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Everything below is true. Mark if you know it and perhaps why it is true.

- (A) Two points determine a line: $mx + b$
- (B) A root of $P(x)$, is a where $P(a) = 0$.
- (C) A degree d polynomial has at most d roots.
- (D) Arithmetic modulo a prime p is a "field".

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(A) If a polynomial has a root at a , $P(x) = Q(x)(x - a)$.

(B) A line has at most one root, if not always zero.

(C) System: $y_1 = mx_1 + b$, $y_2 = mx_2 + b$ has unique solution (m, b) .

(D) Degree of a polynomial $P(x)^2$ is $2d$ if $P(x)$ is degree d .

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(C) may not be true.

The mathematics.

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Arithmetic $(\bmod p) \implies$ work with $O(\log p)$ bit numbers.

Proof sketches.

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Two polynomials: $P(x), Q(x)$, $P(x) - Q(x)$ has too many roots.

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Arithmetic modulo a prime m is a **finite field** denoted by F_m or $GF(m)$.

Intuitively, a field is a set with operations corresponding to addition, multiplication, and division.

Secret Sharing

Modular Arithmetic Fact: Exactly one polynomial degree $\leq d$ over $GF(p)$, $P(x)$, that hits $d + 1$ points.

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3 kids hand out 3 points. Any two know the line.

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(Almost) the same as what is missing: one $P(i)$.

Runtime.

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Runtime: polynomial in k , n , and $\log p$.

1. Evaluate degree $k - 1$ polynomial n times using $\log p$ -bit numbers.
2. Reconstruct secret by solving system of k equations using $\log p$ -bit arithmetic.

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Infinite number for reals, rationals, complex numbers!

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n people, k is enough.

- (A) The modulus needs to be at least $n + 1$.
- (B) The modulus needs to be at least k .
- (C) Use degree k polynomial, hand out n points.
- (D) Use degree n polynomial, hand out k points.
- (E) Use degree $k - 1$ polynomial, hand out n points.
- (F) The modulus needs to be at least 2^s , where s is value of secret.
- (G) The modulus needs to be at least 2^s , where s is size of secret.

Secret Sharing.

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- (A), (B), (E), (F)

Erasure Codes.

Satellite

GPS device

Erasure Codes.

Satellite

3 packet message.

GPS device

Erasure Codes.

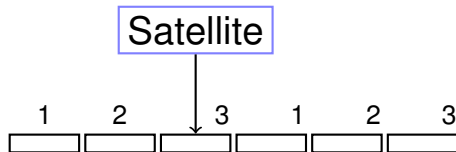
Satellite

3 packet message.

Lose 3 out 6 packets.

GPS device

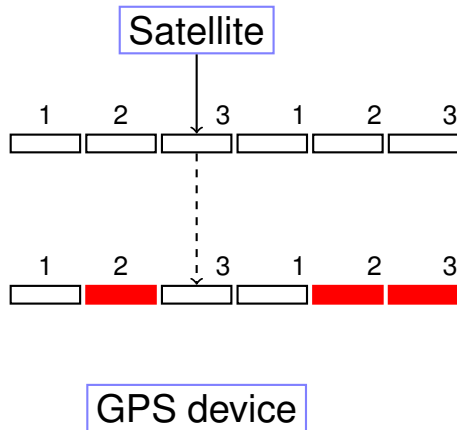
Erasure Codes.



3 packet message. So send 6!

Lose 3 out 6 packets.

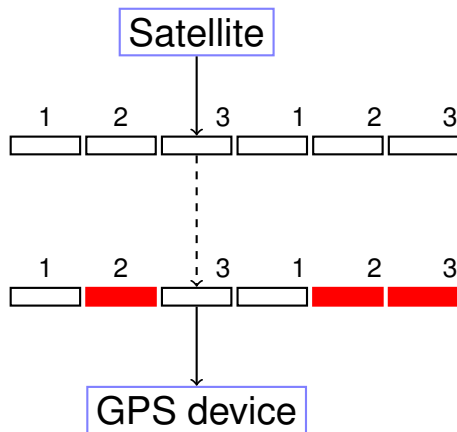
Erasure Codes.



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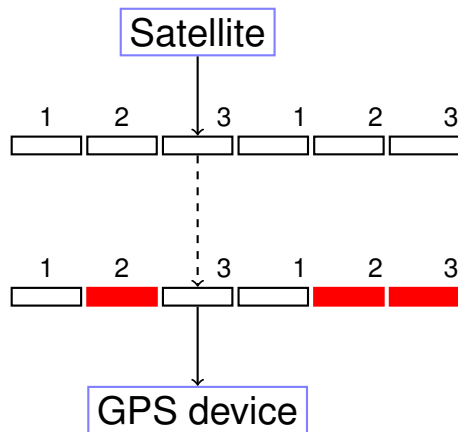
Erasure Codes.



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Lose 3 out 6 packets.

Gets packets 1,1,and 3.

Solution Idea.

n packet message, channel that loses k packets.

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Use polynomials.

The Scheme

Problem: Want to send a message with n packets.

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A degree $n - 1$ polynomial determined by any n points!

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Satellite

GPS device

Erasure Codes.

Satellite

n packet message.

GPS device

Erasure Codes.

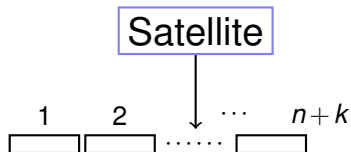
Satellite

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Lose k packets.

GPS device

Erasure Codes.

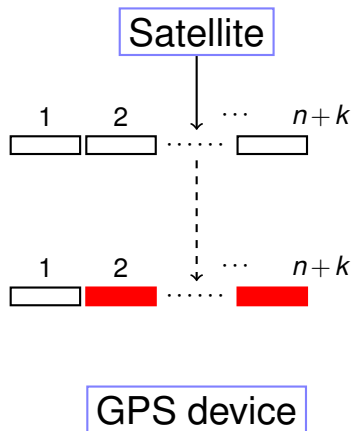


n packet message. So send $n+k$!

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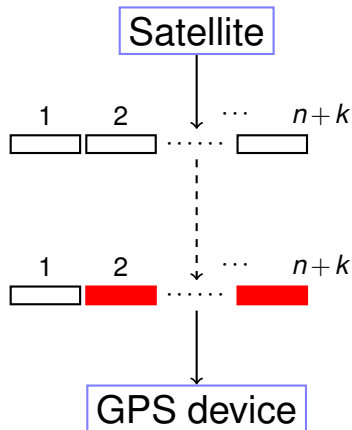
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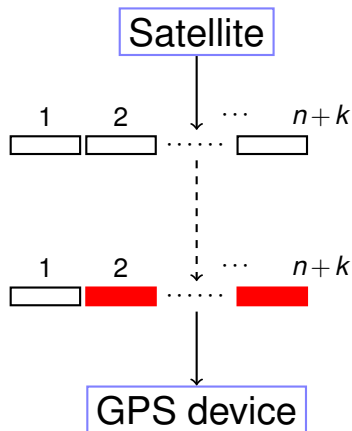
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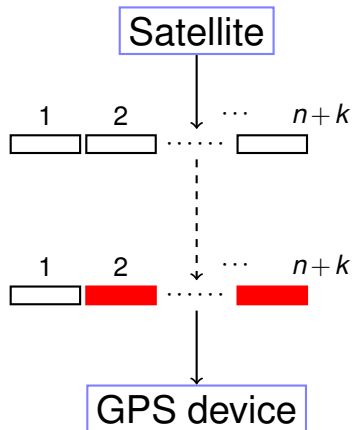


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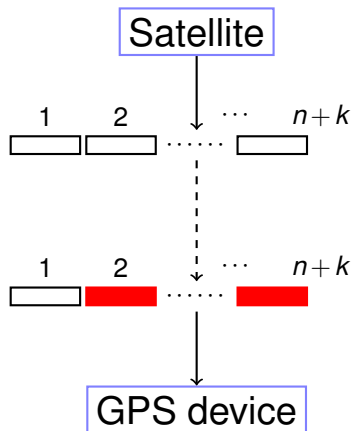
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Send message of 1,4, and 4.

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Make polynomial with $P(1) = 1$, $P(2) = 4$, $P(3) = 4$.

Modulo 7 to accommodate at least 6 packets.

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Notice that packets contain "x-values".

Bad reception!

Send: $(1, 1), (2, 4), (3, 4), (4, 7), (5, 2), (6, 0)$

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Send: $(1, 1), (2, 4), (3, 4), (4, 7), (5, 2), (6, 0)$

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Reconstruct?

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You want to encode a secret consisting of 1,4,4.

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Modulus should be larger than $n + k$ and also larger than 2^b .

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- ▶ ..give Secret Sharing.

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Additional Challenge: Finding **which** packets are corrupt.

Error Correction

Satellite

GPS device

Error Correction

Satellite

3 packet message.

GPS device

Error Correction

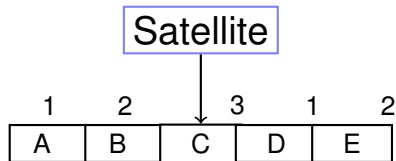
Satellite

3 packet message.

Corrupts 1 packets.

GPS device

Error Correction

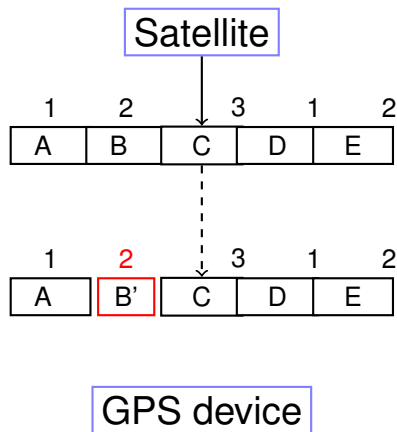


3 packet message. Send 5.

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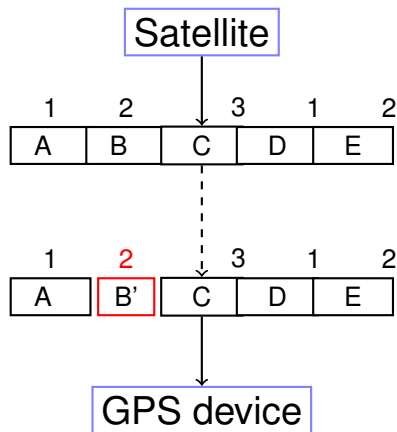
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Problem: Communicate n packets m_1, \dots, m_n on noisy channel that corrupts $\leq k$ packets.

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Properties: proof.

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Proof:

- (1) Sure. Only k corruptions.

Properties: proof.

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- (2) $P(x)$ is unique degree $n - 1$ polynomial that contains $\geq n + k$ received points.

Proof:

(1) Sure. Only k corruptions.

(2) Degree $n - 1$ polynomial $Q(x)$ consistent with $n + k$ points.

Properties: proof.

$P(x)$: degree $n - 1$ polynomial.

Send $P(1), \dots, P(n + 2k)$

Receive $R(1), \dots, R(n + 2k)$

At most k i 's where $P(i) \neq R(i)$.

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(Aside: Message in plain text!)

Receive $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$.

$P(i) = R(i)$ for $n + k = 3 + 1 = 4$ points.

Slow solution.

Brute Force:

For each subset of $n + k$ points

Slow solution.

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Fit degree $n - 1$ polynomial, $Q(x)$, to n of them.

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- For subset of $n + k$ pts where $R(i) = P(i)$,
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- ▶ For subset of $n + k$ pts where $R(i) = P(i)$,
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Reconstructs $P(x)$ and only $P(x)$!!

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Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

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Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

All equations..

$$p_2 + p_1 + p_0 \equiv 3 \pmod{7}$$

$$4p_2 + 2p_1 + p_0 \equiv 1 \pmod{7}$$

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Assume point 1 is wrong and solve..

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How do we find where the bad packets are efficiently?!?!?!?

Ditty...

Oh where, Oh where

Ditty...

Oh where, Oh where
has my little dog gone?

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be
With his ears cut short

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be
With his ears cut short
And his tail cut long

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be
With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone..

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be
With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?
Oh where, oh where do they not fit.

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?
Oh where, oh where do they not fit.

With the polynomial well put

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?
Oh where, oh where do they not fit.

With the polynomial well put
But the channel a bit wrong

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be
With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?
Oh where, oh where do they not fit.
With the polynomial well put
But the channel a bit wrong
Where, oh where do we look?

Where oh where can my **bad** packets be?

$$(p_{n-1} + \cdots p_0) \equiv R(1) \pmod{p}$$

Where oh where can my **bad** packets be?

$$(p_{n-1} + \cdots p_0) \equiv R(1) \pmod{p}$$

$$(p_{n-1}2^{n-1} + \cdots p_0) \equiv R(2) \pmod{p}$$

$$\vdots$$

$$(p_{n-1}(m)^{n-1} + \cdots p_0) \equiv R(n+2k) \pmod{p}$$

Where oh where can my **bad** packets be?

$$\begin{aligned}(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\ (p_{n-1} 2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p} \\ &\vdots \\ (p_{n-1} (m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod{p}\end{aligned}$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.

Where oh where can my **bad** packets be?

$$\begin{aligned}(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\ (p_{n-1} 2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p} \\ &\vdots \\ (p_{n-1} (m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod{p}\end{aligned}$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
Zero times anything is zero!!!!

Where oh where can my **bad** packets be?

$$\begin{aligned}(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\ (p_{n-1} 2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p} \\ &\vdots \\ (p_{n-1} (m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod{p}\end{aligned}$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.

Zero times anything is zero!!!! My love is won.

Where oh where can my **bad** packets be?

$$\begin{aligned} & (p_{n-1} + \cdots p_0) \equiv R(1) \pmod{p} \\ \mathbf{0} \times & \quad (p_{n-1} \mathbf{2}^{n-1} + \cdots p_0) \equiv \mathbf{R(2)} \pmod{p} \\ & \vdots \\ & (p_{n-1} (m)^{n-1} + \cdots p_0) \equiv R(n+2k) \pmod{p} \end{aligned}$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.

Zero times anything is zero!!!! My love is won.

All equations satisfied!!!!

Where oh where can my **bad** packets be?

$$\begin{aligned}(p_{n-1} + \cdots p_0) &\equiv R(1) & (\text{mod } p) \\ (p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2) & (\text{mod } p) \\ &\vdots \\ (p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k) & (\text{mod } p)\end{aligned}$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.

Zero times anything is zero!!!! My love is won.

All equations satisfied!!!!

But which equations should we multiply by 0?

Where oh where can my **bad** packets be?

$$\begin{aligned}(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\ (p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p} \\ &\vdots \\ (p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod{p}\end{aligned}$$

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Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.

Zero times anything is zero!!!! My love is won.

All equations satisfied!!!!

But which equations should we multiply by 0? **Where oh where...??**

We will use a polynomial!!!

Where oh where can my **bad** packets be?

$$\begin{aligned}(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\ (p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p} \\ &\vdots \\ (p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod{p}\end{aligned}$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.

Zero times anything is zero!!!! My love is won.

All equations satisfied!!!!

But which equations should we multiply by 0? **Where oh where...??**

We will use a polynomial!!! That we don't know.

Where oh where can my **bad** packets be?

$$\begin{aligned}(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\ (p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p} \\ &\vdots \\ (p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod{p}\end{aligned}$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.

Zero times anything is zero!!!! My love is won.

All equations satisfied!!!!

But which equations should we multiply by 0? **Where oh where...??**

We will use a polynomial!!! That we don't know. But can find!

Where oh where can my **bad** packets be?

$$\begin{aligned}(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\ (p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p} \\ &\vdots \\ (p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod{p}\end{aligned}$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.

Zero times anything is zero!!!! My love is won.

All equations satisfied!!!!

But which equations should we multiply by 0? **Where oh where...??**

We will use a polynomial!!! That we don't know. But can find!

Errors at points e_1, \dots, e_k . (In diagram above, $e_1 = 2$.)

Where oh where can my **bad** packets be?

$$\begin{aligned}(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\ (p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p} \\ &\vdots \\ (p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod{p}\end{aligned}$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.

Zero times anything is zero!!!! My love is won.

All equations satisfied!!!!

But which equations should we multiply by 0? **Where oh where...??**

We will use a polynomial!!! That we don't know. But can find!

Errors at points e_1, \dots, e_k . (In diagram above, $e_1 = 2$.)

Error locator polynomial: $E(x) = (x - e_1)$

Where oh where can my **bad** packets be?

$$\begin{aligned}(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\ (p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p} \\ &\vdots \\ (p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod{p}\end{aligned}$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.

Zero times anything is zero!!!! My love is won.

All equations satisfied!!!!

But which equations should we multiply by 0? **Where oh where...??**

We will use a polynomial!!! That we don't know. But can find!

Errors at points e_1, \dots, e_k . (In diagram above, $e_1 = 2$.)

Error locator polynomial: $E(x) = (x - e_1)(x - e_2)$

Where oh where can my **bad** packets be?

$$\begin{aligned}(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\ (p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p} \\ &\vdots \\ (p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod{p}\end{aligned}$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.

Zero times anything is zero!!!! My love is won.

All equations satisfied!!!!

But which equations should we multiply by 0? **Where oh where...??**

We will use a polynomial!!! That we don't know. But can find!

Errors at points e_1, \dots, e_k . (In diagram above, $e_1 = 2$.)

Error locator polynomial: $E(x) = (x - e_1)(x - e_2) \dots$

Where oh where can my **bad** packets be?

$$\begin{aligned}(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\ (p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p} \\ &\vdots \\ (p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod{p}\end{aligned}$$

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All equations satisfied!!!!

But which equations should we multiply by 0? **Where oh where...??**

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Errors at points e_1, \dots, e_k . (In diagram above, $e_1 = 2$.)

Error locator polynomial: $E(x) = (x - e_1)(x - e_2) \dots (x - e_k)$.

Where oh where can my **bad** packets be?

$$\begin{aligned}(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\ (p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p} \\ &\vdots \\ (p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod{p}\end{aligned}$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.

Zero times anything is zero!!!! My love is won.

All equations satisfied!!!!

But which equations should we multiply by 0? **Where oh where...??**

We will use a polynomial!!! That we don't know. But can find!

Errors at points e_1, \dots, e_k . (In diagram above, $e_1 = 2$.)

Error locator polynomial: $E(x) = (x - e_1)(x - e_2) \dots (x - e_k)$.

$E(i) = 0$ if and only if $e_j = i$ for some j

Where oh where can my **bad** packets be?

$$E(1)(p_{n-1} + \cdots p_0) \equiv R(1)E(1) \pmod{p}$$

$$E(2)(p_{n-1}2^{n-1} + \cdots p_0) \equiv R(2)E(2) \pmod{p}$$

$$\vdots$$

$$E(m)(p_{n-1}(m)^{n-1} + \cdots p_0) \equiv R(n+2k)E(m) \pmod{p}$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.

Zero times anything is zero!!!! My love is won.

All equations satisfied!!!!

But which equations should we multiply by 0? **Where oh where...??**

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Errors at points e_1, \dots, e_k . (In diagram above, $e_1 = 2$.)

Error locator polynomial: $E(x) = (x - e_1)(x - e_2) \dots (x - e_k)$.

$E(i) = 0$ if and only if $e_j = i$ for some j

Multiply equations by $E(\cdot)$.

Where oh where can my **bad** packets be?

$$\begin{aligned}E(1)(p_{n-1} + \cdots p_0) &\equiv R(1)E(1) \pmod{p} \\E(2)(\textcolor{red}{p}_{n-1}\textcolor{red}{2}^{n-1} + \cdots p_0) &\equiv \textcolor{red}{R}(2)E(2) \pmod{p} \\&\vdots \\E(m)(p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k)E(m) \pmod{p}\end{aligned}$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.

Zero times anything is zero!!!! My love is won.

All equations satisfied!!!!

But which equations should we multiply by 0? **Where oh where...??**

We will use a polynomial!!! That we don't know. But can find!

Errors at points e_1, \dots, e_k . (In diagram above, $e_1 = 2$.)

Error locator polynomial: $E(x) = (x - e_1)(x - e_2) \dots (x - e_k)$.

$E(i) = 0$ if and only if $e_j = i$ for some j

Multiply equations by $E(\cdot)$. (Above $E(x) = (x-2)$.)

Where oh where can my bad packets be?

$$E(1)(p_{n-1} + \cdots p_0) \equiv R(1)E(1) \pmod{p}$$

$$E(2)(p_{n-1}2^{n-1} + \cdots p_0) \equiv R(2)E(2) \pmod{p}$$

\vdots

$$E(m)(p_{n-1}(m)^{n-1} + \cdots p_0) \equiv R(n+2k)E(m) \pmod{p}$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.

Zero times anything is zero!!!! My love is won.

All equations satisfied!!!!

But which equations should we multiply by 0? Where oh where...??

We will use a polynomial!!! That we don't know. But can find!

Errors at points e_1, \dots, e_k . (In diagram above, $e_1 = 2$.)

Error locator polynomial: $E(x) = (x - e_1)(x - e_2) \dots (x - e_k)$.

$E(i) = 0$ if and only if $e_j = i$ for some j

Multiply equations by $E(\cdot)$. (Above $E(x) = (x-2)$.)

All equations satisfied!!

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

Plugin points...

$$(p_2 + p_1 + p_0) \equiv (3) \pmod{7}$$

$$(4p_2 + 2p_1 + p_0) \equiv (1) \pmod{7}$$

$$(2p_2 + 3p_1 + p_0) \equiv (6) \pmod{7}$$

$$(2p_2 + 4p_1 + p_0) \equiv (0) \pmod{7}$$

$$(4p_2 + 5p_1 + p_0) \equiv (3) \pmod{7}$$

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

Plugin points...

$$\begin{aligned}(p_2 + p_1 + p_0) &\equiv (3) && (\text{mod } 7) \\ (4p_2 + 2p_1 + p_0) &\equiv (1) && (\text{mod } 7) \\ (2p_2 + 3p_1 + p_0) &\equiv (6) && (\text{mod } 7) \\ (2p_2 + 4p_1 + p_0) &\equiv (0) && (\text{mod } 7) \\ (4p_2 + 5p_1 + p_0) &\equiv (3) && (\text{mod } 7)\end{aligned}$$

Error locator polynomial: $(x - 2)$.

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

Plugin points...

$$(1 - 2)(p_2 + p_1 + p_0) \equiv (3)(1 - 2) \pmod{7}$$

$$(2 - 2)(4p_2 + 2p_1 + p_0) \equiv (1)(2 - 2) \pmod{7}$$

$$(3 - 2)(2p_2 + 3p_1 + p_0) \equiv (6)(3 - 2) \pmod{7}$$

$$(4 - 2)(2p_2 + 4p_1 + p_0) \equiv (0)(4 - 2) \pmod{7}$$

$$(5 - 2)(4p_2 + 5p_1 + p_0) \equiv (3)(5 - 2) \pmod{7}$$

Error locator polynomial: $(x - 2)$.

Multiply equation i by $(i - 2)$.

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

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$$(5 - 2)(4p_2 + 5p_1 + p_0) \equiv (3)(5 - 2) \pmod{7}$$

Error locator polynomial: $(x - 2)$.

Multiply equation i by $(i - 2)$. All equations satisfied!

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

Plug in points...

$$(1 - 2)(p_2 + p_1 + p_0) \equiv (3)(1 - 2) \pmod{7}$$

$$(2 - 2)(4p_2 + 2p_1 + p_0) \equiv (1)(2 - 2) \pmod{7}$$

$$(3 - 2)(2p_2 + 3p_1 + p_0) \equiv (6)(3 - 2) \pmod{7}$$

$$(4 - 2)(2p_2 + 4p_1 + p_0) \equiv (0)(4 - 2) \pmod{7}$$

$$(5 - 2)(4p_2 + 5p_1 + p_0) \equiv (3)(5 - 2) \pmod{7}$$

Error locator polynomial: $(x - 2)$.

Multiply equation i by $(i - 2)$. All equations satisfied!

But don't know error locator polynomial!

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

Plugin points...

$$(1 - 2)(p_2 + p_1 + p_0) \equiv (3)(1 - 2) \pmod{7}$$

$$(2 - 2)(4p_2 + 2p_1 + p_0) \equiv (1)(2 - 2) \pmod{7}$$

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$$(5 - 2)(4p_2 + 5p_1 + p_0) \equiv (3)(5 - 2) \pmod{7}$$

Error locator polynomial: $(x - 2)$.

Multiply equation i by $(i - 2)$. All equations satisfied!

But don't know error locator polynomial! Do know form:

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

Plugin points...

$$(1 - 2)(p_2 + p_1 + p_0) \equiv (3)(1 - 2) \pmod{7}$$

$$(2 - 2)(4p_2 + 2p_1 + p_0) \equiv (1)(2 - 2) \pmod{7}$$

$$(3 - 2)(2p_2 + 3p_1 + p_0) \equiv (6)(3 - 2) \pmod{7}$$

$$(4 - 2)(2p_2 + 4p_1 + p_0) \equiv (0)(4 - 2) \pmod{7}$$

$$(5 - 2)(4p_2 + 5p_1 + p_0) \equiv (3)(5 - 2) \pmod{7}$$

Error locator polynomial: $(x - 2)$.

Multiply equation i by $(i - 2)$. All equations satisfied!

But don't know error locator polynomial! Do know form: $(x - e)$.

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

Plugin points...

$$(1 - e)(p_2 + p_1 + p_0) \equiv (3)(1 - e) \pmod{7}$$

$$(2 - e)(4p_2 + 2p_1 + p_0) \equiv (1)(2 - e) \pmod{7}$$

$$(3 - e)(2p_2 + 3p_1 + p_0) \equiv (3)(3 - e) \pmod{7}$$

$$(4 - e)(2p_2 + 4p_1 + p_0) \equiv (0)(4 - e) \pmod{7}$$

$$(5 - e)(4p_2 + 5p_1 + p_0) \equiv (3)(5 - e) \pmod{7}$$

Error locator polynomial: $(x - 2)$.

Multiply equation i by $(i - 2)$. All equations satisfied!

But don't know error locator polynomial! Do know form: $(x - e)$.

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

Plugin points...

$$(1 - e)(p_2 + p_1 + p_0) \equiv (3)(1 - e) \pmod{7}$$

$$(2 - e)(4p_2 + 2p_1 + p_0) \equiv (1)(2 - e) \pmod{7}$$

$$(3 - e)(2p_2 + 3p_1 + p_0) \equiv (3)(3 - e) \pmod{7}$$

$$(4 - e)(2p_2 + 4p_1 + p_0) \equiv (0)(4 - e) \pmod{7}$$

$$(5 - e)(4p_2 + 5p_1 + p_0) \equiv (3)(5 - e) \pmod{7}$$

Error locator polynomial: $(x - 2)$.

Multiply equation i by $(i - 2)$. All equations satisfied!

But don't know error locator polynomial! Do know form: $(x - e)$.

4 unknowns (p_0, p_1, p_2 and e),

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

Plugin points...

$$(1 - e)(p_2 + p_1 + p_0) \equiv (3)(1 - e) \pmod{7}$$

$$(2 - e)(4p_2 + 2p_1 + p_0) \equiv (1)(2 - e) \pmod{7}$$

$$(3 - e)(2p_2 + 3p_1 + p_0) \equiv (3)(3 - e) \pmod{7}$$

$$(4 - e)(2p_2 + 4p_1 + p_0) \equiv (0)(4 - e) \pmod{7}$$

$$(5 - e)(4p_2 + 5p_1 + p_0) \equiv (3)(5 - e) \pmod{7}$$

Error locator polynomial: $(x - 2)$.

Multiply equation i by $(i - 2)$. All equations satisfied!

But don't know error locator polynomial! Do know form: $(x - e)$.

4 unknowns (p_0, p_1, p_2 and e), 5 nonlinear equations.

..turn their heads each day,

$$\begin{array}{rcl} (p_{n-1} + \cdots p_0) & \equiv & R(1) \pmod{p} \\ & \vdots & \\ (p_{n-1}i^{n-1} + \cdots p_0) & \equiv & R(i) \pmod{p} \\ & \vdots & \\ (p_{n-1}(n+2k)^{n-1} + \cdots p_0) & \equiv & R(m) \pmod{p} \end{array}$$

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$$\begin{aligned} E(1)(p_{n-1} + \cdots p_0) &\equiv R(1)E(1) \pmod{p} \\ &\vdots \\ E(i)(p_{n-1}i^{n-1} + \cdots p_0) &\equiv R(i)E(i) \pmod{p} \\ &\vdots \\ E(m)(p_{n-1}(n+2k)^{n-1} + \cdots p_0) &\equiv R(m)E(m) \pmod{p} \end{aligned}$$

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and linear in a_i and coefficients of $E(x)$!

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Solving for $Q(x)$ and $E(x)$...

For all points $1, \dots, i, n+2k = m$,

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Solving for $Q(x)$ and $E(x)$...and $P(x)$

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Example.

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$$\begin{array}{r} 1 \ x^2 \\ \hline x - 2 \) \ x^3 + 6 \ x^2 + 6 \ x + 5 \\ \underline{x^3 - 2 \ x^2} \\ 1 \ x^2 + 6 \ x + 5 \end{array}$$

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$$\begin{array}{r}
 \overline{1x^2 + 1x} \\
 x - 2 \overline{) x^3 + 6x^2 + 6x + 5} \\
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$$\begin{array}{r}
 1 x^2 + 1 x \\
 \hline
 x - 2 x^3 + 6 x^2 + 6 x + 5 \\
 x^3 - 2 x^2 \\
 \hline
 1 x^2 + 6 x + 5 \\
 1 x^2 - 2 x \\
 \hline
 x + 5
 \end{array}$$

Example: finishing up.

$$Q(x) = x^3 + 6x^2 + 6x + 5.$$

$$E(x) = x - 2.$$

$$\begin{array}{r} 1 \ x^2 + 1 \ x + 1 \\ \hline x - 2 \) \ x^3 + 6x^2 + 6x + 5 \\ x^3 - 2x^2 \\ \hline 1 \ x^2 + 6x + 5 \\ 1 \ x^2 - 2x \\ \hline x + 5 \\ x - 2 \end{array}$$

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Message is $P(1) = 3, P(2) = 0, P(3) = 6$.

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What is $\frac{x-2}{x-2}$? 1

Except at $x = 2$? Hole there?

Error Correction: Berlekamp-Welsh

Message: m_1, \dots, m_n .

Sender:

1. Form degree $n - 1$ polynomial $P(x)$ where $P(i) = m_i$.
2. Send $P(1), \dots, P(n + 2k)$.

Receiver:

1. Receive $R(1), \dots, R(n + 2k)$.
2. Solve $n + 2k$ equations, $Q(i) = E(i)R(i)$ to find $Q(x) = E(x)P(x)$ and $E(x)$.
3. Compute $P(x) = Q(x)/E(x)$.
4. Compute $P(1), \dots, P(n)$.

Check your understanding.

You have error locator polynomial!

Check your understanding.

You have error locator polynomial!

Where oh where have my packets gone **wrong**?

Check your understanding.

You have error locator polynomial!

Where oh where have my packets gone **wrong**?

Factor?

Check your understanding.

You have error locator polynomial!

Where oh where have my packets gone **wrong**?

Factor? Sure.

Check your understanding.

You have error locator polynomial!

Where oh where have my packets gone **wrong**?

Factor? Sure.

Check all values?

Check your understanding.

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Where oh where have my packets gone **wrong**?

Factor? Sure.

Check all values? Sure.

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Check all values? Sure.

Check your understanding.

You have error locator polynomial!

Where oh where have my packets gone **wrong**?

Factor? Sure.

Check all values? Sure.

Efficiency?

Check your understanding.

You have error locator polynomial!

Where oh where have my packets gone **wrong**?

Factor? Sure.

Check all values? Sure.

Efficiency? Sure.

Check your understanding.

You have error locator polynomial!

Where oh where have my packets gone **wrong**?

Factor? Sure.

Check all values? Sure.

Efficiency? Sure. Only $n+2k$ values.

Check your understanding.

You have error locator polynomial!

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Factor? Sure.

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See where it is 0.

Hmmm...

Is there one and only one $P(x)$ from Berlekamp-Welsh procedure?

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Is there one and only one $P(x)$ from Berlekamp-Welsh procedure?

Existence: there is a $P(x)$ and $E(x)$ that satisfy equations.

Unique solution for $P(x)$

Uniqueness: any solution $Q'(x)$ and $E'(x)$ have

$$\frac{Q'(x)}{E'(x)} = \frac{Q(x)}{E(x)} = P(x). \quad (1)$$

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We claim

$$Q'(x)E(x) = Q(x)E'(x) \text{ on } n+2k \text{ values of } x. \quad (2)$$

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$E(x)$ and $E'(x)$ have at most k zeros each.

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Cross multiplying gives equality in fact for these points. □

Points to polynomials, have to deal with zeros!

Example: dealing with $\frac{x-2}{x-2}$ at $x = 2$.

Yaay!!

Berlekamp-Welsh algorithm decodes correctly when k errors!

Poll

Say you sent a message of length 4, encoded as $P(x)$ where one sends packets $P(1), \dots, P(8)$.

You receive packets $R(1), \dots, R(8)$.

Packets 1 and 4 are corrupted.

(A) $R(1) \neq P(1)$

(B) The degree of $P(x)E(x) = 3 + 2 = 5$.

(C) The degree of $E(x)$ is 2.

(D) The number of coefficients of $P(x)$ is 4.

(E) The number of coefficients of $P(x)Q(x)$ is 6.

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- (A) $E(x) = (x - 1)(x - 4)$
- (B) The number of coefficients in $E(x)$ is 2.
- (C) The number of unknown coefficients in $E(x)$ is 2.
- (D) $E(x) = (x - 1)(x - 2)$
- (E) $R(4) \neq P(4)$
- (F) The degree of $R(x)$ is 5.

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(A), (C), (E).

Summary. Error Correction.

Communicate n packets, with k erasures.

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How many packets?

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How to encode? With polynomial, $P(x)$.

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Recover? Reconstruct $P(x)$ with any n points!

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How to encode? With polynomial, $P(x)$.

Of degree? $n - 1$

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k changes to make diff. messages overlap

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Cool.

Really Cool!