RECURSION AND TREE RECURSION

CS 61A SMALL GROUP TUTORING

July 6 and 7, 2020

1 Recursion

There are three steps to writing a recursive function:

1. Create base case(s)
2. Reduce your problem to a smaller subproblem and call your function recursively on the smaller subproblem
3. Figure out how to get from the smaller subproblem back to the larger problem

Real World Analogy for Recursion

Imagine that you're in line for boba, but the line is really long, so you want to know what position you're in. You decide to ask the person in front of you how many people are in front of them. That way, you can take their response and add 1 to it. Now, the person in front of you is faced with the same problem that you were trying to solve, with one less person in front of them than you. They decide to take the same approach that you did, by asking the person in front of them. This continues until the very first person in line is asked. At this point, the person at the front knows that there are 0 people in front of them, so they can tell the person behind them that there are 0 people in front. Now, the second person can figure out that there is 1 person in front of them, and can relay that back to the person behind them, and so on, until the answer reaches you.

Looking at this example, we see that we have broken down the problem of "how many people are there in front of me?" to 1 + "how many people are there in front of the person in front of me"? This problem will terminate with the person at the front of the line (with 0 people in front of them). Putting this into more formal terms, we are breaking down the problem into a **recurrence relationship**, and the termination case is called the **base case**.

1. (Spring 2015 MT1 Q3C) Implement the `combine` function, which takes a non-negative integer `n`, a two-argument function `f`, and a number `result`. It applies `f` to the first digit of `n` and the result of combining the rest of the digits of `n` by repeatedly applying `f` (see the doctests). If `n` has no digits (because it is zero), `combine` returns `result`.

```
def combine(n, f, result):
    """
    Combine the digits in non-negative integer n using f.

    >>> combine(3, mul, 2) # mul(3, 2)
    6
    >>> combine(43, mul, 2) # mul(4, mul(3, 2))
    24
    >>> combine(6502, add, 3) # add(6, add(5, add(0, add(2, 3))))
    16
    >>> combine(239, pow, 0) # pow(2, pow(3, pow(9, 0))))
    8
    """
    if n == 0:
        return result
    else:
        return combine(n//10, f, f(n%10, result))
```

2. Implement the function `replace`, which takes in 3 integers, `n`, `old`, and `new` and returns a number that is identical to `n` except that all instances of `old` in `n` are replaced with `new`. Assume that `old` and `new` are both positive integers and less than 10.

```
def replace(n, old, new):
    """
    >>> replace(1294, 9, 3)
    1234
    >>> replace(555, 5, 8)
    888
    >>> replace(1, 1, 2)
    2
    >>> replace(0, 1, 2)
    0
    >>> replace(12345123, 6, 9)
    12345123
    """
    if n == 0:
        return 0
    last = n % 10
    rest = n // 10
    if last == old:
        return replace(rest, old, new) * 10 + new
    else:
        return replace(rest, old, new) * 10 + last
```

1 2 9 4
1 2 3 4

replace(1249, 9, 3)
1243
1249 // 10
124 * 10
1240 + 3
1243

Base

Recursive

2 Tree Recursion

Tree Recursion vs Recursion

In most recursive problems we've seen so far, the solution function contains only one call to itself. However, some problems will require multiple recursive calls – we colloquially call this type of recursion "tree recursion," since the propagation of function frames reminds us of the branches of a tree. "Tree recursive" or not, these problems are still solved the same way as those requiring a single function call: a base case, the recursive leap of faith on a subproblem, and solving the original problem with the solution to our subproblems. The difference? We simply may need to use multiple subproblems to solve our original problem.

Tree recursion will often be needed when solving counting problems (how many ways are there of doing something?) and optimization problems (what is the maximum or minimum number of ways of doing something?), but remember there are all sorts of problems that may need multiple recursive calls! Always come back to the recursive leap of faith.

Two rules that are often useful in solving counting problems:

1. If there are x *ways* of doing something and y *ways* of doing another thing, there are xy *ways* of doing **both** at the same time.
2. If there are x *ways* of doing one thing and y *ways* of doing another, but we can't do both things at the same time, there are $x + y$ ways of doing either the first thing **or** the second thing.

H/T . H/T

2 . 2 = 4

Advice

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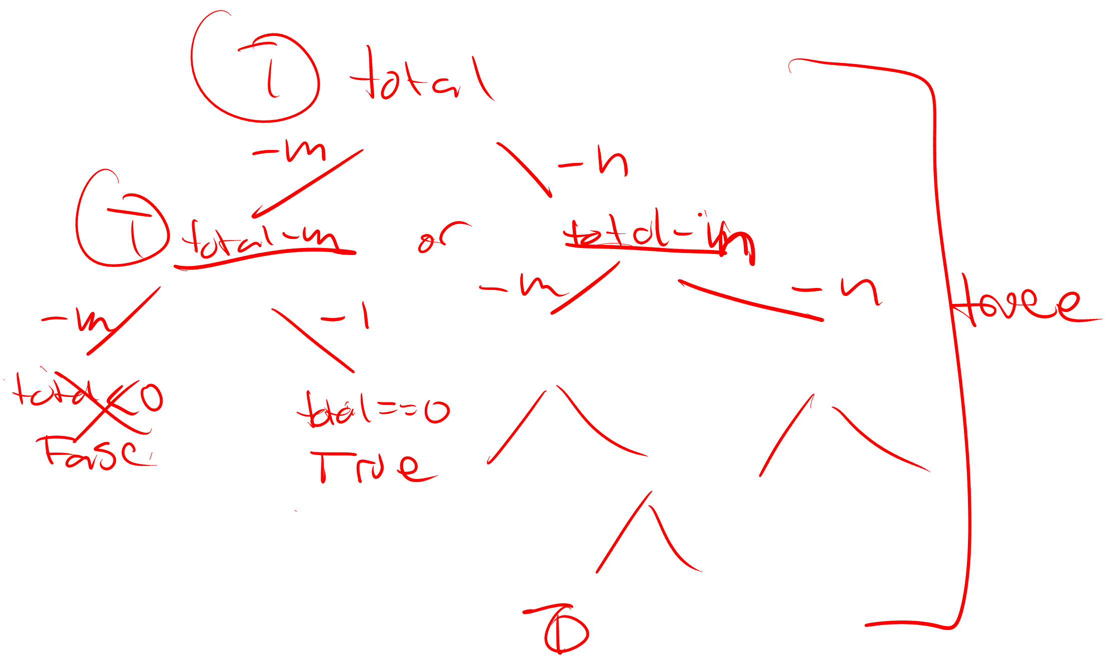
1. Mario needs to jump over a series of Piranha plants, represented as a string of 0's and 1's. Mario only moves forward and can either *step* (move forward one space) or *jump* (move forward two spaces) from each position. How many different ways can Mario traverse a level without stepping or jumping into a Piranha plant? Assume that every level begins with a 1 (where Mario starts) and ends with a 1 (where Mario must end up).
   Hint: Does it matter whether Mario goes from left to right or right to left? Which one is easier to check?

```
def mario_number(level):
    """
    Return the number of ways that Mario can traverse the
    level, where Mario can either hop by one digit or two
    digits each turn. A level is defined as being an integer
    with digits where a 1 is something Mario can step on and
    0 is something Mario cannot step on.
    >>> mario_number(10101)
    1
    >>> mario_number(11101)
    2
    >>> mario_number(100101)
    0
    """
    if ____________:

        _________

    elif ___________________:

        _________

    else:

        ____________________________________________________________
```

2. James wants to print this week's discussion handouts for all the students in CS 61A. However, both printers are broken! The first printer only prints multiples of `n` pages, and the second printer only prints multiples of `m` pages. Help James figure out whether or not it's possible to print exactly `total` number of handouts!

```
def has_sum(total, n, m):
    """
    >>> has_sum(1, 3, 5)
    False
    >>> has_sum(5, 3, 5) # 0 * 3 + 1 * 5 = 5
    True
    >>> has_sum(11, 3, 5) # 2 * 3 + 1 * 5 = 11
    True
    """
    if ________________________________________:
        return ______________________________
    elif ______________________________________:
        return ______________________________
    return ________________________________________
```



3. The next day, the printers break down even more! Each time they are used, the first printer prints a random `x` copies $50 \leq x \leq 60$, and the second printer prints a random `y` copies $130 \leq y \leq 140$. James also relaxes his expectations: he's satisfied as long as there's at least `lower` copies so there are enough for everyone, but no more than `upper` copies to prevent waste.

```
def sum_range(lower, upper):
    """
    >>> sum_range(45, 60) # Printer 1 prints within this range
    True
    >>> sum_range(40, 55) # Printer 1 can print a number 56-60
    False
    >>> sum_range(170, 201) # Printer 1 + 2 will print between 180 and 200
       copies total
    True
    """
    def copies(pmin, pmax):
        if ____________________________________________________:

            return ____________________________________________

        elif ____________________________________________________:

            return ____________________________________________

        return ____________________________________________________

    return copies(0, 0)
```