Math 110 Homework 11

Tarang Srivastava

April 14, 2020

1 Exercises 7.B

Problem 3: Define the linear map T as follows for each $z = (z_1, z_2, z_3) \in \mathbb{C}^3$.

$$Tz = (2z_1, 3z_2, 2z_1)$$

Note the switch up in the last spot. We can check that T is indeed closed under addition and scalar multiplication, thus a linear map. Firstly, $\lambda = 2$ is an eigenvalue with the eigenvector $w = (1, 0, 1)$.

$$T(1, 0, 1) = (2, 0, 2) = 2(1, 0, 1)$$

Similarly, $\lambda = 3$ is an eigenvalue with the eigenvector $w = (0, 1, 0)$.

$$T(0, 1, 0) = (0, 3, 0) = 3(0, 1, 0)$$

Finally, $(T^2 - 5T + 6I)z \neq 0$ if $z_3 \neq 6z_1$. So given we have values that are nonzero after that linear map, $T^2 - 5T + 6I \neq 0$.

Problem 4: Given that $\mathbb{F} = \mathbb{C}$ by the complex spectral theorem T is normal if and only if T has a diagonal matrix with respect to some orthonormal basis. Since T is diagonalizable if and only if V is equal to the sum of the eigenspaces of distinct eigenvalues, we have shown the last case. Also by the complex spectral theorem T is normal if and only if V has an orthonormal basis consisting of eigenvectors of T. That completes the proof. (Ask Alex if there's some part that guarantees that we have distinct eigenvalues, but it is not that difficult to show that if all the basis eigenvectors are orthogonal they must be associated with distinct eigenvalues.)

Problem 6: