1 Exercises 5.B

Problem 1: We wish to show that

$$(I - T)^{-1} = I + T + ... + T^{n-1}$$

So, we can multiply $I - T$ to both sides to get

$$I = (I - T)(I + T + ... + T^{n-1})$$

Then, we can distribute and see we get

$$I = I - T + T - T^2 + T^2 + ... - T^{n-1} + T^{n-1} + T^n$$

After cancelling out all the similar terms we are left with

$$I = I + T^n = I$$

Since, $T^n = 0$. So, to prove the statement we do the following operations

$$I = I + T^n = I$$

$$I = I - T + T - T^2 + T^2 + ... - T^{n-1} + T^{n-1} + T^n$$

$$I = (I - T)(I + T + ... + T^{n-1})$$

Then, we multiply both sides by $(I - T)^{-1}$ to get

$$(I - T)^{-1} = I + T + ... + T^{n-1}$$

as desired.