**Problem 7:** Since, $p$ is in the standard basis of $\mathcal{P}_m(\mathbb{R})$ it has only one term to consider. Namely, a term $x^j$ where $j \in \{1, ..., m\}$. Then, if $p$ has a degree strictly greater than $j$ it will have some $x$ term for $p^{(j)}$. Thus, evaluated at 0, it will equal 0. If $p$ has a degree strictly less than $j$ then $p^{(j)} = 0$ by the power rule form calculus. So, when $p$ has degree exactly $j$ we have that it will evaluate to some constant by the power rule. This constant will be exactly $j!$, again by the power rule. So, we normalize it by dividing it by $j!$, so $\varphi(j) = 1$. Then, we have a valid $\varphi_j$ such that it only evaluates to 1 for one of the polynomials in the standard basis and 0 for all other. Thus, a valid dual basis.

**Problem 9:** Since, $\psi \in V'$ it is equivalent to the following linear combination with scalars $a_1, .., a_n$. That is,

$$\psi = a_1 \varphi_1 + ... + a_n \varphi_n$$

Then for some $v_j$ in the basis of $V$ w ehave that

$$\psi(v_j) = a_j \varphi_j(v_j) = a_j$$

Since, all the other ones are 0, since it is evaluated at a basis vector Then, we simply make direct substitutions for each $j \in \{1, ..., n\}$ and get the desired result.

**Problem 11:** This is pretty simple. For the $\Longleftarrow$ direction. All $d_j$ are some arbitrary costants in $\mathbb{F}$. So, we construct $A_{j,k}$ such that each column is just a scalar multiple of each other. Since, we are multiplying the same $(c_1, ..., c_m$ by arbitrary constants $d_j$. Then, clearly if all the vectors are scalar multiples of each other we must remove all of them except 1 to get a linearly independent list. Therefore, the dimension of the row space is 1 and thus rank is 1. For the $\Longrightarrow$ direction we argue that since $A$ has rank 1, then the column space has dimesnion 1. So, it follows that the columns are constructed as follows for some vectors $d_1, ..., d_n$. That is they are scalar multiples of each other.

**Problem 19:** $\Longrightarrow$ . If $U = V$, then the $\varphi$ such that $\varphi(u) = 0 = \varphi(v)$ for all $u \in U$ and all $v \in V$ is just 0, since 0 is unique in a vector space.
$\Longleftarrow$ . We know $V$ is finite-dimensional, so

$$\dim U + \dim U^0 = \dim V$$

But, the annhilator has dimension 0 so $\dim U = \dim V$ since, $U$ is a subspace of $V$ it implies that

$$U = V$$