

**Problem 1.** [MATLAB/Python/C] Implement two (possibly more, if you choose to) functions that provide all the computational kinematics quantities that are associated with the basic GCons  $\Phi^{DP2}$  and  $\Phi^D$ . Stick with the  $\mathbf{r} - \mathbf{p}$  formulation. Specifically, your code should be able to return any or all of the following quantities:

- (i) The value of the expression of the constraint
- (ii) The right-hand side of the velocity equation  $\nu$
- (iii) The right-hand side of the acceleration equation  $\gamma$
- (iv) The expression of the partial derivatives  $\Phi_{\mathbf{r}}$  and  $\Phi_{\mathbf{p}}$

You might decide to have one function for  $\Phi^{DP2}$  and yet another function for  $\Phi^D$  to produce for these two GCons the quantities in (i) through (iv) above. Since you don't need all these quantities all the time, you should devise a methodology (perhaps using flags) to instruct the subroutine what quantities are actually needed.

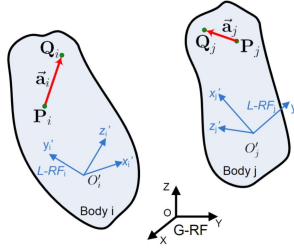
$$\Phi^{DP1}(i, \mathbf{a}_i, j, \mathbf{a}_j, f(t)) = \mathbf{a}_i^T \mathbf{A}_i^T \mathbf{A}_j \mathbf{a}_j - f(t) = 0$$

$$\dot{\gamma}^{DP1}(i, \mathbf{a}_i, j, \mathbf{a}_j, f(t)) = -\mathbf{a}_i^T \mathbf{B}(\mathbf{p}_j, \mathbf{a}_j) \dot{\mathbf{p}}_j - \mathbf{a}_j^T \mathbf{B}(\mathbf{p}_i, \mathbf{a}_i) \dot{\mathbf{p}}_i - 2\dot{\mathbf{a}}_i^T \mathbf{a}_j + \dot{f}(t)$$

$$\nu^{DP1} = -\frac{\partial \Phi^{DP1}}{\partial t} = \frac{\partial f}{\partial t}$$

$$\Phi_{\mathbf{q}}^{DP1} = \begin{bmatrix} \frac{\partial \Phi^{DP1}}{\partial \mathbf{r}_i} & \frac{\partial \Phi^{DP1}}{\partial \mathbf{p}_i} & \frac{\partial \Phi^{DP1}}{\partial \mathbf{r}_j} & \frac{\partial \Phi^{DP1}}{\partial \mathbf{p}_j} \end{bmatrix}$$

$$\Phi_{\mathbf{q}}^{DP1} = \begin{bmatrix} \mathbf{0}_{1 \times 3} & \mathbf{a}_j^T \mathbf{B}(\mathbf{p}_i, \mathbf{a}_i) & \mathbf{0}_{1 \times 3} & \mathbf{a}_i^T \mathbf{B}(\mathbf{p}_j, \mathbf{a}_j) \end{bmatrix}$$



$$\Phi^{DP2}(i, \mathbf{a}_i, \mathbf{s}_i^P, j, \mathbf{s}_j^Q, f(t)) = \mathbf{a}_i^T \mathbf{A}_i^T \mathbf{d}_{ij} - f(t) = \mathbf{a}_i^T \mathbf{A}_i^T (\mathbf{r}_j + \mathbf{A}_j \mathbf{s}_j^Q - \mathbf{r}_i - \mathbf{A}_i \mathbf{s}_i^P) - f(t) = 0$$

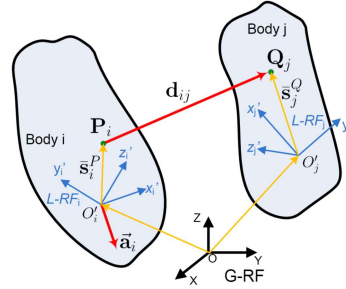
$$\dot{\gamma}^{DP2}(i, \mathbf{a}_i, \mathbf{s}_i^P, j, \mathbf{s}_j^Q, f(t)) = -\mathbf{a}_i^T \mathbf{B}(\mathbf{p}_j, \mathbf{s}_j^Q) \dot{\mathbf{p}}_j + \mathbf{a}_i^T \mathbf{B}(\mathbf{p}_i, \mathbf{s}_i^P) \dot{\mathbf{p}}_i - \mathbf{d}_{ij}^T \mathbf{B}(\mathbf{p}_i, \mathbf{a}_i) \dot{\mathbf{p}}_i - 2\dot{\mathbf{a}}_i^T \mathbf{d}_{ij} + \dot{f}(t)$$

$$\nu^{DP2} = -\frac{\partial \Phi^{DP2}}{\partial t} = \frac{\partial f}{\partial t}$$

$$\Phi_{\mathbf{q}, \mathbf{q}_i}^{DP2}(\mathbf{a}_i, \mathbf{d}_{ij}) = \mathbf{a}_i^T (\mathbf{d}_{ij})_{\mathbf{q}_i, \mathbf{q}_j} + \mathbf{d}_{ij}^T (\mathbf{a}_i)_{\mathbf{q}_i, \mathbf{q}_j}$$

$$= \mathbf{a}_i^T \begin{bmatrix} -\mathbf{I}_3 & -\mathbf{B}(\mathbf{p}_i, \mathbf{s}_i^P) & \mathbf{I}_3 & \mathbf{B}(\mathbf{p}_j, \mathbf{s}_j^Q) \end{bmatrix} + \mathbf{d}_{ij}^T \begin{bmatrix} \mathbf{0} & \mathbf{B}(\mathbf{p}_i, \mathbf{a}_i) & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

$$= \begin{bmatrix} -\mathbf{a}_i^T & \mathbf{d}_{ij}^T \mathbf{B}(\mathbf{p}_i, \mathbf{a}_i) - \mathbf{a}_i^T \mathbf{B}(\mathbf{p}_i, \mathbf{s}_i^P) & \mathbf{a}_i^T & \mathbf{a}_i^T \mathbf{B}(\mathbf{p}_j, \mathbf{s}_j^Q) \end{bmatrix}$$



$$\Phi^D(i, \mathbf{s}_i^P, j, \mathbf{s}_j^Q, f(t)) = \mathbf{d}_{ij}^T \mathbf{d}_{ij} - f(t) = (\mathbf{r}_j + \mathbf{A}_j \mathbf{s}_j^Q - \mathbf{r}_i - \mathbf{A}_i \mathbf{s}_i^P)^T (\mathbf{r}_j + \mathbf{A}_j \mathbf{s}_j^Q - \mathbf{r}_i - \mathbf{A}_i \mathbf{s}_i^P) - f(t) = 0$$

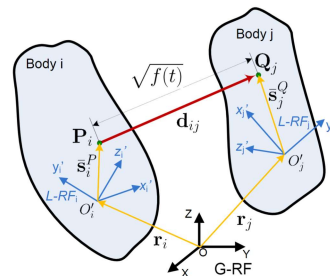
$$\dot{\gamma}^D(i, \mathbf{s}_i^P, j, \mathbf{s}_j^Q, f(t)) = -2\mathbf{d}_{ij}^T \mathbf{B}(\mathbf{p}_j, \mathbf{s}_j^Q) \dot{\mathbf{p}}_j + 2\mathbf{d}_{ij}^T \mathbf{B}(\mathbf{p}_i, \mathbf{s}_i^P) \dot{\mathbf{p}}_i - 2\dot{\mathbf{d}}_{ij}^T \mathbf{d}_{ij} + \dot{f}(t)$$

$$\nu^D = -\frac{\partial \Phi^D}{\partial t} = \frac{\partial f}{\partial t}$$

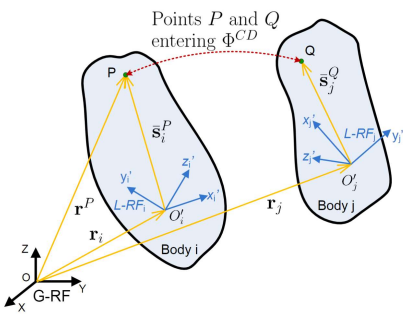
$$\Phi_{\mathbf{q}, \mathbf{q}_i}^D = (\mathbf{d}_{ij}^T \mathbf{d}_{ij})_{\mathbf{q}_i, \mathbf{q}_j} = 2\mathbf{d}_{ij}^T [\mathbf{d}_{ij}]_{\mathbf{q}_i, \mathbf{q}_j}$$

$$= 2\mathbf{d}_{ij}^T \begin{bmatrix} -\mathbf{I}_3 & -\mathbf{B}(\mathbf{p}_i, \mathbf{s}_i^P) & \mathbf{I}_3 & \mathbf{B}(\mathbf{p}_j, \mathbf{s}_j^Q) \end{bmatrix}$$

$$= \begin{bmatrix} -2\mathbf{d}_{ij}^T & -2\mathbf{d}_{ij}^T \mathbf{B}(\mathbf{p}_i, \mathbf{s}_i^P) & 2\mathbf{d}_{ij}^T & 2\mathbf{d}_{ij}^T \mathbf{B}(\mathbf{p}_j, \mathbf{s}_j^Q) \end{bmatrix}$$



$$\begin{aligned} \Phi^{CD}(\mathbf{c}, i, \bar{\mathbf{s}}_i^P, j, \bar{\mathbf{s}}_j^Q, f(t)) &= \mathbf{c}^T \mathbf{d}_{ij} - f(t) = \mathbf{c}^T (\mathbf{r}_j + \mathbf{A}_j \bar{\mathbf{s}}_j^Q - \mathbf{r}_i - \mathbf{A}_i \bar{\mathbf{s}}_i^P) - f(t) = 0 \\ \dot{\gamma}^{CD}(\mathbf{c}, i, \bar{\mathbf{s}}_i^P, j, \bar{\mathbf{s}}_j^Q, f(t)) &= \mathbf{c}^T \mathbf{B}(\mathbf{p}_i, \bar{\mathbf{s}}_i^P) \dot{\mathbf{p}}_i - \mathbf{c}^T \mathbf{B}(\mathbf{p}_j, \bar{\mathbf{s}}_j^Q) \dot{\mathbf{p}}_j + \dot{f}(t) \\ \nu^D &= -\frac{\partial \Phi^D}{\partial t} = \frac{\partial f}{\partial t} \\ \Phi_{\mathbf{q}_i, \mathbf{q}_j}^{CD} &= (\mathbf{c}^T \mathbf{d}_{ij})_{\mathbf{q}_i, \mathbf{q}_j} = \mathbf{c}^T [\mathbf{d}_{ij}]_{\mathbf{q}_i, \mathbf{q}_j} \\ &= \mathbf{c}^T \begin{bmatrix} -\mathbf{I}_3 & -\mathbf{B}(\mathbf{p}_i, \bar{\mathbf{s}}_i^P) & \mathbf{I}_3 & \mathbf{B}(\mathbf{p}_j, \bar{\mathbf{s}}_j^Q) \end{bmatrix} \\ &= \begin{bmatrix} -\mathbf{c}^T & -\mathbf{c}^T \mathbf{B}(\mathbf{p}_i, \bar{\mathbf{s}}_i^P) & \mathbf{c}^T & \mathbf{c}^T \mathbf{B}(\mathbf{p}_j, \bar{\mathbf{s}}_j^Q) \end{bmatrix} \end{aligned}$$



$$\mathbf{A} = [(2e_0^2 - 1)\mathbf{I} + 2(\mathbf{e}\mathbf{e}^T + e_0\tilde{\mathbf{e}})] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = 2 \begin{bmatrix} e_0^2 + e_1^2 - \frac{1}{2} & e_1e_2 - e_0e_3 & e_1e_3 + e_0e_2 \\ e_1e_2 + e_0e_3 & e_0^2 + e_2^2 - \frac{1}{2} & e_2e_3 - e_0e_1 \\ e_1e_3 - e_0e_2 & e_2e_3 + e_0e_1 & e_0^2 + e_3^2 - \frac{1}{2} \end{bmatrix}$$

$$\mathbf{B}(\mathbf{p}, \bar{\mathbf{a}}) \equiv 2 \begin{bmatrix} (e_0 \mathbf{I}_3 + \tilde{\mathbf{e}}) \bar{\mathbf{a}} & \mathbf{e} \bar{\mathbf{a}}^T - (e_0 \mathbf{I}_3 + \tilde{\mathbf{e}}) \tilde{\bar{\mathbf{a}}} \end{bmatrix}$$

$$[\mathbf{A}(\mathbf{p}) \bar{\mathbf{a}}]_{\mathbf{p}} = \mathbf{a}_{\mathbf{p}} = \mathbf{B}(\mathbf{p}, \bar{\mathbf{a}}) \in \mathbb{R}^{3 \times 4}$$

$$\mathbf{p} = \begin{bmatrix} e_0 \\ e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

$$\begin{aligned} \mathbf{a}_i &= \mathbf{A}_i \bar{\mathbf{a}}_i \Rightarrow \dot{\mathbf{a}}_i = \mathbf{B}(\mathbf{p}_i, \bar{\mathbf{a}}_i) \dot{\mathbf{p}}_i \Rightarrow \ddot{\mathbf{a}}_i = \mathbf{B}(\dot{\mathbf{p}}_i, \bar{\mathbf{a}}_i) \dot{\mathbf{p}}_i + \mathbf{B}(\mathbf{p}_i, \bar{\mathbf{a}}_i) \ddot{\mathbf{p}}_i \\ \dot{\mathbf{d}}_{ij} &= \dot{\mathbf{r}}_j + \mathbf{B}(\mathbf{p}_j, \bar{\mathbf{s}}_j^Q) \dot{\mathbf{p}}_j - \dot{\mathbf{r}}_i - \mathbf{B}(\mathbf{p}_i, \bar{\mathbf{s}}_i^P) \dot{\mathbf{p}}_i \\ \ddot{\mathbf{d}}_{ij} &= \ddot{\mathbf{r}}_j + \mathbf{B}(\mathbf{p}_j, \bar{\mathbf{s}}_j^Q) \ddot{\mathbf{p}}_j + \mathbf{B}(\dot{\mathbf{p}}_j, \bar{\mathbf{s}}_j^Q) \dot{\mathbf{p}}_j - \ddot{\mathbf{r}}_i - \mathbf{B}(\mathbf{p}_i, \bar{\mathbf{s}}_i^P) \ddot{\mathbf{p}}_i - \mathbf{B}(\dot{\mathbf{p}}_i, \bar{\mathbf{s}}_i^P) \dot{\mathbf{p}}_i \end{aligned}$$

$$aside: \tilde{\tilde{\mathbf{a}}}_i = \begin{bmatrix} 0 & -a_x & a_y \\ a_x & 0 & -a_y \\ -a_y & a_x & 0 \end{bmatrix}$$

**Problem 2.** [MATLAB/Python/C] In an input file "revJoint.mdl" (or in your driver.py file, if you chose to hardcode the GCon attributes), define your first mechanism model that will have one pendulum hanging as shown in Figure 1. The pendulum has  $L = 2$ , it is symmetric as shown in the Figure 1, and it is attached to ground at its tip (point Q). The revolute joint on the ground is at point O of location (0,0,0); that is, the origin of the G-RF. Note that the pendulum moves in the global OYZ plane; also note that the O'x'y' plane of the L-RF is parallel to the OYZ plane of the G-RF. The pendulum is subjected to a motion specified as  $f(t) = \frac{\pi}{4} \cos(2t)$ . One way to implement this motion is through a DPl basic geometric constraint.

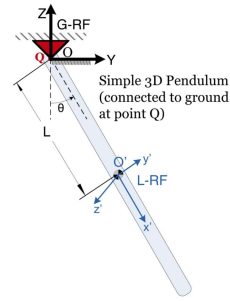


Figure 1: Pendulum with revolute joint.

Your code should be able to provide the following information at  $t = 0$ : value of  $\Phi(\mathbf{q}, t)$ ,  $\Phi_{\mathbf{q}}$ ,  $\nu$ , and  $\gamma$ . All units used are SI. The program that will be executed for this problem should be named "simEngine3D-A6P2" followed by the filename extension of your programming language.

joint : revolute  $\Rightarrow$   $3 \times \text{C0}$   
 $2 \times \text{DPl}$

$$\text{C0}_1 : \mathbf{z} \cdot (\dot{\mathbf{r}} - \dot{\mathbf{r}}^a) = 0 \quad \text{C0}_2 : \hat{\mathbf{j}} \cdot (\dot{\mathbf{r}} - \dot{\mathbf{r}}^a) = 0 \quad \text{C0}_3 : \hat{\mathbf{k}} \cdot (\dot{\mathbf{r}} - \dot{\mathbf{r}}^a) = 0$$

$\hat{\mathbf{z}}^a = (-2, 0, 0)^T \Rightarrow f(t) = 0$

$$\text{DPl}_1 : \hat{\mathbf{z}}' \cdot \hat{\mathbf{x}} = 0 \quad \text{DPl}_2 : \hat{\mathbf{y}}' \cdot \hat{\mathbf{x}} = 0 \quad \Rightarrow f(t) = 0$$

$$\text{Driving : } \hat{\mathbf{x}}' \cdot -\hat{\mathbf{z}} = |\hat{\mathbf{x}}'| |\hat{\mathbf{z}}| \cos(\theta(t)) = \cos\left(\frac{\pi}{4} \cos(2t)\right)$$

⤴ Note: I had a problem in prob. 3 using this; I would get the absolute val. of the answer for y position of the origin  
 $\Rightarrow$  I switch this to:

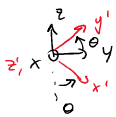
$$\hat{\mathbf{y}}' \cdot (-\hat{\mathbf{z}}) = 1 \cdot 1 \cdot \cos\left(\theta(t) + \frac{\pi}{2}\right) = \cos\left(\frac{\pi}{4} \cos(2t) + \frac{\pi}{2}\right)$$

$$\Rightarrow f(t) = \cos\left(\frac{\pi}{4} \cos(2t) + \frac{\pi}{2}\right)$$

$$f'(t) = \frac{\pi}{2} \sin(2t) \cos\left(\frac{\pi}{4} \cos(2t)\right)$$

$$f''(t) = \pi \cos(2t) \cos\left(\frac{\pi}{4} \cos(2t)\right) + \frac{\pi^2}{4} \sin^2(2t) \sin\left(\frac{\pi}{4} \cos(2t)\right)$$

initial conditions



$$\begin{aligned} \hat{\mathbf{x}}' &= 0 \hat{\mathbf{x}} + \sin \theta \hat{\mathbf{y}} - \cos \theta \hat{\mathbf{z}} \\ \hat{\mathbf{y}}' &= 0 \hat{\mathbf{x}} + \cos \theta \hat{\mathbf{y}} + \sin \theta \hat{\mathbf{z}} \\ \hat{\mathbf{z}}' &= 1 \hat{\mathbf{x}} + 0 \hat{\mathbf{y}} + 0 \hat{\mathbf{z}} \end{aligned}$$

$$\Rightarrow \mathbf{A} = \begin{bmatrix} \hat{\mathbf{x}}' & \hat{\mathbf{y}}' & \hat{\mathbf{z}}' \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ \sin \theta & \cos \theta & 0 \\ -\cos \theta & \sin \theta & 0 \end{bmatrix}$$

$$\theta(0) = \frac{\pi}{4} \cos(2 \cdot 0) = \frac{\pi}{4}$$

$$e_0 = \left( \frac{\text{tr}(\mathbf{A}) + 1}{4} \right)^{1/2} = \left( \frac{\cos(\frac{\pi}{4}) + 1}{4} \right)^{1/2} = 0.453281 \dots$$

• Step 1: Compute  $e_0^2$  as

$$e_0^2 = \frac{\text{tr}(\mathbf{A}) + 1}{4}$$

$$e_0 = \left( \frac{4}{1} \right)^{1/2} = 2$$

$$e_1 = \left( \frac{2(0) - \cos(\frac{\pi}{4}) + 1}{4} \right)^{1/2} = 0.27 \dots$$

$$e_2 = .5 \dots$$

$$e_3 = 0.27 \dots$$

$$\vec{r}_{n,i} = A \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1.41 \\ -1.41 \end{bmatrix}$$

use to find the analytical solns

$$\mathbf{r}^P = \mathbf{r} + \mathbf{s}^P = \mathbf{r} + \mathbf{A}\mathbf{s}^P$$

$$\dot{\mathbf{v}}^P = \frac{d\dot{\mathbf{r}}^P}{dt} = \dot{\mathbf{r}} + \dot{\mathbf{s}}^P = \dot{\mathbf{r}} + \dot{\boldsymbol{\omega}} \times \mathbf{s}^P$$

$$\ddot{\mathbf{a}}^P \equiv \frac{d^2\dot{\mathbf{r}}^P}{dt^2} = \ddot{\mathbf{r}} + \dot{\boldsymbol{\omega}} \times \dot{\mathbf{s}}^P + \ddot{\boldsymbol{\omega}} \times \mathbf{s}^P + \dot{\boldsymbol{\omega}} \times \dot{\mathbf{s}}^P$$

$$\dot{\mathbf{r}}^P = \dot{\mathbf{r}} + \dot{\mathbf{s}}^P = \dot{\mathbf{r}} + \dot{\mathbf{A}}\mathbf{s}^P = \dot{\mathbf{r}} + \dot{\boldsymbol{\omega}}\mathbf{A}\mathbf{s}^P = \dot{\mathbf{r}} + \dot{\boldsymbol{\omega}}\mathbf{s}^P$$

$$\mathbf{a}^P \equiv \ddot{\mathbf{r}}^P = \ddot{\mathbf{r}} + \ddot{\boldsymbol{\omega}}\mathbf{A}\mathbf{s}^P + \dot{\boldsymbol{\omega}}\mathbf{A}\dot{\mathbf{s}}^P = \ddot{\mathbf{r}} + \ddot{\boldsymbol{\omega}}\mathbf{s}^P + \dot{\boldsymbol{\omega}}\mathbf{s}^P$$

$$\vec{r}^Q = \vec{r}^O + \vec{a}^Q$$

$$\vec{a}^Q = \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{a}_i = \mathbf{A}_i \ddot{\mathbf{a}}_i \Rightarrow \ddot{\mathbf{a}}_i = \mathbf{B}(\mathbf{p}_i, \ddot{\mathbf{a}}_i) \ddot{\mathbf{p}}_i \Rightarrow \ddot{\mathbf{a}}_i = \mathbf{B}(\dot{\mathbf{p}}_i, \ddot{\mathbf{a}}_i) \ddot{\mathbf{p}}_i + \mathbf{B}(\mathbf{p}_i, \ddot{\mathbf{a}}_i) \ddot{\mathbf{p}}_i$$

construct a global  $\Phi, \nu, \delta, \Phi_p$

$$\Phi^K(\mathbf{q}, t) = \begin{bmatrix} \Phi^G(\mathbf{q}) \\ \Phi^D(\mathbf{q}, t) \end{bmatrix} = \mathbf{0}_{6nb} \quad \Phi^F(\mathbf{q}, t) = \begin{bmatrix} \Phi^G(\mathbf{q}) \\ \Phi^D(\mathbf{q}, t) \\ \Phi^P(\mathbf{p}) \end{bmatrix} = \mathbf{0}_{7nb}$$

$$\nu^P = \mathbf{0}_{nb} \quad \& \quad \gamma^P = \begin{bmatrix} -2\dot{\mathbf{p}}_1^T \dot{\mathbf{p}}_1 \\ \dots \\ -2\dot{\mathbf{p}}_{nb}^T \dot{\mathbf{p}}_{nb} \end{bmatrix}$$

$$\Phi_p^p = \begin{bmatrix} 2\rho_i \\ \dots \end{bmatrix}$$

$$\Phi_q^F(\mathbf{q}^{(k)}) \Delta \mathbf{q}^{(k+1)} = \Phi(\mathbf{q}^{(k)}, t)$$

$$\mathbf{q}^{(k+1)} = \mathbf{q}^{(k)} - \Delta \mathbf{q}^{(k+1)}$$

solve for jacobian global  $\Rightarrow$  newton  
raphson to get  $\mathbf{q}$  at each time step

$$\Phi_q \dot{\mathbf{q}} = \gamma_{7nb}$$

$$\Phi_q \dot{\mathbf{q}} = \nu_{7nb}$$

use these to solve for  $\dot{\mathbf{q}}, \ddot{\mathbf{q}}$

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e0_ini =
0.6533
e1_ini =
0.2706
e2_ini =
0.5000
e3_ini =
0.2706
r_ini =
0
1.4142
-1.4142

Phi
ans =
1.0e-15 *
-0.3053
0
0
0.6106
0
0
0
-0.1110

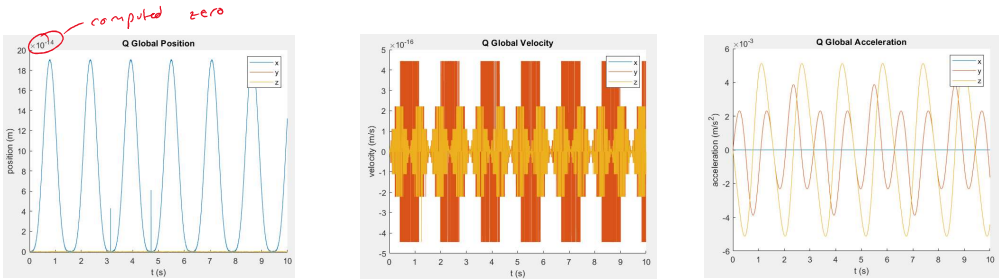
nu
ans =
0
0
0
0
0
0
0
0
0

gamma
ans =
0
0
0
0
0
0
2.2214
0
Jacobian
ans =
0 0 0 1.3066 0.5412 -1.3066 -0.5412
0 0 0 -0.5412 1.3066 0.5412 -1.3066
1.0000 0 0 -2.6131 -1.0824 2.6131 1.0824
0 1.0000 0 -1.0824 -2.6131 -1.0824 -2.6131
0 0 1.0000 2.6131 -1.0824 2.6131 -1.0824
0 0 0 -0.5412 -1.3066 -0.5412 -1.3066
0 0 0 1.3066 0.5412 1.3066 0.5412

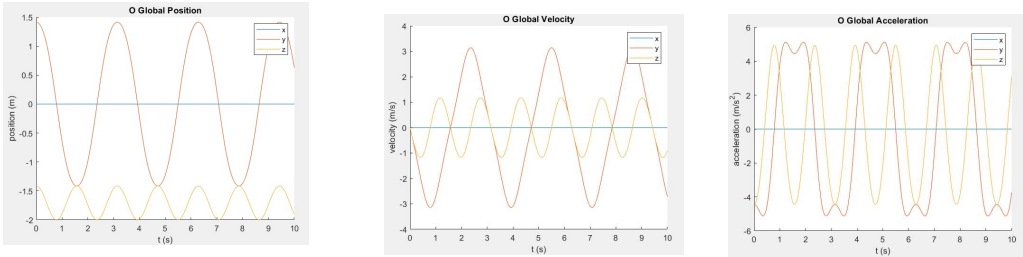
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**Problem 3.** [MATLAB/Python/C] This problem builds on Problem 2. The schematic of the mechanism is shown in Figure 1. The rigid body is subjected to a motion specified as  $f(t) = \frac{\pi}{4} \cos(2t)$ .

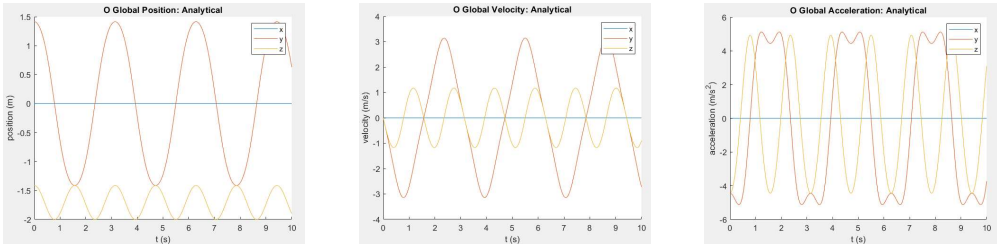
You will need to carry out a Kinematics Analysis for the mechanism for 10 seconds of its evolution. To this end, use a time grid with time steps of  $\Delta t = 10^{-3}$ . Add **six plots** to your report including location/velocity/acceleration of point  $O'/Q$  in the G-RF as a function of time, and discuss the results that you obtained for point  $Q$ . The program that will be executed for this problem should be named “simEngine3D-A6P3” followed by the filename extension of your programming language.



these show approx. computed zeros or noise w/in approx. order of the tolerance  $\Rightarrow$  they are  $\approx 0$ . Note: the acceleration term is slightly larger  $\Rightarrow$  could have bug



solved very close to analytical



analytical