Math 110 Homework 1

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1 Chapter 1.A

a. Problem 11

Explain why there does not exits $\lambda \in \mathbb{C}$ such that

$$\lambda(2-3i, 5+4i, -6+7i) = (12-5i, 7+22i, -32-9i)$$

By the definition of multiplication of scalars and lists we know that λ must satisfy all the following equations.

$$\lambda(2-3i) = 12-5i$$
$$\lambda(5+4i) = 7+22i$$
$$\lambda(-6+7i) = -32-9i$$

Solving for the first two equations gives us that

$$\lambda = 3+2i$$

But for the last equation we find that this value for λ does not work. Therefore no such λ exists that satisfies all three equations and therefore the larger equation.

2 Chapter 1.B

a. Problem 1

Prove that $-(-v) = v$ for every $v \in V$

By the existence of an additive inverse for all v we have

$$-(-v) + (-v) = 0$$

adding v to both sides

$$-(-v) + (-v) + v = 0 + v$$

by associativity

$$-(-v) + ((-v) + v) = v$$
$$-(-v) + 0 = v$$

$-(-v) = v$ as desired.

b. Problem 6

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No, $\mathbb{R} \cup \{\infty\} \cup \{-\infty\}$ is not a vector space, because it does not follow all the properties of a vector space. Specifically, consider associativity. It must be true that,

$$((-\infty) + (-\infty)) + \infty = (-\infty) + ((-\infty) + \infty)$$
$$0 \neq (-\infty)$$

Therefore associativity does not hold and it is not a vector space.

3 Chapter 1.C

a. Problem 1(c)

Determine whether it is a subspace

No. Consider the following counter example. Let $a = (1,1,0)$. This is in the subspace since $1 \cdot 1 \cdot 0 = 0$ and let $b = (1,0,1)$. This is in the subspace since $1 \cdot 0 \cdot 1 = 0$ For the subspace to be closed under addition $a+b$ must be in the subspace, but $a+b = (1,1,1)$ which is not in the subspace since $1 \cdot 1 \cdot 1 = 1$.

b. Problem 1(d)

Determine whether it is a subspace

Yes. It is closed under vector addition and scalar multiplication. The additive identitiy can be shown from these two as noted in lecture.

Closure under addition:

Let $x = (x_1, x_2, x_3)$ and $y = (y_1, y_2, y_3)$ such that they are in the subspace. For $x + y$ to be in the subspace, we need to show that $x_1 + y_1 = 5(x_3 + y_3)$. Which follows from the fact that $x_1 + y_1 = 5x_3 + 5y_3 = 5(x_3 + y_3)$ by factoring out the 5.

Closure under multiplication: