Section 1.A

Complex Numbers: A complex number is an ordered pair (a, b), where $a, b \in \mathbf{R}$, but we will write this as $a + bi$
The set of all complex numbers is denoted by $\mathbf{C}$:

$$\mathbf{C} = \{a + bi : a, b \in \mathbf{R}\}$$

Addition and multiplication on $\mathbf{C}$ are defined by

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

$$(a + bi)(c + di) = (ac - bd) + (ad + bc)i$$

here $a, b, c, d \in \mathbf{R}$

Properties of complex arithmetic:
commutativity

$$\alpha + \beta = \beta + \alpha \text{ and } \alpha\beta = \beta\alpha \text{ for all } \alpha, \beta \in \mathbf{C}$$

associativity

$$(\alpha + \beta) + \lambda = \alpha + (\beta + \lambda) \text{ and } (\alpha\beta)\lambda = \alpha(\beta\lambda) \text{ for all } \alpha, \beta, \lambda \in \mathbf{C}$$

identities

$$\lambda + 0 = \lambda \text{ and } \lambda 1 = \lambda \text{ for all } \lambda \in \mathbf{C}$$

additive inverse for every $\alpha \in \mathbf{C}$, there exists a unique $\beta \in \mathbf{C}$ such that $\alpha + \beta = 0$
multiplicative inverse for every $\alpha \in \mathbf{C}$ with $\alpha \neq 0$, there exists a unique $\beta \in \mathbf{C}$ such that $\alpha\beta = 1$
distributive property

$$\lambda(\alpha + \beta) = \lambda\alpha + \lambda\beta \text{ for all } \lambda, \alpha, \beta \in \mathbf{C}$$

$-\alpha$, subtraction, $1/\alpha$, division: Let $\alpha, \beta \in \mathbf{C}$ · Let $-\alpha$ denote the additive inverse of α. Thus $-\alpha$ is the unique complex number such that

$$\alpha + (-\alpha) = 0$$

· Subtraction on C is defined by

$$\beta - \alpha = \beta + (-\alpha)$$

· For $\alpha \neq 0$, let $1/\alpha$ denote the multiplicative inverse of α. Thus $1/\alpha$ is the unique complex number such that

$$\alpha(1/\alpha) = 1$$

· Division on $\mathbf{C}$ is defined by

$$\beta/\alpha = \beta(1/\alpha)$$

list, length: Suppose n is a nonnegative integer. A list of length n is an ordered collection of n elements (which might be numbers, other lists, or more abstract entities) separated by commas and surrounded by parentheses. A list of length n looks like this:

$$(x_1, \ldots, x_n)$$

Two lists are equal if and only if they have the same length and the same elements in the same order.
$\mathbb{F}^n$: $\mathbf{F}^n$ is the set of all lists of length n of elements of $\mathbf{F}$:

$$\mathbf{F}^n = \left\{(x_1, \ldots, x_n) : x_j \in \mathbf{F} \text{ for } j = 1, \ldots, n\right\}$$

For $(x_1, \ldots, x_n) \in \mathbf{F}^n$ and $j \in \{1, \ldots, n\}$, we say that x_j is the j^{th} coordinate of $(x_1, \ldots, x_n)$
addition in $\mathbb{F}^n$: Addition in $\mathbf{F}^n$ is defined by adding corresponding coordinates:

$$(x_1, \ldots, x_n) + (y_1, \ldots, y_n) = (x_1 + y_1, \ldots, x_n + y_n)$$

Commutativity of addition in $\mathbb{F}^n$: If $x, y \in \mathbf{F}^n$, then $x + y = y + x$
0: Let 0 denote the list of length n whose coordinates are all 0 :

$$0 = (0, \ldots, 0)$$

additive inverse in $\mathbb{F}^n$: For $x \in \mathbf{F}^n$, the additive inverse of x, denoted $-x$, is the vector $-x \in \mathbf{F}^n$ such that $x + (-x) = 0$ In other words, if $x = (x_1, \ldots, x_n)$, then $-x = (-x_1, \ldots, -x_n)$
scalar multiplication in $\mathbb{F}^n$: The product of a number λ and a vector in $\mathbf{F}^n$ is computed by multiplying each coordinate of the vector by λ :

$$\lambda\,(x_1, \ldots, x_n) = (\lambda x_1, \ldots, \lambda x_n)$$

here $\lambda \in \mathbf{F}$ and $(x_1, \ldots, x_n) \in \mathbf{F}^n$