# MUSA 74 Homework 2

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**a.** Homework 1.35
I will not take this offer. After the 60 minutes has passed the Devil will have all the bills, and I will be left with none. This is simply because the set of his bills are $\mathbf{N}$ and therefor where all bills are labeled by $n \in \mathbf{N}$, I will be left with no more bills.

**b.** Homework 1.36
Proof. If an $x \in A \cap B$, then it follows that $f(x) \in A \cap B$ since $A \cap B \subseteq X$. Therefore $f(x) \in A$ and $f(x) \in B$ for all $x$. So by definition the claim follows.
Similarly, for a $x \in A \cup B$, $f(x) \in B$ since $A \cup B \subseteq X$. So it follows that for all $f(x)$ is in $A$ or $B$, and the claim follows.
We can show a counterexample for disproving $f(A \cap B) = f(A) \cap f(B)$. We have already shown that $f(A \cap B) \subseteq f(A) \cap f(B)$, so we must show that it is not the case that $f(A) \cap f(B) \subseteq f(A \cap B)$. Consider the transformation $f : Z \to Z^+$. Where $Z$ is the integers and $Z^+$ are all the positive integers. The transformation is as follows $f(x) = x^2$. If $A$ is all the positives and $B$ is all the negatives. $f(A) \cap f(B)$ is all the squares $1, 4, 9, 16, ....$ But $f(AcapB)$ is the empty set since the intersection of the positives and negatives is the empty set. It is therefore false that the squares is a subset of the empty set.

**c.** Homework 1.37
$\mathcal{P}(X)$ has the cardinality of $2^n$ if the cardinality of $X$ is $n$. This is because each element of $X$ has two options, to be either in or not in of any arbitrary subset of $X$. Since there are 2 options over $n$ elements it leads to $2^n$. For a similar reason the cardinality of $B(X)$ is also $2^x$, because the "choice" of an element being inside or outside of an arbitrary set can be represented by a $0, 1$.

**d.** Homework 5.7
Firstly, $\forall n \in N$, $n < \forall a \in \omega$. And by definition of naturals $n_{k+1} = n_k + 1$, all naturals are successors. So, if there does exist a limit it must not be in $N$. By definition of $2\omega$ $\omega_{k+1} = \omega_k + 1$, all of them are successors and there exists no limit.