

The future in this course.

What's to come?

The future in this course.

What's to come? Probability.

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A bag contains:

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A bag contains:



What is the chance that a ball taken from the bag is blue?

The future in this course.

What's to come? Probability.

A bag contains:



What is the chance that a ball taken from the bag is blue?

Count blue.

The future in this course.

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A bag contains:



What is the chance that a ball taken from the bag is blue?

Count blue. Count total.

The future in this course.

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A bag contains:



What is the chance that a ball taken from the bag is blue?

Count blue. Count total. Divide. **Chances?**

The future in this course.

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A bag contains:



What is the chance that a ball taken from the bag is blue?

Count blue. Count total. Divide. **Chances?**

- (A) Red Probability is $3/8$
- (B) Blue probability is $3/9$
- (C) Yellow Probability is $2/8$
- (D) Blue probability is $3/8$

The future in this course.

What's to come? Probability.

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- (A) Red Probability is $3/8$
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Today:

The future in this course.

What's to come? Probability.

A bag contains:



What is the chance that a ball taken from the bag is blue?

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- (A) Red Probability is $3/8$
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Today: Counting!

Outline: basics

1. Counting.
2. Tree
3. Rules of Counting
4. Sample with/without replacement where order does/doesn't matter.

Probability is soon..but first let's count.

Count?

How many outcomes possible for k coin tosses?

How many poker hands?

How many handshakes for n people?

How many diagonals in a n sided convex polygon?

How many 10 digit numbers?

How many 10 digit numbers without repetition?

How many ways can I divide up 5 dollars among 3 people?

Using a tree..

How many 3-bit strings?

Using a tree..

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How many different sequences of three bits from $\{0, 1\}$?

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How would you make one sequence?

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How many different ways to do that making?

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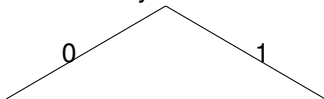
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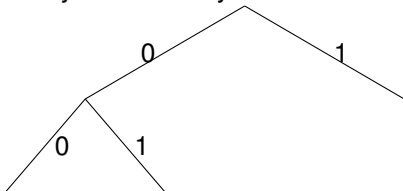
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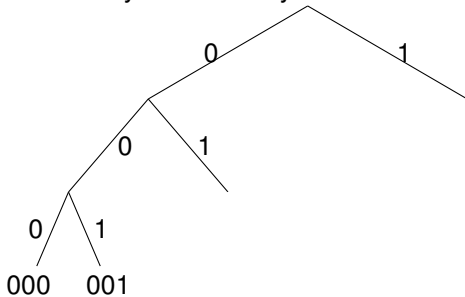
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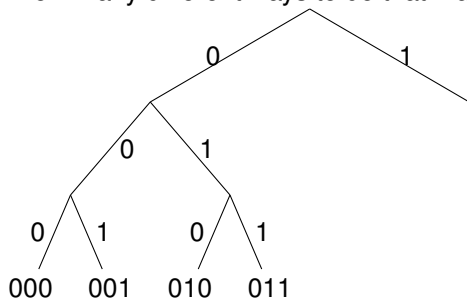
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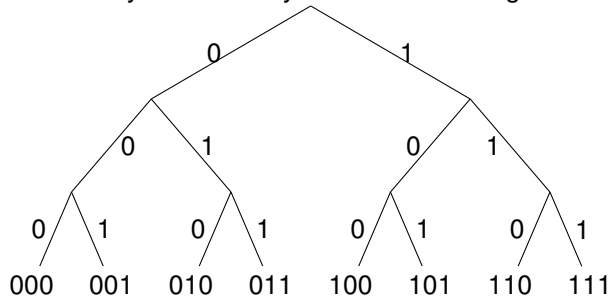
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8 leaves which is $2 \times 2 \times 2$.

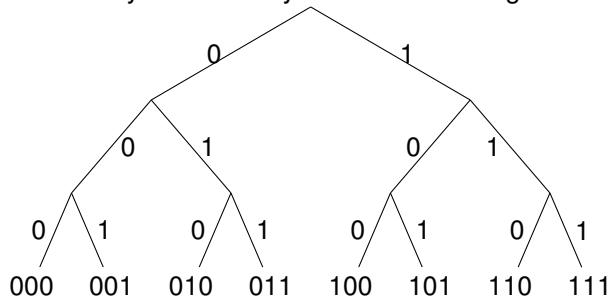
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8 leaves which is $2 \times 2 \times 2$. One leaf for each string.

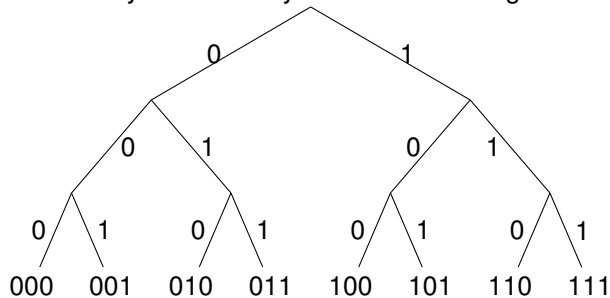
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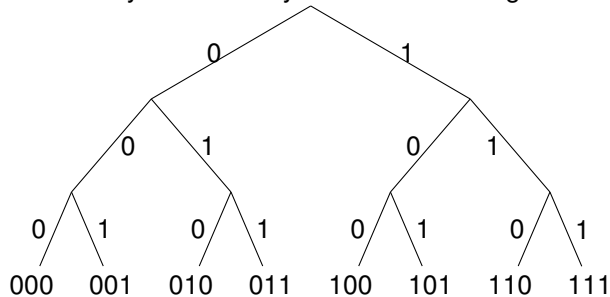
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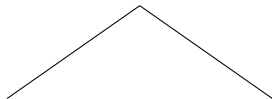
8 3-bit strings!

First Rule of Counting: Product Rule

Objects made by choosing from n_1 , then n_2 , ..., then n_k
the number of objects is $n_1 \times n_2 \cdots \times n_k$.

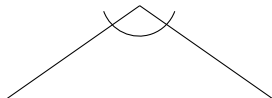
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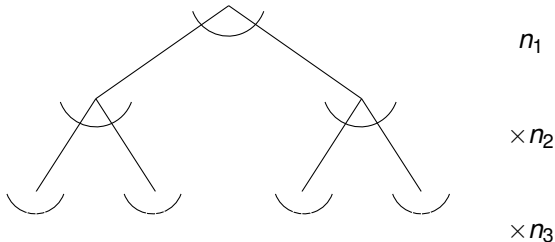
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n_1

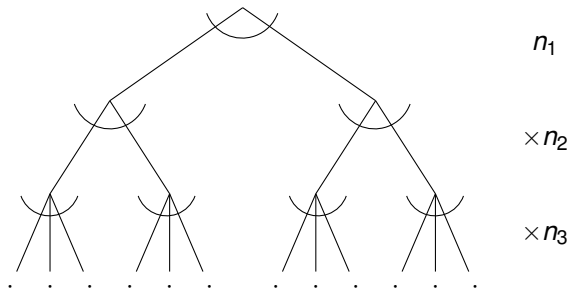
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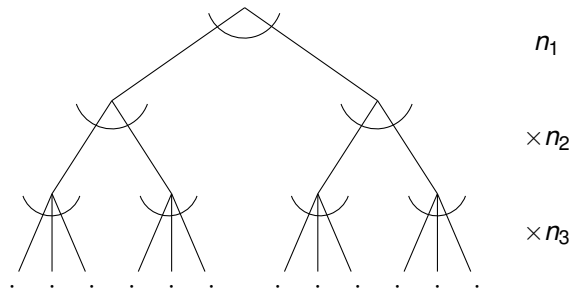
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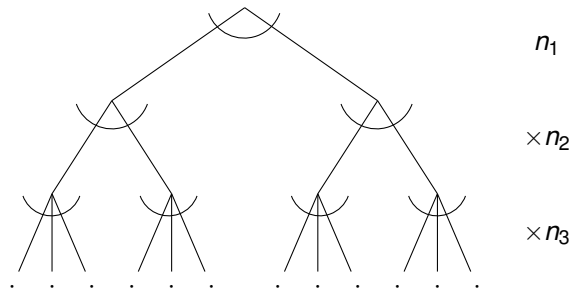
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In picture, $2 \times 2 \times 3 = 12!$

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the number of objects is $n_1 \times n_2 \cdots \times n_k$.



In picture, $2 \times 2 \times 3 = 12!$

Poll

Mark whats corect.

Poll

Mark whats corect.

(A) $|10 \text{ digit numbers}| = 10^1 0$

(B) $|k \text{ coin tosses}| = 2^k$

(C) $|10 \text{ digit numbers}| = 9 * 10^9$

(D) $|n \text{ digit base } m \text{ numbers}| = m^n$

(E) $|n \text{ digit base } m \text{ numbers}| = (m - 1)m^n$

Poll

Mark whats corect.

(A) $|10 \text{ digit numbers}| = 10^{10}$

(B) $|k \text{ coin tosses}| = 2^k$

(C) $|10 \text{ digit numbers}| = 9 * 10^9$

(D) $|n \text{ digit base } m \text{ numbers}| = m^n$

(E) $|n \text{ digit base } m \text{ numbers}| = (m - 1)m^n$

(A) or (C)? (D) or (E)? (C) are correct.

Using the first rule..

How many outcomes possible for k coin tosses?

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice,

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...

2

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...

$$2 \times 2$$

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...

$2 \times 2 \dots$

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...

$$2 \times 2 \cdots \times 2$$

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...

$$2 \times 2 \cdots \times 2 = 2^k$$

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...

$$2 \times 2 \cdots \times 2 = 2^k$$

How many 10 digit numbers?

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How many 10 digit numbers?

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How many 10 digit numbers?

10 ways for first choice, 10 ways for second choice, ...

$$10 \times 10 \cdots \times 10$$

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How many n digit base m numbers?

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(Is 09, a two digit number?)

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$$10 \times 10 \cdots \times 10 = 10^k$$

How many n digit base m numbers?

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(Is 09, a two digit number?)

If no. Then $(m - 1)m^{n-1}$.

Functions, polynomials.

How many functions f mapping S to T ?

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$|T|$ ways to choose for $f(s_1)$,

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How many polynomials of degree d modulo p ?

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p values for first point,

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Questions?

Permutations.

¹By definition: $0! = 1$.

Permutations.

How many 10 digit numbers **without repeating a digit**?

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How many 10 digit numbers **without repeating a digit**?

10 ways for first,

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Permutations.

How many 10 digit numbers **without repeating a digit**?

10 ways for first, 9 ways for second,

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Permutations.

How many 10 digit numbers **without repeating a digit**?

10 ways for first, 9 ways for second, 8 ways for third,

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... $10 * 9 * 8 \cdots * 1 = 10!$.¹

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How many different samples of size k from n numbers **without replacement**.

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Permutations of n objects.

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One-to-One Functions.

How many one-to-one functions from $|S|$ to $|S|$.

$|S|$ choices for $f(s_1)$, $|S| - 1$ choices for $f(s_2)$, ...

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$|S|$ choices for $f(s_1)$, $|S| - 1$ choices for $f(s_2)$, ...

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A one-to-one function is a permutation!

Counting sets..when order doesn't matter.

How many poker hands?

²When each unordered object corresponds equal numbers of ordered objects.

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How many poker hands?

$$52 \times 51 \times 50 \times 49 \times 48$$

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Are $A, K, Q, 10, J$ of spades
and $10, J, Q, K, A$ of spades the same?

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Are $A, K, Q, 10, J$ of spades
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Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.²

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Number of orderings for a poker hand: "5!"

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Number of orderings for a poker hand: " $5!$ "
(The " $!$ " means factorial, not Exclamation.)

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Counting sets..when order doesn't matter.

How many poker hands?

$$52 \times 51 \times 50 \times 49 \times 48 \text{ ???}$$

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Number of orderings for a poker hand: "5!"

Can write as...

$$\frac{52 \times 51 \times 50 \times 49 \times 48}{5!}$$
$$\frac{52!}{5! \times 47!}$$

²When each unordered object corresponds equal numbers of ordered objects.

Counting sets..when order doesn't matter.

How many poker hands?

$$52 \times 51 \times 50 \times 49 \times 48 ???$$

Are $A, K, Q, 10, J$ of spades
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Number of orderings for a poker hand: "5!"

$$\begin{array}{r} 52 \times 51 \times 50 \times 49 \times 48 \\ \hline 5! \\ \hline 52! \\ \hline 5! \times 47! \end{array}$$

Can write as...

Generic: ways to choose 5 out of 52 possibilities.

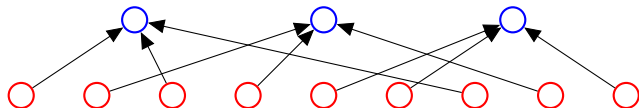
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Ordered to unordered.

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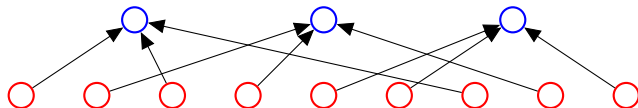
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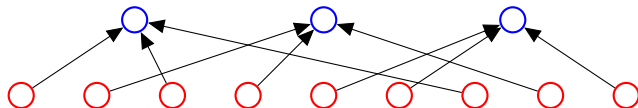
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How many red nodes (ordered objects)?

Ordered to unordered.

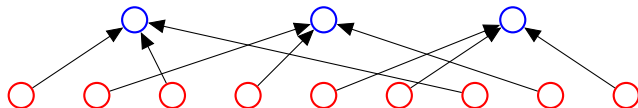
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Ordered to unordered.

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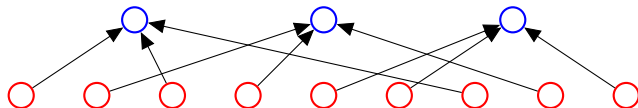


How many red nodes (ordered objects)? 9.

How many red nodes mapped to one blue node?

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.

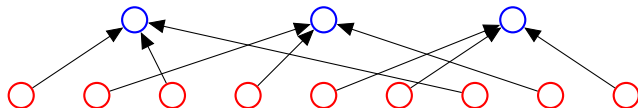


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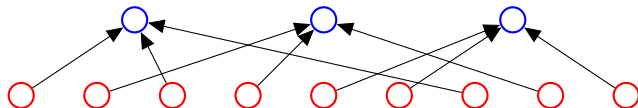
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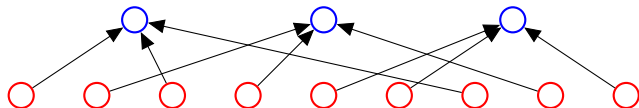
How many red nodes (ordered objects)? 9.

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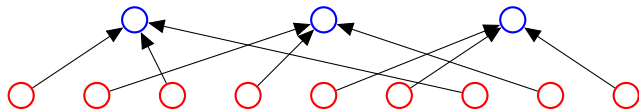
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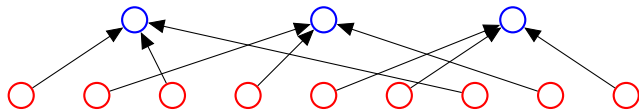
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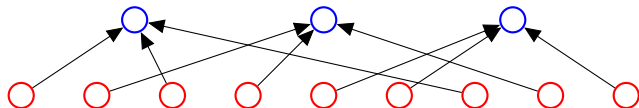
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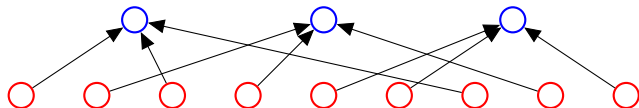
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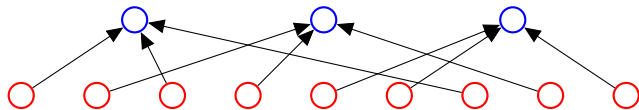
How many poker deals? $52 \cdot 51 \cdot 50 \cdot 49 \cdot 48$.

How many poker deals per hand?

Map each deal to ordered deal:

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.



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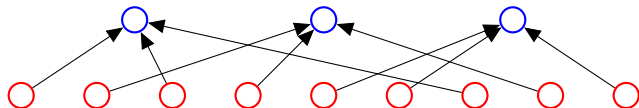
How many poker deals? $52 \cdot 51 \cdot 50 \cdot 49 \cdot 48$.

How many poker deals per hand?

Map each deal to ordered deal: 5!

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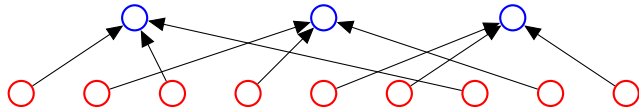
How many poker deals per hand?

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How many poker hands?

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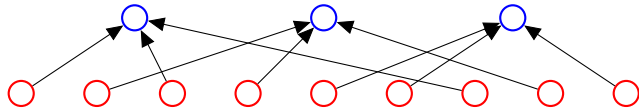
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Map each deal to ordered deal: $5!$

How many poker hands? $\frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5!}$

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Questions?

..order doesn't matter.

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Choose 2 out of n ?

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$$\underline{n \times (n - 1)}$$

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Choose 2 out of n ?

$$\frac{n \times (n - 1)}{2}$$

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$$\frac{n \times (n-1)}{2} = \frac{n!}{(n-2)! \times 2}$$

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Notation: $\binom{n}{k}$ and pronounced “ n choose k .”

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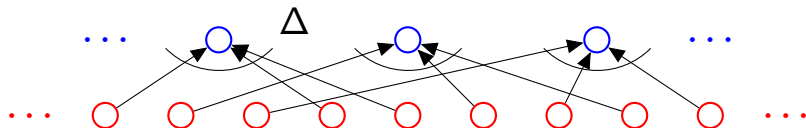
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Familiar? Questions?

Example: Visualize the proof..

First rule: $n_1 \times n_2 \cdots \times n_3$. **Product Rule.**

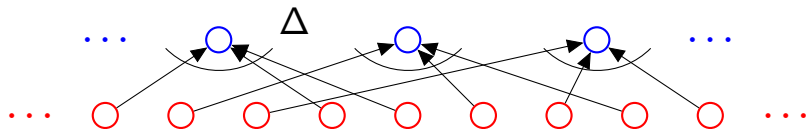
Second rule: when order doesn't matter divide...



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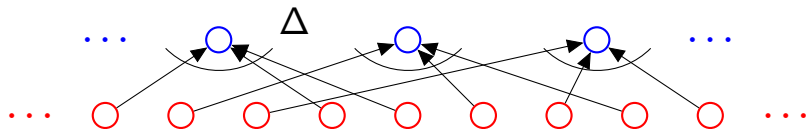


3 card Poker deals: 52

Example: Visualize the proof..

First rule: $n_1 \times n_2 \cdots \times n_3$. **Product Rule.**

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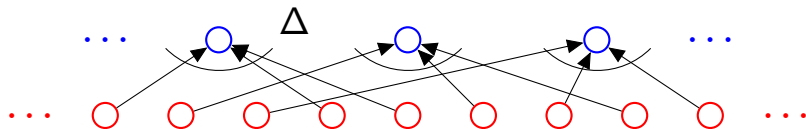


3 card Poker deals: 52×51

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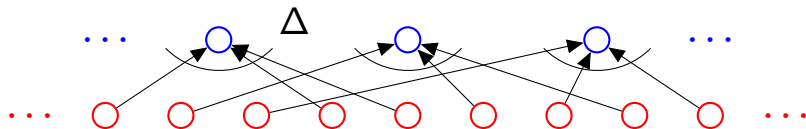


3 card Poker deals: $52 \times 51 \times 50$

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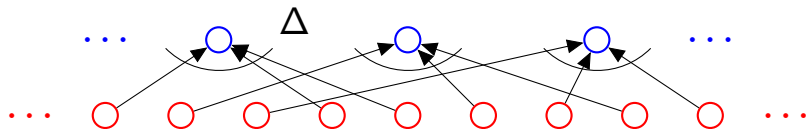


3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$.

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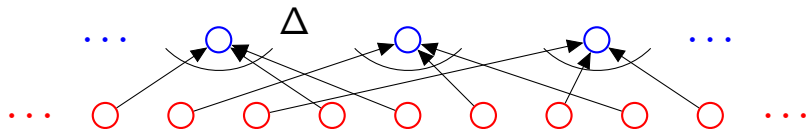


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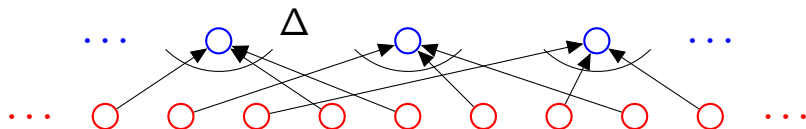
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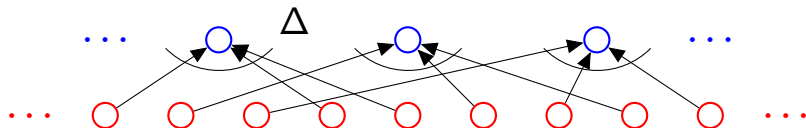
Poker hands: Δ ?

Hand: Q, K, A.

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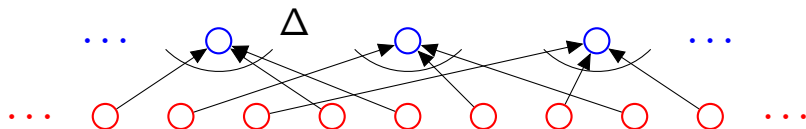
Hand: Q, K, A .

Deals: Q, K, A :

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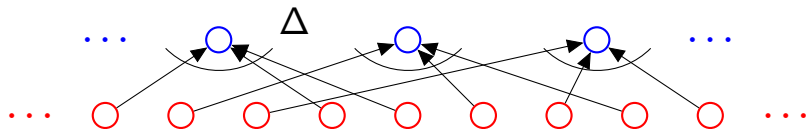
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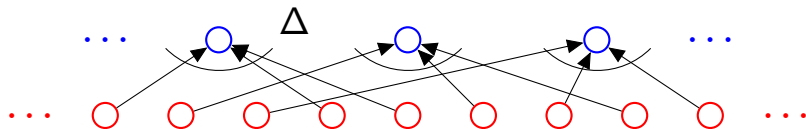
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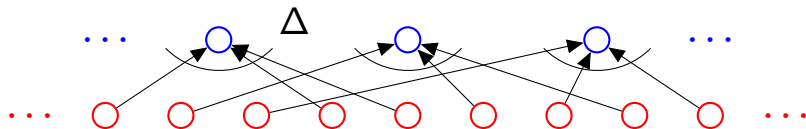
Deals: Q, K, A : Q, A, K : K, A, Q : K, A, Q : A, K, Q : A, Q, K.

$\Delta = 3 \times 2 \times 1$

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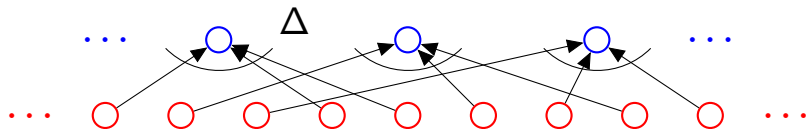
Deals: Q, K, A : Q, A, K : K, A, Q : K, A, Q : A, K, Q : A, Q, K.

$\Delta = 3 \times 2 \times 1$ First rule again.

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Poker hands: Δ ?

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Deals: Q, K, A : Q, A, K : K, A, Q : K, A, Q : A, K, Q : A, Q, K.

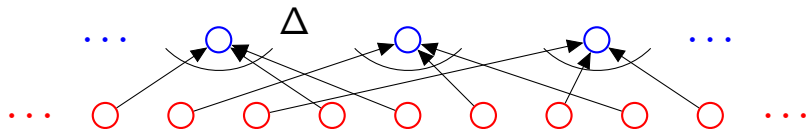
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Total:

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Poker hands: Δ ?

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Deals: Q, K, A : Q, A, K : K, A, Q : K, A, Q : A, K, Q : A, Q, K.

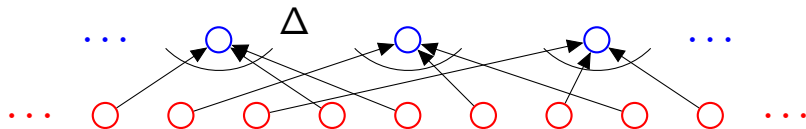
$\Delta = 3 \times 2 \times 1$ First rule again.

Total: $\frac{52!}{49!3!}$

Example: Visualize the proof..

First rule: $n_1 \times n_2 \cdots \times n_3$. **Product Rule.**

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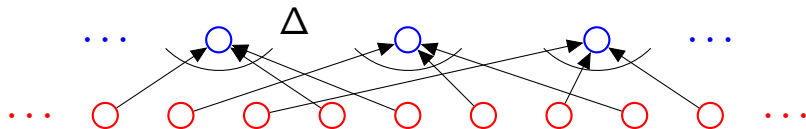
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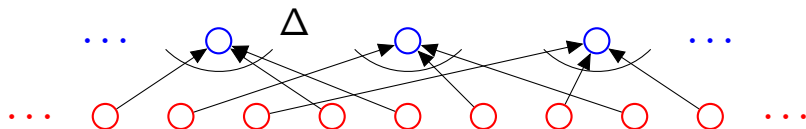
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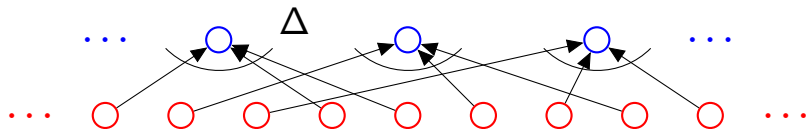
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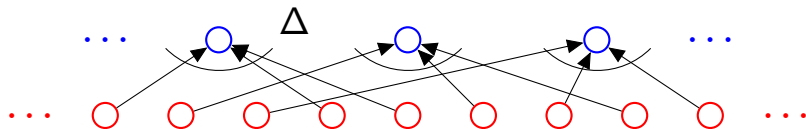
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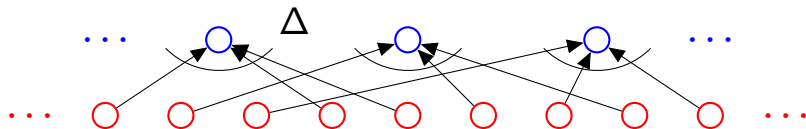
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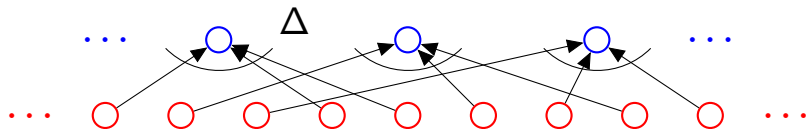
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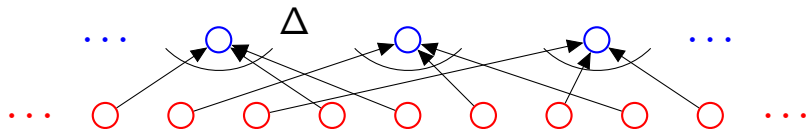
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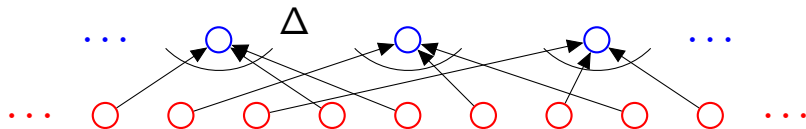
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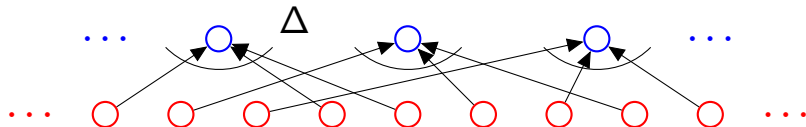
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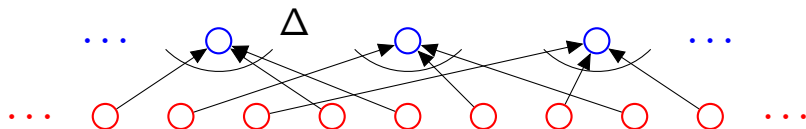
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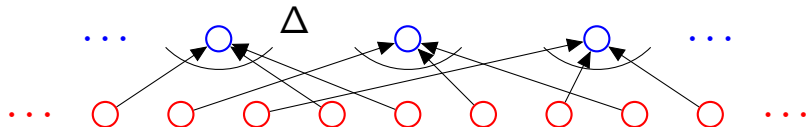


Orderings of ANAGRAM?

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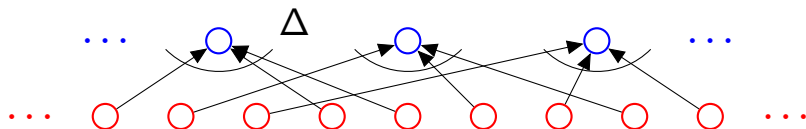
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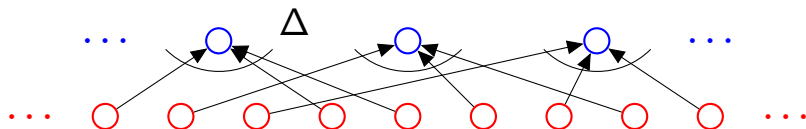
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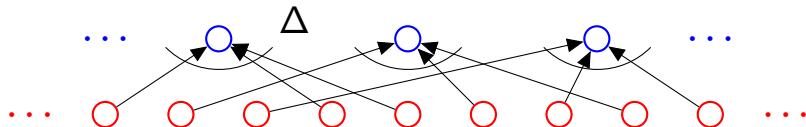
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A's are the same!

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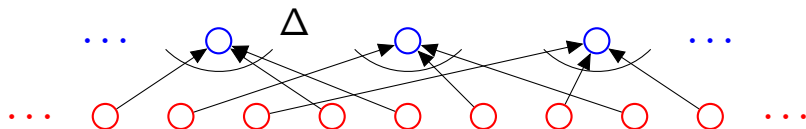
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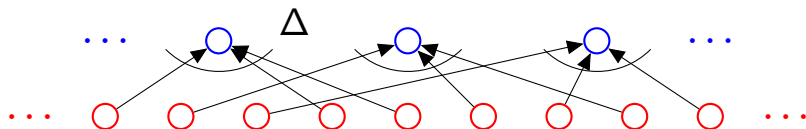
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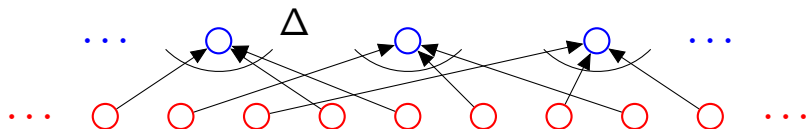
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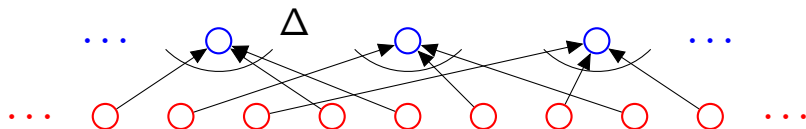
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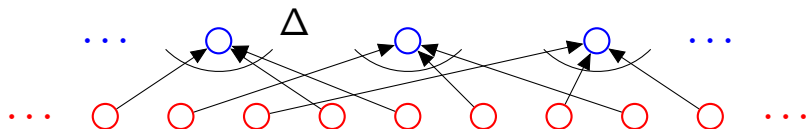
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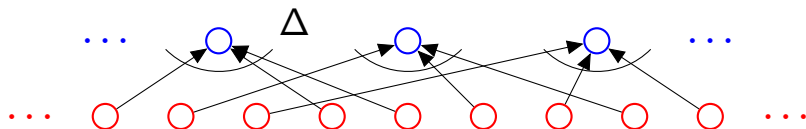
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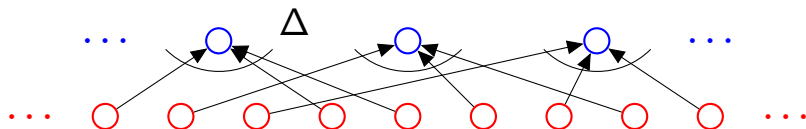
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$\Delta = 3 \times 2 \times 1 = 3!$

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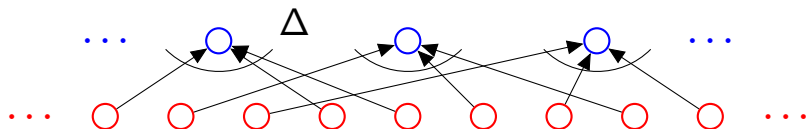
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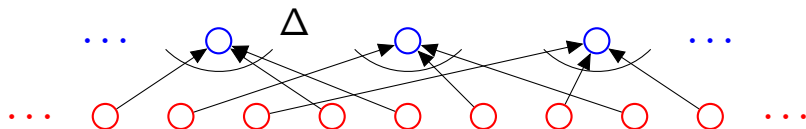
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Poll

Mark what's correct.

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- (A) |Poker hands| = $\binom{52}{5}$
- (B) Orderings of ANAGRAM = $7!/3!$
- (C) Orderings of "CAT". = $3!$
- (D) Orders of MISSISSIPPI = $11!/4!4!2!$
- (E) Orderings of ANAGRAM = $7!/4!$
- (F) Orders of MISSISSIPPI = $11!/10!$

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- (A) $|\text{Poker hands}| = \binom{52}{5}$
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- (E) Orderings of ANAGRAM = $7!/4!$
- (F) Orders of MISSISSIPPI = $11!/10!$
- (A)-(E) are correct.

Some Practice.

How many orderings of letters of CAT?

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3 ways to choose first letter, 2 ways for second, 1 for last.

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total orderings of 7 letters.

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How many orderings of MISSISSIPPI?

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Total orderings? $\frac{7!}{3!}$

How many orderings of MISSISSIPPI?

4 S’s, 4 I’s, 2 P’s.

11 letters total.

Some Practice.

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How many orderings of MISSISSIPPI?

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11 letters total.

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11 letters total.

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Sampling...

Sample k items out of n

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Without replacement:

Sampling...

Sample k items out of n

Without replacement:

Order matters:

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n - 1 \times n - 2 \dots$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n - 1 \times n - 2 \dots \times n - k + 1$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n - 1 \times n - 2 \dots \times n - k + 1 = \frac{n!}{(n-k)!}$

Order does not matter:

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n - 1 \times n - 2 \dots \times n - k + 1 = \frac{n!}{(n-k)!}$

Order does not matter:

Second Rule: divide by number of orders

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Sample k items out of n

Without replacement:

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Second Rule: divide by number of orders – “ $k!$ ”

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Sample k items out of n

Without replacement:

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Order does not matter:

Second Rule: divide by number of orders – “ $k!$ ”

$$\implies \frac{n!}{(n-k)!k!}.$$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \dots \times n-k+1 = \frac{n!}{(n-k)!}$

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“ n choose k ”

With Replacement.

Order matters: $n \times n$

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“ n choose k ”

With Replacement.

Order matters: $n \times n \times \dots n$

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Unordered elt: 1, 2, 3

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How do we deal with this mess??

Splitting up some money....

How many ways can Bob and Alice split 5 dollars?

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For each of 5 dollars pick Bob or Alice(2^5), divide out order

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For each of 5 dollars pick Bob or Alice (2^5), divide out order ???

5 dollars for Bob and 0 for Alice:

one ordered set: (B, B, B, B, B) .

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“Sorted” way to specify, first Alice’s dollars, then Bob’s.

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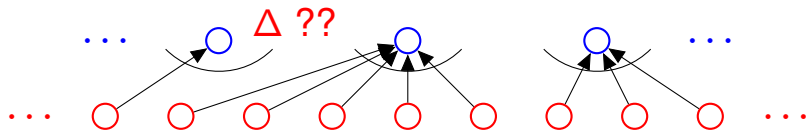
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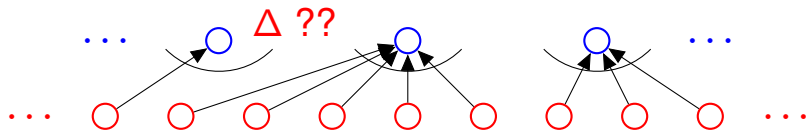
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How many ways can Bob and Alice split 5 dollars?

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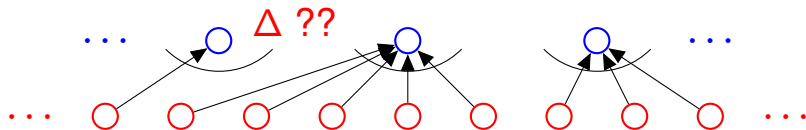
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(A, B, B, B, B) : 5: $(A, B, B, B, B), (B, A, B, B, B), (B, B, A, B, B), \dots$

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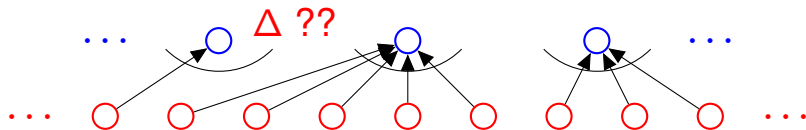
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(A, A, B, B, B) : $\binom{5}{2}$; $(A, A, B, B, B), (A, B, A, B, B), (A, B, B, A, B), \dots$

and so on.



Second rule of counting is no good here!

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How many ways can Alice, Bob, and Eve split 5 dollars.

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Alice gets 3, Bob gets 1, Eve gets 1: (A, A, A, B, E) .

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Stars and Bars: $**|*|**$.

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Alice: 0, Bob: 1, Eve: 4.

Stars and Bars: $|*|****$.

Each split “is” a sequence of stars and bars.

Each sequence of stars and bars “is” a split.

Counting Rule: if there is a one-to-one mapping between two sets they have the same size!

Stars and Bars.

How many different 5 star and 2 bar diagrams?

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| * | * * * *.

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7 positions in which to place the 2 bars.

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Bars in first and third position.

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How many different 5 star and 2 bar diagrams?

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7 positions in which to place the 2 bars.

— — — — — — —

Alice: 0; Bob 1; Eve: 4

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Bars in first and third position.

Alice: 1; Bob 4; Eve: 0

* | * * * * |.

Bars in second and seventh position.

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7 positions in which to place the 2 bars.

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Alice: 1; Bob 4; Eve: 0

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Bars in second and seventh position.

$\binom{7}{2}$ ways to do so and

$\binom{7}{2}$ ways to split 5 dollars among 3 people.

Stars and Bars.

Ways to add up n numbers to sum to k ?

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In general, k stars $n - 1$ bars.

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$n + k - 1$ positions from which to choose $n - 1$ bar positions.

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$n + k - 1$ positions from which to choose $n - 1$ bar positions.

$$\binom{n+k-1}{n-1}$$

Or: k unordered choices from set of n possibilities with replacement.

Sample with replacement where order doesn't matter.

Summary.

First rule: $n_1 \times n_2 \cdots \times n_3$.

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Sample k times from n objects with replacement and order doesn't matter: $\binom{k+n-1}{n-1}$.

Poll

Mark whats correct.

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(A) ways to split k dollars among n : $\binom{k+n-1}{n-1}$

(B) ways to split n dollars among k : $\binom{n+k-1}{k-1}$

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