MUSA 74 Homework 7

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No feedback needed. Thank you for the great semester! The p-adic stuff was super hard :(

a. Homework 3.11
Given that f and g are continuous for $f+g$

$$|x-y| \leq \delta,$$
$$|f(x)-f(y)| \leq \epsilon/2$$
$$|g(x)-g(y)| \leq \epsilon/2$$

Then it follows directly that

$$|f(x)+g(x)-f(y)-g(y)| \leq |f(x)-f(y)|+|g(x)-g(y)|$$
$$\leq \epsilon$$

by the triangle inequality

To show fg is continuous ...
For $|f|$ consider that

$$||x|-|y|| \leq |x-y| \leq \delta$$

Then the definiton follows with

$$||f(x)|-|f(y)|| \leq |f(x)-f(y)| \leq \epsilon$$

b. Homework 3.12
Let x_n be the decimal expansion for r. We can form such a sequence for any real number r. Additionally, each x_i is a raitonal number since we know that it is just an arbitrary real number over 10 to some power.

c. For contradiction assume that f is not constant. Then there must exist x, y such that $x > y$ and $f(x) > f(y)$ and $f(x)-f(y) = 1$. We know this is true since f is a mapping to $\mathbb{Z}$. Therefore, $f(x) > f(x)-0.5 > f(y)$. But clearly $f(x)-0.5$ is not in $\mathbb{Z}$. So, we have a contradiction since by the IVT there does not exist a x such that $f(x)-0.5$ but that would make f not continuous. So it must be constant.

d. Homework 3.26B
Consider $f = \sqrt{x}$. Then, let $y \in f([a,b])$ for when $y = \sqrt{\sqrt{2}}$. There is not $x \in \mathbb{Q}$ such that $f(x) = y$.

e. Homework 5.46
We can show that d_p is ultrametric when considering that for rational numbers $|x-y|_p$ results in $x = p^n(a/b)$. So we can take the rational numbers as $x = p/q$ and $y = r/s$. therefore we have $|x-y|_p = |\frac{ps-rq}{qs}$. We must find the p that satisifies such conditions. Intuitively, the maximum of three such numbers for when we consider the ultrametric will be depended on this numberator and denominator form.

f. Homework 5.48
Let x_n be a sequence such that

$$x_n = a_0 + a_1 p + .. + a_n p^n$$

Then we can show that it holds $|x_{n+k} - x_n|_p = a_{n+k}p^{n+k} + ... + a_n p^n$. Since it is p-adic we know that this must hold the definition for a Cauchy sequence since for all ϵ we can find a follwoing k.

g. Homework 5.49
Cauchy equivalence is a equivalence relation based on the fact that. If a given sequence is Cauchy equivalent then it must be true that it is equivalent to itself. Let x_n be a Cauchy sequence then its equivalence relation to y_n means that...

h. Homework 5.51
We can treat addition as our previous understanding of addition but with p in mind. That is if each class can be written as $a_0 + a_1 p + ...$ then addition between that and $b_0 + b_1 p + ...$ would just result in $(a_0+b_0)+(a_1+b_1)p + ...$ Similarily we can define multiplication to be this pointwise multiplication where the resulting product would be $(a_0 b_0) + (a_1 b_1)p + ...$