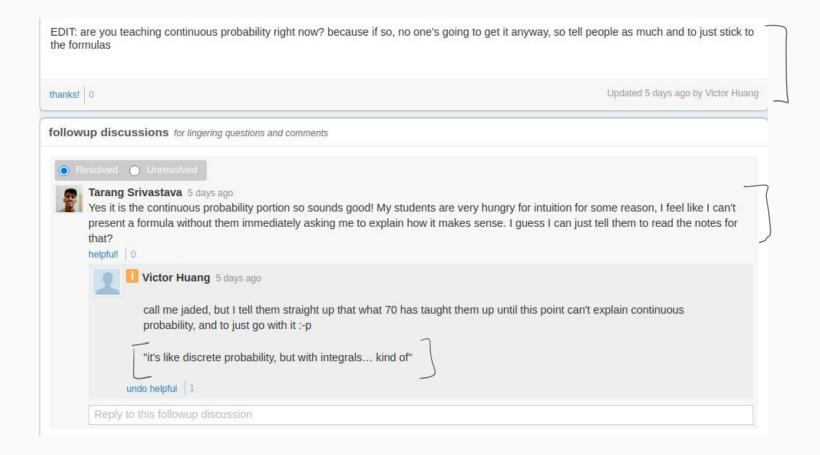
## Discussion 7A

Tarang Srivastava - CS 70 Summer 2020

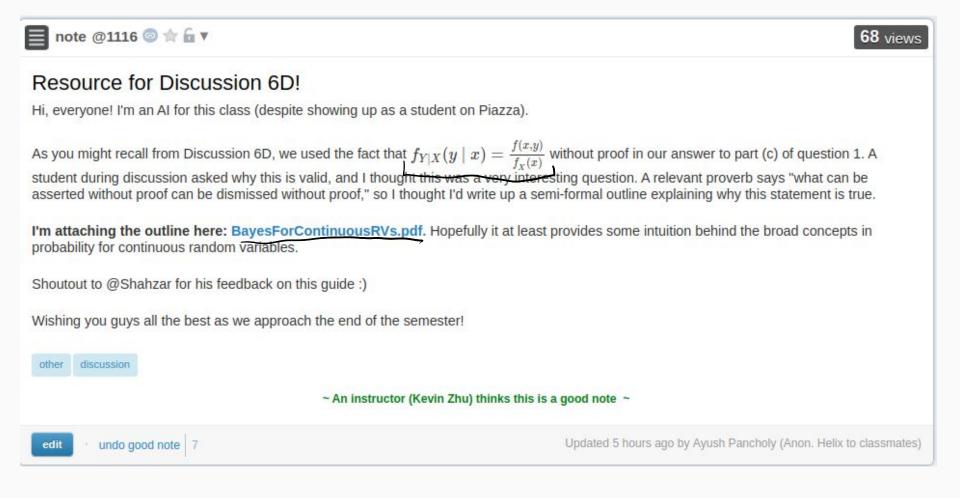
## Mini Review

#### In light of Continuous Probability



For context this is a Piazza thread between my instructor and I for CS 375, the class on how to teach computer science. Also Victor has been like a 3 time TA and 2 time Head TA for CS 70 so I'm not doubting he knows how to best teach CS70 for success.

#### Maybe I'm just naive, but there's more to the class than the grade...



Stay curious and question everything, not every rabbit hole will end up benefiting your CS70 grade. In fact, most probably won't, but there's more to all this than just the class. Here's to the final stretch!

#### Lecture Highlights

If X is a nonnegative random variable with finite mean and a > 0, then the probability that X is at least a is at most the expectation of X divided by a.  $P(X \ge a) \le \mathbb{E}[X] \underbrace{\text{Magn}}_{X} = e_{X} \underbrace{\text{postation}}_{X}$ 

Consider tossing a fair coin n times and let X denote the number of heads observed. What is the probability of observing more than 0.75\*n heads? (Note 17 pg. 3)

P[
$$X \ge \frac{3}{4}$$
 $\sqrt{1} \le \frac{\frac{5}{2}}{\frac{3}{4}}$  $\sqrt{1}$  XN Bhomial ( $N, \frac{1}{2}$ )

E[ $X$ ] =  $Np$  =  $\frac{9}{2}$ 

Weak est  $|| loose|| loose|| loose|| loose|| loose||  $|| loose||$  |  $|| loose||$$ 

If X is any random variable with finite mean and a > 0, then for any r > 0:

$$P(|X| \ge a) \le \frac{\mathbb{E}[|X|^r]}{a^r}$$

Marion & Society

#### **Lecture Highlights**

If X is a random variable with finite mean  $\mu$  and a>0, then the probability that X is at least c away from its mean is at most the variance of X divided by  $c^2$ .

$$P(|X - \mu| \ge c) \le \frac{\operatorname{Var}[X]}{c^2}$$

$$F[X]$$

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the moon

For any random variable X with finite expectation  $\mathbb{E}[X] = \mu$  and finite standard deviation  $\sigma = \sqrt{\operatorname{Var}[X]}$ ,

$$\Pr[|X - \mu| \ge k\sigma] \le \frac{1}{k^2},$$

for any constant k > 0.

Consider tossing a fair coin n times and let X denote the number of heads observed. What is the probability of observing more than 0.75\*n heads? (Note 17 pg. 3)

$$P[x \ge \frac{3}{4}n] = 7$$

$$P[x-\frac{1}{2}n \geq \frac{1}{4}n]$$

### Question 3 (Zoom Poll)

(a) A fair coin is tossed multiple times and you win a prize if there are more than 60% heads. Which number of tosses would you prefer: 10 tosses or 100 tosses? Explain.



(b) A fair coin is tossed multiple times and you win a prize if there are more than 40% heads. Which number of tosses would you prefer: 10 tosses or 100 tosses? Explain.



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(c) A fair coin is tossed multiple times and you win a prize if there are between 40% and 60% heads. Which number of tosses would you prefer: 10 tosses or 100 tosses? Explain.

100 /

(d) A fair coin is tossed multiple times and you win a prize if there are exactly 50% heads. Which number of tosses would you prefer: 10 tosses or 100 tosses? Explain.



 $\left(\begin{array}{c}10\\5\end{array}\right)\frac{1}{2}$ 

7

 $\begin{pmatrix} 100 \\ 50 \end{pmatrix} \stackrel{1}{2} 100$ 

n Samples

n-760

2

1000 con

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500 47 500

- (a) 10 tosses. By LLN, the sample mean should have higher probability to be close to the population mean as *n* increases. Therefore the average proportion of coins that are heads should be closer to 0.50, and has a lower chance of being greater than 0.60 if there are 100 tosses (compared with 10 tosses).
- (b) 100 tosses. Again, by LLN, the sample mean should have higher probability to be close to the population mean as *n* increases. Therefore the average proportion of coins that are heads should be closer to 0.50, and has a lower chance of being smaller than 0.40 if there are 100 tosses. A lower chance of being smaller than 0.40 is the desired result.
- (c) 100 tosses. Again, by LLN, the average proportion of coins that are heads should be closer to 0.50, and has a lower chance of being both smaller than 0.40 if there are 100 tosses. Similarly, there is a lower chance of being larger than 0.60 if there are 100 tosses. Lower chances of both of these events is desired if we want the fraction of heads to be between 0.4 and 0.6.
- (d) tosses. Compare the probability of getting equal number of heads and tails between 2n and

2n+2 tosses.

$$\mathbb{P}[n \text{ heads in } 2n \text{ tosses}] = \binom{2n}{n} \frac{1}{2^{2n}}$$

$$\mathbb{P}[n+1 \text{ heads in } 2n+2 \text{ tosses}] = \binom{2n+2}{n+1} \frac{1}{2^{2n+2}} = \frac{(2n+2)!}{(n+1)!(n+1)!} \cdot \frac{1}{2^{2n+2}}$$

$$= \frac{(2n+2)(2n+1)2n!}{(n+1)(n+1)n!n!} \cdot \frac{1}{2^{2n+2}}$$

$$= \frac{2n+2}{n+1} \cdot \frac{2n+1}{n+1} \binom{2n}{n} \cdot \frac{1}{2^{2n+2}} < \left(\frac{2n+2}{n+1}\right)^2 \binom{2n}{n} \cdot \frac{1}{2^{2n+2}}$$

$$= 4\binom{2n}{n} \cdot \frac{1}{2^{2n+2}} = \binom{2n}{n} \frac{1}{2^{2n}} = \mathbb{P}[n \text{ heads in } 2n \text{ tosses}]$$

As we increment n, the probability will always decrease. Therefore, the larger n is, the less probability we'll get exactly 50% heads.

A random variable X has variance Var(X) = 9 and expectation  $\mathbb{E}[X] = 2$ . Furthermore, the value of X is never greater than 10. Given this information, provide either a proof or a counterexample for the following statements.

- (a)  $\mathbb{E}[X^2] = 13$ .
- (b)  $\mathbb{P}[X=2] > 0$ . A piscole
- (c)  $\mathbb{P}[X \ge 2] = \mathbb{P}[X \le 2]$ .
- (d)  $\mathbb{P}[X \le 1] \le 8/9$ .
- (e)  $\mathbb{P}[X \ge 6] \le 9/16$ .

$$(a) \mathbb{E}[X^2] = 13.$$
Two

(b) 
$$\mathbb{P}[X=2] > 0$$
.

$$P[x = -\int_{-\frac{1}{2}}^{\infty}$$

(c) 
$$\mathbb{P}[X \geq 2] = \mathbb{P}[X \leq 2]$$
.

False:

$$V_{m}(x) = E[x^{2}] - E[x]^{2}$$

$$9 = V - 3^{2}$$

$$E[x^{2}] = 9 + 4 = 13$$

$$a_{rbi}v_{m}v_{j}$$

$$P[x = a] = \frac{1}{2}$$

$$E[x^{2}] = a + b$$

$$P[x = b] = \frac{1}{2}$$

$$E[x^{2}] = a + b$$

$$P[x = b] = \frac{1}{2}$$

$$A = b$$

Question 1

$$|-P[\times \times 1]| \leq 8$$

$$|-P[\times 1]| = 8$$

$$|-P[\times$$

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A friend tells you about a course called "Laziness in Modern Society" that requires almost no work. You hope to take this course next semester to give yourself a well-deserved break after working hard in CS 70. At the first lecture, the professor announces that grades will depend only on two homework assignments. Homework 1 will consist of three questions, each worth 10 points, and Homework 2 will consist of four questions, also each worth 10 points. He will give an A to any student who gets at least 60 of the possible 70 points.

However, speaking with the professor in office hours you hear some very disturbing news. He tells you that, in the spirit of the class, the GSIs are very lazy, and to save time the grading will be done as follows. For each student's Homework 1, the GSIs will choose an integer randomly from a distribution with mean  $\mu = 5$  and variance  $\sigma^2 = 1$ . They'll mark each of the three questions with that score. To grade Homework 2, they'll again choose a random number from the same distribution, independently of the first number, and will mark all four questions with that score.

If you take the class, what will the mean and variance of your total class score be? Use Chebyshev's inequality to conclude that you have less than a 5% chance of getting an A when the grades are randomly chosen this way.

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