ME 608: Homework 3

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1 Problem 1

1.1 Part a

The following plots show the resultant CFL for the minimum time step corresponding to the maximum CFL being 0.85. The corresponding time step duration is 0.001423. α comes out to be while 0.013593 and the convection speed is chosen as $3x\bar{i} + 4y\bar{j}$

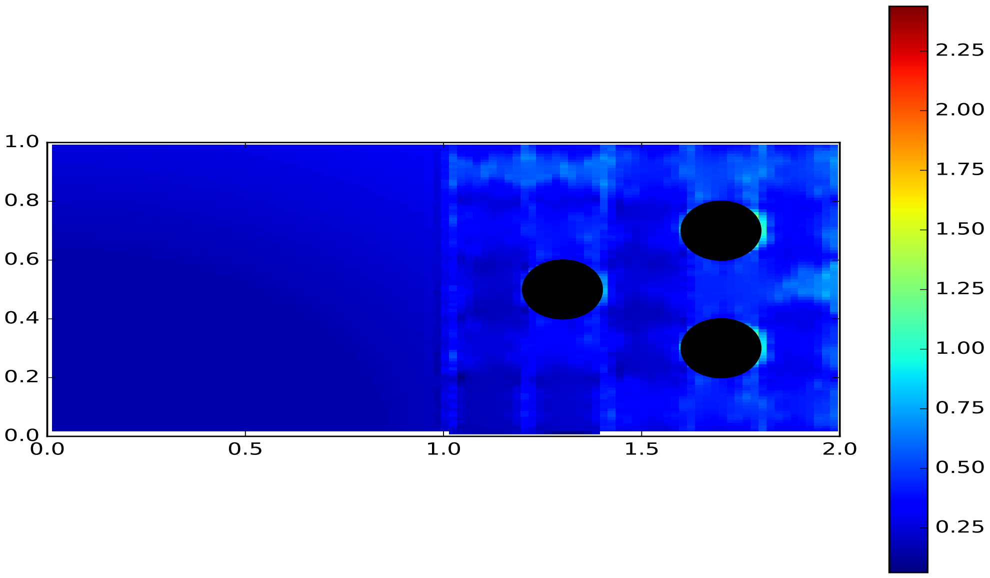

Figure 1: CFL variation

1.2 Part b

The following plots show the time evolution for a purely diffusive problem using explicit RK2 time advancement. The cases were run for $t = 1$ and the CFL in this case is 0.597

Steady state can be achieved at around t=10 (based on visual inspection of the solution)

The solution blows up for a CFL of ≈ 23.3 and the corresponding time step is 0.039. This is much higher as compared to the estimated value of 0.001423. However, it should be noted that this simplistic view of the CFL solely based on the area may not be a good

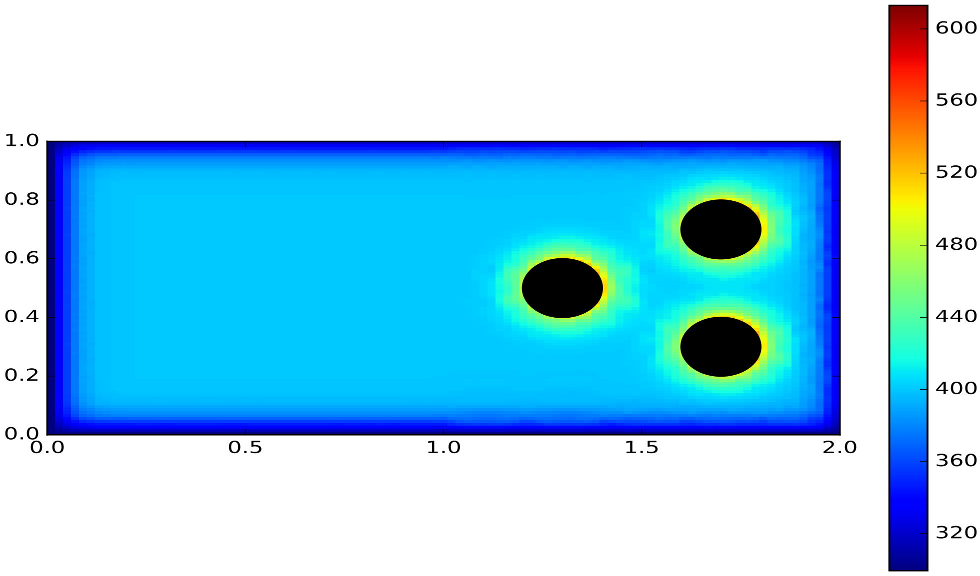

Figure 2: t=0.099

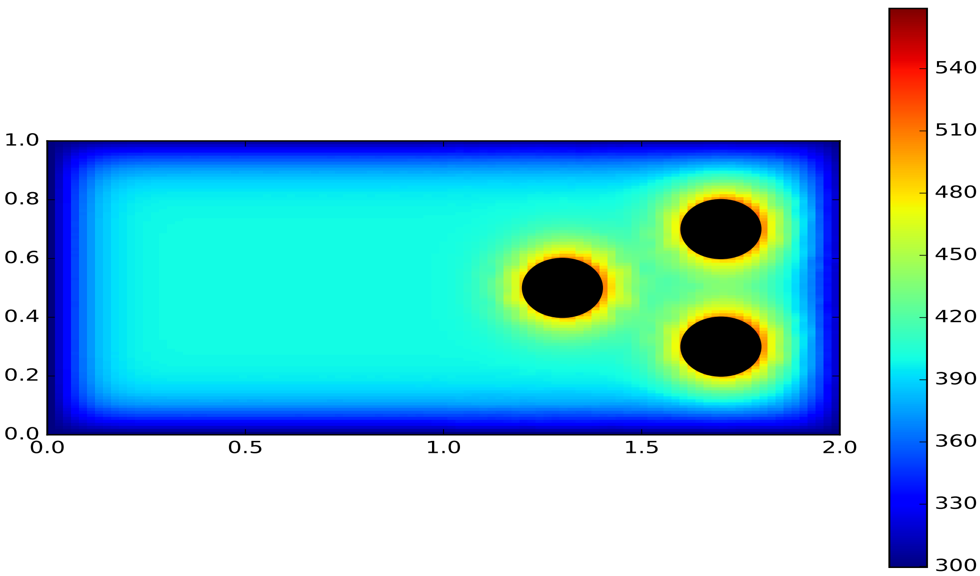

Figure 3: t=0.299

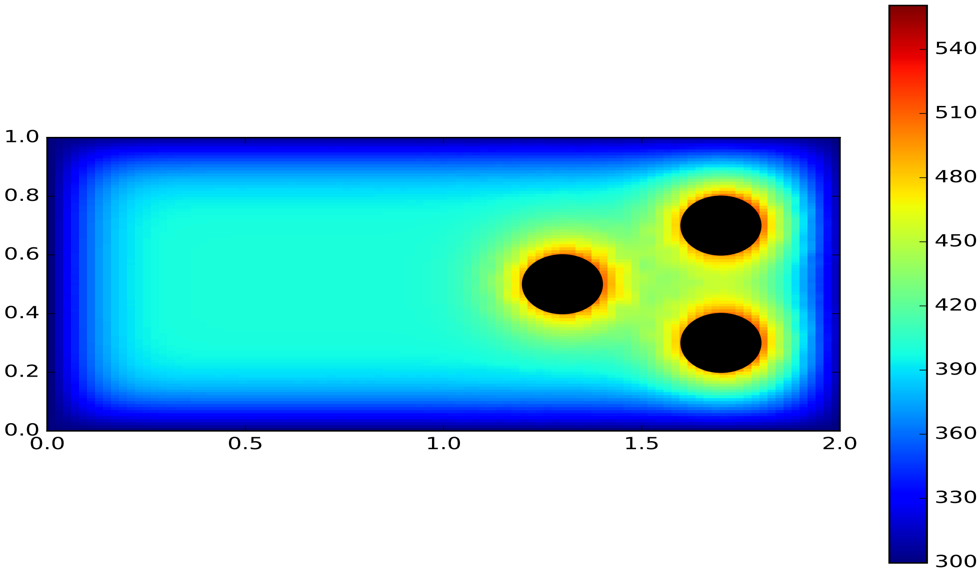

Figure 4: t=0.499

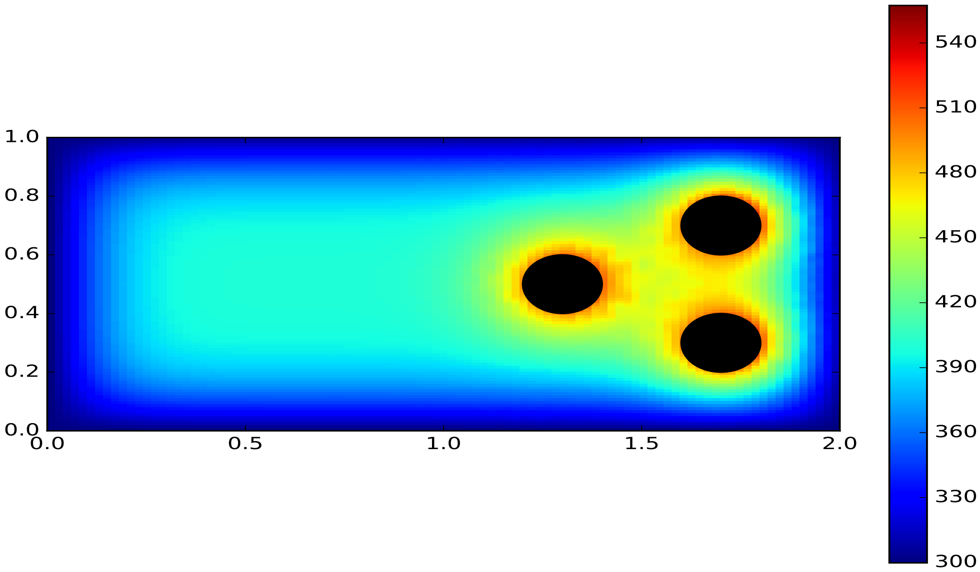

Figure 5: t=0.799

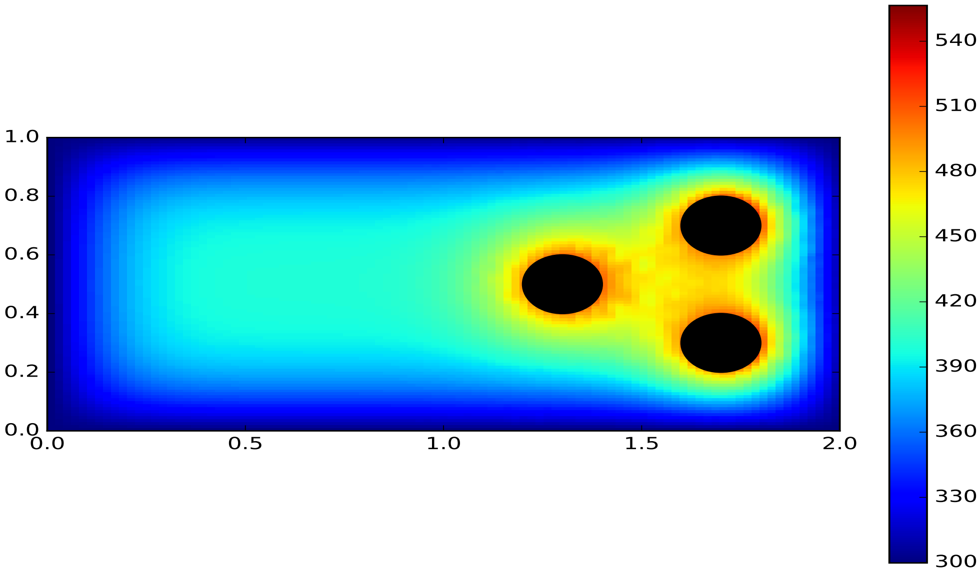

Figure 6: t=0.999

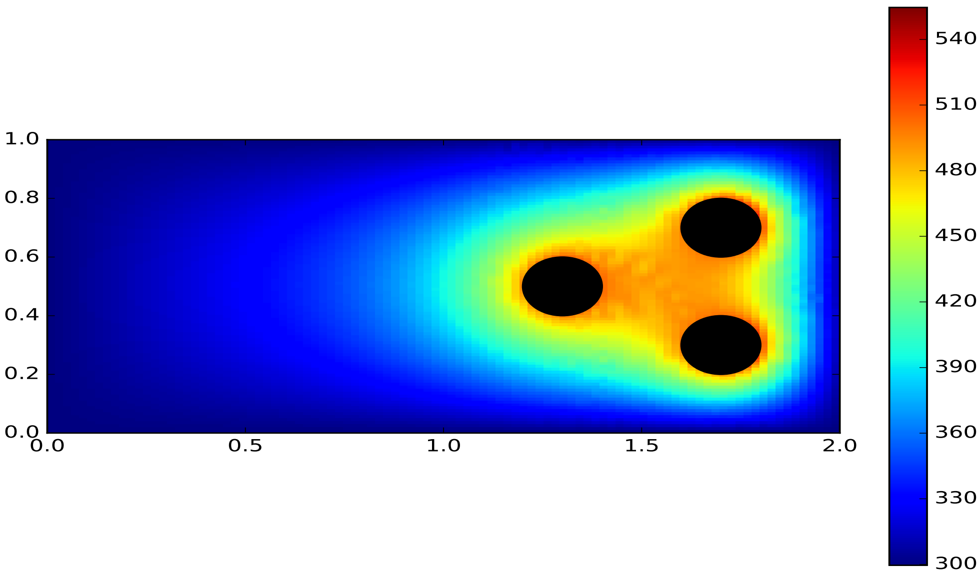

Figure 7: Near steady-state solution

1.3 Part c

The following plot shows the final solution at t=1 for various CFL values for the purely convective case using explicit RK2 time advancement. As expected, the scheme is unstable, displaying instability to CFL values as low as 0.015

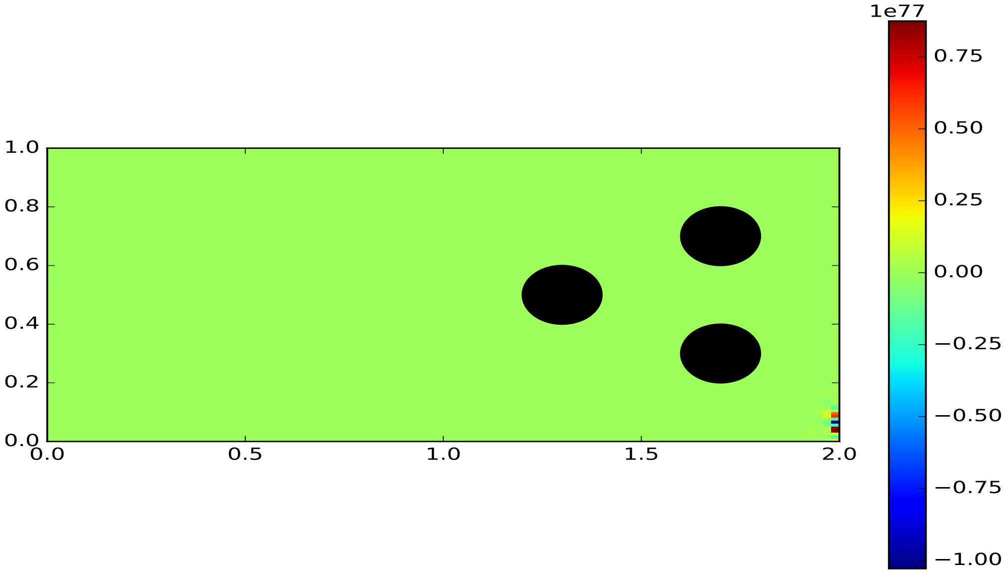

Figure 8: Pure Convection, CFL=0.015

Instead of using explicit time advancement, one can use a Semi-Lagrangian approach. The following plot shows the solution at $t = 1$ using a Semi-Lagrange formulation for the gradient.Once again, the convection velocity used is $c = 3x\bar{i} + 4y\bar{j}$

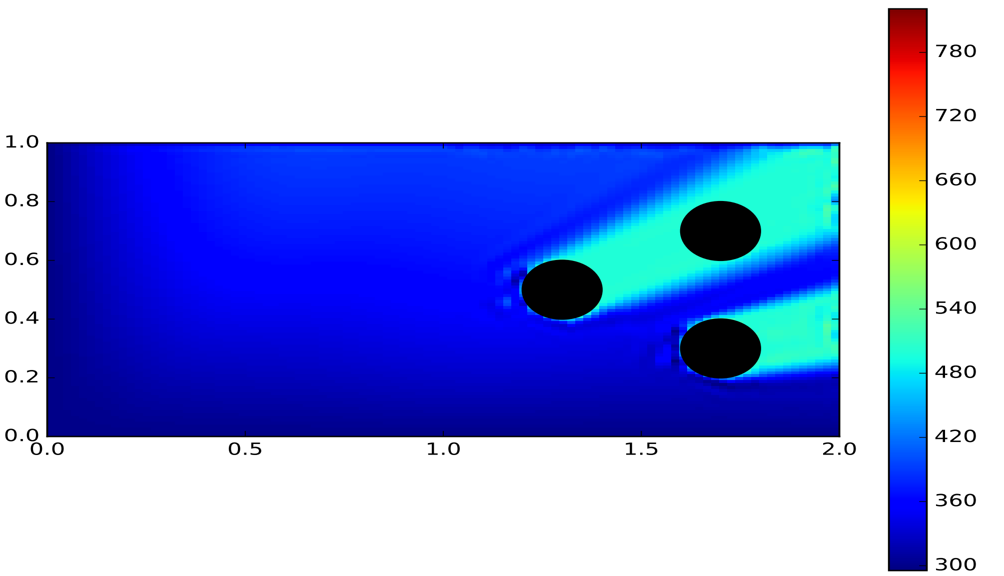

Figure 9: Solution at t=1 using Semi-Lagrange formulation

1.4 Part d

For subpart d, a coarser version of the same mesh "Mesh_2" has been used. The following plots show the time evolution for a time step of 0.01. The problem is stable upto a time step size of almost 0.1, where some local overshoots beyond the upper bound of 500 can be observed

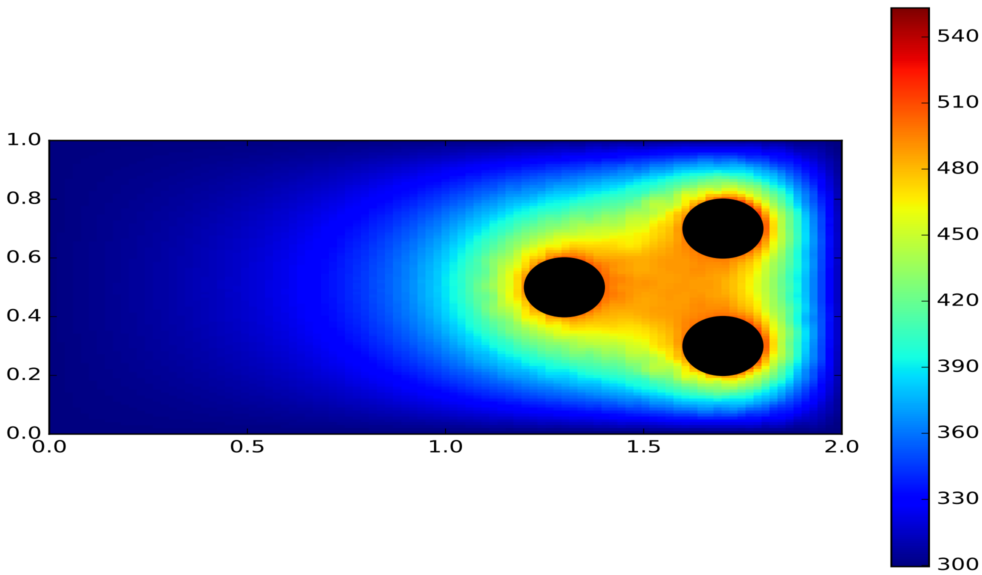

Figure 10: Solution at t=0.29

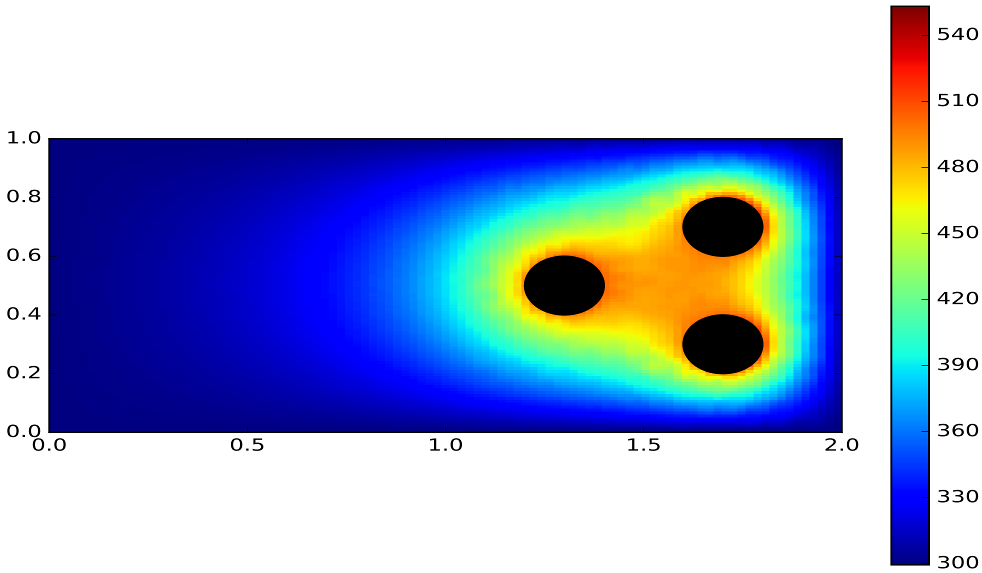

Figure 11: Solution at t=0.49

2 Problem 2

The domain considered is a unit domain (0,1) $\times$ (0,1).

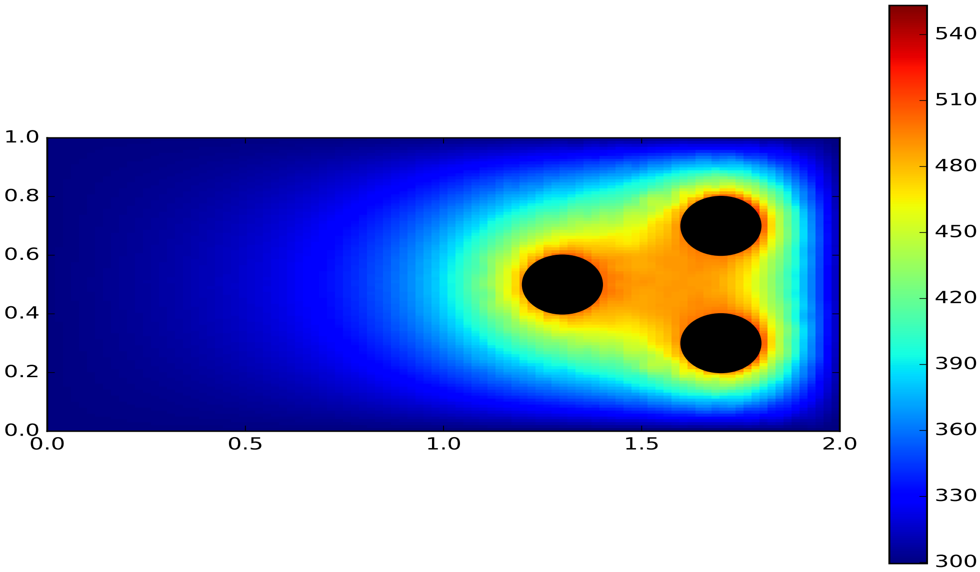

Figure 12: Solution at t=0.79

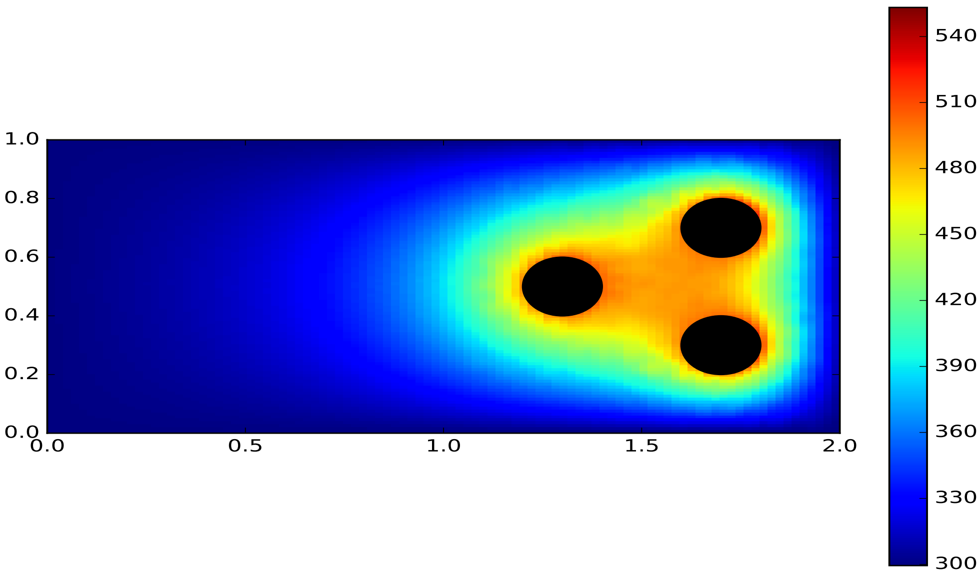

Figure 13: Solution at t=1

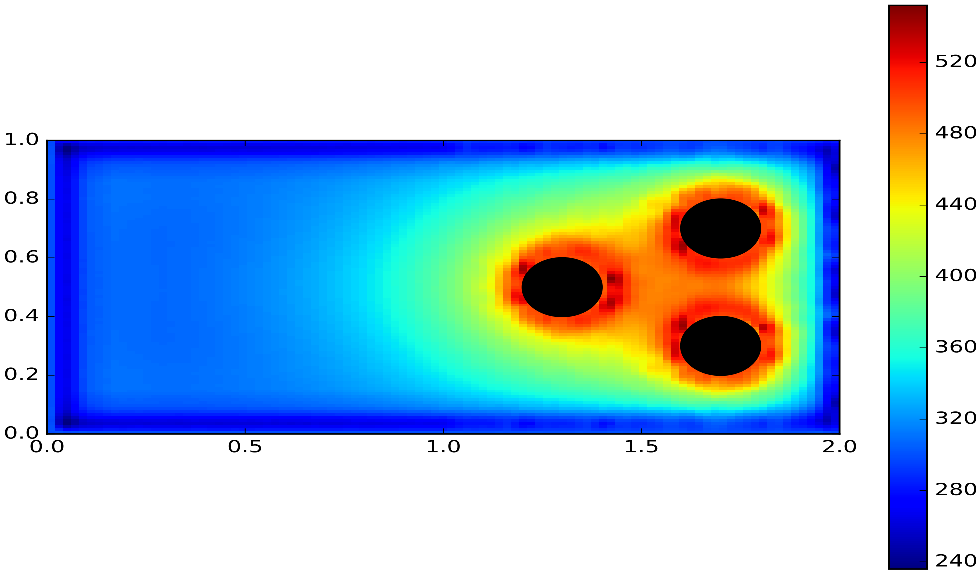

Figure 14: Solution for $\Delta t = 0.1$

The verification of the first and second derivatives can be printed from the code. For the 1st derivative validation, the functions chosen are $f(x) = x$ and $f(y) = y$, which give the derivative output as 1 (can be seen in the output as the first two 2D arrays.). $f(x) = x^2$ and $f(y) = y^2$ was chosen for the second verification, which accordingly yields a 2 through the 2nd derivative operator (output as two 2D arrays containing 2s by the code). The following plot shows the solution and flux points on a unit domain

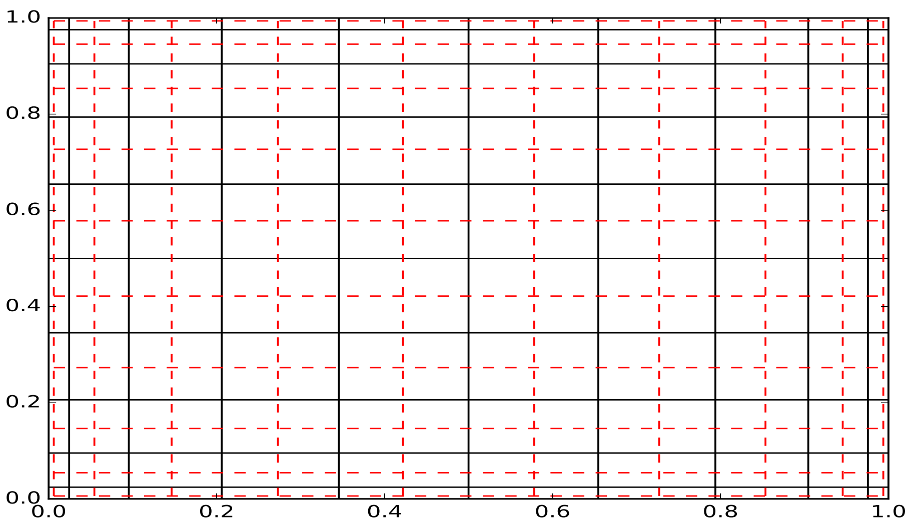

Figure 15: Solution and flux points for a unit domain

2.1 Part a

The following plot shows the solution to the diffusion equation on the unit domain. A value of 0 is given within the domain with a boundary value of 500. The final time is 100 and the solution is almost at steady state (as seen from the contour legend)

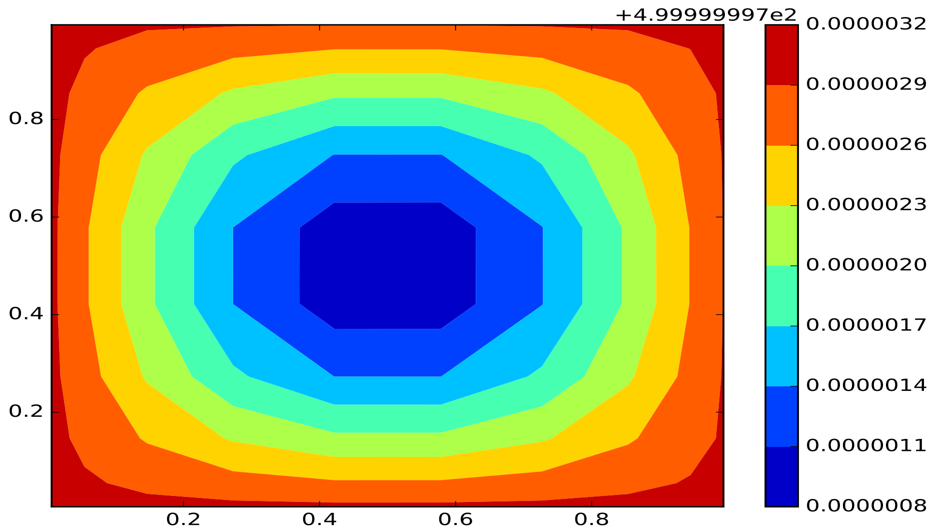

Figure 16: Solution for pure diffusion on a unit domain

2.2 Part b

The following figure shows the solution for the purely advective case. It can be seen that the value throughout the cell is 500, which is expected.

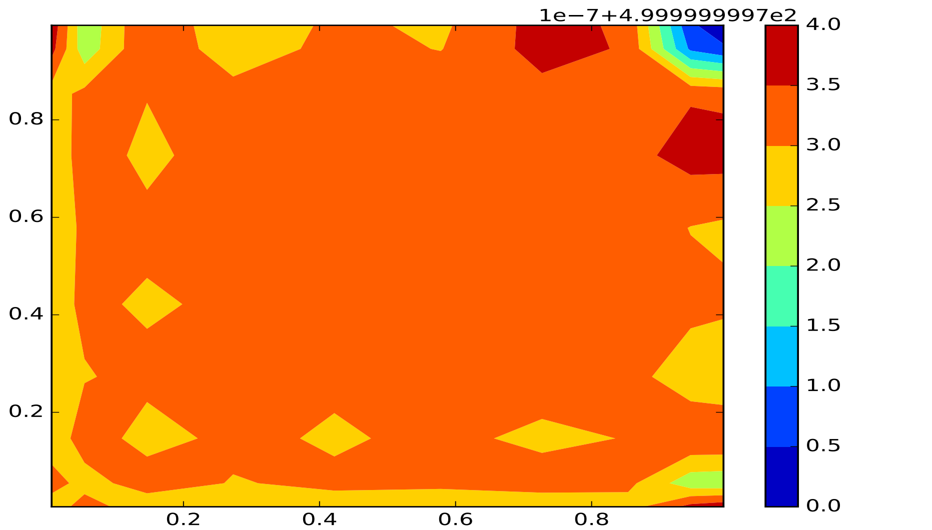

Figure 17: Solution for pure advection on a unit domain