DECA End of Chapter 6 Quiz

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You are allowed to use a Laplace transform table to solve these problems. (Page 321)

1. Find the solution of the differential equation using the Laplace Transform

$$y'' + y = \sin 2t$$

satisfying the initial condition

$$y(0) = 2, \qquad y'(0) = 1$$

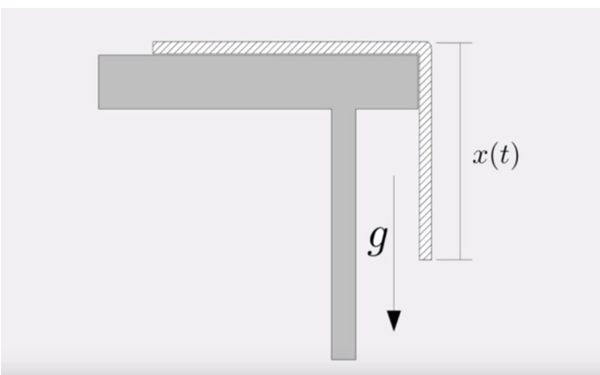
2. Find the inverse transform of

$$F(s) = \frac{1 - e^{-2s}}{s^2}$$

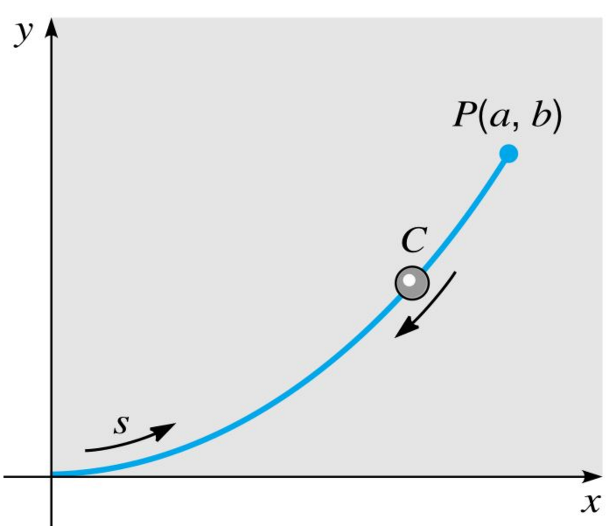
3. Find the inverse transform of

$$H(s) = \frac{a}{s^2(s^2 + a^2)}$$

4. Consider a rope with mass M and of length L. Solve for the equation of the displacement of the rope with respect to time



5. The Tautochrone - the curve down which a particle will slide freely under gravity alone, reaching the bottom in time regardless of its starting point on the curve.
A figure of the curve is shown on the side. The starting point is $P(a, b)$ and the end point is the origin.



a. Given that the Arc length s is measured from the origin and $f(y)$ gives the rate of change of s with respect to y.

$$f(y) = \frac{ds}{dy} = \left[1 + \left(\frac{dx}{dy}\right)^2\right]^{1/2}$$

b. Using conservation of energy the time $T(b)$ required for a particle to slide from P to the origin is (Show this step!!)

$$T(b) = \frac{1}{\sqrt{2g}} \int_0^b \frac{f(y)}{\sqrt{b-y}} dy$$

c. By the property of a Tautochrone the value of $T(b) = T_0$ is a constant for each b. Taking the Laplace transform show that

$$F(s) = \sqrt{\frac{2g}{\pi}} \frac{T_0}{\sqrt{s}}$$

d. Then show that

$$f(y) = \frac{\sqrt{2g}}{\pi} \frac{T_0}{\sqrt{y}}$$

e. Using the previous answers show

$$\frac{dx}{dy} = \sqrt{\frac{2\alpha - y}{y}}$$

where $\alpha = gT_0^2/\pi^2$

f. Use the substitution $y = 2\alpha \sin^2(\theta/2)$ and show

$$x = \alpha(\theta + \sin\theta)$$

$$y = \alpha(1 - \cos\theta)$$

Which is a parametric equation of a cycloid, the solution to the problem