

SVM

① 画图，找 support vector

(最近且 $y_i(w^T x_i + w_0) = 1$) 或 距离 (离 hyper plane)

$$\left. \begin{array}{l} \text{support vector } y_i(w^T x_i + w_0) = 1 \\ \text{non-support } y_i > 1 \end{array} \right\} \begin{array}{l} \lambda_i \neq 0 \\ \lambda_i = 0 \end{array}$$

② 分类
③ 分类后，重新算 center.

hyper plane: $w^T x + w_0 = 0$

$\lambda_i = 0 \rightarrow$ correct classified

$x_i \neq 0 \rightarrow$ $\left\{ \begin{array}{l} \text{on margin} \\ \text{misclassified} \end{array} \right.$

K-Means

k-means:

class 1 center 1, class 2 center 2.

input x 到两 center 分类。

③ 分类后，重新算 center.

- ① RBF
- ② hidden layer 1 次。
pick $n_H \uparrow$ \propto $\frac{1}{n_H}$

$$\begin{aligned} Z &= w Y \\ W &= Z Y^+ \end{aligned}$$

$$③ \rho_{max} = \max \{ \|c_j - c_i\| \}$$

$$④ \sigma_j = \frac{\rho_{max}}{\sqrt{2 \cdot n_H}}$$

$$⑤ y_1(x_i) = e^{-\frac{\|x_i - c_1\|^2}{2\sigma_1^2}}, i = 1, 2, 3, 4$$

$$y_2(x_i) = e^{-\frac{\|x_i - c_2\|^2}{2\sigma_2^2}}$$

perceptron $a = [w_0, w]$

- sequential: ① $g(x) = a^t x$ $a = [0, w_1, w_2]$

~~if just one~~

- ② assign label ($g(x) \geq 0 \rightarrow y_k = 1$)
 $g(x) < 0 \rightarrow -1$)

2.

- multiclass perceptron:

$$\text{① } \hat{y}_k = \arg \max_i g_i(x_k)$$

$$g_i(x_k) = a_i^t x_k \leftarrow \text{augment } [1 \dots 1]$$

- ③ if $y_k \neq t_k$ (target label t_k) (misclassified)

$\rightarrow \text{update } a \leftarrow a + \gamma \frac{(t_k - y_k)}{\|x_k\|} x_k$

- ④ until all samples in correct class

$$\text{⑤ if } y_k \neq t_k \text{ mis}$$

$$a_{\text{right}} = a_{\text{right}} + \gamma \cdot x_k$$

$$\text{wrong} = \text{wrong} - \gamma x_k$$

sequential

~~if Aug + normal~~

- ① $g(x) = a^t x$

$$\begin{bmatrix} 1 & 1 & 1 & -1 & -1 \\ -x_1 & -x_2 \\ -x_3 & -x_4 \end{bmatrix} \quad \text{② if } g(x) < 0 \rightarrow x_k \text{ is misclassified.}$$

$\rightarrow \text{update } a \leftarrow a + \gamma x_k$

batch

- ① $g(x) = a^t x$

~~Aug + normal~~

- ② if $g(x) < 0 \rightarrow x_k$ misclassified

$$\text{sum-misclassified} = \sum x_k$$

until no change \uparrow margin

- ③ update $a \leftarrow a + \gamma \sum x_k$

epoch 2:

$$\text{seq: } w \leftarrow w + \gamma \left[t_k - H(w x_k) \right] x_k^t \leftarrow \text{Aug.}$$

$$\text{batch: } w \leftarrow w + \gamma \sum (t_k - H(w x_k)) x_k^t$$

$$\text{① } g(x) = a^t x$$

- ② assign label ($g(x) \geq 0 \rightarrow y_k = 1$)
 $g(x) < 0 \rightarrow y_k = -1$)

- ③ if $y_k \neq t_k$ (misclassified)

$$\text{④ } a \leftarrow a + \gamma \sum t_k \cdot x_k$$

$$\text{batch: } w \leftarrow w + \gamma \sum (t_k - H(w x_k)) x_k^t$$

Lecture 2.

y_k correctly classified: $a^t y_k > 0$ & y_k labelled w_2

→ Discriminant Function:

$$\left\{ \begin{array}{l} \text{Linear Discriminant} \\ \text{Generalised Linear} \end{array} \right\}$$

perception learning.

← learning driven by misclassified

← by gradient descent

← multi-class learning.

MSE learning.

← learning driven by all data

← by gradient descend (~~widow-Hoff~~ Widrow-Hoff).

Gradient descent: $\left\{ \begin{array}{l} \text{batch} \\ \text{sequential} \end{array} \right\}$

②. y_k is correctly classified $\Leftrightarrow a^t y_k > 0$.

margin. b

$$a^t y_k > b$$

• Gradient descent:

$$\text{cost: } J(a)$$

$$a \leftarrow a - \eta J(a)$$

η : learning rate, step size.

- perceptron learning \leftarrow multiple input, one output
- $a^t y$ ↓ boundary function.

$$J_p = \sum_{\text{misclassified}} (-a^t y)$$

• ▲ batch perceptron

$$\nabla J_p(a) = \sum_{\text{misclassified}} (-y)$$

sample normalisation

- △ $g(x) = a^t y$ $\left\{ \begin{array}{l} a = [w_0, w_1^T] \\ y = [1, x]^T \end{array} \right.$ boundary.
- △ generalised linear Discriminant Function:

$$g(x) = w_0 + w_1 x_1 + w_2 x_2 + \dots + w_n x_n$$

$$= a^t y$$

$$\left\{ \begin{array}{l} a^t = [w_0, w_1, \dots, w_n, w_2, \dots] \\ y^t = [1, x_1, x_2, \dots, x_n, \dots] \end{array} \right.$$

y_k correctly classified: $a^t y_k > 0$ & y_k labelled w_2

at $y_k < 0$ & y_k labelled w_2

0 $y \leftarrow -y$ $\forall y \in w_2$.

y_k is correctly classified $\Leftrightarrow a^t y_k > 0$.

margin. b

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$$= a^t y$$

$$\left\{ \begin{array}{l} a^t = [w_0, w_1, \dots, w_n, w_2, \dots] \\ y^t = [1, x_1, x_2, \dots, x_n, \dots] \end{array} \right.$$

• D multiclass perception

n.

initialise a_c for each class
 for each sample (x^k, y^k)
 \rightarrow classify : $c = \arg \max g_c(x)$

if y^k is misclassified $c' \neq c$:

$$a_{ck} \leftarrow a_{ck} + \gamma y^k$$

$$a_{c'} \leftarrow a_{c'} - \gamma y^k$$

until a not change

MSE : All sample used

$$J_s(a) = \|Ya - b\|^2 \quad J_s(a) = (aty - bk)^2$$

Widrow-Hoff (LMS):

$$\nabla J_s(a) = 2Y^t(Ya - b)$$

$$a \leftarrow a - \gamma Y^t(Ya - b) \quad \text{batch}$$

$$\text{sequential: } a \leftarrow a - \gamma (a^t y^k - b_k) y^k$$

until

k-NN : k nearest neighbour

- ① new sample
- ② existing samples (with label)
- ③ compute distance between new sample & existing ones.

- ④ choose first k labels

margin b.

$$Ya = b$$

↓ weight

avg + normalised

Lecture 3.

Linear Threshold Units.

→ Delta learning

Competitive network

Negative Feedback Network

Autoencoder Network.

Delta learning:

- initialise w, y .
- For each sample (x_k, t_k)
 - update weight.

$$w \leftarrow w + \gamma [t_k - H(w^T x_k)] x_k^t$$

until all samples correctly classified.

Negative feedback

- Activation
- Initial. y_0, w .

$y = x - w^T y$

$$y = y + \alpha w$$

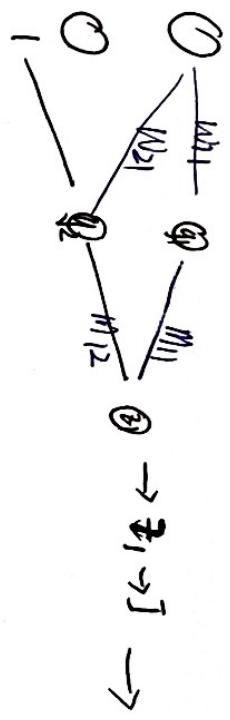
Learning: $w \leftarrow w + \gamma y e^t$

Back propagation

$$\Delta w_{j0} = -\gamma \frac{\partial J}{\partial w_{j0}}$$

$$= -\gamma \frac{\partial J}{\partial z_1} \frac{\partial z_1}{\partial net_{z_1}} \frac{\partial net_{z_1}}{\partial y_2} \frac{\partial y_2}{\partial net_{y_2}} \frac{\partial net_{y_2}}{\partial w_{j0}}$$

$$w_{j0} \leftarrow w_{j0} + \Delta w_{j0}$$



$$w = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{12} \end{bmatrix} \quad m = [m_1 \quad m_0]$$

$$z = m \cdot (w \cdot x + w_0) + m_0$$