Differential Equations

A Concise Review of Elementary Differential Equations

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1 Introduction

1.1 Direction Fields

Equations containing derivatives are differential equations. Differential Equations can be modeled by directional field (slope field). Directional fields are given in the form

$$\frac{dy}{dt} = f(t, y)$$

¡Insert image here¿ Directional fields can be used to find the equilibrium solution.

1.2 Solutions of Some Differential Equations

Consider the following differential equation

Example 1.

$$\frac{dp}{dt} = \frac{p - 900}{2} \tag{1}$$

$$\frac{dp/dt}{p - 900} = \frac{1}{2} \tag{2}$$

$$\frac{d}{dt} \ln |p - 900| = \frac{1}{2} \tag{3}$$

Integrate both sides

$$\ln |p - 900| = t/2 + C \tag{4}$$

$$|p - 900| = e^C e^{t/2} \tag{5}$$

$$p - 900 = \pm e^C e^{t/2} \tag{6}$$

Note how e^C is simply a constant c and the $\pm$ can be replaced by the c

$$p = 900 + ce^{t/2} \tag{7}$$

The solution to this differential equation is given as such where c is any non zero constant

The general solution can be found using a similar method

Definition 1.1. General Solution

given a general differential equation

$$\frac{dy}{dt} = ay - b \tag{8}$$

with initial condition

$$y(0) = y_0 \tag{9}$$

$$\frac{dy}{dt} = (y - b/a)a \tag{10}$$

$$\frac{dy/dt}{y - b} = a \tag{11}$$

Integrating both sides

$$ln|y - (b/a)| = at + C \tag{12}$$
$$y = b/a + ce^{at} \tag{13}$$

Given the initial condition C can be replaced, giving

$$y = (b/a) + [y_0 - (b/a)]e^{at} \tag{14}$$