Math 110 Homework 1

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Chapter 1.A

Problem 11: *By the definition of multiplication of scalars and lists we know that λ must satisfy all the following equations.*

$$\lambda(2-3i) = 12-5i$$
$$\lambda(5+4i) = 7+22i$$
$$\lambda(-6+7i) = -32-9i$$

Solving for the first two equations gives us that

$$\lambda = 3+2i$$

But for the last equation we find that this value for λ does not work. Therefore no such λ exists that satisfies all three equations and therefore the larger equation.

Chapter 1.B

Problem 1: *By the existence of an additive inverse for all v we have*

$$-(-v)+(-v) = 0$$

adding v to both sides

$$-(-v)+(-v)+v = 0+v$$

by associativity

$$-(-v)+((-v)+v) = v$$
$$-(-v)+0 = v$$

$-(-v) = v$ *as desired.*

Problem 6: *No, $\mathbb{R}\cup\{\infty\}\cup\{-\infty\}$ is not a vector space, because it does not follow all the properties of a vector space. Specifically, consider associativity. It must be true that,*

$$((-\infty)+(-\infty))+\infty = (-\infty)+((-\infty)+\infty)$$
$$0 \neq (-\infty)$$

Therefore associativity does not hold and it is not a vector space.

Chapter 1.C

Problem 1(c): *No. Consider the following counter example. Let $a=(1,1,0)$. This is in the subspace since $1\cdot 1\cdot 0 = 0$ and let $b=(1,0,1)$. This is in the subspace since $1\cdot 0\cdot 1 = 0$ For the subspace to be closed under addition $a+b$ must be in the subspace, but $a+b=(1,1,1)$ which is not in the subspace since $1\cdot 1\cdot 1 = 1$.*

Problem 1(d): *Yes. It is closed under vector addition and scalar multiplication. The additive identitiy can be shown from these two as noted in lecture.*
Closure under addition:
Let $x=(x_1,x_2,x_3)$ and $y=(y_1,y_2,y_3)$ such that they are in the subspace. For $x+y$ to be in the subspace, we need to show that $x_1+y_1 = 5(x_3+y_3)$. Which follows from the fact that $x_1+y_1 = 5x_3+5y_3 = 5(x_3+y_3)$ by factoring out the 5. Also note that $x_1+y_1\in\mathbb{F}$ so that part is definitely in the subspace.
Closure under multiplication:
Let $x=(x_1,x_2,x_3)$ and $c\in\mathbb{F}$ it holds that $cx_1 = 5cx_3$ which is then just $x_1 = 5x_3$ therefore it is closed under scalar multiplication. Also, note that $cx_1\in\mathbb{F}$ so that condition is met as well for all the elements.

Problem 3: *Yes. The following is a subspace.*
Clousure under addition:
Let $f,g\in\mathbb{R}^{(4,4)}$ such that they are differentiable and $f'(-1) = 3f(2)$ and $g(-1) = 3g(2)$. Firstly, the sum of differentiable functions is differentiable. Adding the two we get

$$f'(-1)+g'(-1) = 3f(2)+3g(2)$$

We can factor out the 3 and then use the definition for function addition

$$(f'+g')(-1) = 3((f+g)(2))$$

which then by the definition of scalar multiplication is

$$(f'+g')(-1) = (3(f+g))(2)$$

as desired.
Closure under multiplication:
A scalar multiple of a differentiable function is still differentiable. Given

$$\lambda f'(-1) = 3\lambda f(2)$$

using the definition of scalar multiplication we have

$$(\lambda f')(-1) = (\lambda 3f)(2)$$

as desired.

Problem 5: *No. The complex vector space is a vector space over $\mathbb{C}$ so it is possible to multiply for example with a scalar i, in which case $\mathbb{R}^2$ is not closed under multiplication.*

Problem 7: *Let $U=\{(x,y)\in\mathbb{R}^2 : x^2+y^2=1\}$. That is U the unit circle. We can always find an additive inverse, namely, $(-x,-y)$ for some $x,y\in U$. It is not a subspace because it does not have the additive identitiy.*

Problem 8: *From question 1 part c. Let $U=\{(x,y)\in\mathbb{R}^2 : xy=0\}$ Clearly, and $c\in\mathbb{R}$ times the $u\in U$ will still be in the subspace. Since, if $xy=0 \implies cxy=0$. It is not closed under addtion as shown in Problem 1(c).*

2.A

Problem 3: $t=2$