ME 614: Homework 3

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14/04/2017

1 Problem 1

1.1 Part a

The spy plots for ∇^2 operator on a 5×5 grid for homogeneous Dirichlet and Neumann conditions has been plotted

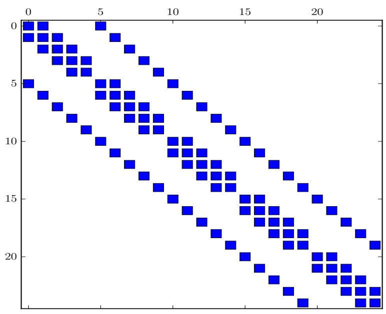

Figure 1: Homogeneous Dirichlet

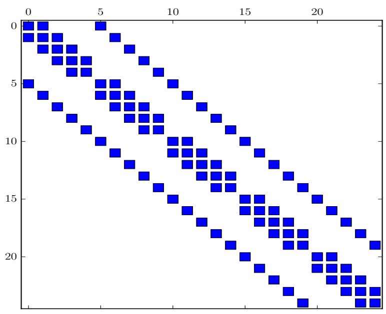

Figure 2: Homogeneous Neumann

1.2 Part b

For the given ϕ, f(x,y) is:

$$f(x,y) = 2\pi n sin(2\pi ny)[cos(2\pi nx) + \frac{4\pi n sin(2\pi nx)}{Re}]$$

The given equation has been solved using the Gauss-Seidel method. The following plots display the number of iterations needed for each value of ω. 3 grid sizes i.e. 15×15, 30×30 and 45×45 have been chosen.

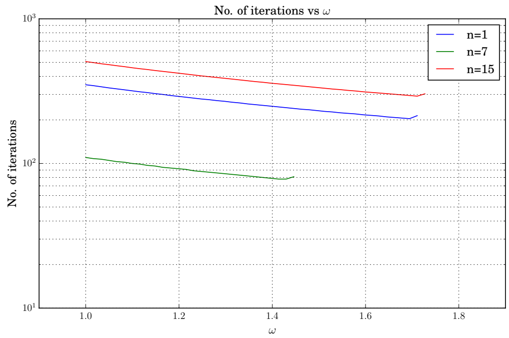

Figure 3: Optimum ω for 15×15

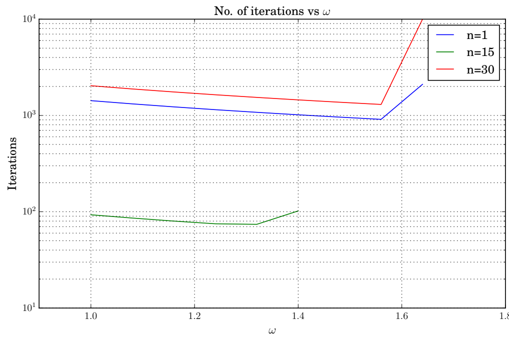

Figure 4: Optimum ω for 30×30

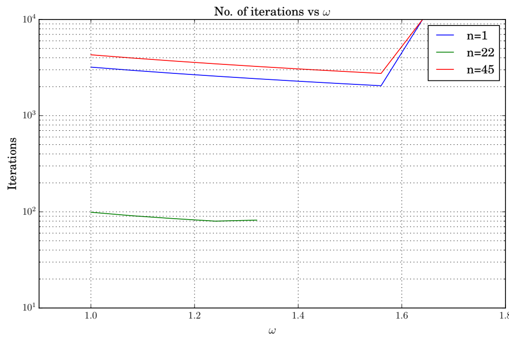

Figure 5: Optimum ω for 45×45

As seen from the above plots, the optimum ω is the location where the number of iterations is the lowest. The corresponding value of wavenumber n has been shown in the legend for each plot.

The following plots show the variation of the absolute value of the residual $||r_k||$ vs the number of iterations (k).

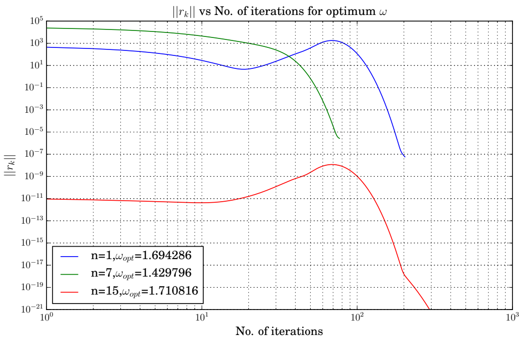

Figure 6: Residual history for ω_{opt} for 15×15

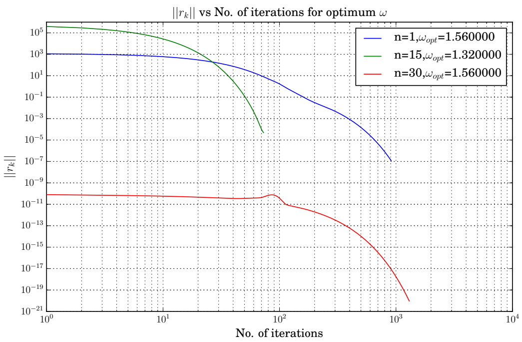

Figure 7: Residual history for ω_{opt} for 30×30

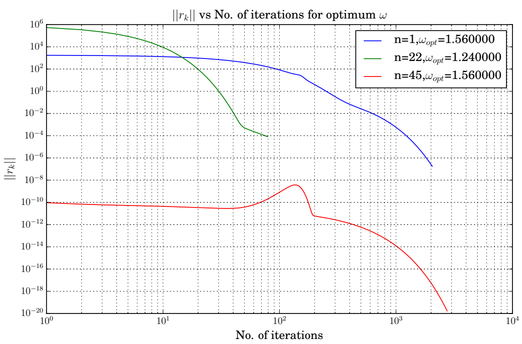

Figure 8: Residual history for ω_{opt} for 45×45

1.3 Part c

The following plot shows the variation of $||r_k||$ with iteration number (k) for the optimal ω on a 256×256 grid. A wavenumber of 128 has been chosen. The convergence criteria is 10^{-10}. The optimum ω has been found in a way similar to part one (optimum ω mentioned in a legend of the plots). A first order scheme has been used for the $\frac{\partial}{\partial x}$ operator (keeping in mind considerations from the previous part).

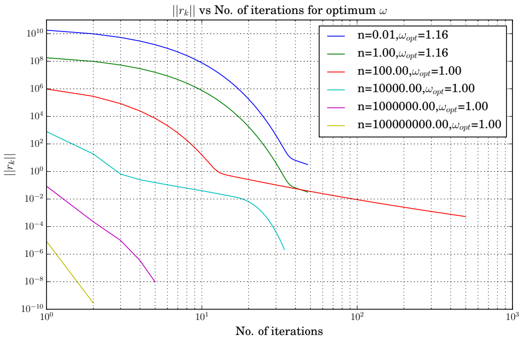

Figure 9: Residual history at ω_{opt} for 256×256, n=128

This scheme does not always meeting the convergence criteria.

1.4 Part d

For the non-linear equation, f(x,y) for the same class of solutions is:

$$f(x,y) = 2\pi n sin(2\pi ny)sin(2\pi ny[cos(2\pi nx)sin(2\pi ny + \frac{4\pi n}{Re}]$$

The plot shown below displays the number of iterations needed with and without LU pre-factorization for method (a) i.e. leaving the Laplacian operator on the RHS and method (b) i.e. by moving the Laplacian part to the RHS together

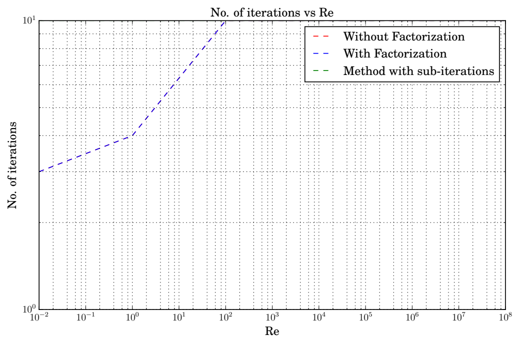

Figure 10: Iterations required for different types of factorization, with and without LU factorization

As seen, the method (a) converges for low Re (i.e. 0.01 and 1) and method (b) fails to converge even for a convergence criteria of 10^{-6}. The residuals for each iteration are found to be substantially high. A possible reason for that may be the use of a second order central scheme for $\frac{\partial}{\partial x}$, which has problems with the matrix diagonal in the inversion step (or the method being used by spsolve).

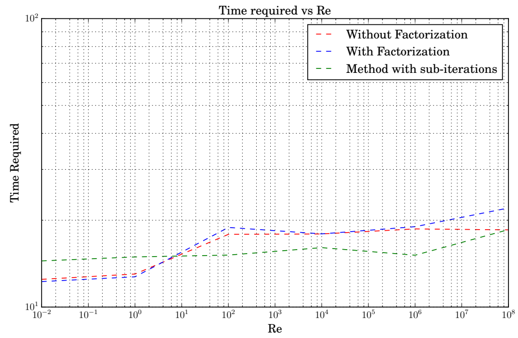

Figure 11: Performance of each method. Method (a)-with and without pre-factorization

From the plot for the performance of part (a) with and without LU factorization, it can be seen that LU factorization speeds up the solution for low-Re (i.e. where convergence is obtained). For higher RE, the time calculations are irrelevant, since the schemes diverge anyway.

2 Problem 2

Here, a solution to the Taylor-Green vortex has been calculated at t=1 using explicit time advancement and the fractional step method for pressure-velocity coupling. For time advancement, the following schemes are used (only u equations mentioned for brevity, v equations are the same):
RK1:

$$u^{n+1} = u^n + \Delta t R^n$$

RK2:

$$u^1 = u^n + \Delta t R^n$$
$$u^{n+1} = u^n + \frac{\Delta t}{2}(R^n + R^1)$$

RK3:

$$u^1 = u^n + \frac{\Delta}{4}R^n$$
$$u^2 = u^n + \frac{2\Delta}{3}R^n$$
$$u^3 = u^1 + \frac{3\Delta}{20}R^2$$
$$u^4 = u^1 + \frac{5\Delta}{12}R^2$$
$$u^{n+1} = u^3 + \frac{3\Delta t}{5}R^4$$

RK4:

$$u^{n+1} = u^n + \frac{\Delta t}{6}(R^n + 2R^1 + 2R^2 + R^3$$

where

$$u^1 = u^n + \frac{\Delta t}{2}R^n$$
$$u^2 = u^n + \frac{\Delta t}{2}R^1$$
$$u^3 = u^n + \Delta t R^2$$

The figure below shows the plot of the RMS error with grid spacing.

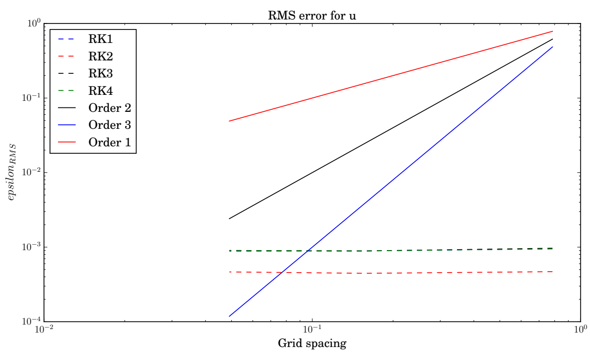

Figure 12: RMS error for u

As seen, the time-advancement scheme does not influence the spatial order of accuracy. From the values of the RMS error, it can be seen that the final solution is a good estimate of the final solution (O(RMS error) 0.01. However, the RMS error seems to be fairly constant rather than second order. In the code, u and v have been interpolated (two sets of simple averages) onto v and u respectively to calculate the

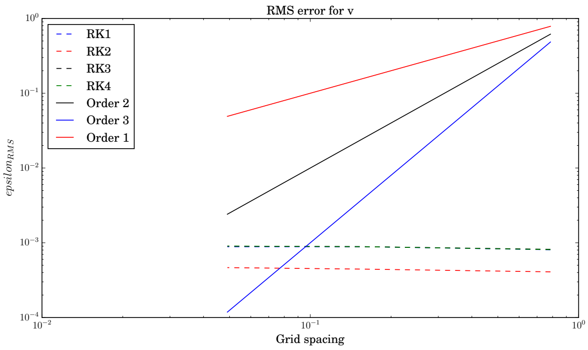

Figure 13: RMS error for v

convective terms for each time advancement sub-step. The reduction in the order of accuracy can be attributed to this interpolation polluting the order of accuracy of the solution. (The solution is fairly accurate, but the velocity interpolations greatly overwhelm the decay of the numerical error)