DECA Chapter 7 Test

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1. Find the general solution of the system

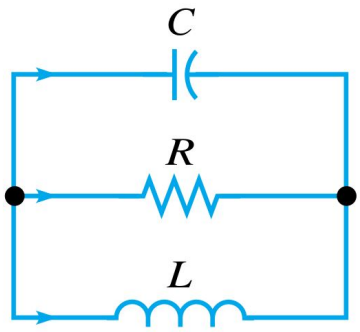
$$\mathbf{x}' = \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix} \mathbf{x}$$

2. Consider the following system for the following question

$$\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix} \mathbf{x}$$

1. Find the general solution of the system
2. Find the fundamental matrix $\Psi(t)$
3. Find the fundamental matrix Φ such that $\Phi(0) = \mathbf{I}$
4. Find the transformation matrix $\mathbf{T}$ and its inverse $\mathbf{T}^{-1}$
5. Using the transformation matrix above find the diagonal matrix $\mathbf{D}$
6. Using the diagonal matrix reduce the original system to the form $\mathbf{y}' = \mathbf{D}\mathbf{y}$
7. Using the exponential diagonal and the transformation matrix find the fundamental matrix Ψ

3. Consider the following circuit where L is inductance, C is capacitance and R is resistance



Let I_1, I_2, I_3 be the currents through the capacitor, resistor, and inductor, respectively. Likewise, let $V_1, V_2, and V_3$ be the corresponding voltage drops. The arrows denote an arbitrary positive direction in which current and voltage will be positive.

1. Applying Kirchoffs second law to the upper loop in the circuit show that

$$V_1 - V_2 = 0$$

2. In a similar way show that

$$V_2 - V_3 = 0$$

3. Applying Kirchhoffs first law to either node in the circuit show that

$$I_1 + I_2 + I_3 = 0$$

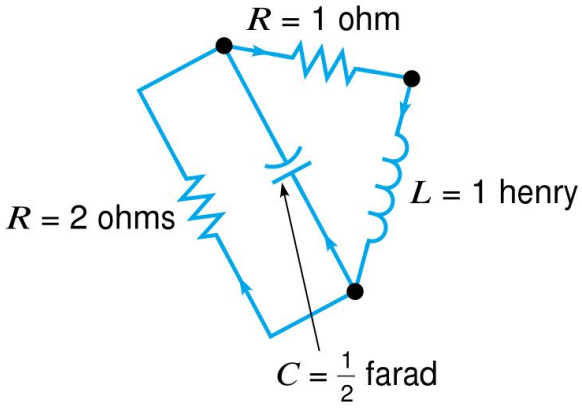
4. Use the current voltage relation to obtain the following

$$CV_1' = I_1 \qquad V_2 = RI_2 \qquad LI_3' = V_3$$

5. Eliminate V_2, V_3, I_1, I_2 to obtain

$$CV_1' = -I_3 - \frac{V_1}{R} \qquad LI_3' = V_1$$

4. Consider the following circuit



I will request that you make an honest attempt to do the first step without looking at the second step (cover it up now!!)

1. Express this circuit in terms of a differential system of the form

$$\mathbf{x}' = \mathbf{A}\mathbf{x}$$

2. The answer to the previous question, and the system is given by

$$\frac{d}{dt}\begin{pmatrix} I \\ V \end{pmatrix} = \begin{pmatrix} -1 & -1 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} I \\ V \end{pmatrix}$$

Suppose that at time $t = 0$ the current is 2 amperes and the voltage drop is 2 volts. Find $I(t)$ and $V(t)$ at any time

5. Return back to the circuit we discussed in Question (3)

1. express that circuit in terms of a differential system, similar to the one in Question (4)
2. show that the eigenvalues are real and equal if $L = 4R^2C$
3. Suppose that $R = 1\Omega, C = 1F, L = 4H$ Suppose also that $I(0) = 1A$ and $V(0) = 2V$ find $I(t)$ and $V(t)$