Math 110 Homework 3

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1 Exercise 2.A

Problem 8: The statement holds. Assume for contradiction that $\lambda v_1, .., \lambda v_m$ is linearly dependent. That is, for $\lambda \neq 0$, $\lambda v_1 + ... + \lambda v_m = 0$. Then we have a contradiction, since we claimed that $v_1, ..., v_m$ are linearly independent, and we have an example of a linear combination with non-zero coefficients that is equal to zero. Then, $\lambda v_1, .., \lambda v_m$ must be linearly independent.

Problem 9: The statement is false. Consider the counterexample where $w_i = -v_i$ for $i \in \{1, ..., m\}$. We know that $w_i \in V$, since the additive inverse must exist in V for v_i. Then, for non-zero coefficients the linear combination $(v_1 + w_1) + ... + (v_m + w_m) = 0$ which shows that it is linearly dependent.

Problem 10: To show that $w \in \text{span}(v_1, ..., v_m)$ we just have to show there exists a linear combination such that

$$w = a_1 v_1 + ... + a_m v_m$$

By linear dependence we have $a_1(v_1 + w) + ... + a_1(v_m + w) = 0$ for nonzero coefficients. We can collect that w terms and bring it to the other side of the equal sign and divide by the coefficients, again because they are not all zero, to get

$$a_1 v_1 + ... + a_m v_m = (a_1 + ... + a_m) w$$

$$\frac{a_1}{(a_1 + ... + a_m)} v_1 + ... + \frac{a_m}{(a_1 + ... + a_m)} v_m = w$$

Thus, showing that w is in the span of $v_1, ..., v_m$.

Problem 11: