Improved Approximation Algorithms for Clustered TSP and Subgroup Planning

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Abstract

In the Clustered TSP (CTSP), we are given an edge-weighted graph satisfying the triangle inequality property, and a family of pairwise disjoint vertex groups. The goal is to find a minimum weight tour that includes all vertices, ensuring that the vertices within each group appear consecutively on the tour. The subgroup planning problem (SGPP) is an extension of CTSP by relaxing some triangle inequality requirements on edge weights. CTSP and SGPP have plentiful applications in AI and robotics. In this paper, we design three improved approximation algorithms for SGPP and CTSP. First, we propose a polynomial-time 2.167-approximation algorithm for SGPP, improving the previous ratio of 3 (IJCAI 2017). Second, we give an FPT 2.072-approximation algorithm for SGPP parameterized by the maximum group size, improving the previous ratio of 2.5 (IJCAI 2017). Third, we prove an FPT $(\beta < 1.5)$-approximation algorithm for SGPP parameterized by the number of groups, which even improves the previous ratio 1.667 for CTSP (ORL 1999). We also conduct experiments to evaluate the performance of our algorithms.

Introduction

The famous traveling salesman problem (TSP) as well as some other path planning problems, are going to find a route of a vehicle to visit a group of locations under some requirements. These problems play an important role in operations research and have also attracted certain attention in AI and robotics (Nash, Koenig, and Likhachev 2009; Jaillet and Porta 2013; Ninomiya et al. 2015; Surynek 2015).

One application scenario for these path planning problems is that the group of locations can be divided into smaller subgroups based on different properties, and each subgroup has its own rules and requirements for the route. Motivated by warehouse routing and production planning, the Clustered TSP (CTSP) was proposed (Chisman 1975; Lokin 1979). In CTSP, the group of locations is divided into several subgroups, and locations within the same subgroup must be visited consecutively (in any order).

To answer more application scenarios, CTSP was further generalized to the subgroup path planning problem (SGPP) in the literature (Sumita et al. 2017). The goal of SGPP is still to find a minimum weight tour visiting all vertices (locations) such that the vertices in the same subgroup are visited consecutively, as in CTSP. An illustration of SGPP, along with CTSP is shown in Figure 1. However, the edge weight function in SGPP is more general. In CTSP, the distances among any three locations always satisfy the triangle inequality relationship. However, in SGPP, the edge weight function w *partially* satisfies the triangle inequality. Specifically, it requires that $w(a,b) + w(b,c) \geq w(a,c)$ holds only if a, b, c belong to the same subgroup or a, c do not belong to the same subgroup. This property is called the *partial metric*, which was defined to meet the needs of AI applications (Sumita et al. 2017). Note that the targets in different subgroups may be different and then the routing cost (edge weight) in each subgroup may be different. We may even add other expenses caused when executing tasks in each subgroup to the edge weights in the subgroup. Thus, in most application scenarios, the weight of edges within each subgroup may be much higher than that of other edges, which may lead to the partial metric. Although many applications of SGPP and CTSP have been mentioned in the literature (Sumita et al. 2017; Laporte and Palekar 2002), we introduce one more application for SGPP below.

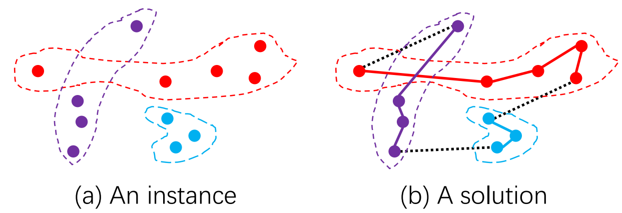

Figure 1: An example of SGPP and CTSP, where (a) represents three subgroups of vertices, and (b) represents a feasible tour

In electronic printing, a robotic arm needs to process various types of holes, such as through-holes, blind vias, and buried vias, which can be considered as distinct subgroups of vertices. To reduce switching costs and minimize error accumulation, the robotic arm must process holes of the same type consecutively. Moreover, different types of holes require varying levels of precision, resulting in higher routing costs within groups containing more complex or precision-

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