1   Exercises 5.A

Problem 1: The argument is as follows.

(a) Let u be an arbitrary vector $u \in U$. If $U \subset \text{null } T$, then $u \in \text{null } T$. So, $Tu = 0$. Since, U is a vector space, it must be that $0 \in U$, so $Tu \in U$. Thus, U is invariant under T given the condition.

(b) By definition we have $Tu \in \text{range } T$ for all $u \in U$. Since, $\text{range } T \subset U$ we have that for all u, $Tu \in U$. Thus, U is invariant under T given the condition.