Math 110 Homework 1

Tarang Srivastava

1 Chapter 1.A

a. Problem 11

Explain why there does not exits $\lambda \in \mathbb{C}$ such that

$$\lambda(2-3i, 5+4i, -6+7i) = (12-5i, 7+22i, -32-9i)$$

By the definition of multiplication of scalars and lists we know that λ must satisfy all the following equations.

$$\lambda(2-3i) = 12-5i$$
$$\lambda(5+4i) = 7+22i$$
$$\lambda(-6+7i) = -32-9i$$

Solving for the first two equations gives us that

$$\lambda = 3+2i$$

But for the last equation we find that this value for λ does not work. Therefore no such λ exists that satisfies all three equations and therefore the larger equation.

2 Chapter 1.B

a. Problem 1

Prove that $-(-v) = v$ for every $v \in V$

By the existence of an additive inverse for all v we have

$$-(-v) + (-v) = 0$$

adding v to both sides

$$-(-v) + (-v) + v = 0 + v$$

by associativity

$$-(-v) + ((-v) + v) = v$$
$$-(-v) + 0 = v$$

$-(-v) = v$ as desired.

b. Problem 6

page 17 Axler

No, $\mathbb{R} \cup \{\infty\} \cup \{-\infty\}$ is not a vector space, because it does not follow all the properties of a vector space. Specifically, consider associativity. It must be true that,

$$((-\infty) + (-\infty)) + \infty = (-\infty) + ((-\infty) + \infty)$$
$$0 \neq (-\infty)$$

Therefore associativity does not hold and it is not a vector space.

3 Chapter 1.C

a. Problem 1(c)

Determine whether it is a subspace

No. Consider the following counter example. Let $a = (1,1,0)$. This is in the subspace since $1 \cdot 1 \cdot 0 = 0$ and let $b = (1,0,1)$. This is in the subspace since $1 \cdot 0 \cdot 1 = 0$ For the subspace to be closed under addition $a+b$ must be in the subspace, but $a+b = (1,1,1)$ which is not in the subspace since $1 \cdot 1 \cdot 1 = 1$.

b. Problem 1(d)

Determine whether it is a subspace

Yes. It is closed under vector addition and scalar multiplication. The additive identitiy can be shown from these two as noted in lecture.

Closure under addition:

Let $x = (x_1, x_2, x_3)$ and $y = (y_1, y_2, y_3)$ such that they are in the subspace. For $x+y$ to be in the subspace, we need to show that $x_1 + y_1 = 5(x_3 + y_3)$. Which follows from the fact that $x_1 + y_1 = 5x_3 + 5y_3 = 5(x_3 + y_3)$ by factoring out the 5. Also note that $x_1 + y_1 \in \mathbb{F}$ so that part is definitely in the subspace.

Closure under multiplication:

Let $x = (x_1, x_2, x_3)$ and $c \in \mathbb{F}$ it holds that $cx_1 = 5cx_3$ which is then just $x_1 = 5x_3$ therefore it is closed under scalar multiplication. Also, note that $cx_1 \in \mathbb{F}$ so that condition is met as well for all the elements.

c. Problem 3

Yes. The following is a subspace.

Clousure under addition:

Let $f, g \in \mathbb{R}^{(4,4)}$ such that they are differentiable and $f'(-1) = 3f(2)$ and $g(-1) = 3g(2)$. Firstly, the sum of differentiable functions is differentiable. Adding the two we get

$$f'(-1) + g'(-1) = 3f(2) + 3g(2)$$

We can factor out the 3 and then use the definition for function addition

$$(f' + g')(-1) = 3((f+g)(2))$$

which then by the definition of scalar multiplication is

$$(f' + g')(-1) = (3(f+g))(2)$$

as desired.

Closure under multiplication:

A scalar multiple of a differentiable function is still differentiable. Given

$$\lambda f'(-1) = 3\lambda f(2)$$

using the definition of scalar multiplication we have

$$(\lambda f')(-1) = (\lambda 3 f)(2)$$

as desired.

d. Problem 5

No. The complex vector space is a vector space over $\mathbb{C}$ so it is possible to multiply for example with a scalar i, in which case $\mathbb{R}^2$ is not closed under multiplication.

e. Problem 7

Let $U = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$. That is U the unit circle. We can always find an additive inverse, namely, $(-x,-y)$ for some $x, y \in U$. It is not a subspace because it does not have the additive identitiy.

f. Problem 8

From question 1 part c. Let $U = \{(x, y) \in \mathbb{R}^2 : xy = 0\}$ Clearly, and $c \in \mathbb{R}$ times the $u \in U$ will still be in the subspace. Since, if $xy = 0 \implies cxy = 0$. It is not closed under addtion as shown in Problem 1(c).