6 Algorithmic Journeys with Concepts

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Rails Reactor / Giphy

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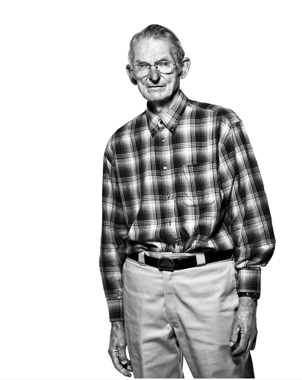
The Software Industry is Not Industrialized

Software components (routines), to be widely applicable to different machines and users, should be available in families arranged according to precision, robustness, generality and time-space performance.



A Familiar Example. Douglas McIlroy about sin

Dimensions along which we wish to have variablity:

1. precision, for which perhaps ten different approximating functionsmight suffice
2. floating vs fixed computation
3. argument ranges $[0, \pi/2]$, $[0, 2pi]$, also $[-\pi/2, pi/2]$, $[-\pi, \pi]$, $[-big, +big]$
4. robustness - ranging from no argument validation through signaling of complete loss of significance, to signaling of specified range violations

Periodic Table

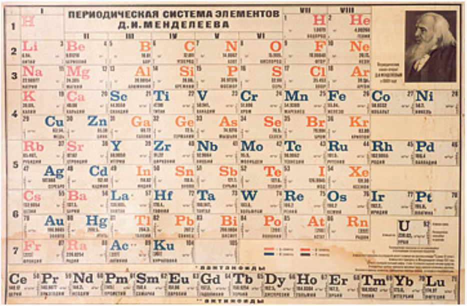
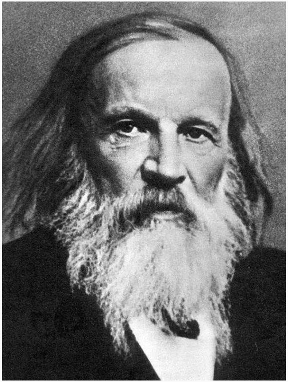


Figure 1: A Russian periodic table based on Dmitri Mendeleyev's original table of 1869.



Lists every species of plant known at the time, classified into genera. It is the first work to consistently apply binomial names and was the starting point for the naming of plants.

Elements

1. Definitions
2. Postulates
3. Common notions

Common Notions

1. Things which are equal to the same thing are also equal to one other.
2. If equals be added to equals, the wholes are equal.
3. If equals be subructed from equals, the remainders are equal.
4. Things which coincide with one another are equal to one another.
5. The whole is greater than the part.

Basic idea

The essence of generic programming lies in the idea of concepts. A concept is a way of describing a family of related object types.

Natural Science	**Mathematics**	**Programming**	**Programming Examples**
genus	theory	concept	Integral, Character
species	model	type or class	uint8_t, char
individual	element	instance	01000001(65, 'A')

Definitions

1. Datum
2. Value
3. Value type
4. Object
5. Object type

Datum

Definition
A **datum** is a sequence of bits.

Example
01000001 is an example of a datum.

Value

Definition
A **value is** a **datum** together with its interpretation.

Example
The **datum** 01000001 might have the interpretation of the integer 65, or the character “A”.

Explanation
Every **value** must be associated with a **datum** in memory; there is no way to refer to disembodied **values** in modern programming languages.

Value type

Definition
A **value type** is a set of values sharing a common interpretation.

Object

Definition
An **object** is a collection of bits in memory that contain a **value** of a given **value type**.

Explanation
An **object** is immutable if the value never changes, and mutable otherwise. An object is unrestricted if it can contain any **value** of its **value type**.

Object type

Definition

An **object type** is a uniform method of storing and retrieving **values** of a given **value type** from a particular **object** when given its address.

Programming with concepts

Regular type

Operation

1. Copy construction
2. Assignment
3. Equality
4. Destruction

Semantic

$$\forall a\ \forall b\ \forall c : T\ a(b) \implies (b = c \implies a = c)$$

$$\forall a\ \forall b\ \forall c : a \leftarrow b \implies (b = c \implies a = c)$$

$$\forall f \in RegularFunction : a = b \implies f(a) = f(b)$$

Semiregular type

Operation

1. Copy construction
2. Assignment
3. Destruction

Semantic

$$\forall a\ \forall b\ \forall c : T\ a(b) \implies (b = c \implies a = c)$$

$$\forall a\ \forall b\ \forall c : a \leftarrow b \implies (b = c \implies a = c)$$

$$\forall f \in RegularFunction : a = b \implies f(a) = f(b)$$

Concepts

$$
\begin{aligned}
\mathit{Readable}(T) \triangleq\ & \mathit{Regular}(T) \land \\
& \mathit{ValueType} : \mathit{Readable} \rightarrow \mathit{Regular} \land \\
& \mathit{source} : T \rightarrow \mathit{ValueType}(T) \land
\end{aligned}
$$

Concepts

$$
\begin{aligned}
Writable(T) \triangleq\ & Regular(T) \land \\
& ValueType : Writable \to Regular \land \\
& (\forall x \in T)(\forall v \in ValueType(T))\ sink(x) \leftarrow v \\
& \quad is\ a\ well-formed\ statement \\
& source : T \to ValueType(T) \land
\end{aligned}
$$

Concepts

$$
\begin{aligned}
Iterator(T) \triangleq\ & Regular(T) \wedge \\
& DistanceType : Iterator \to Integer \wedge \\
& successor : T \to T \wedge \\
& successor\ is\ not\ necessarily - regular
\end{aligned}
$$

Concepts

$$\mathit{ForwardIterator}(T) \triangleq \mathit{Iterator}(T) \land \mathit{regular\_unary\_function}(\mathit{successor})$$

Concepts

$$
\begin{aligned}
\mathit{BidirectionalIterator}(T) \triangleq\ & \mathit{ForwardIterator}(T) \wedge \\
& \mathit{predecessor} : T \rightarrow T \wedge \\
& \mathit{predecessor\ takes\ constant\ type} \wedge \\
& (\forall i \in T) \mathit{successor}(i) \mathit{isdefined} \implies \\
& \quad \mathit{predecessor}(\mathit{successor}(i))\ \mathit{is\ defined} \\
& \quad \mathit{and\ equals\ to}\ i \wedge \\
& (\forall i \in T) \mathit{predecessor}(i)\ \mathit{is\ defined} \implies \\
& \quad \mathit{successor}(\mathit{predecessor}(i))\ \mathit{is\ defined} \\
& \quad \mathit{and\ equals\ to}\ i
\end{aligned}
$$

Concepts

$$
\begin{aligned}
\mathit{IndexedIterator}(T) \triangleq\ & \mathit{ForwardIterator}(T) \land \\
& + : T \text{ } x \text{ } \mathit{DifferenceType}(T) \rightarrow T \land \\
& - : T \text{ } x \text{ } T \rightarrow \mathit{DifferenceType}(T) \land \\
& + \textit{ takes constant time} \\
& - \textit{ takes constant time}
\end{aligned}
$$

Concepts

$$
\begin{aligned}
\mathit{RandomAccessIterator}(T) \triangleq\ & \mathit{BidirectionalIterator}(T) \land \\
& \mathit{IndexedIterator}(T) \land \\
& \mathit{TotallyOrdered}(T) \land \\
& (\forall i, j \in T) i < j \iff i \prec j \land \\
& \mathit{DifferenceType} : \\
& \quad \mathit{RandomAccessIterator} \rightarrow \mathit{Integer} \land \\
& + : T\ x\ \mathit{DifferenceType}(T) \rightarrow T \land \\
& - : T\ x\ \mathit{DifferenceType}(T) \rightarrow T \land \\
& - : T\ x\ T \rightarrow \mathit{DifferenceType}(T) \land \\
& < \textit{takes constant time} \land \\
& - \textit{between and iterator and an integer} \\
& \quad \textit{takes constant time}
\end{aligned}
$$

Concepts

$$
\begin{aligned}
\mathit{FunctionalProcedure}(F) \triangleq{}& F \textit{ is a regular procedure defined on regular} \\
& \textit{types}: \textit{ replacing its inputs with equal objects} \\
& \textit{results in equal output objects.}
\end{aligned}
$$

$$
\begin{aligned}
\mathit{UnaryFunction}(F) \triangleq{}& \mathit{FunctionalProcedure}(F) \wedge \mathit{Arity}(F) = 1 \\
& \wedge\ \mathit{Domain} : \mathit{UnaryFunction} \to \mathit{Regular} \\
& F \mapsto \mathit{InputType}(F, 0)
\end{aligned}
$$

$$
\begin{aligned}
\mathit{HomogeneousFunction}(F) \triangleq{}& \mathit{FunctionalProcedure}(F) \wedge \mathit{Arity}(F) > 0 \\
& \wedge (\forall i, j \in \mathbb{N})(i, j < \mathit{Arity}(F)) \implies (\mathit{InputType}(F, i) = \mathit{InputType}(F, j)) \\
& \wedge \mathit{Domain} : \mathit{HomogeneousFunction} \to \mathit{Regular} \\
& F \implies \mathit{InputType}(F, 0)
\end{aligned}
$$

Concepts

$$Predicate(P) \triangleq FunctionalProcedure(F) \land Codomain(P) = bool$$

$$HomogeneousPredicate(P) \triangleq Predicate(P) \land HomogeneousFunction(P)$$

$$Relation(R) \triangleq HomogeneousPredicate(R) \land Arity(R) = 2$$

$$TotallyOrdered(T) \triangleq Regular(T) \land <: T\ x\ T \to bool \land total\_ordering(<)$$

$$
\begin{aligned}
&\mathit{property}(R : \mathit{Relation}) \\
&\mathit{total\_ordering} : R \\
r \mapsto \mathit{transitive}(r) \ \wedge (\forall a, b \in \mathit{Domain}(R))\ &\mathit{exactly\ one\ of\ following\ holds} : \\
&r(a, b),\ r(b, a),\ \mathit{or}\ a = b
\end{aligned}
$$

Journey 1

1. min
2. max

Journey #1

```
int min(int x, int y) {
    if (y < x) {
        return y;
    }
    return x;
}
```

Journey #1

```
int min(int x, int y) {
    if (y < x) {
        return y;
    }
    return x;
}


double min(double x, double y) {
    if (y < x) {
        return y;
    }
    return x;
}
```

Journey #1

```
template<typename T>
T min(T x, T y) {
    if (y < x) {
        return y;
    }
    return x;
}
```

Dealing with large objects

```
template<typename T>
const T& min(const T& x, const T& y) {
    if (y < x) {
        return y;
    }
    return x;
}
```

Journey #1

```
template<typename T, typename P>
const T& min(const T& x, const T& y, P pred) {
    if (pred(y, x)) {
        return y;
    }
    return x;
}
```

Journey #1

```
struct employee {
    std::string full_name;
    int64_t salary;
};

void usage() {
  employee e0{"Bjarne␣Stroustrup", 9999999ll};
  employee e1{"Alex␣Stepanov", 9999999ll};
  min(e0, e1, [](const auto& x, const auto& y) {
    return x.salary < y.salary;
  }).salary += 10000ll;
}
```

Journey #1

```
template<typename T, typename P>
T& min(T& x, T& y, P pred) {
    if (pred(y, x)) {
        return y;
    }
    return x;
}
```

Journey #1

```
template<Regular T, Relation r>
const T& min(const T& x, const T& y, Relation r) {
    if (r(y, x)) { return y; }
    return x;
}

template<Regular T, Relation r>
T& min(T& x, T& y, Relation r) {
    if (r(y, x)) { return y; }
    return x;
}
```

Journey #1

```
template<TotallyOrdered T>
const T& min(const T& x, const T& y) {
    return min(x, y, std::less<T>());
}

template<TotallyOrdered T>
T& min(T& x, T& y) {
    return min(x, y, std::less<T>());
}
```

Journey #1

Journey #1

```
namespace cppcon {

template<totally_ordered T>
const T& min(const T& x, const T& y) {
    if (y < x) {
        return y;
    }
    return x;
}

template<totally_ordered T>
T& min(T& x, T& y) {
    if (y < x) {
        return y;
    }
    return x;
```

Journey #2

1. unique
2. unique_count

Journey #2

```
template<forward_iterator It, relation<ValueType(It)> R>
It unique(It first, It last, R r) {
    if (first == last) { return last; }
    It result = first; ++first;
    while (first != last) {
        if (*result == *first) {
           ++first;
        } else {
            ++result;
            *result = *first;
            ++first;
        }
    }
    ++result;
    return result;
}
```

Journey #3

1. frequencies
2. transform_subgroups
3. squash_subgroups

Journey #4

1. split
2. transform_splits

Journey #5

1. remove_if
2. partition_semistable

More examples of concepts

1. Regular Type
2. Semiegular Type
3. Functional Procedure
4. Homogeneous Function
5. Homogeneous Predicate
6. Semiring
7. Sequence
8. Totally Ordered
9. Input Iterator
10. Forfward Iterator
11. Bidirectional Iterator

Properties

1. Associative
2. Distributive
3. Transitive
4. Semiegular Type
5. Functional Procedure

Techniques

1. Transformation-action duality
2. Operation-accumulation procedure duality
3. Memory adaptivity
4. Reduction to constrained subproblem

Conclusion

Conclusion

1. Concreteness costs
2. Abstracting algorithms to their most general setting without losing efficiency
3. Know your algorithms
4. If y