

HIGHER-ORDER FUNCTIONS AND RECURSION

COMPUTER SCIENCE MENTORS CS 61A

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1 Higher-Order Functions

1. What is the difference between lambda functions and def statements? What is one of the purposes of higher order functions?

2. Draw the environment diagram that results from running the code.

```
x = 20
def foo(y):
    x = 5
    def bar():
        return lambda y: x - y
    return bar
```

```
y = foo(7)
z = y()
print(z(2))
```

3. Draw the environment diagram that results from running the code.

```
apple = 4
def orange(apple):
    apple = 5
    def plum(x):
        return lambda plum: plum * 2
    return plum

orange(apple)("hihi")(4)
```

def y

4. Write a higher-order function that passes the following doctests.

Challenge: Write the function body in one line.

```
def mystery(f, x):
    """
    >>> from operator import add, mul
    >>> a = mystery(add, 3)
    >>> a(4) # add(3, 4)
    7
    >>> a(12) # add(3, 12)
    15
    >>> b = mystery(mul, 5)
    >>> b(7) # mul(5, 7)
    35
    >>> b(1)
    5
    >>> c = mystery(lambda x, y: x * x + y, 4)
    >>> c(5)
    21
    >>> c(7)
    23
    """
```

hints

is a function

returns a function

def mystery
def ...:

return f(x)

lambda y: f(x, y)

def h(y):
 return f(x, y)

5. What would Python display?

```
>>> foo = mystery(lambda a, b: a(b), lambda c: 5 + square(c))
>>> foo(-2)
```

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2 Recursion

Every Recursive function has three things.

1. One or more base cases
2. One or more ways to break the problem down into a smaller problem
 - E.g. Given a number as input, we need to break it down into a smaller number
3. Solve the smaller problem recursively; from that, form a solution to the original problem

→ Base Case

Recursion

1. What is wrong with the following function? How can we fix it?

```
def factorial(n):
    return n * factorial(n)
```

Factorial(n) = $n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot (2) \cdot (1)$

Factorial(4) = $4 \cdot 3 \cdot 2 \cdot 1 = 24$

↳ Base case?

def Factorial(n):

if $n \geq 1$:

return 1

else:

return $n * \text{Factorial}(n-1)$

2. Write a function is_sorted that takes in an integer n and returns true if the digits of that number are nondecreasing from right to left.

```
def is_sorted(n):
    """
    >>> is_sorted(2)
    True
    >>> is_sorted(22222)
    True
    >>> is_sorted(9876543210)
    True
    >>> is_sorted(9087654321)
    False
    """
```

to get last digit of n

$n \% 10$

to iterate to next digit

$n // 10$

right-digit = $n \% 10$
rest = $n // 10$

if $n < 10$;
return True

else:

{ return right-digit \leq rest $\%$ 10
and is_sorted(rest)

if $n < 10$:

return True $\rightarrow (n // 10) \% 10$

elif right digit > rest $\%$ 10:

return False

else:

return is_sorted(rest)

recursion →

"Short-circuit"

False and True
True or ...