Modern Physics

A Collection of Notes and Problems

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Contents

Chapter 1

The Basics of Relativity

This chapter deals with unaccelerated motion, special relativity. General relativity will be revisited later.

1.1 Background

Example 1.1.1. Imagine a coordinate system S' fixed to a train moving with speed v on a track while a second coordinate system S is fixed to the track. We assume that the track is at rest relative to the "ether". Suppose that at time $t = 0$ lightning strikes at points $x = -L$ and $+L$ on the track. At this instant the train has its center, $x' = 0$, at the origin of the S-system , $x = 0$. Then an observer on the ground at $x = 0$ will see the lightning flashes arrive from either side at a time $t = L/c$; that is they will arrive simultaneously. Will this be true for an observer stationed at the center of the train? If not, what is the time interval between the arrival of the flashes?
If the train was not moving, then, yes, the lightning would appear to have struck simultaneously. Since, the train is moving with speed v, lets say in the positive direction. Then, the lightning at $+L$ would have appeared to have happened before.
Going back to the question, the calculation for $t = L/c$ comes from simply using velocity and distance. If we consider the lightning at $-L$ the total distance it has to travel is $L+vt$. Where vt comes from the train moving away from $-L$. Since, $\Delta x = \bar{v}t$, we can express

$$L + vt = ct$$

solving for t

$$t = L/(c - v)$$

. We can similarly find the time for the right lightning. Since, the distance for the right one is $L - vt$

$$t = L/(c + v)$$

The time difference is then

$$\begin{aligned}\Delta t &= L/(c - v) - L/(c + v) \\ &= \frac{2L}{c(1 - v^2/c^2)}\end{aligned}$$

The purpose of this example is to consider the assumption we made: and observer in a particular frame can measure the time that and event happens in his or her frame.

1.2 Einstein's Special Theory of Relativity

Einstein's Postulates

1. The principal of relativity. The test of a physical law by any experiment carried out in a uniformly moving frame of reference does not depend of the speed of that frame relative to any other frame moving uniformly with respect to it.

2. The priciple of constancy. There exists a frame of reference S (call it the rest frame) with respect to which the speed of light is c. The speed is then also c in every other frame of reference moving uniformly with respect to S. The implies that the speed of light is independent of the motion of the source.

A length *in its own rest frame* is called the **proper length**. The period of a clock in frame in which it is resting is known as **proper time**.

Time Dilation

$$t = t' \frac{1}{\sqrt{1 - v^2/c^2}}$$

Example 1.2.1. A beam of muons have a lifetime of $2.2 \times 10^{-6}s$ are measured to move with a speed of $0.99c$. (a) Without time dilation how far would they travel before they decay? With time dilation?

1. $x = vt = 0.99ct = 0.99 \times 3 \times 10^8 \times 2.2 \times 10^{-6} = 650m$

2. using
$$t = t' \frac{1}{\sqrt{1 - v^2/c^2}}$$
we multiply $650 \times 1/\sqrt{1 - .99^2} = 4620$

1.3 Lorentz Transformations

Lorentz transformations take the four coordinates (x, y, z, t) that describe an "event," something that occurs at a definite point in space and at a definite time.
The factor $1/\sqrt{q - v^2/c^2}$ is used so often we will define

$$\gamma \equiv \frac{1}{\sqrt{1 - v^2/c^2}}$$

We can express the **spatial** Lorentz transformation as
$$x' = \gamma(x - vt)$$

The Lorentz transformation for **time** is

$$t' = \gamma \left[t - \frac{v}{c^2} x \right]$$

Example 1.3.1. Consider a spaceship of proper length 100m that moves along the positive $x-$axis at $0.9c$ with respect to the ground. If S is a coordinate frame fixed to the ground and S' is a coordinate frame fixed to the ship, the origins are set so that at $t = t' = 0$ the front of the ship is at $x = x' = 0$. Where is the back of the ship at $t = 0$, according to an observer on the ground, and at $t = 0$ what time T' will a clock fixed to the back of the ship read, according to a ground-based observer? Do the problem (a) classically and (b) according to special relativity.

Classically, the answer is quite obvious. When the tip of the ship crosses $x = 0$ and time, $t = 0$ then the time at the back will also be $T' = 0$.

According to special relativity, these events do not happen at the same time. We can look at the distinct events that occur, and see we need to use both the time and spatial Lorentz transformation.

$$x' = \gamma(x - v(0)) = \gamma x$$
$$x = x'/\gamma$$

Now to use the time transformation

$$t' = \gamma\left[0 - \frac{v}{c^2}x\right] = \gamma\left[0 - \frac{v}{c^2}\frac{x'}{\gamma}\right]$$
$$t' = \frac{0.9}{c}x'$$
$$t' = \frac{0.9}{3\times 10^8}100$$
$$T' = 3\times 10^{-7}\text{s}$$

1.4 Simultaneity and Relativity

Simply put, simultaneity refers to the idea that two spatially separated events occur at the same time - is not absolute, but depends on the observer's reference frame.

Example 1.4.1. Consider two observers one on the train the other on the platform.

A flash of light is given off at the center of the traincar just as the two observers pass each other. For the observer on board the train, the front and back of the traincar are at fixed distances from the light source and as such, according to this observer, the light will reach the front and back of the traincar at the same time.

For the observer standing on the platform, on the other hand, the rear of the traincar is moving (catching up) toward the point at which the flash was given off, and the front of the traincar is moving away from it. As the speed of light is finite and the same in all directions for all observers, the light headed for the back of the train will have less distance to cover than the light headed for the front. Thus, the flashes of light will strike the ends of the traincar at different times.

This is unexpected because in classical physics we would have assumed that light hits either side of the traincar irrespective of the frame, but such is not the case because of the strange behavior of light: light speed is constant in all frames.

1.5 The Relativistic Doppler Shift

When dealing with *length contraction* and *time dilation* it is reasonable to consider the affect these have on light, specifically light's

wavelength and frequency.
Time dilation takes the equation

$$t = \gamma t'$$

and in classical physics the angular dependent Doppler shift for the frequence f takes the form

$$f' = f(1 - \cos(\theta)v/c)$$

thus in special relativity it becomes

$$f' = f\gamma\left[1 - \cos(\theta)v/c\right]$$

Note that $\theta = 0$ is when the source and detector are moving apart, and $\theta = \pi$ is when they are moving towards each other.

Example 1.5.1. At what speed would a motorist in a very fast car have to go so that he or she would see a red traffic light as green? We assume that the light looks red when the motorist is at rest. Take a wavelength of 650 nm for red light and 530nm for green.

We can use

$$f' = f\gamma\left[1 - \cos(\theta)v/c\right]$$

and since the motorist is moving towards the traffic light $\theta = \pi$ thus

$$\frac{f}{f'} = \frac{1 + v/c}{\sqrt{1 - v^2/c^2}} = \sqrt{\frac{1 + v/c}{1 - v/c}}$$

we can use $v = f\lambda$ to square both sides and find

$$\frac{v}{c} = \frac{(\lambda/\lambda')^2 - 1}{(\lambda/\lambda')^2 + 1} = \frac{(650/530)^2 - 1}{(650/530)^2 + 1} = 0.20$$

1.6 Relativistic Velocity Addition

The usual Galilean addition of velocity follows (note the primes are not derivatives, but velocity in the S' frame.)

$$u' = \frac{dx'}{dt'} = \frac{dx'}{dt}\frac{dt}{dt'} = u - v$$

but it is expected that this will change in special relativity.
if we use the Lorentz transformations:

- $x' = \lambda(v)(x - vt)$
- $t' = \lambda(v)[t - (v/c^2)x]$

First,

$$\frac{dx'}{dt} = \lambda(v)(u - v)$$

remember $u = dx/dt$
and

$$\frac{dt'}{dt} = \lambda(v)[1 - (v/c^2)u]$$

so

$$u' = \frac{u - v}{1 - (v/c^2)u}$$

where $u' = dx'/dt'$ is the speed in the S' frame.

Chapter 2

Consequences of Relativity

2.1 Time Dilation and the Decay of Unstable Particles

Unstable particle decay after a certain time, but by observing how long they take to decay when traveling at relativistic speeds we can observe the effects of time dilation.
Statistically according to the exponential-decay law

$$N(t) = N(0)e^{-t/\tau}$$

where $N(t)$ is the number of particles present at time t, given that $N(0)$ is the number present at the initial time $t = 0$.
To an observer moving at relativistic velocity v in the $x-$direction with respect to S, time will be

$$t' = \gamma(v)t$$

. If we consider the fraction

$$N(t)/N(0) = e^{-t/\tau}$$

both t and τ will be dilated, in other words

$$\tau' = \gamma\tau$$

so the fraction of the particles remaining is the same as viewed by both the rest and moving observer.
We can see the effects of time dilation on the **muon** particle, μ.

Example 2.1.1. A beam of muons is injected in a storage ring, a device that uses electromagnetic fields to maintain the muons in uniform circular motion. The ring's radius is 60m, and the muons are injected with a velocity such that $\gamma = 15$. How many revolutions of the ring will an "average" muon make before it decays? The proper lifetime of a muon is 2.2×10^{-6}s.
Solution. We can find the speed of the muons using γ.

$$\frac{v}{c} = \sqrt{\frac{\gamma^2 - 1}{\gamma^2}} = \sqrt{\frac{15^2 - 1}{15^2}} = 0.998$$

and the distance it travels in its lifetime is

$$x = yv\tau = 15 \times (0.998)\left(3 \times 10^8 m/s\right)$$
$$\left(2.2 \times 10^{-6} s\right) = 9.88 \times 10^3\text{m}$$

2.2 Relativistic Doppler Shift

We discussed this earlier with light, frequency and wavelength.
The frequency in the source's rest frame is called **proper frequency**. We know these are connected by

$$\lambda = cT$$

Now consider the situation where the observer is moving with velocity v away from the source. First, the observer moved away a distance vT' and second the original distance λ will have Lorentz contracted to

$$\lambda\sqrt{1 - v^2/c^2}$$

Putting this together the total distance the light would have traveled between pulses is

$$cT' = vT' + \lambda\sqrt{1 - v^2/c^2}$$

replacing λ and solving for T'

$$T' = T\sqrt{\frac{1 + v/c}{1 - v/c}}$$

Since frequency is the inverse of period

$$f' = f\sqrt{\frac{1 - v/c}{1 + v/c}}$$

Now consider the source is in motion, and to observer is not. So, the distance the light travels, cT', is given by

$$cT' = \gamma T \times (c + v)$$

which finally gives us

$$cT' = \gamma T(c+v) = cT\frac{1 + v/c}{\sqrt{1 - v^2/c^2}} = cT\sqrt{\frac{1 + v/c}{1 - v/c}}$$

Imagine the use of this in *Doppler Radar* that can not only tell us position through time delay but also frequency shift and give information about the objects motion!

2.3 Mass, Momentum and Energy

What happens when you try to go faster and faster to the speed of light? It gets harder to accelerate because, as

$$v \to c, \gamma \to \infty$$

A weird thing to consider is that massless objects like photons do travel at the speed of light, and so may other massless objects.
The quick result for momentum with relativity is

$$\vec{\mathrm{p}} = \gamma m \vec{\mathrm{v}}$$

We know classically

$$W_{net} = \Delta KE$$

thus we can simply find that

$$W_{net} = mc^2(\lambda - 1)$$

This derivation is a solution in the test for this chapter.
We can also find the energy by

$$E = \sqrt{p^2c^2 + m^2c^4}$$

dropping mass for massless objects this becomes

$$E = pc$$

2.4 The Equivalence of Mass and Energy

2.5 Forces in Relativity

The one equation we need is

$$F = \frac{dp}{dp}$$

where

$$p \equiv \gamma m v$$

A special case for **constant force**. Suppose that a constant force of magnitude F_0 acts in the x-direction, starting at $t = 0$, when the object is at rest.

$$F_0 = \frac{d}{dt}(\gamma(u) m u)$$

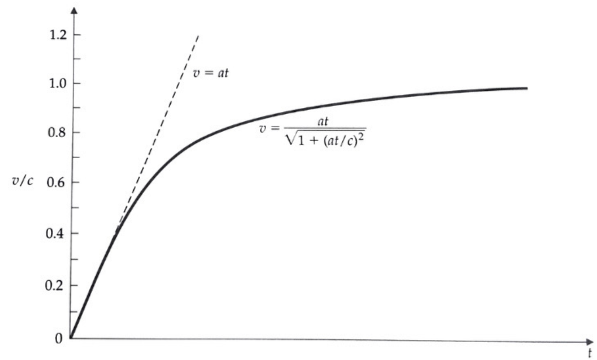
for the object's speed u, where $dx/dt = u$. Since F_0 is constant integrating with respect to t gives us

$$(F_0 t)/m = \gamma(u) u + C$$

consider initial conditions, $u = 0$ and $t = 0$, then $C = 0$. Solving for u we get

$$u = \frac{(F_0 t/m)}{\sqrt{1 + \frac{F_0^2 t^2}{m^2 c^2}}}$$

which has the property that sets the speed limit of c.



Example 2.5.1. Find the position of an object of mass m on which the constant force acts as a function of time.
If we integrate

$$u = \frac{(F_0 t/m)}{\sqrt{1 + \frac{F_0^2 t^2}{m^2 c^2}}}$$

with respect to time, we will get position. Lets say $x, t = 0$ we get

$$x(t) = \frac{mc^2}{F_0}\left(\sqrt{1 + \frac{F_0^2 t^2}{m^2 c^2}} - 1\right)$$

Chapter 3

Waves As Particles and Particles As Waves

3.1 Nature of Photons

Particles that make up radiation are called photons.
Photons travel in the speed of light, as a quanta. The energy is given by

$$E = pc$$

Energy is also related to the frequency

$$E = hf$$

where h is

$$h \approx 6.63 \times 10^{-34}\text{J}\cdot\text{ s}$$

Combining the equations we get

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$$

we can also express it with angular frequency where

$$\omega = 2\pi f$$

so

$$E = \hbar\omega$$

where

$$\hbar = h/2\pi \approx 1.05 \times 10^{-34}\text{J} \cdot \text{s}$$

is what is usually called Planck's constant.

Example 3.1.1. Suppose that a 60W bulb radiates light primarily of wavelength 1000nm. Find the number of photons emitted per second.
We can divide the total energy release per second, Watts, by the energy per photon, to find photons per second.

$$n = \frac{60\text{W}}{hf} = \frac{60\text{W}}{(6.63 \times 10^{-34}\text{J} \cdot \text{s})\,(3 \times 10^{14}\text{s}^{-1})}$$

$n = 3 \times 10^{20}$photons/s

3.2 Photoelectric Effect