Discussion 2C — More Graphs

Makeup @ 7p today
optional

Agenda

- no discussion Friday
- Question 1 & 2
- BR
- Question 3/4

Review

G is planar & connected $\Rightarrow V - E + F = 2$

Definition of a Tree

1. G is connected and contains no cycles.
2. G is connected and has $n-1$ edges (where $n = |V|$ is the number of vertices).
3. G is connected, and the removal of any single edge disconnects G.
4. G has no cycles, and the addition of any single edge creates a cycle.

1 Short Answers - Graphs

(a) Bob removed a degree 3 node from an n-vertex tree. How many connected components are there in the resulting graph?

3

(b) Given an n-vertex tree, Bob added 10 edges to it and then Alice removed 5 edges. If the resulting graph has 3 connected components, how many edges must be removed in order to remove all cycles from the resulting graph?

7

Notes:

- Between any two vertices in a tree there exists a unique path

- A tree with n vertices has exactly $n-1$ edges

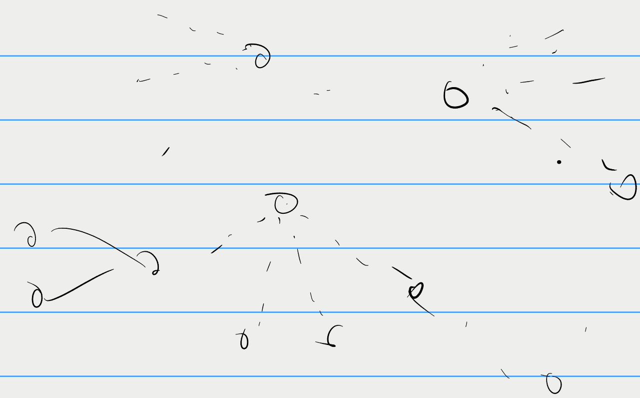
a) 3

Let v be the removed vertex

v has 3 neighbors, n_1, n_2, n_3

...

~~at least~~ 3 components



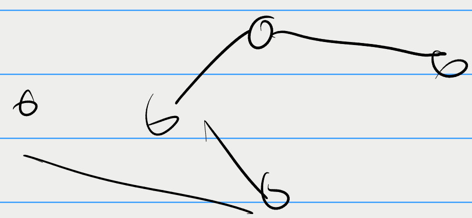
b) 7

vertices

$n = n_1 + n_2 + n_3$ # vertices in comp.

$n_1 - 1 \quad n_2 - 1 \quad n_3 - 1$

$\boxed{n-3}$ we want



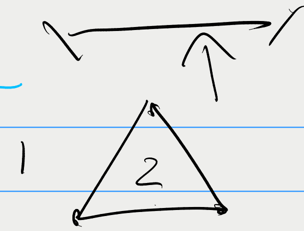
Edges $(n-1) + 10 - 5 = n + 4$

7

2 Triangular Faces

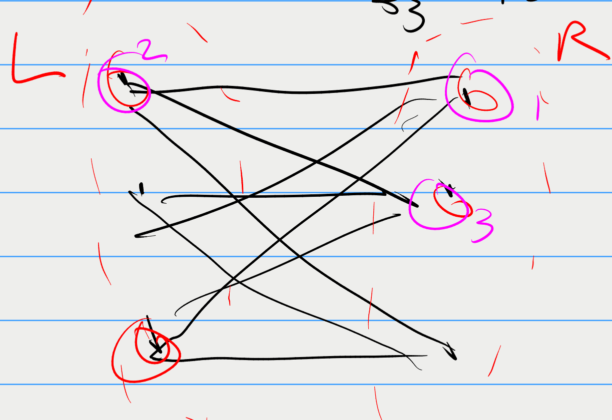
Suppose we have a connected planar graph G with v vertices and e edges such that $e = 3v - 6$. Prove that in any planar drawing of G, every face must be a triangle; that is, prove that every face must be incident to exactly three edges of G. Hint: If a graph is planar, then it has $\leq 3v - 6$ edges.

$3f \leq 2e$



Proceed b

a) Prove that K_{33} is non planar

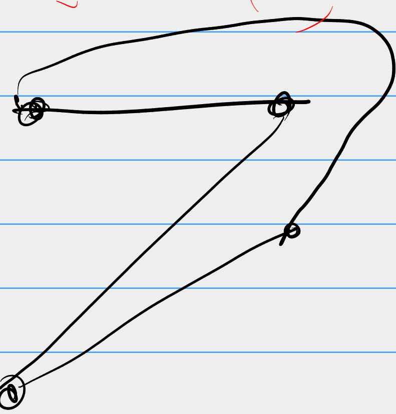


$v = 6$

$e = 9$

Assume K_{33} is Planar

$\boxed{4f \leq 2e}$



$v - e + f = 2$

$6 - 9 + f = 2$

$f = 5$

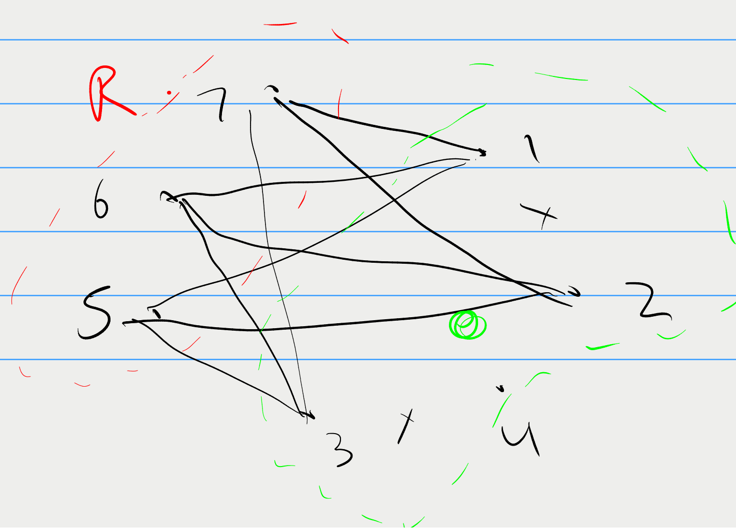
$4(5) \leq 2(e)$

$20 \cancel{\leq} 18$

b) Assume that G is planar

G has at least 7. Consider any 5

K_5 v_1 and v_2



K_{33}

3 Graph Coloring

Prove that a graph with maximum degree at most k is $(k+1)$-colorable.

5

Base

Inductive Step $n = k$

$k+1$

$\rightarrow k$

$k+1$

4 Hypercubes

The vertex set of the n-dimensional hypercube $G = (V, E)$ is given by $V = \{0,1\}^n$ (recall that $\{0,1\}^n$ denotes the set of all n-bit strings). There is an edge between two vertices x and y if and only if x and y differ in exactly one bit position. These problems will help you understand hypercubes.

(a) Draw 1-, 2-, and 3-dimensional hypercubes and label the vertices using the corresponding bit strings.

(b) Show that for any $n \geq 1$, the n-dimensional hypercube is bipartite.