Oh where
has my little dog gone?

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone..

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?
Oh where, oh where do they not fit.

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?
Oh where, oh where do they not fit.

With the polynomial well put

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?
Oh where, oh where do they not fit.

With the polynomial well put
But the channel a bit wrong

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?
Oh where, oh where do they not fit.

With the polynomial well put
But the channel a bit wrong
Where, oh where do we look?

Where oh where can my bad packets be?

$$(p_{n-1} + \cdots p_0) \quad \equiv \quad R(1) \qquad \pmod{p}$$

Where oh where can my bad packets be?

$$
\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod p \\
(p_{n-1} 2^{n-1} + \cdots p_0) &\equiv R(2) \pmod p \\
&\vdots \\
(p_{n-1} (m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod p
\end{aligned}
$$

Where oh where can my bad packets be?

$$\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
(p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p} \\
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\end{aligned}$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.

Where oh where can my bad packets be?

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\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
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\end{aligned}
$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.

Zero times anything is zero!!!!!

Where oh where can my bad packets be?

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\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
(p_{n-1} 2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p} \\
&\vdots \\
(p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod{p}
\end{aligned}
$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
Zero times anything is zero!!!!! My love is won.

Where oh where can my bad packets be?

$$
\begin{array}{rrcll}
 & (p_{n-1} + \cdots p_0) & \equiv & R(1) & \pmod{p} \\
\mathbf{0}\times & (p_{n-1}2^{n-1} + \cdots p_0) & \equiv & R(2) & \pmod{p} \\
 & & \vdots & & \\
 & (p_{n-1}(m)^{n-1} + \cdots p_0) & \equiv & R(n+2k) & \pmod{p}
\end{array}
$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
Zero times anything is zero!!!!! My love is won.
All equations satisfied!!!!!

Where oh where can my bad packets be?

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\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
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(p_{n-1} (m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod{p}
\end{aligned}
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Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.

Zero times anything is zero!!!!! My love is won.

All equations satisfied!!!!!

But which equations should we multiply by 0?

Where oh where can my bad packets be?

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\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
(p_{n-1} 2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p} \\
&\vdots \\
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\end{aligned}
$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
Zero times anything is zero!!!!! My love is won.
All equations satisfied!!!!!

But which equations should we multiply by 0? Where oh where...

Where oh where can my bad packets be?

$$(p_{n-1} + \cdots p_0) \equiv R(1) \pmod{p}$$

$$(p_{n-1}2^{n-1} + \cdots p_0) \equiv R(2) \pmod{p}$$

$$\vdots$$

$$(p_{n-1}(m)^{n-1} + \cdots p_0) \equiv R(n+2k) \pmod{p}$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
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Where oh where can my bad packets be?

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&\vdots \\
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\end{aligned}$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
Zero times anything is zero!!!!! My love is won.
All equations satisfied!!!!!

But which equations should we multiply by 0? Where oh where...??

We will use a polynomial!!!

Where oh where can my bad packets be?

$$
\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
(p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p} \\
&\vdots \\
(p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod{p}
\end{aligned}
$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
Zero times anything is zero!!!!! My love is won.
All equations satisfied!!!!!

But which equations should we multiply by 0? Where oh where...??

We will use a polynomial!!! That we don't know.

Where oh where can my bad packets be?

$$\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod p \\
(p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2) \pmod p \\
&\vdots \\
(p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod p
\end{aligned}$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
Zero times anything is zero!!!!! My love is won.
All equations satisfied!!!!!

But which equations should we multiply by 0? Where oh where...??

We will use a polynomial!!! That we don't know. But can find!

Where oh where can my bad packets be?

$$
\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod p\\
(p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2) \pmod p\\
&\vdots\\
(p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod p
\end{aligned}
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Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
Zero times anything is zero!!!!! My love is won.
All equations satisfied!!!!!

But which equations should we multiply by 0? Where oh where...??

We will use a polynomial!!! That we don't know. But can find!

Errors at points $e_1, \ldots, e_k$. (In diagram above, $e_1 = 2$.)

Where oh where can my bad packets be?

$$
\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
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Errors at points $e_1, \ldots, e_k$. (In diagram above, $e_1 = 2$.)

Error locator polynomial: $E(x) = (x - e_1)$

Where oh where can my bad packets be?

$$\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) && \pmod{p} \\
(p_{n-1} 2^{n-1} + \cdots p_0) &\equiv R(2) && \pmod{p} \\
&\vdots \\
(p_{n-1} (m)^{n-1} + \cdots p_0) &\equiv R(n+2k) && \pmod{p}
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Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
Zero times anything is zero!!!!! My love is won.
All equations satisfied!!!!!

But which equations should we multiply by 0? Where oh where...??

We will use a polynomial!!! That we don't know. But can find!

Errors at points $e_1, \ldots, e_k$. (In diagram above, $e_1 = 2$.)

Error locator polynomial: $E(x) = (x - e_1)(x - e_2)$

Where oh where can my bad packets be?

$$
\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
(p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p} \\
&\vdots \\
(p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod{p}
\end{aligned}
$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
Zero times anything is zero!!!!! My love is won.
All equations satisfied!!!!!

But which equations should we multiply by 0? Where oh where...??

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Errors at points $e_1, \ldots, e_k$. (In diagram above, $e_1 = 2$.)

Error locator polynomial: $E(x) = (x - e_1)(x - e_2) \ldots$

Where oh where can my bad packets be?

$$\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) && \pmod{p} \\
(p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2) && \pmod{p} \\
&\vdots \\
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Zero times anything is zero!!!!! My love is won.
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Errors at points $e_1, \ldots, e_k$. (In diagram above, $e_1 = 2$.)

Error locator polynomial: $E(x) = (x - e_1)(x - e_2)\ldots(x - e_k)$.

Where oh where can my bad packets be?

$$\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
(p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p} \\
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Errors at points $e_1, \ldots, e_k$. (In diagram above, $e_1 = 2$.)

Error locator polynomial: $E(x) = (x - e_1)(x - e_2)\ldots(x - e_k)$.

$E(i) = 0$ if and only if $e_j = i$ for some j

Where oh where can my bad packets be?

$$
\begin{aligned}
E(1)(p_{n-1} + \cdots p_0) &\equiv R(1)E(1) \pmod{p} \\
E(2)(p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2)E(2) \pmod{p} \\
&\vdots \\
E(m)(p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k)E(m) \pmod{p}
\end{aligned}
$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
Zero times anything is zero!!!!! My love is won.
All equations satisfied!!!!!

But which equations should we multiply by 0? Where oh where...??

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Errors at points $e_1, \ldots, e_k$. (In diagram above, $e_1 = 2$.)

Error locator polynomial: $E(x) = (x - e_1)(x - e_2)\ldots(x - e_k)$.

$E(i) = 0$ if and only if $e_j = i$ for some j

Multiply equations by $E(\cdot)$.

Where oh where can my bad packets be?

$$
\begin{aligned}
E(1)(p_{n-1} + \cdots p_0) &\equiv R(1)E(1) \pmod p \\
E(2)(p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2)E(2) \pmod p \\
&\vdots \\
E(m)(p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k)E(m) \pmod p
\end{aligned}
$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
Zero times anything is zero!!!!! My love is won.
All equations satisfied!!!!!

But which equations should we multiply by 0? Where oh where...??

We will use a polynomial!!! That we don't know. But can find!

Errors at points $e_1, \ldots, e_k$. (In diagram above, $e_1 = 2$.)

Error locator polynomial: $E(x) = (x - e_1)(x - e_2)\ldots(x - e_k)$.

$E(i) = 0$ if and only if $e_j = i$ for some j

Multiply equations by $E(\cdot)$. (Above E(x) = (x-2).)

Where oh where can my bad packets be?

$$
\begin{aligned}
E(1)(p_{n-1} + \cdots p_0) &\equiv R(1)E(1) \pmod{p} \\
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&\vdots \\
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\end{aligned}
$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
Zero times anything is zero!!!!! My love is won.
All equations satisfied!!!!!

But which equations should we multiply by 0? Where oh where...??

We will use a polynomial!!! That we don't know. But can find!

Errors at points $e_1, \ldots, e_k$. (In diagram above, $e_1 = 2$.)

Error locator polynomial: $E(x) = (x - e_1)(x - e_2)\ldots(x - e_k)$.

$E(i) = 0$ if and only if $e_j = i$ for some j

Multiply equations by $E(\cdot)$. (Above E(x) = (x-2).)

All equations satisfied!!

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2 x^2 + p_1 x + p_0$ that contains $n + k = 3 + 1$ points.

Plugin points...

$$
\begin{aligned}
(p_2 + p_1 + p_0) &\equiv (3) \pmod 7 \\
(4p_2 + 2p_1 + p_0) &\equiv (1) \pmod 7 \\
(2p_2 + 3p_1 + p_0) &\equiv (6) \pmod 7 \\
(2p_2 + 4p_1 + p_0) &\equiv (0) \pmod 7 \\
(4p_2 + 5p_1 + p_0) &\equiv (3) \pmod 7
\end{aligned}
$$

Example.

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(2p_2 + 4p_1 + p_0) &\equiv (0) && \pmod 7 \\
(4p_2 + 5p_1 + p_0) &\equiv (3) && \pmod 7
\end{aligned}$$

Error locator polynomial: $(x - 2)$.

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2 x^2 + p_1 x + p_0$ that contains $n + k = 3 + 1$ points.

Plugin points...

$$
\begin{aligned}
(1-2)(p_2 + p_1 + p_0) &\equiv (3)(1-2) \pmod 7 \\
(2-2)(4p_2 + 2p_1 + p_0) &\equiv (1)(2-2) \pmod 7 \\
(3-2)(2p_2 + 3p_1 + p_0) &\equiv (6)(3-2) \pmod 7 \\
(4-2)(2p_2 + 4p_1 + p_0) &\equiv (0)(4-2) \pmod 7 \\
(5-2)(4p_2 + 5p_1 + p_0) &\equiv (3)(5-2) \pmod 7
\end{aligned}
$$

Error locator polynomial: $(x - 2)$.

Multiply equation i by $(i - 2)$.

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

Plugin points...

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(1-2)(p_2 + p_1 + p_0) &\equiv (3)(1-2) \pmod 7 \\
(2-2)(4p_2 + 2p_1 + p_0) &\equiv (1)(2-2) \pmod 7 \\
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(5-2)(4p_2 + 5p_1 + p_0) &\equiv (3)(5-2) \pmod 7
\end{aligned}$$

Error locator polynomial: $(x-2)$.

Multiply equation i by $(i-2)$. All equations satisfied!

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(5-2)(4p_2 + 5p_1 + p_0) &\equiv (3)(5-2) \pmod 7
\end{aligned}
$$

Error locator polynomial: $(x - 2)$.

Multiply equation i by $(i - 2)$. All equations satisfied!

But don't know error locator polynomial!

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2 x^2 + p_1 x + p_0$ that contains $n + k = 3 + 1$ points.

Plugin points...

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(2-2)(4p_2 + 2p_1 + p_0) &\equiv (1)(2-2) \pmod{7} \\
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(5-2)(4p_2 + 5p_1 + p_0) &\equiv (3)(5-2) \pmod{7}
\end{aligned}$$

Error locator polynomial: $(x-2)$.

Multiply equation i by $(i-2)$. All equations satisfied!

But don't know error locator polynomial! Do know form:

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

Plugin points...

$$
\begin{aligned}
(1-2)(p_2 + p_1 + p_0) &\equiv (3)(1-2) \pmod 7 \\
(2-2)(4p_2 + 2p_1 + p_0) &\equiv (1)(2-2) \pmod 7 \\
(3-2)(2p_2 + 3p_1 + p_0) &\equiv (6)(3-2) \pmod 7 \\
(4-2)(2p_2 + 4p_1 + p_0) &\equiv (0)(4-2) \pmod 7 \\
(5-2)(4p_2 + 5p_1 + p_0) &\equiv (3)(5-2) \pmod 7
\end{aligned}
$$

Error locator polynomial: $(x - 2)$.

Multiply equation i by $(i - 2)$. All equations satisfied!

But don't know error locator polynomial! Do know form: $(x - e)$.

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

Plugin points...

$$\begin{aligned}
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Error locator polynomial: $(x - 2)$.

Multiply equation i by $(i - 2)$. All equations satisfied!

But don't know error locator polynomial! Do know form: $(x - e)$.

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2 x^2 + p_1 x + p_0$ that contains $n + k = 3 + 1$ points.

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4 unknowns (p_0, p_1, p_2 and e), 5 nonlinear equations.

..turn their heads each day,

$$
\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
&\vdots \\
\color{red}{(p_{n-1} i^{n-1} + \cdots p_0)} &\equiv \color{red}{R(i)} \pmod{p} \\
&\vdots \\
(p_{n-1}(n+2k)^{n-1} + \cdots p_0) &\equiv R(m) \pmod{p}
\end{aligned}
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and linear in a_i and coefficients of $E(x)$!

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Solving for $Q(x)$ and $E(x)$...

For all points $1, \ldots, i, n+2k = m$,

$$Q(i) = R(i)E(i) \pmod{p}$$

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Solve for coefficients of $Q(x)$ and $E(x)$.

Solving for $Q(x)$ and $E(x)$...and $P(x)$

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a_3 + 2a_2 + 4a_1 + a_0 &\equiv 0(4 - b_0) \pmod 7 \\
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$a_3 = 1$, $a_2 = 6$, $a_1 = 6$, $a_0 = 5$ and $b_0 = 2$.

$Q(x) = x^3 + 6x^2 + 6x + 5.$

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

$Q(x) = E(x)P(x) = a_3x^3 + a_2x^2 + a_1x + a_0$

$E(x) = x - b_0$

$Q(i) = R(i)E(i)$.

$$\begin{aligned}
a_3 + a_2 + a_1 + a_0 &\equiv 3(1 - b_0) \pmod{7} \\
a_3 + 4a_2 + 2a_1 + a_0 &\equiv 1(2 - b_0) \pmod{7} \\
6a_3 + 2a_2 + 3a_1 + a_0 &\equiv 6(3 - b_0) \pmod{7} \\
a_3 + 2a_2 + 4a_1 + a_0 &\equiv 0(4 - b_0) \pmod{7} \\
6a_3 + 4a_2 + 5a_1 + a_0 &\equiv 3(5 - b_0) \pmod{7}
\end{aligned}$$

$a_3 = 1$, $a_2 = 6$, $a_1 = 6$, $a_0 = 5$ and $b_0 = 2$.

$Q(x) = x^3 + 6x^2 + 6x + 5$.

$E(x) = x - 2$.

Example: finishing up.

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```
           ------------------
 x - 2 ) x^3  + 6 x^2 + 6 x + 5
```

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$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
                  1 x^2
          -----------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
        x^3  - 2 x^2
```

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$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
                   1 x^2
          -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                 1 x^2 + 6 x + 5
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
                  1 x^2 + 1 x
         -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                  1 x^2 + 6 x + 5
                  1 x^2 - 2 x
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                  1 x^2 + 1 x
          -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                  1 x^2 + 6 x + 5
                  1 x^2 - 2 x
                  ---------------
                            x + 5
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                1 x^2 + 1 x + 1
         -----------------
 x - 2 ) x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                1 x^2 + 6 x + 5
                1 x^2 - 2 x
                ---------------
                          x + 5
                          x - 2
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
                 1 x^2 + 1 x + 1
        -----------------
  x - 2 ) x^3  + 6 x^2 + 6 x + 5
          x^3  - 2 x^2
          ----------
                 1 x^2 + 6 x + 5
                 1 x^2 - 2 x
                 ---------------
                           x + 5
                           x - 2
                           -----
                               0
```

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```
                1 x^2 + 1 x + 1
         -----------------
 x - 2 ) x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                1 x^2 + 6 x + 5
                1 x^2 - 2 x
                ---------------
                          x + 5
                          x - 2
                          -----
                              0
```

$P(x) = x^2 + x + 1$

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
                1 x^2 + 1 x + 1
         -----------------
  x - 2 ) x^3  + 6 x^2 + 6 x + 5
          x^3  - 2 x^2
          ----------
                1 x^2 + 6 x + 5
                1 x^2 - 2 x
                ---------------
                          x + 5
                          x - 2
                          -----
                              0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                  1 x^2 + 1 x + 1
          -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                  1 x^2 + 6 x + 5
                  1 x^2 - 2 x
                  ---------------
                            x + 5
                            x - 2
                            -----
                                0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

What is $\frac{x-2}{x-2}$?

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                 1 x^2 + 1 x + 1
          -----------------
 x - 2 ) x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                 1 x^2 + 6 x + 5
                 1 x^2 - 2 x
                 ---------------
                           x + 5
                           x - 2
                           -----
                               0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

What is $\frac{x-2}{x-2}$? 1

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
                   1 x^2 + 1 x + 1
          -----------------
    x - 2 ) x^3  + 6 x^2 + 6 x + 5
            x^3  - 2 x^2
            ----------
                   1 x^2 + 6 x + 5
                   1 x^2 - 2 x
                   ---------------
                             x + 5
                             x - 2
                             -----
                                 0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

What is $\frac{x-2}{x-2}$? 1
Except at $x = 2$?

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
                  1 x^2 + 1 x + 1
         -----------------
    x - 2 ) x^3  + 6 x^2 + 6 x + 5
            x^3  - 2 x^2
            ----------
                  1 x^2 + 6 x + 5
                  1 x^2 - 2 x
                  ---------------
                            x + 5
                            x - 2
                            -----
                                0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

What is $\frac{x-2}{x-2}$? 1
Except at $x = 2$? Hole there?

Error Correction: Berlekamp-Welsh

Message: $m_1, \ldots, m_n$.
Sender:

1. Form degree $n-1$ polynomial $P(x)$ where $P(i) = m_i$.
2. Send $P(1), \ldots, P(n+2k)$.

Receiver:

1. Receive $R(1), \ldots, R(n+2k)$.
2. Solve $n+2k$ equations, $Q(i) = E(i)R(i)$ to find $Q(x) = E(x)P(x)$ and $E(x)$.
3. Compute $P(x) = Q(x)/E(x)$.
4. Compute $P(1), \ldots, P(n)$.

Check your undersanding.

You have error locator polynomial!

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor?

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values?

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Efficiency?

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Efficiency? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Efficiency? Sure. Only $n+2k$ values.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Efficiency? Sure. Only $n+2k$ values.
See where it is 0.

Hmmm...

Is there one and only one $P(x)$ from Berlekamp-Welsh procedure?

Hmmm...

Is there one and only one $P(x)$ from Berlekamp-Welsh procedure?

Existence: there is a $P(x)$ and $E(x)$ that satisfy equations.

Unique solution for $P(x)$

Uniqueness: any solution $Q'(x)$ and $E'(x)$ have

$$\frac{Q'(x)}{E'(x)} = \frac{Q(x)}{E(x)} = P(x). \tag{1}$$

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Proof:
We claim

$$Q'(x)E(x) = Q(x)E'(x) \text{ on } n+2k \text{ values of } x. \tag{2}$$

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Equation 2 implies 1:

$Q'(x)E(x)$ and $Q(x)E'(x)$ are degree $n+2k-1$
and agree on $n+2k$ points

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$Q'(x)E(x)$ and $Q(x)E'(x)$ are degree $n+2k-1$
and agree on $n+2k$ points
$E(x)$ and $E'(x)$ have at most k zeros each.

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Last bit.

Fact: $Q'(x)E(x) = Q(x)E'(x)$ on $n+2k$ values of x.

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If $E(i) = 0$, then $Q(i) = 0$. If $E'(i) = 0$, then $Q'(i) = 0$.
$\implies$ $Q(i)E'(i) = Q'(i)E(i)$ holds when $E(i)$ or $E'(i)$ are zero.

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Cross multiplying gives equality in fact for these points.

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Points to polynomials, have to deal with zeros!

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$$\frac{Q'(i)}{E'(i)} = \frac{Q(i)}{E(i)} = R(i).$$

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Points to polynomials, have to deal with zeros!

Example: dealing with $\frac{x-2}{x-2}$ at $x = 2$.

Yaay!!

Berlekamp-Welsh algorithm decodes correctly when k errors!

Poll

Say you sent a message of length 4, encoded as $P(x)$ where one sends packets $P(1),...P(8)$.

You recieve packets $R(1),...R(8)$.

Packets 1 and 4 are corrupted.

(A) $R(1) \neq P(1)$
(B) The degree of $P(x)E(x) = 3+2 = 5$.
(C) The degree of $E(x)$ is 2.
(D) The number of coefficients of $P(x)$ is 4.
(E) The number of coefficients of $P(x)Q(x)$ is 6.

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all true.

(A) $E(x) = (x-1)(x-4)$
(B) The number of coefficents in $E(x)$ is 2.
(C) The number of unknown coefficents in $E(x)$ is 2.
(D) $E(x) = (x-1)(x-2)$
(E) $R(4) \neq P(4)$
(F) The degree of $R(x)$ is 5.

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(C) The degree of $E(x)$ is 2.
(D) The number of coefficients of $P(x)$ is 4.
(E) The number of coefficients of $P(x)Q(x)$ is 6.

all true.

(A) $E(x) = (x-1)(x-4)$
(B) The number of coefficents in $E(x)$ is 2.
(C) The number of unknown coefficents in $E(x)$ is 2.
(D) $E(x) = (x-1)(x-2)$
(E) $R(4) \neq P(4)$
(F) The degree of $R(x)$ is 5.

(A), (C), (E).

Summary. Error Correction.

Communicate n packets, with k erasures.

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Communicate n packets, with k erasures.

How many packets?

Summary. Error Correction.

Communicate n packets, with k erasures.

How many packets? $n+k$

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Communicate n packets, with k erasures.

How many packets? $n+k$
How to encode?

Summary. Error Correction.

Communicate n packets, with k erasures.

How many packets? $n+k$
How to encode? With polynomial, $P(x)$.

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Communicate n packets, with k erasures.

How many packets? $n+k$
How to encode? With polynomial, $P(x)$.
Of degree?

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How many packets? $n+k$
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Summary. Error Correction.

Communicate n packets, with k erasures.

How many packets? $n+k$
How to encode? With polynomial, $P(x)$.
Of degree? $n-1$
Recover?

Summary. Error Correction.

Communicate n packets, with k erasures.

How many packets? $n+k$
How to encode? With polynomial, $P(x)$.
Of degree? $n-1$
Recover? Reconstruct $P(x)$ with any n points!

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Why?

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Why?
k changes to make diff. messages overlap

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How to encode?

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How to encode? With polynomial, $P(x)$.

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    - Nonlinear equations.

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Reconstruct $E(x)$ and $Q(x) = E(x)P(x)$.

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Reed-Solomon codes.

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Reed-Solomon codes. Welsh-Berlekamp Decoding.

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Reed-Solomon codes. Welsh-Berlekamp Decoding. Perfection!

Cool.

Really Cool!