Automated Detection of Voids in Redshift Surveys

Hagai El-Ad[1], Tsvi Piran[1], and Luiz Nicolaci da Costa[2,3]

ABSTRACT

We present a new void search algorithm for automated detection of voids in three-dimensional redshift surveys. Based on a model in which the main features of the LSS of the Universe are voids and walls, we classify the galaxies into wall galaxies and field galaxies and we define voids as continuous volumes that are devoid of any wall galaxies. Field galaxies are allowed within the voids. The algorithm makes no assumptions regarding the shapes of the voids and the only constraint that is imposed is that the voids are always thicker than a preset limit, thus eliminating connections between adjacent voids through small breaches in the walls. By appropriate scaling of the parameters with the selection function this algorithm can be used to analyze flux-limited surveys. We test the algorithm on Voronoi tessellations and apply it to the SSRS2 redshift survey to derive the spectrum of void sizes and other void properties. We find that the average diameter of a void is $37\pm8h^{-1}$ Mpc. We suggest the usage of this fully automated algorithm to derive a quantitative description of the voids, as another tool in describing the large scale structure of the Universe and for comparison with numerical simulations.

Subject headings: cosmology: observations — galaxies: statistics — large-scale structure of the universe

[1]Racah Institute for Physics, The Hebrew University, Jerusalem, 91904 Israel

[2]European Southern Observatory, Karl-Schwarzschild Straße 2, D-85748 Garching b. München, Germany

[3]Observatório Nacional, Rua José Cristino 77, Rio de Janeiro, Brazil

1. Introduction

Perhaps one of the most intriguing findings of dense and complete nearby redshift surveys has been the discovery of large voids on scales of $\sim 50h^{-1}$ Mpc and that such large voids appear to be a common feature of the galaxy distribution (Geller & Huchra 1989, da Costa et al. 1994). Deeper surveys support these findings (Landy et al. 1996) with some even suggesting the presence of larger voids (Broadhurst et al. 1990). Since the discovery of the large Boötes void it has been recognized that inhomogeneities on such scales could impose strong constraints on theoretical models for the formation of large scale structure. However, if galaxy-voids reflect true voids in the underlying mass distribution, then despite the potential impact that this could have in constraining the power spectrum of mass perturbations (e.g., Blumenthal et al. 1992, Piran et al. 1993) or in our understanding of the biasing processes in galaxy formation, the voids have been largely ignored in recent work.

The major obstacle here has been the difficulty of developing proper tools to identify voids in an objective manner and to quantify their properties. We present a new void search technique for detection of voids and the determination of their properties in an automated way, not biased by the human eye. We test this algorithm using a Voronoi tessellation and we apply it to the SSRS2 survey.

2. The Void Searching Algorithm

Our algorithm is based on a model in which the main features of the LSS of the Universe are voids and walls. *Walls* are generally thin, 2D structures characterized by a high density of galaxies. Although coherent over large scales the walls are not homogeneous, and like our eyes we would like to ignore small breaches in the walls when identifying the individual voids. Galaxies within walls are hereafter labeled *wall galaxies*. The walls constitute boundaries between under-dense regions, generally ellipsoidal in shape. These under-dense regions – the so called *voids* – are not totally empty: there are a few galaxies scattered here and there, and we will name these *field galaxies*.

Previous algorithms have used various definitions for voids: Pellegrini et al. (1989) examined ensembles of contiguous cells with densities below a given threshold. Kauffmann & Fairall (1991) used empty cubes, to which adjacent faces could be attached. Lindner et al. (1995) looked for single spheres, that are devoid of a certain type of galaxies. The latter two definitions constrain the shapes of the voids to be spherical or roughly ellipsoidal. We define a void as a continuous volume that does not contain any wall galaxies and is nowhere thinner than a given diameter. In other words, one can freely move a sphere with the minimal

diameter all through the void. This definition does not determine the shape of the void: it can be a single sphere, an ellipsoid, or have a more complex shape, including a concave one.

An important feature in our definition is that voids may contain field galaxies. A stiffer requirement, such that voids would be completely empty volumes, is too restrictive for it would imply that a single field galaxy located in the middle of what we would like to recognize as a void might prevent its identification. However, for this definition to be practical we must be able to identify the field galaxies before we can start locating the voids.

Our algorithm is divided into two steps. First the *Wall Builder* identifies the wall galaxies and the field galaxies. Then the *Void Finder* finds the voids in the wall galaxy distribution. All together, our method incorporates three parameters, which will be defined below. Two of these parameters (n and β) are used to locate the field galaxies; the third parameter (ξ) is used during the void search. Specific values for these parameters were chosen after trials-and-errors with various simulations, to give results that resemble as much as possible eye-estimates of the voids. A detailed parametric study of the dependence of the algorithm on the values of these parameters will be published elsewhere.

A wall galaxy is required to have at least n other wall galaxies within a sphere of radius L around it. Every galaxy that does not satisfy this condition is classified as a field galaxy. Note that the above requirement must be applied in a recursive manner until all the field galaxies are found. Let the distance to the n'th closest neighbor of a galaxy be l_n. For the sample this quantity has an average value $\bar{l}_n$ and a standard deviation σ_n. The radius L is defined as $L \equiv \bar{l}_n + \beta\sigma_n$. We have chosen here $n = 3$ and $\beta = 1.5$. Note that $n = 3$ is the minimal value that enables filtering of long thin chains of galaxies, generally preferring 2D structures that we would like to recognize as walls. As a side-bonus of this procedure we identify the walls, by connecting together all of the wall galaxies. These connections are not used in the next step but provide us with another visual tool to examine our results (see Fig. 1, left panel). Hence this part of the algorithm is called the *Wall Builder*. Note that smoothing is not required here since the statistics used are based on a galaxy count.

The *Void Finder* searches for spheres that are devoid of any wall galaxies. These spheres are used as building blocks for the voids. A single void is composed of as many (or as few) superimposing spheres as required for covering all of its volume. The limits of the distribution are treated as rigid boundaries, thus causing some distortion in the voids found close to the boundaries: these voids tend to be smaller than the rest. For a void with a maximal sphere of a diameter $d_{\max}$ we consider only spheres with diameters larger than $\xi d_{\max}$, where ξ is the "thinness parameter". If the void is composed of more than one sphere (as is usually the case), then each sphere must intersect at least one another sphere with a circle wider than the minimal diameter $\xi d_{\max}$. We have taken $\xi = 8/9$, which allows for enough flexibility

without connecting distinct voids. A lower ξ reduces the total number of the voids, with a slow increase in their total volume.

A major problem encountered is that of keeping apart neighboring voids. The walls separating voids often lack a few bricks, and thus a gap is created through which finger-shaped voids might find their way into a neighboring void, in this way connecting two that should have been kept separate. We use the following iterative procedure to overcome this problem. First, we look for the largest voids, using a large maximal sphere. Thus, voids found during the i'th iteration are composed of spheres in the diameter range $\xi d_i < d < d_i$, where d_i denotes the maximal diameter d_{max} required in the i'th iteration (voids containing spheres with a larger diameter having been detected during former iterations). Following iterations search for smaller voids, avoiding regions in which voids were found by former iterations. Once a void is found in a certain iteration, it is not tampered with afterwards. Thus, voids found in later iterations can not send "thin fingers" to previously identified voids.

The iterative nature of this method naturally brings up the question of when to stop: we could go on until we mark every empty space as a part of a void. The criterion we use here is based on comparisons with Poisson distributions with the same geometry and the same number of galaxies. Averaging together many such random simulations one can derive N_d, the expectation value N for the number of voids found containing at least one sphere with a diameter larger than d. As long as the actual number $\hat{N}_d$ of voids found in the original distribution exceeds N_d ($\hat{N}_d \gg N_d$), the voids are considered significant and we proceed to the next iteration.

The parameter determining the iteration in which a void is detected is the radius of the largest sphere contained in that void and not its total volume. Of course the total volume of a void and the radius of the largest sphere contained in it are correlated, since voids with a bigger largest-sphere tend to be bigger voids. However, this may not always be the case: a void composed of a single large sphere will be detected earlier – and hence considered more significant – than a *larger* void composed of several spheres all having smaller radii. This is in agreement with the fact that clear spherical voids are more prominent when a galaxy distribution is inspected by eye. In retrospect this choice is also justified by the theoretical expectation that voids become more spherical with cosmological time (Blumenthal et al. 1992).

3.   Applications to Mock Surveys

We have used Voronoi tessellations (Voronoi 1908) as a test bed for our algorithm. By construction Voronoi tessellations have the desired characteristics consisting of large empty regions and wall galaxies. To those we randomly add galaxies and an additional degree of stochasticity arises from the realization. The location and number of the Voronoi cells (the would-be voids), the spread of the wall galaxies and the amount of random galaxies are all known in advance. Therefore, we can test how well our algorithm performs in recovering the voids in the tessellation.

Figure 1 shows a single slice cut through a cubic Voronoi tessellation. This tessellation was created using 8 nuclei for the Voronoi cells, with 3000 galaxies of which 10% were located randomly all over the volume. The designated wall galaxies were positioned randomly on the boundaries between the Voronoi cells, with a Gaussian displacement in the distance from the cell boundary. The left panel depicts the original Voronoi cells, over which we superpose the simulated galaxy distribution. Note the walls highlighted along the boundaries between the Voronoi cells. The right panel shows the voids, as identified by the Void Finder. Note how the field galaxies were filtered, thus allowing for a good fit between the Voronoi cells and the voids found.

4.   Voids in the SSRS2 Survey

We have applied our algorithm to the recently completed SSRS2 survey (da Costa et al. 1994) which consists of ~ 3600 galaxies in the region $-40° < \delta < -2.5°$ and $b \leq -40°$. In order to apply the algorithm to actual redshift surveys, one must take into account the geometry and selection effects for a magnitude-limited sample. Our algorithm can process any survey-like geometry defined in spherical coordinates, treating the limits of the survey as rigid boundaries.

In a magnitude-limited redshift survey like the SSRS2 the average galaxy number density decreases with depth since only the more luminous galaxies are visible at larger distances. If not corrected, this selection effect will interfere with the algorithm: field galaxies will occur more frequently, and the derived size of the voids will be larger because of the decrease in the mean density. All together, systematically larger voids will be found at larger distances.

To minimize these effects we have considered a semi-volume limited sample consisting of galaxies brighter than $M_o \leq -19$, corresponding to a depth $r_o = 79.5h^{-1}$ Mpc. We have

computed the selection function:

$$\phi(r) = \frac{\Gamma(x_M, 1 - \alpha)}{\Gamma(x_{M_o}, 1 - \alpha)} \tag{1}$$

where $x_M = 10^{0.4(M_* - M)}$, using a Schechter luminosity function (Schechter 1976) with $M_* = -19.6$ and $\alpha = 1.2$ as derived for the SSRS2 (da Costa 1995).

We apply corrections based on the selection function to both phases of the algorithm. In the Wall Builder phase, we consider larger spheres when counting the neighbors of more distant galaxies. The volumes of the counting-spheres are:

$$V = \begin{cases} V_0 & r < r_o \\ V_0/\phi(r) & r_o < r < r_{\max} \end{cases} \tag{2}$$

where $V_0 = 4\pi L^3/3$. The radius L is calculated here using only galaxies with $r < r_o$. Our final semi-volume limited sample consists of 1898 galaxies, extending out to $r_{\max} = 130h^{-1}$ Mpc where ϕ has dropped to 17%.

A similar correction is applied to the Void Finder. Voids of a certain size found in a low density environment are less significant than voids of the same size found in a high density environment. In order that all the voids found by the Void Finder in a given iteration will be equally significant we adjust the algorithm so that at a given iteration only relatively larger voids are accepted, if located at $r > r_o$. Thus we determine an initial diameter d_{io} for the void search within the volume-limited region, and scale it by the selection function at larger redshifts in the same way as we change the volumes of the counting spheres. During consecutive iterations this initial value is reduced both in the volume-limited region as well as at large redshifts, again properly scaled by the selection function.

We identified twelve voids within the volume probed by the SSRS2. These are the most significant voids found, all of which having been detected during initial iterations for which voids were rarely found in equivalent random distributions. Five additional voids are still quite large, having volumes that exceed those of a sphere with a diameter of $25h^{-1}$ Mpc, but these were found during later iterations and therefor are less significant and were not considered in the calculations below. The location and the characteristics of the identified voids are given in Table 1. The top part of the table includes the 12 voids found in the survey; the bottom part includes the 5 additional voids. Column (1) lists the values of d_{io} used while detecting each void. The diameters given in column (2) are of a sphere with the same volume as the whole void, as is listed in column (3). The center of the void given in columns (4)-(6) is defined as its center-of-(no)-mass. The density contrast estimate listed in column (7) was corrected for the average galaxy density at the same distance as the center of

the void. Finally, in column (8) we give the fraction of the total volume of the void covered by the single largest sphere contained in it. This value is typically $\sim 50\%$ of the total volume of the void.

The average size of the voids in the SSRS2 survey as estimated from the equivalent diameters is $\bar{d} = 37 \pm 8 h^{-1}$ Mpc, lower but consistent with eye estimates of Geller & Huchra (1989) and da Costa et al. (1994), which set this figure at $50 h^{-1}$ Mpc, and similar to the value of $38 h^{-1}$ Mpc obtained from the first zero-crossing of the correlation function (Goldwirth et al. 1995). The largest void found in the SSRS2 survey has an equivalent diameter $d = 56.4 h^{-1}$ Mpc, making it comparable in volume to the large void found in the Boötes (Kirshner et al. 1981). The shape of this void approaches that of an ellipsoid whose major axis is perpendicular to the line of sight, located at: $85 h^{-1}$ Mpc $< r < 130 h^{-1}$ Mpc; $-25.0° < \delta < -2.5°$; $21^h < \alpha < 23.5^h$. This void might actually be larger, since it is bounded by the limits of the SSRS2 survey. The average under-density within the voids was found to be $\delta\rho/\rho \approx -0.9$, a quite remarkable result showing how empty voids are of bright galaxies.

Figure 2 depicts a 3D representation of the voids in the SSRS2 survey. The figure is striking and it unmistakably shows the propriety of using the picture of a void-filled Universe to describe the observed galaxy distribution. The twelve voids comprise 41% of the survey's volume. An additional 9% is covered by the five less significant voids, and more volume is covered by still smaller voids. We estimate that the walls occupy less than 25% of the volume of the Universe. It is important to point out that because of the paucity of large clusters and the small amplitude of peculiar motions in the volume surveyed by the SSRS2, redshift distortions are small (da Costa 1995) and the properties derived here should reflect those of voids in real space.

Figure 3 shows a histogram of two distributions of equivalent-diameter spheres: the distribution of the 12 voids found in the SSRS2 survey, compared to several random distributions, averaged together. Most of the voids found in the random distributions have diameters smaller than $25 h^{-1}$ Mpc. We used a χ^2 test to compare the distribution of the SSRS2 survey voids' diameters with that of the random distributions. The probability that the two originate from the same distribution is $P = 0.087$. Note that in any case the probability could not be smaller than $P = 0.035$ (the result of the χ^2 test comparing these 12 voids to a null histogram, with the same number of degrees of freedom). We expect that when larger surveys are made available then (with more voids) the χ^2 test will yield $P < 0.01$. For example, a survey with a volume twice as large (i.e., with 24 voids) would yield $P = 0.001$.

The method we applied, of volume-limiting a part of the survey, prevents us from directly benefiting from the excess information hidden with the faint galaxies that we have eliminated. However, we can use these galaxies to re-examine our results. There are 1264

such galaxies in the SSRS2 survey; obviously, they are all located at $r < r_o$. Almost 50% of this region is covered by voids – but only 10% of the faint galaxies are found within them. This verification is important, since the volume-limiting process chooses only the brighter galaxies. It is interesting to compare the percentage of faint galaxies within voids, to that of the brighter galaxies $M_o \leq -19$ found within the voids: only 5% of the later kind are contained in the voids. This result indicates that the population of faint galaxies within voids is larger than the population of bright galaxies.

5. Summary

We have presented a new algorithm for the identification of voids in redshift surveys. The algorithm mimics the human eye in identifying the voids, focusing on the voids and disregarding small breaches on the walls. This feature, together with the flexibility in void shapes, the recognition and filtering of the field galaxies and the ability to correct for selection effects, makes the algorithm powerful and of interest for cosmological applications.

We have applied the algorithm to the SSRS2 data sample and found 12 voids with $\bar{d} = 37 \pm 8h^{-1}$ Mpc, in good agreement with visual interpretation of these maps. However, the algorithm is automated and objective and it identifies voids and computes their properties in a quantitative way. The formalism can be applied also to mock surveys generated from N-body simulations. Therefore it should be considered a new tool in our arsenal for investigating the nature of clustering in the Universe. We hope that the results obtained in this way will prove useful in future comparisons between theory and data, and would allow the most remarkable feature of the observed galaxy distribution to be used in a quantitative way to constrain theories of large scale structure.

Table 1. Properties of the voids in the SSRS2 survey

Largest Diameter[a] $[h^{-1}\mathrm{Mpc}]$ (1)	Equivalent Diameter $[h^{-1}\mathrm{Mpc}]$ (2)	Total Volume $[h^{-3}\mathrm{KMpc}^3]$ (3)	Location of Center			Void Under-density (7)	Largest Sphere's Fraction (8)
			r $[h^{-1}\mathrm{Mpc}]$ (4)	$r.a.$ [hours] (5)	$dec.$ [degrees] (6)		
35.1	42.1	39.3	76.5	$1^h 50^m$	-16°47′	-0.89	0.54
29.9	43.4	42.8	90.8	3 43	-30 04	-0.89	0.44
28.9	56.4	94.0	108.7	22 21	-13 08	-0.91	0.31
27.6	33.3	19.2	70.2	21 40	-13 56	-0.90	0.50
26.0	32.2	17.7	53.0	23 48	-24 39	-0.94	0.49
26.0	30.4	14.9	56.1	3 48	-20 19	-0.91	0.55
23.7	25.2	8.3	77.2	3 17	-11 40	-0.91	0.73
22.6	27.8	11.3	85.3	23 17	-12 19	-0.94	0.59
21.6	39.8	33.4	115.8	3 08	-14 18	-0.92	0.50
20.8	38.2	29.3	102.7	0 35	-9 28	-0.73	0.30
20.8	29.4	13.2	86.2	0 44	-28 52	-0.98	0.35
19.8	42.6	40.5	115.1	0 24	-29 03	-0.89	0.31
19.2	33.0	19.0	114.5	$2^h\ 02^m$	-9°09′	-0.73	0.58
19.2	26.8	10.0	73.4	22 57	-32 07	-0.97	0.34
19.2	31.7	16.9	114.4	2 42	-33 03	-0.75	0.71
18.5	33.8	20.4	112.2	4 17	-15 27	-0.96	0.43
16.9	27.6	11.0	116.1	21 24	-33 17	-0.75	0.84

[a]Identical values in several rows imply that these voids were all identified during the same iteration.

REFERENCES

Blumenthal, G. R. et al. 1992, ApJ, 388, 234

Broadhurst, T. J. et al. 1990, Nature, 343, 726

da Costa, L. N. et al. 1994, ApJ, 424, L1

da Costa, L. N. 1995, in preparation.

Geller, M. J., and Huchra, J. P. 1989, Science, 246, 897

Goldwirth, D. S., da Costa, L. N., and Van de Weygaert, R. 1995, MNRAS, 275, 1185 (astro-ph/9503002)

Kauffmann, G., and Fairall, A. P. 1991, MNRAS, 248, 313

Kirshner, R. P. et al. 1981, ApJ, 248, L57

Landy, S. D. et al. 1996, ApJ, 456, L1 (astro-ph/9510146)

Lindner, U. et al. 1995, A&A, 301, 329 (astro-ph/9503044)

Pellegrini, P. S., da Costa, L. N., and de Carvalho, R. R. 1989, ApJ, 339, 595

Piran, T. et al. 1993, MNRAS, 265, 681 (astro-ph/9305019)

Schechter, P. 1976, ApJ, 203, 297
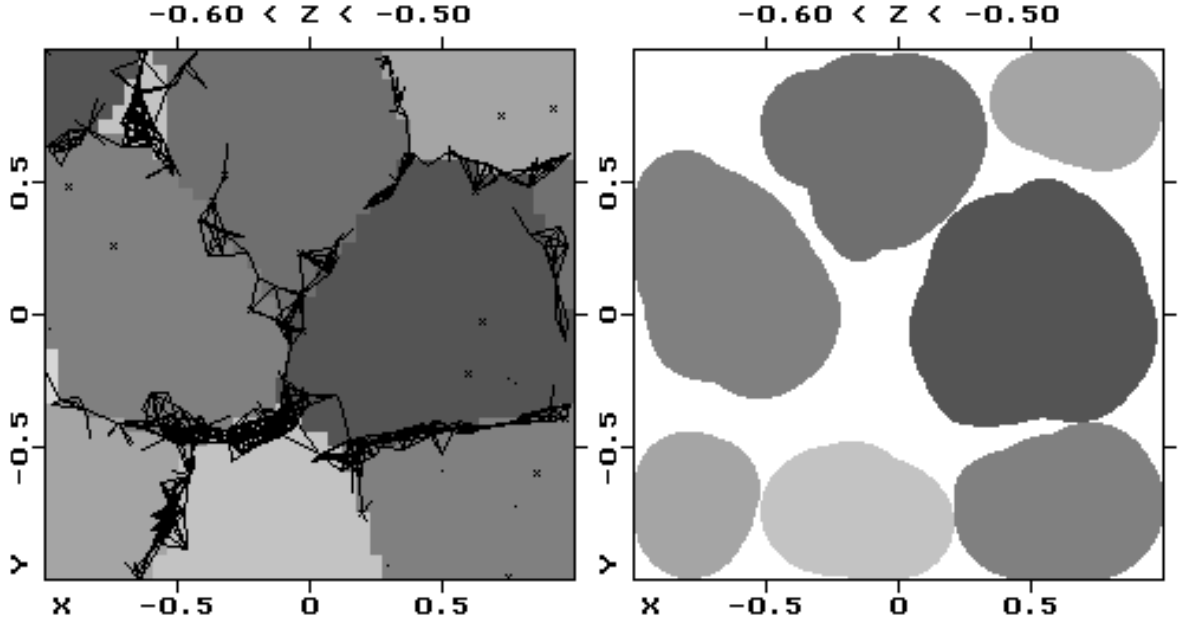
Voronoi, G. 1908, J. reine angew. Math, 134, 198

Fig. 1.— A slice through a Voronoi tessellation. The width of the slice shown here is 5% of the whole cube. Left panel: the original Voronoi cells; each cell is drawn with a different color, in the background. On top of the cells is the simulated galaxy distribution. The walls are highlighted by connecting (black lines) the galaxies that the Wall Builder has identified as wall galaxies. The remaining, unconnected galaxies, are the field galaxies. '+' marks the galaxies distributed around the Voronoi boundaries. '×' marks galaxies distributed randomly all over the volume. Right panel: the voids as found by our algorithm. Here each shaded area is a cut through a void at the middle of the slice. See text for more details.
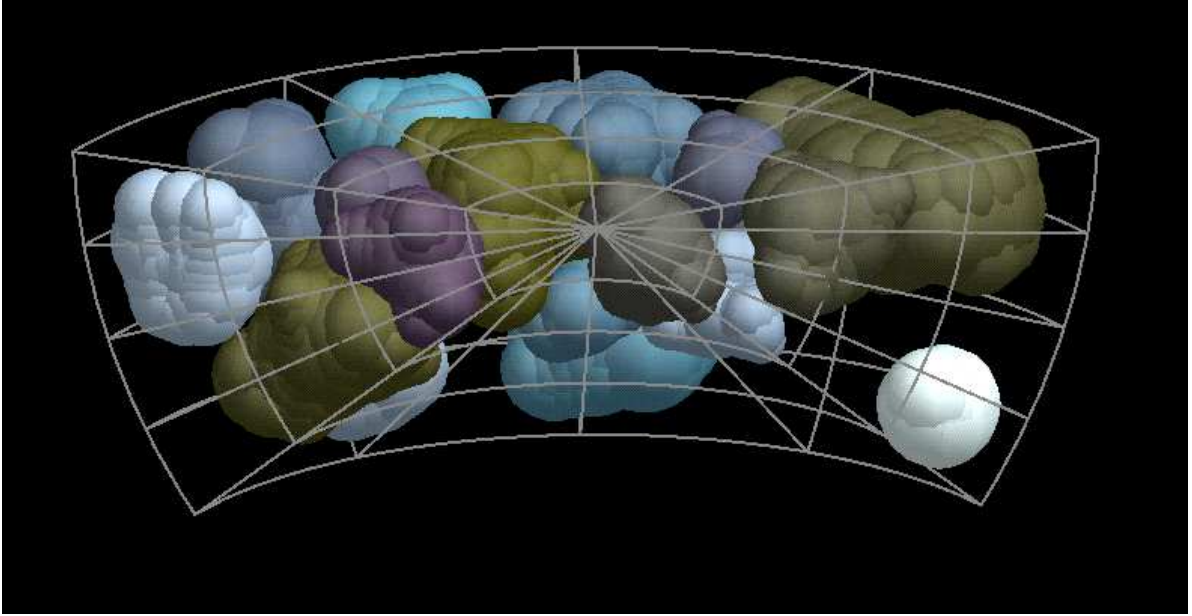
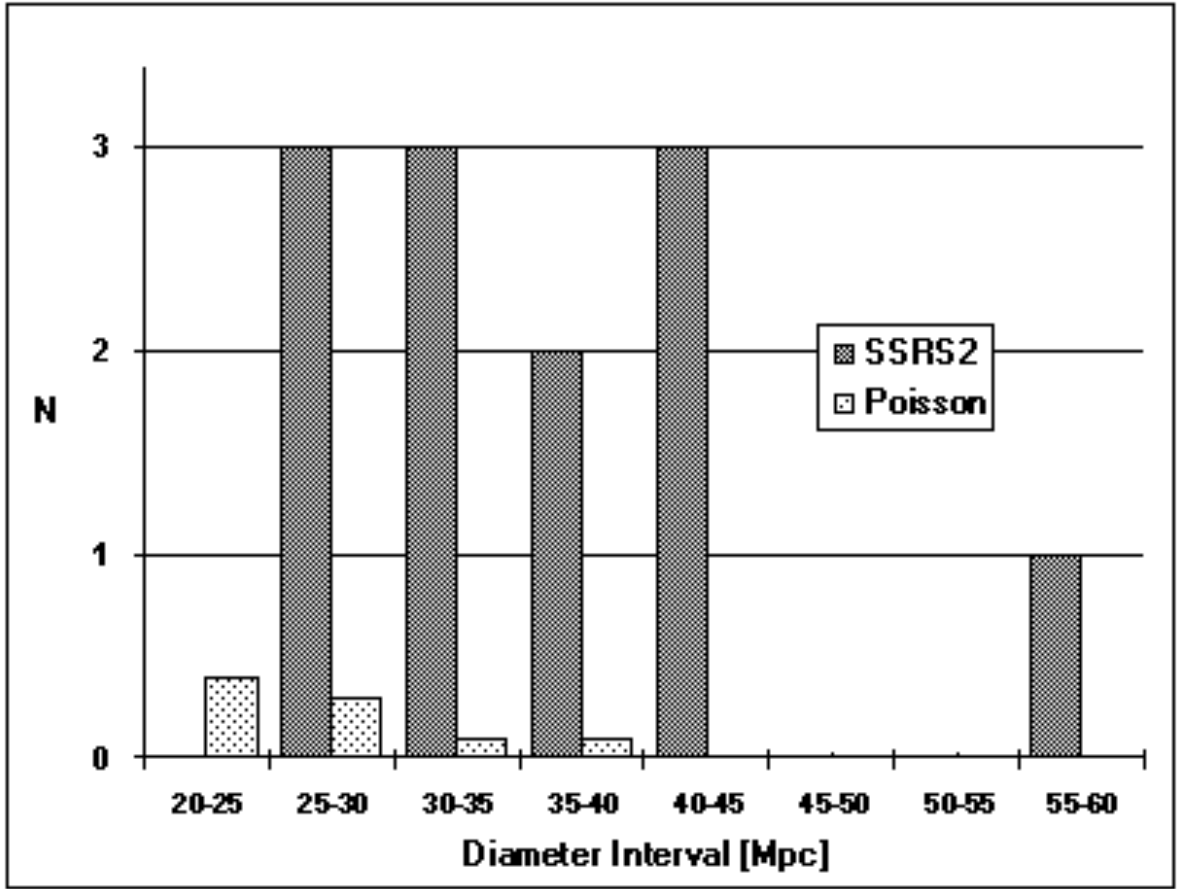Fig. 2.— 3D view of the voids in the SSRS2 survey.

Fig. 3.— Spectrum of void sizes in the SSRS2 survey, compared to several equivalent random distributions, averaged together. The equivalent distributions have the same number of galaxies, the same geometry and the same selection function as the SSRS2.