Discussion 2B

Announcements

- Please consider making a collection of definitions for the graph theory topic

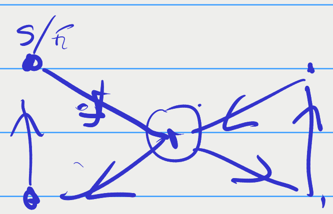
- The key for graphs is having all the definitions down first

tour

a walk with no repeating edges that starts and ends on the same vertex

cycle

a tour the only repeated vertex is the start or end



Definitions for G to be a tree

1. G is connected and contains no cycles.

2. G is connected and has $n-1$ edges (where $n = |V|$ is the number of vertices).

3. G is connected, and the removal of any single edge disconnects G.

4. G has no cycles, and the addition of any single edge creates a cycle.

1 True or False

(a) Any pair of vertices in a tree are connected by exactly one path.

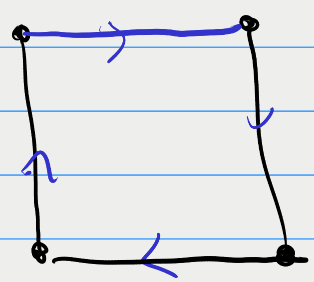
(b) Adding an edge between two vertices of a tree creates a new cycle.

(c) Adding an edge in a connected graph creates exactly one new cycle.

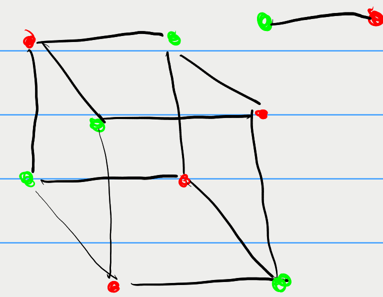
a) True. Assume that theres two, then cycle, contradiction

b) True. From definition

c) False.



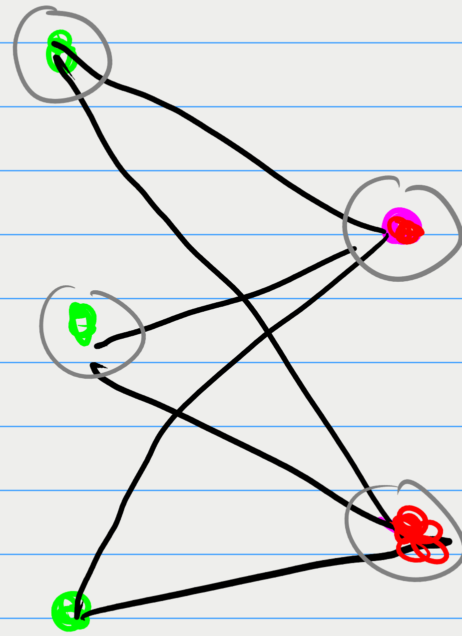
Examples of Bipartite Graphs



2 Bipartite Graph

A bipartite graph consists of 2 disjoint sets of vertices (say L and R), such that no 2 vertices in the same set have an edge between them. For example, here is a bipartite graph (with $L = \{$green vertices$\}$ and $R = \{$red vertices$\}$), and a non-bipartite graph.

Prove that a graph has no tours of odd length if it is a bipartite (This is equivalent to proving that, a graph G being a bipartite implies that G has no tours of odd length).



Suppose there is a tour

$n_0 \in L$, $n_1 \in R$

n_0, n_1 | 0^{th}

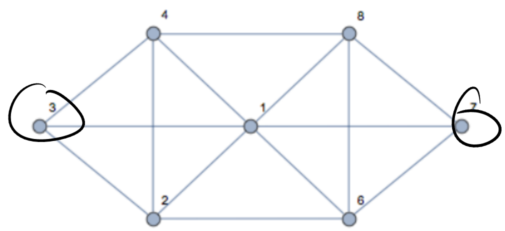
n_1, n_2 | 1^{st}

$(2k+1)^{th}$ edge

n_{2k}, n_{2k+1}

$2k+1, 2k+2$] even edge

3 Eulerian Tour and Eulerian Walk

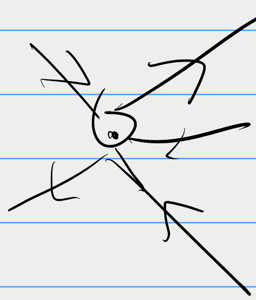


(a) Is there an Eulerian tour in the graph above? If no, give justification. If yes, provide an example.

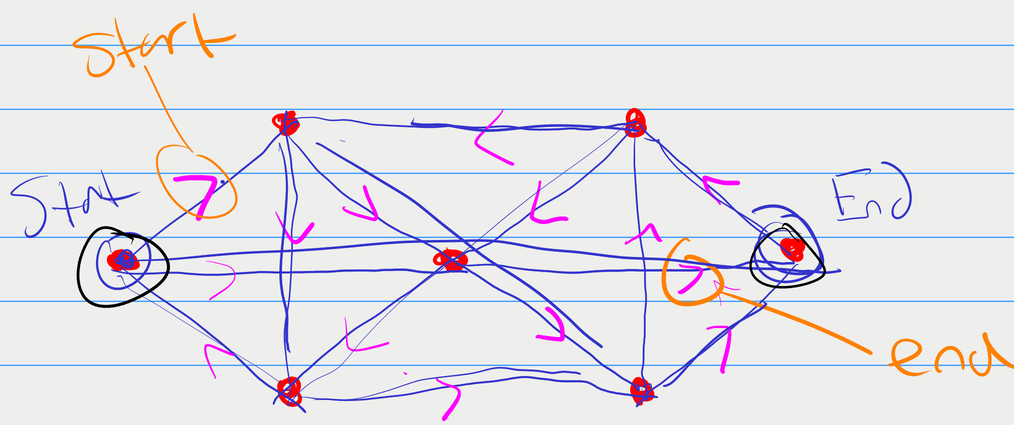
(b) Is there an Eulerian walk in the graph above? An Eulerian walk is a walk that uses each edge exactly once. If no, give justification. If yes, provide an example.

(c) What is the condition that there is an Eulerian walk in an undirected graph? Briefly justify your answer.

(a) no, because $\exists$ a vertex w/ odd degree



(b) Yes,



(c) G has Eulerian walk $\iff$ connected
two odd degree vertices

exactly each edge once

4 Odd Degree Vertices

Good Exercise to do

Claim: Let $G = (V, E)$ be an undirected graph. The number of vertices of G that have odd degree is even.

Prove the claim above using:

(i) Direct proof (e.g., counting the number of edges in G). *Hint: in lecture, we proved that* $\sum_{v \in V} \deg v = 2|E|$.

(ii) Induction on $m = |E|$ (number of edges)

(iii) Induction on $n = |V|$ (number of vertices)

(i) $|E| = m$

$$\underbrace{\sum_{v \in V_{odd}} \deg v}_{\text{must have even \# of terms}} = \underbrace{2m - \sum_{v \notin V_{odd}} \deg v}_{\text{all terms are even}}$$