Math 110 Homework 3

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1 Exercises 2.C

Problem 1: Since, U is a subspace there exists a basis B such that $|B| = \dim U = \dim V$. We have that B is a linearly independent list in V and of the right length therefore it must be a basis of V. So, $U = \operatorname{span}(B) = V$.

Problem 7:

Problem 8:

Problem 9:

Problem 10: We begin by showing that $p_0, ..., p_m$ is a linearly independent list. Observe, that any linear combination of $p_0, ..., p_m$ is a polynomial of degree m. So, the only polynomial of degree m that is the 0 polynomial is the one with all zero coefficients. Thus, $p_0, ..., p_m$ is a linearly independent list. Since, it is of the right length $m + 1$, and for all $0 \leq i \leq m$, p_i is in $\mathcal{P}_m(\mathbb{F})$. Then, it must be a basis for $\mathcal{P}_m(\mathbb{F})$.

: Assume for contradiction that $U \cap W = \{0\}$. Then, $U + W$ is a direct sum, and $U, W \subset V$. So, it must be that $U \oplus W = V$. Thereofre, $\dim V = 9 = \dim W + \dim U - \dim W \cap U = 5 + 5 - 0$. Here we have a contradiction, thus, $U \cap W \neq \{0\}$.