Exoplanet Atmospheres Problem Set – Solutions

Section I: Life & Habitability

1a. Calculating the Habitable Zone (HZ) Boundaries

The equilibrium temperature of a planet:

$$T_{eq} = \left(\frac{L_\star(1-A)}{16\pi\sigma a^2}\right)^{1/4}$$

Given:

- Bond albedo $A = 0.3$
- Stellar luminosity $L_\star = 3.827 \times 10^{26}\ \mathrm{W}$
- Stefan-Boltzmann constant $\sigma = 5.670 \times 10^{-8}\ \mathrm{W\ m^{-2}K^{-4}}$
- Water freezing point: $T = 273\,\mathrm{K}$
- Water boiling point: $T = 373\,\mathrm{K}$

Rewriting equation for distance: $a = \left(\frac{L_\star(1-A)}{16\pi\sigma T_{eq}^4}\right)^{1/2}$

For inner edge (373 K): $a_{inner} = \left(\frac{3.827\times10^{26}(1-0.3)}{16\pi(5.670\times10^{-8})(373)^4}\right)^{1/2} \approx 0.75\,\mathrm{AU}$

For outer edge (273 K): $a_{outer} = \left(\frac{3.827\times10^{26}(1-0.3)}{16\pi(5.670\times10^{-8})(273)^4}\right)^{1/2} \approx 1.30\,\mathrm{AU}$

Final Answer: The habitable zone lies approximately between **0.75 AU and 1.30 AU**.

1b. Is the Model an Overestimate or Underestimate?

The toy model is a **slight underestimate** of the habitable zone's actual extent.

Because it ignores atmospheric greenhouse effects and earth's actual surface temperature (\~288 K) is higher than its calculated equilibrium temperature (\~255 K).

Hence, atmospheric warming expands the habitable zone beyond the basic blackbody model.

Section II: Interpreting Atmospheric Absorption Spectra

2. Labeling Absorption Features in Earth's Atmosphere

Using known spectral features:

- **H2O (Water Vapor):** Broad absorption across IR (0.9–1.2 μm, 1.3–1.5 μm, 1.8–2.0 μm)

- **CO_2 (Carbon Dioxide):** Strong feature near 4.3 μm and 15 μm
- **O_2 (Molecular Oxygen):** Band near 0.76 μm
- **O_3 (Ozone):** Chappuis band (0.55–0.65 μm), strong UV band < 0.3 μm
- **CH_4 (Methane):** Bands near 1.6 μm and 2.3 μm

Note: Students would typically annotate these on a provided spectrum, but here are the wavelengths for labeling.

Section III: Characterizing Atmospheric Loss

3a. Escape Velocity of Terra II

Escape velocity: $v_e = \sqrt{\frac{2GM}{R}}$

Given:

- $M = 4M_\oplus = 4 \times 5.972 \times 10^{24} = 2.389 \times 10^{25}\ \mathrm{kg}$
- $R = 2R_\oplus = 2 \times 6.378 \times 10^{6} = 1.2756 \times 10^{7}\ \mathrm{m}$
- $G = 6.6743 \times 10^{-11}\ \mathrm{m^3 kg^{-1} s^{-2}}$

$$v_e = \sqrt{\frac{2\times 6.6743\times 10^{-11}\times 2.389\times 10^{25}}{1.2756\times 10^{7}}} \approx 15,830\,\mathrm{m/s} = 15.83\,\mathrm{km/s}$$

3b. Atmosphere Retention on Terra II

Given:

- Escape velocity: **15.83 km/s**
- XUV Flux: **100× Earth's**

From the Cosmic Shoreline, high XUV flux can strip atmospheres unless the escape velocity is very high. For 100× XUV, the required escape velocity is roughly 20 km/s.

So, Terra II likely does not retain a substantial atmosphere and would lie *below* the shoreline.