Section 1.A

Complex Numbers: A complex number is an ordered pair (a, b), where $a, b \in \mathbf{R}$, but we will write this as $a + bi$
The set of all complex numbers is denoted by $\mathbf{C}$:

$$\mathbf{C} = \{a + bi : a, b \in \mathbf{R}\}$$

Addition and multiplication on $\mathbf{C}$ are defined by

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

$$(a + bi)(c + di) = (ac - bd) + (ad + bc)i$$

here $a, b, c, d \in \mathbf{R}$

Properties of complex arithmetic:
commutativity

$$\alpha + \beta = \beta + \alpha \text{ and } \alpha\beta = \beta\alpha \text{ for all } \alpha, \beta \in \mathbf{C}$$

associativity

$$(\alpha + \beta) + \lambda = \alpha + (\beta + \lambda) \text{ and } (\alpha\beta)\lambda = \alpha(\beta\lambda) \text{ for all } \alpha, \beta, \lambda \in \mathbf{C}$$

identities

$$\lambda + 0 = \lambda \text{ and } \lambda 1 = \lambda \text{ for all } \lambda \in \mathbf{C}$$

additive inverse for every $\alpha \in \mathbf{C}$, there exists a unique $\beta \in \mathbf{C}$ such that $\alpha + \beta = 0$
multiplicative inverse for every $\alpha \in \mathbf{C}$ with $\alpha \neq 0$, there exists a unique $\beta \in \mathbf{C}$ such that $\alpha\beta = 1$
distributive property

$$\lambda(\alpha + \beta) = \lambda\alpha + \lambda\beta \text{ for all } \lambda, \alpha, \beta \in \mathbf{C}$$

$-\alpha$, subtraction, $1/\alpha$, division: Let $\alpha, \beta \in \mathbf{C}$ · Let $-\alpha$ denote the additive inverse of α. Thus $-\alpha$ is the unique complex number such that

$$\alpha + (-\alpha) = 0$$

· Subtraction on $\mathbf{C}$ is defined by

$$\beta - \alpha = \beta + (-\alpha)$$

· For $\alpha \neq 0$, let $1/\alpha$ denote the multiplicative inverse of α. Thus $1/\alpha$ is the unique complex number such that

$$\alpha(1/\alpha) = 1$$

· Division on $\mathbf{C}$ is defined by

$$\beta/\alpha = \beta(1/\alpha)$$

list, length: Suppose n is a nonnegative integer. A list of length n is an ordered collection of n elements (which might be numbers, other lists, or more abstract entities) separated by commas and surrounded by parentheses. A list of length n looks like this:

$$(x_1, \ldots, x_n)$$

Two lists are equal if and only if they have the same length and the same elements in the same order.
$\mathbb{F}^n$: $\mathbf{F}^n$ is the set of all lists of length n of elements of $\mathbf{F}$:

$$\mathbf{F}^n = \left\{(x_1, \ldots, x_n) : x_j \in \mathbf{F} \text{ for } j = 1, \ldots, n\right\}$$

For $(x_1, \ldots, x_n) \in \mathbf{F}^n$ and $j \in \{1, \ldots, n\}$, we say that x_j is the j^{th} coordinate of $(x_1, \ldots, x_n)$
addition in $\mathbb{F}^n$: Addition in $\mathbf{F}^n$ is defined by adding corresponding coordinates:

$$(x_1, \ldots, x_n) + (y_1, \ldots, y_n) = (x_1 + y_1, \ldots, x_n + y_n)$$

Commutativity of addition in $\mathbb{F}^n$: If $x, y \in \mathbf{F}^n$, then $x + y = y + x$
0: Let 0 denote the list of length n whose coordinates are all 0 :

$$0 = (0, \ldots, 0)$$

additive inverse in $\mathbb{F}^n$: For $x \in \mathbf{F}^n$, the additive inverse of x, denoted $-x$, is the vector $-x \in \mathbf{F}^n$ such that $x + (-x) = 0$ In other words, if $x = (x_1, \ldots, x_n)$, then $-x = (-x_1, \ldots, -x_n)$

scalar multiplication in $\mathbb{F}^n$: The product of a number λ and a vector in $\mathbf{F}^n$ is computed by multiplying each coordinate of the vector by λ :

$$\lambda(x_1, \ldots, x_n) = (\lambda x_1, \ldots, \lambda x_n)$$

here $\lambda \in \mathbf{F}$ and $(x_1, \ldots, x_n) \in \mathbf{F}^n$

Section 1.B

addition, scalar multiplication: · An addition on a set V is a function that assigns an element $u + v \in V$ to each pair of elements $u, v \in V$ · A scalar multiplication on a set V is a function that assigns an element $\lambda v \in V$ to each $\lambda \in \mathbf{F}$ and each $v \in V$
Vector Space: A vector space is a set V along with an addition on V and a scalar multiplication on V such that the following properties hold: commutativity

$$u + v = v + u \text{ for all } u, v \in V$$

associativity $(u + v) + w = u + (v + w)$ and $(ab)v = a(bv)$ for all $u, v, w \in V$
and all $a, b \in \mathbf{F}$

additive identity there exists an element $0 \in V$ such that $v + 0 = v$ for all $v \in V$ additive inverse for every $v \in V$, there exists $w \in V$ such that $v + w = 0$ multiplicative identity $1v = v$ for all $v \in V$ distributive properties

$$a(u+v) = au+av \text{ and } (a+b)v = av+bv \text{ for all } a, b \in \mathbf{F} \text{ and}$$

all $u, v \in V$
vector, point: Elements of a vector space are called vectors or points.
real vector space, complex vector space: • A vector space over $\mathbf{R}$ is called a real vector space. · A vector space over $\mathbf{C}$ is called a complex vector space.
$\mathbb{F}^S$: · If S is a set, then $\mathbf{F}^S$ denotes the set of functions from S to $\mathbf{F}$ · For $f, g \in \mathbf{F}^S$, the sum $f + g \in \mathbf{F}^S$ is the function defined by

$$(f + g)(x) = f(x) + g(x)$$

for all $x \in S$ • For $\lambda \in \mathbf{F}$ and $f \in \mathbf{F}^S$, the product $\lambda f \in \mathbf{F}^S$ is the function defined by

$$(\lambda f)(x) = \lambda f(x)$$

for all $x \in S$
Unique Additive Identity: A vector space has a unique additive identity
Unique additive inverse: Every element in a vector space has a unique additive inverse.
The number 0 times a vector: $0v = 0$ for every $v \in V$
A number times the vector 0: $a0 = 0$ for every $a \in \mathbf{F}$
The number -1 times a vector: $(-1)v = -v$ for every $v \in V$