

1 Exercises 5.A

Problem 1: The argument is as follows.

- (a) Let u be an arbitrary vector $u \in U$. If $U \subset \text{null } T$, then $u \in \text{null } T$. So, $Tu = 0$. Since, U is a vector space, it must be that $0 \in U$, so $Tu \in U$. Thus, U is invariant under T given the condition.
- (b) By definition we have $Tu \in \text{range } T$ for all $u \in U$. Since, $\text{range } T \subset U$ we have that for all u , $Tu \in U$. Thus, U is invariant under T given the condition.

Problem 3: We wish to show that for all $u \in \text{range } S$ we have that $Tu \in \text{range } S$. Let $v \in V$, then $STv \in \text{range } S$ by definition. Given $ST = TS$, we have that $STv = TSv$. So, $TSv \in \text{range } S$. Let $u \in \text{range } S$, then there exists some $v \in V$ such that $Sv = u$. Since, $TSv \in \text{range } S$, we have $Tu \in \text{range } S$.