ME 608: Homework 2

Tanmay C. Shidhore

December 16, 2017

1 Problem 1

1.1 Part1

Figures 1,2,3,4,5,6 show the validation for the bivariate fit for N>4. A mesh perturbation of 10^{-8} was used in the provided test script. The maximum number of surrounding stencil points was found to be 9.

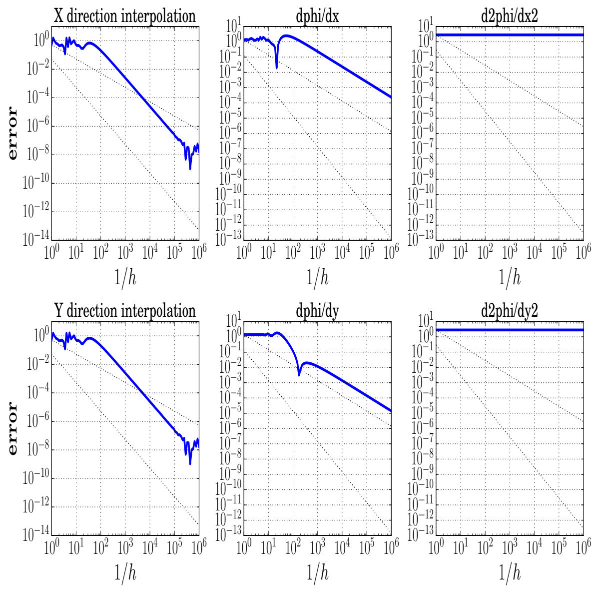

Figure 1: N=4

1.2 Part 2

The following plots 7,8,9,10 show the spy plot of Poisson operator A for 4 with progressive mesh refinement (Mesh 4 is most refined, mesh 1 is least refined)

As seen from the spy plot for A, the operator is mostly diagonally dominant and a solution using spsolve can be expected.

1.3 Part 3

The solution for the Poisson equation for a unit source (i.e. $\nabla^2 u = -1$) using spsolve is plotted in figures 11, 12, 13 and 14 using griddata. A mask has been implemented to cover up the spurious interpolations done by grid data inside the holes in the domain.

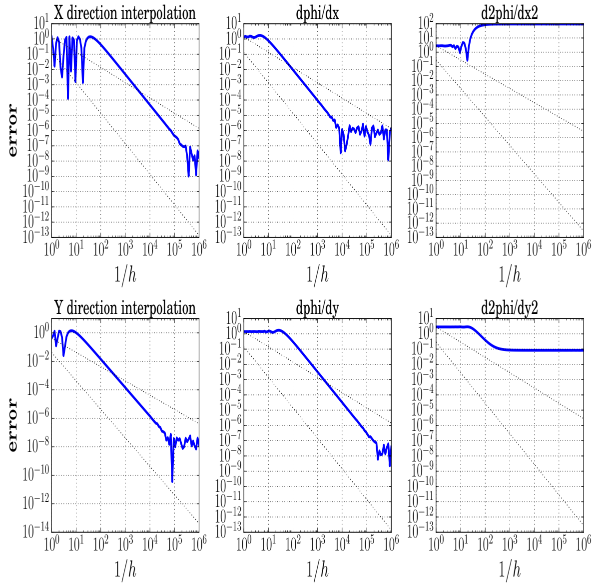

Figure 2: N=5

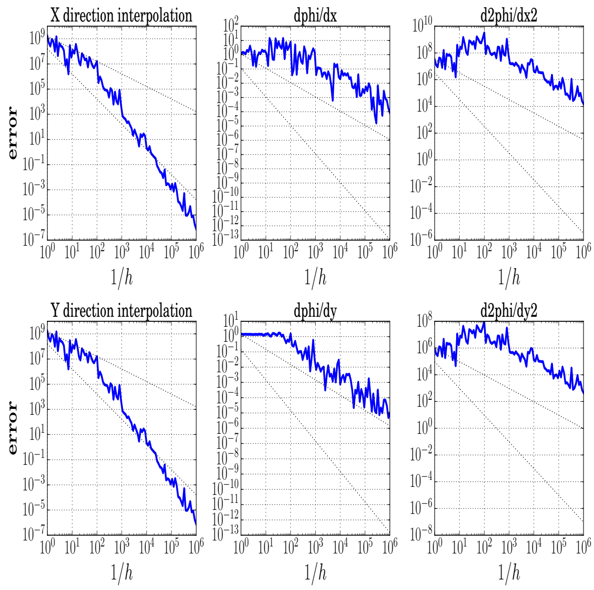

Figure 3: N=6

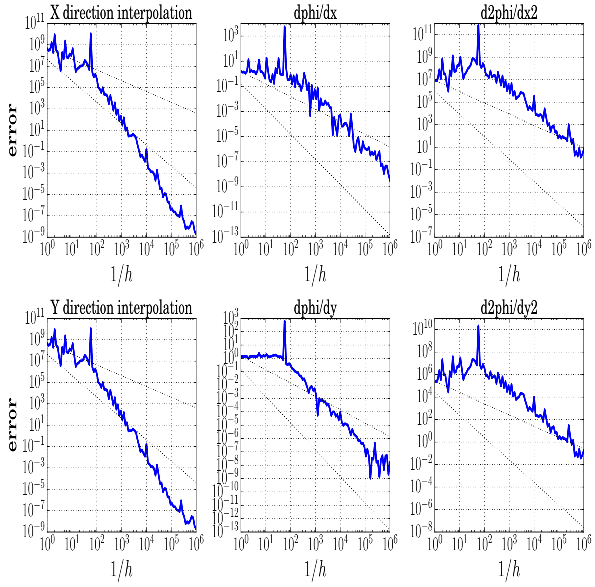

Figure 4: N=7

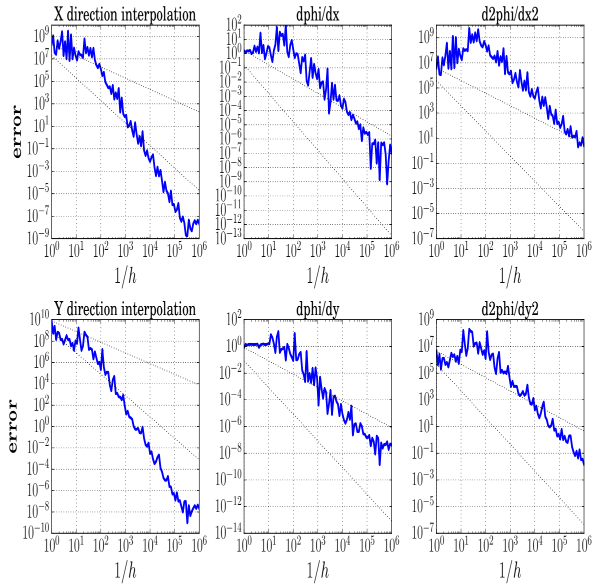

Figure 5: N=8

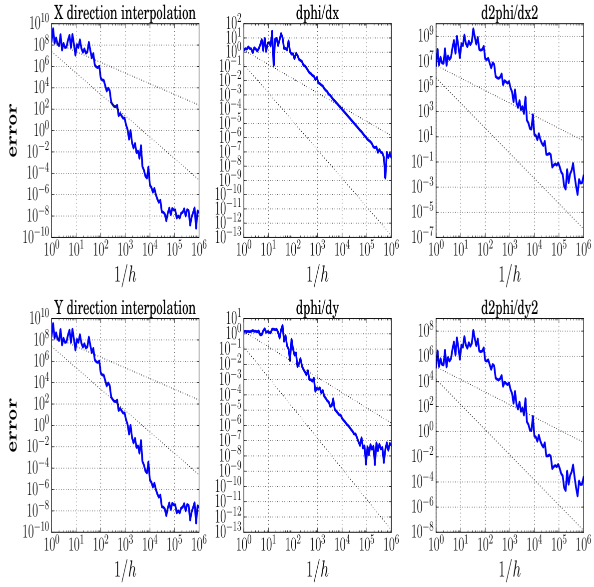

Figure 6: N=9

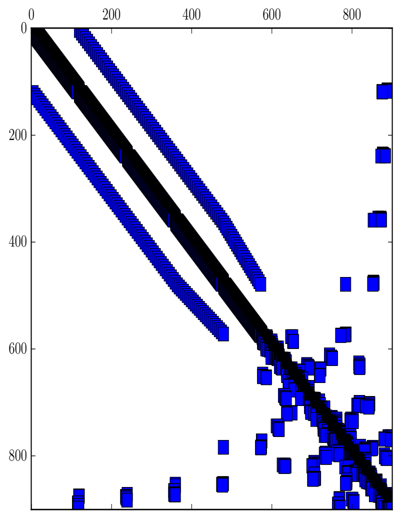

Figure 7: Mesh 1

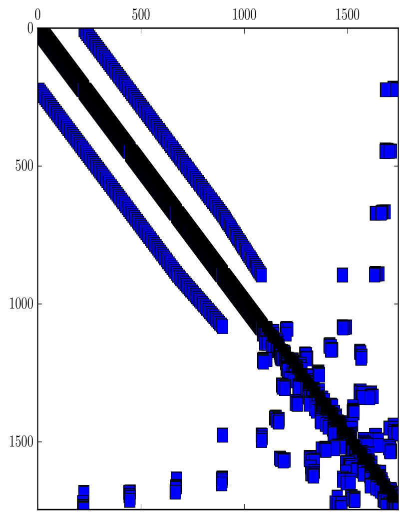

Figure 8: Mesh 2

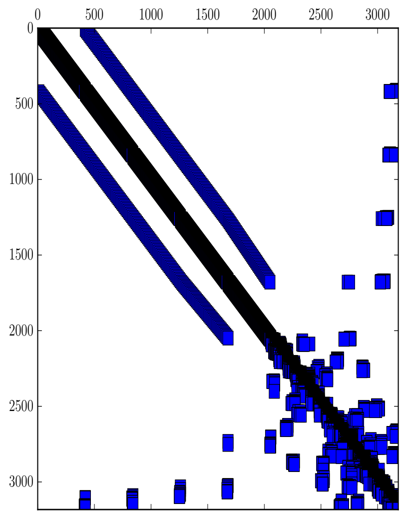

Figure 9: Mesh 3

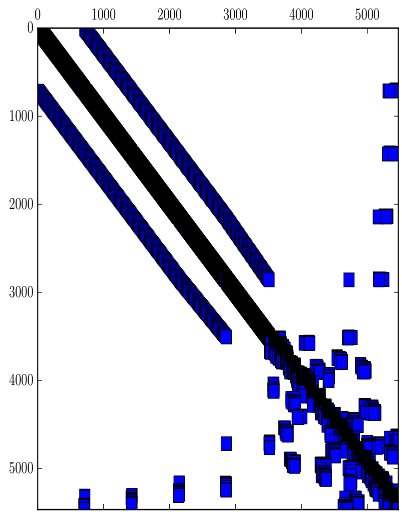

Figure 10: Mesh 4

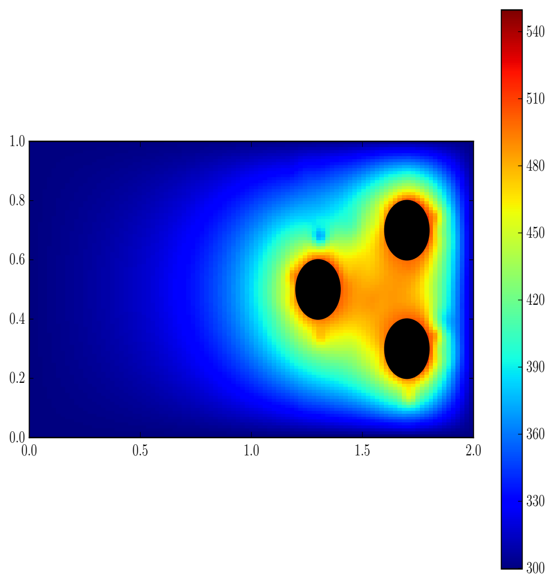

Figure 11: Mesh 1:Final Solution field

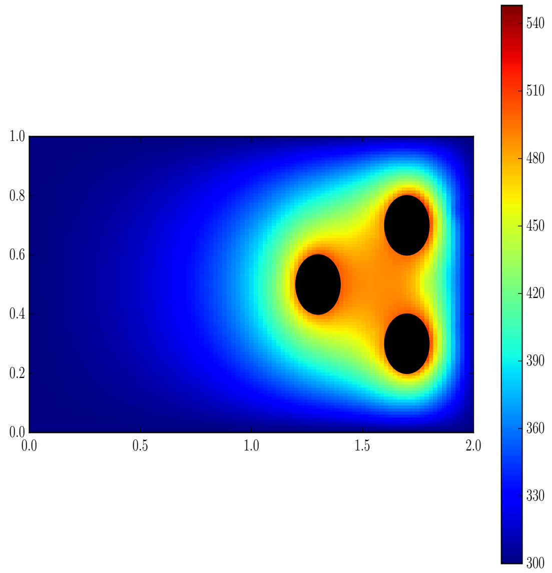

Figure 12: Mesh 2:Final Solution field

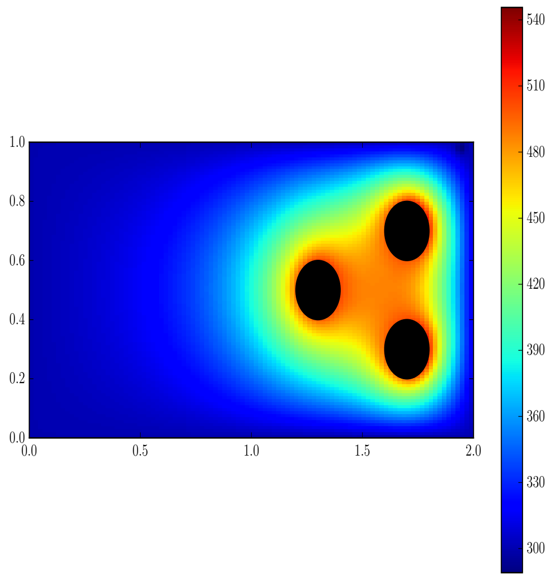

Figure 13: Mesh 3:Final Solution field

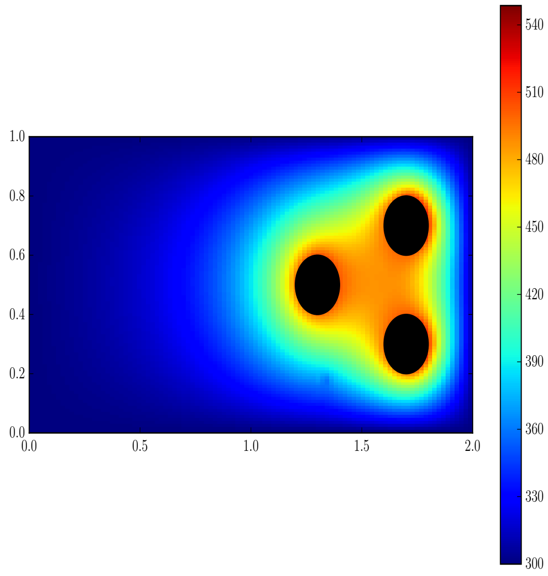

Figure 14: Mesh 4:Final Solution field

1.4 Part 4

The earlier solutions were also checked for grid convergence. Two slices were considered and the solution along those slices was plotted as a function of the array index of the slice (see figure 15)(the slice was chosen to have 51 points(as we are interested only in the trend of the actual data rather than the exact co-ordinate location at which the values are taken at):

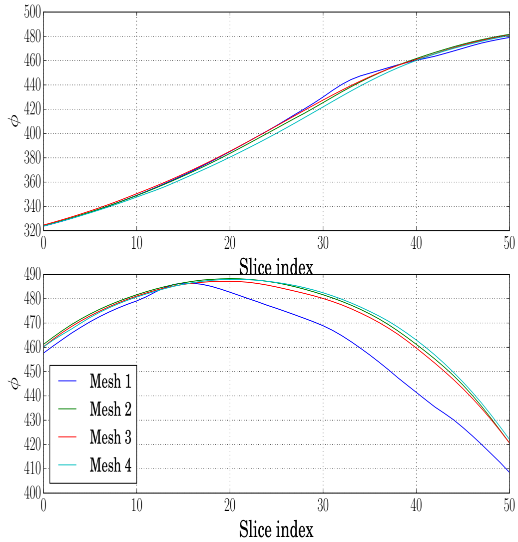

Figure 15: Grid Convergence for the solution over 4 meshes

The solutions for grids 3 and 4 over the two slices are in good agreement, showing that any further refinement is not expected to change the solution dramatically.

1.5 Part 5

Even though grid convergence is a good measure of improvement in solution, there is no certainty that the solver produces solutions close to the analytical solution on the domain. In order to ascertain this, the solver was tested for a circular domain with homogeneous dirichlet conditions at the circumference. The solution for this case is given by:

$$u(x, y) = \frac{1 - x^2 - y^2}{4} \tag{1}$$

Figure 16 shows the numerical solution for 4 meshes (mesh 1 trough 4 in the increasing order of refinement) on a slice from (0.5,0.1) to (0.7,0.4). The analytical solution on the slice is also shown for reference:

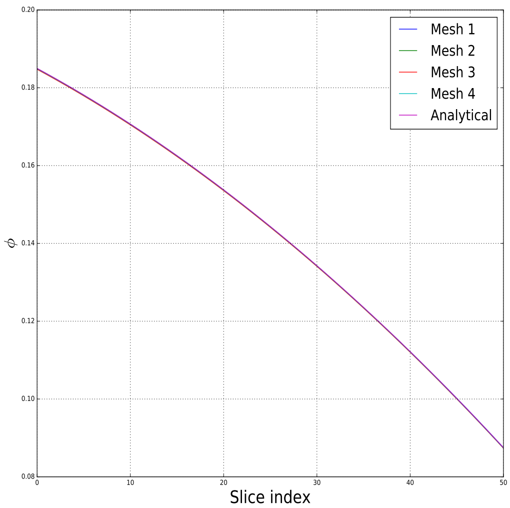

Figure 16: Grid Convergence for the slice within the circular domain

2 Problem 2

2.1 Part a

The second finest mesh was chosen and four meshes of deteriorated quality were obtained keeping the number of CVs roughly constant. Figures 17, 18, 19 and 20 show the quality contour of the meshes:

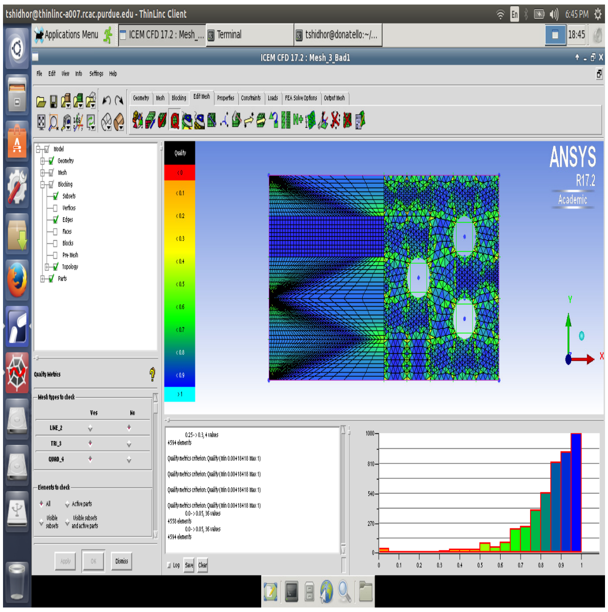

Figure 17: First bad quality mesh

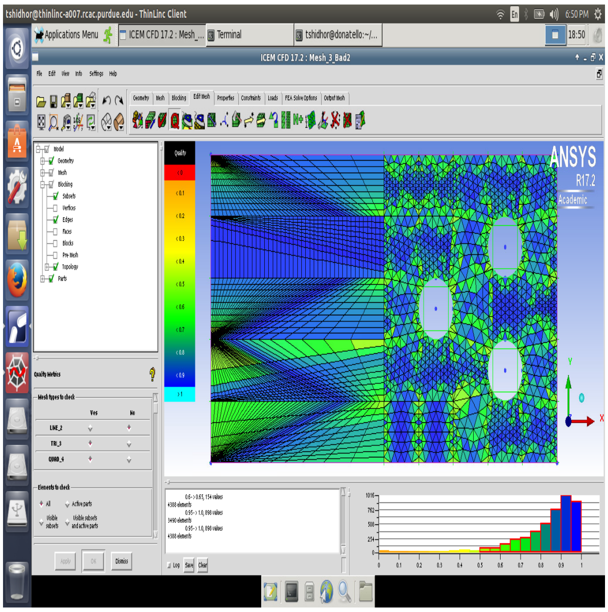

Figure 18: Second bad quality mesh

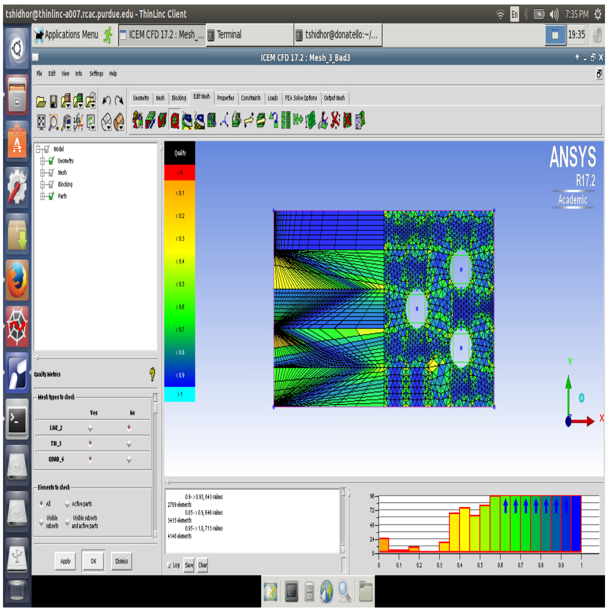

Figure 19: Third bad quality mesh

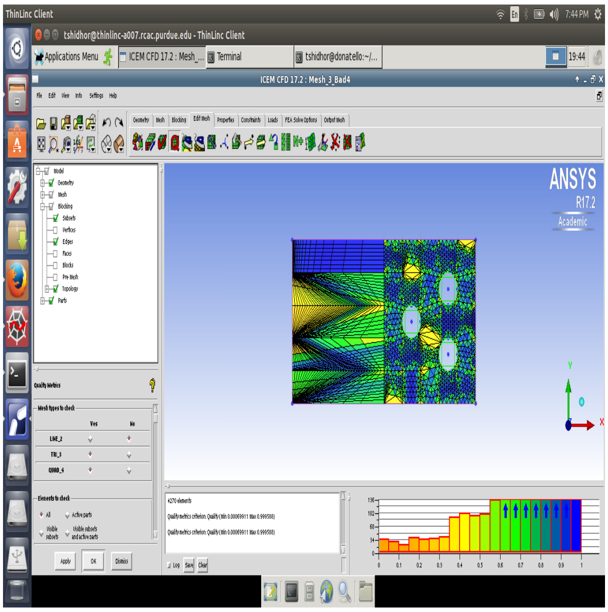

Figure 20: Fourth bad quality mesh

2.2 Part b

The iterative solver fails to converge on all the four good quality meshes considered in problem 1. figure 21 shows the spectral radius of the modified iteration matrix C with the relaxation parameter ω, where C = Bω + (1-ω)I, where I is the identity matrix, B = $A^-1_1 A_2$

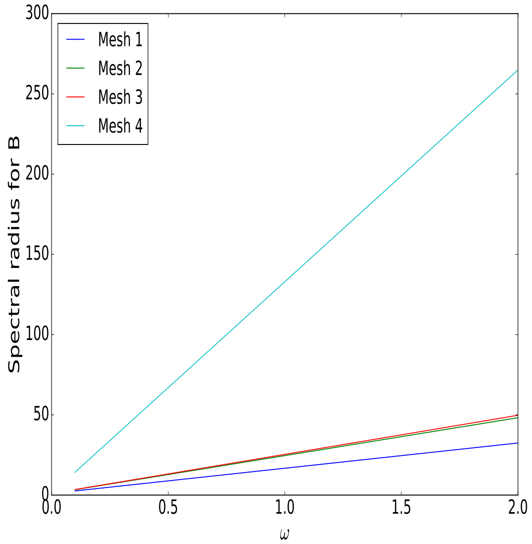

Figure 21: Variation in spectral radius for modified matrix C with ω

As seen from the graph, the spectral radius never falls below 1, supporting the earlier blow-up of the iterative solver.

2.3 Part c

As opposed to Gauss-Seidel algorithm, spsolve gives a (not exactly accurate but a fairly accurate) solution.

3 Problem 3

3.1 Part a

The following figures (22 through 29) show the spy plots for the x and y gradient for the four meshes from problem 1:

The function $f(x, y) = x^2y^2$ was chosen to verify the gradient operator. The gradient of f(x,y) is:

$$\vec{\nabla} f = 2xy^2\vec{i} + 2x^2y\vec{j} \tag{2}$$

The following quiver plots (figures 30 through 33) show the computed (red) and analytical gradients for all the four meshes:

3.2 Part b

The following figures (34 through 41) show the spy plots for the x and y divergence for the four meshes from problem 1:

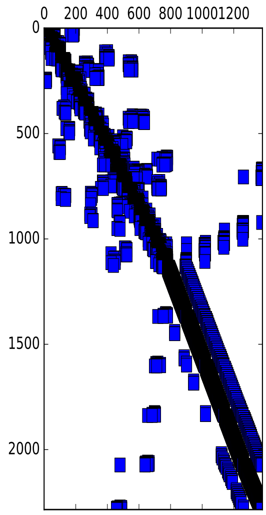

Figure 22: Mesh 1: Spy for Gx

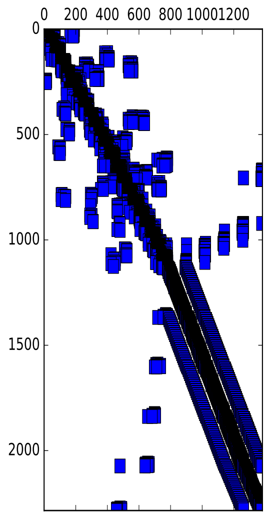

Figure 23: Mesh 1: Spy for Gy

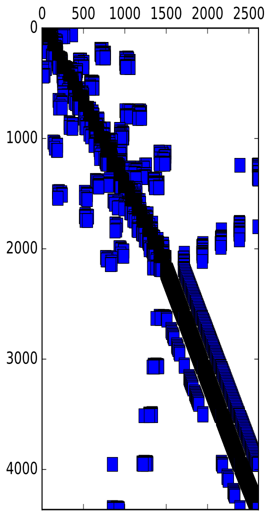

Figure 24: Mesh 2: Spy for Gx

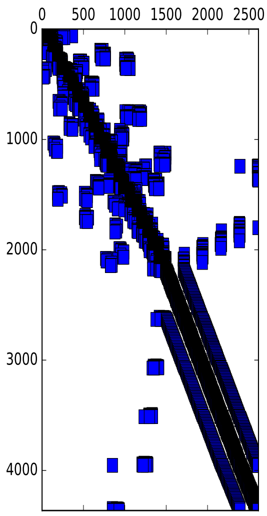

Figure 25: Mesh 2: Spy for Gy

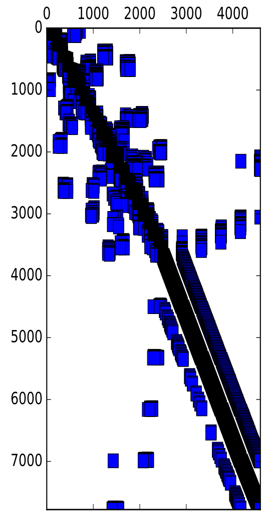

Figure 26: Mesh 3: Spy for Gx

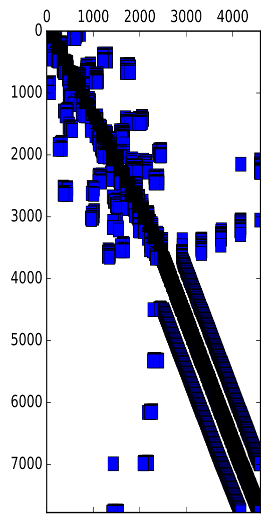

Figure 27: Mesh 3: Spy for Gy

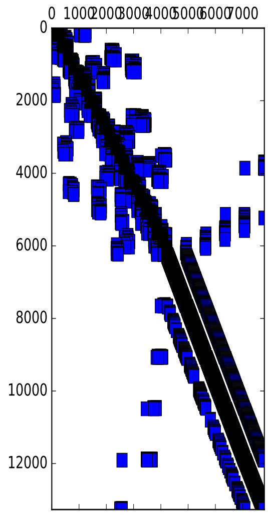

Figure 28: Mesh 4: Spy for Gx

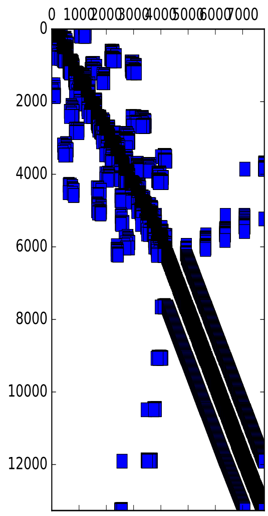

Figure 29: Mesh 4: Spy for Gy

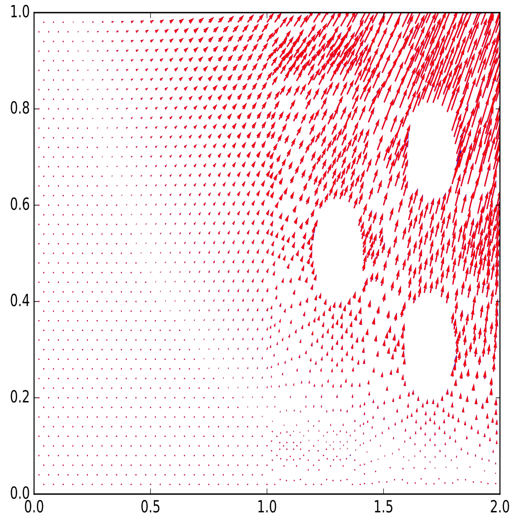

Figure 30: Mesh1: Quiver plot for gradient

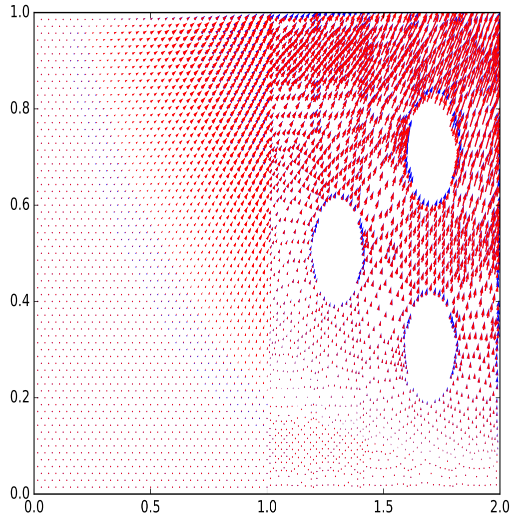

Figure 31: Mesh2: Quiver plot for gradient

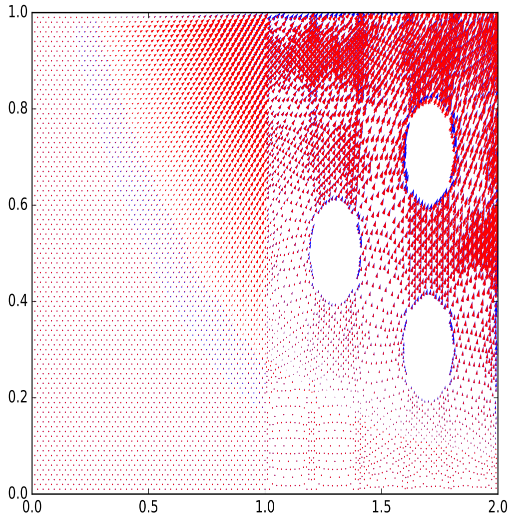

Figure 32: Mesh3: Quiver plot for gradient

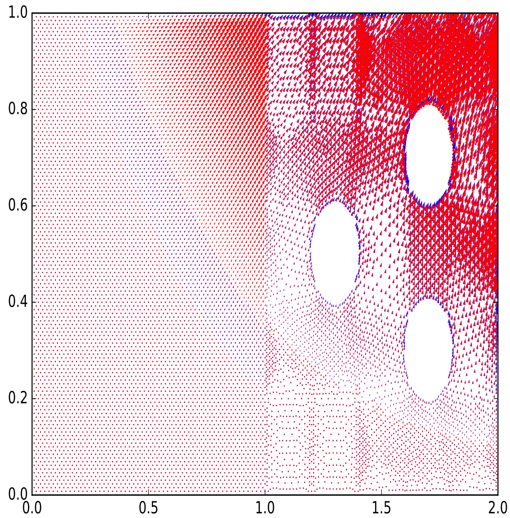

Figure 33: Mesh4: Quiver plot for gradient

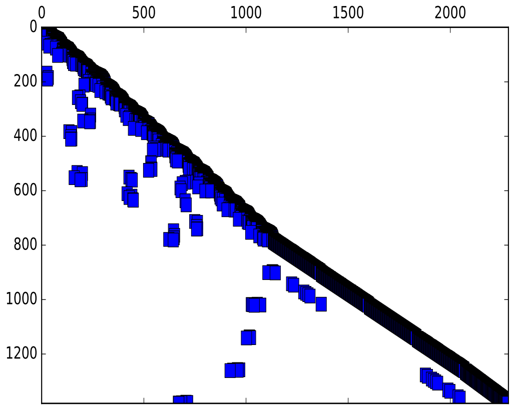

Figure 34: Mesh 1: Spy for Dx

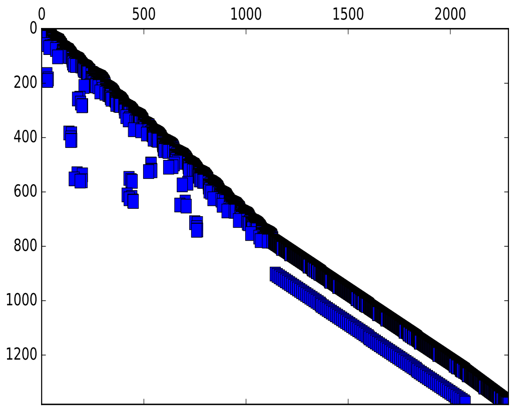

Figure 35: Mesh 1: Spy for Dy

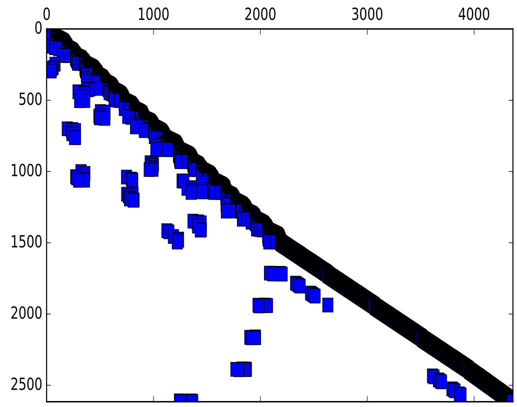

Figure 36: Mesh 2: Spy for Dx

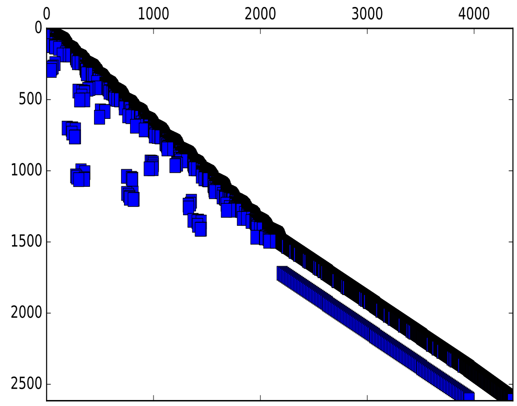

Figure 37: Mesh 2: Spy for Dy

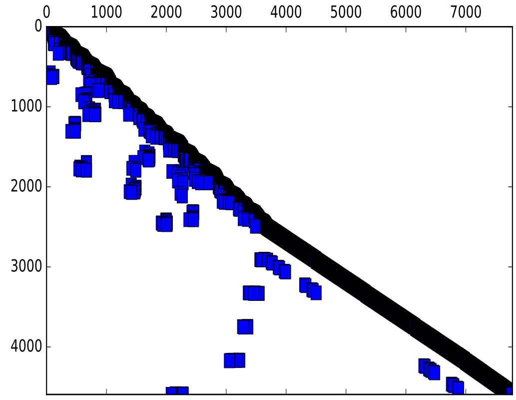

Figure 38: Mesh 3: Spy for Dx

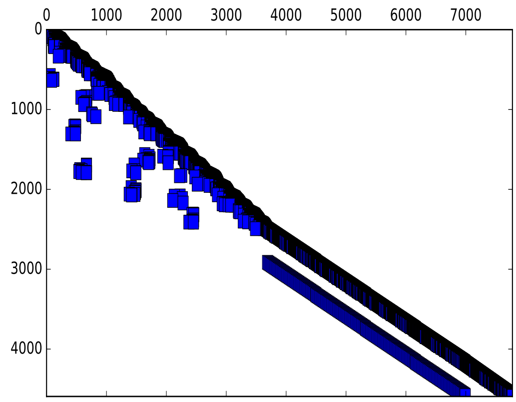

Figure 39: Mesh 3: Spy for Dy

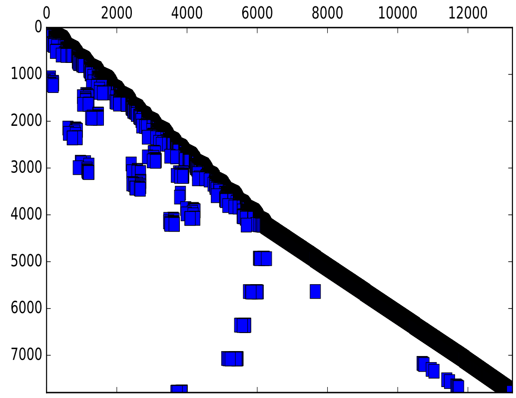

Figure 40: Mesh 4: Spy for Dx

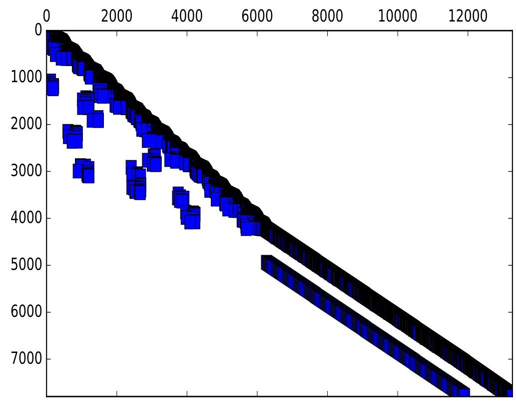

Figure 41: Mesh 4: Spy for Dy

The divergence less velocities $u(x,y) = xy^2$ and $v(x,y) = -x^2y$ was chosen to verify the divergence operator.

The following contour plots (42 through 45) show the computed divergence for all the four meshes. Note that here, since the considered field is divergenceless, the analytical value of the divergence is uniformly zero. It can be seen from figures 42, 43, 44, 45 that the divergence values are very close to 1.

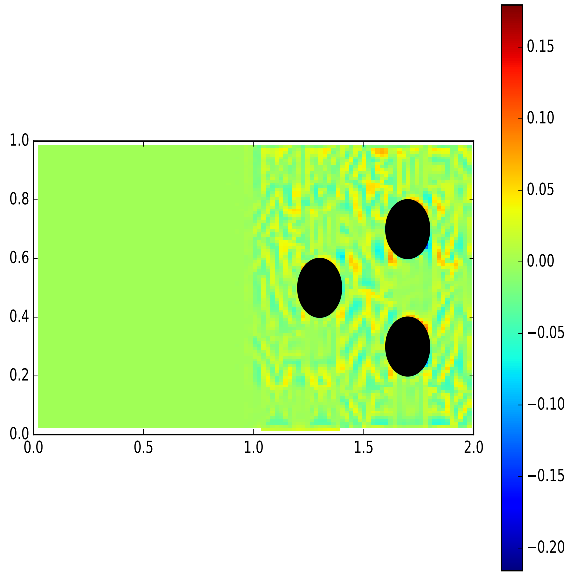

Figure 42: Mesh1: Contour plot for divergence

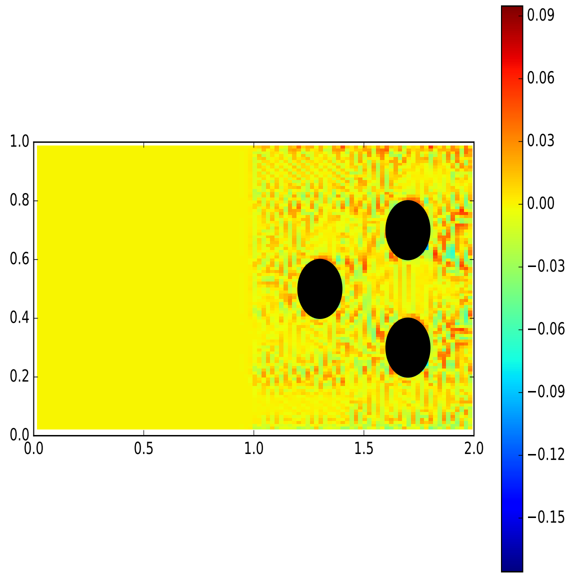

Figure 43: Mesh2: Contour plot for divergence

3.3 Part c

For this part, the same analytical (divergence) less velocities $u(x,y) = xy^2$ and $v(x,y) = -x^2y$ are considered (shown in red in figures 46, 48, 50, 52). A small perturbation of $u_{perturb} = v_{perturb} = 0.1x$

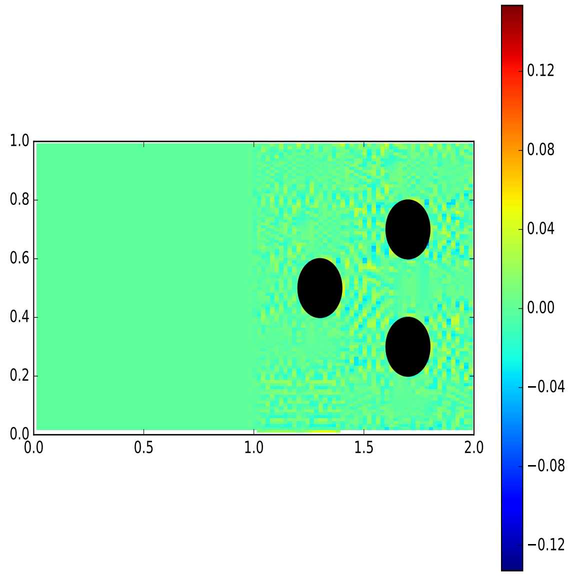

Figure 44: Mesh3: Contour plot for divergence

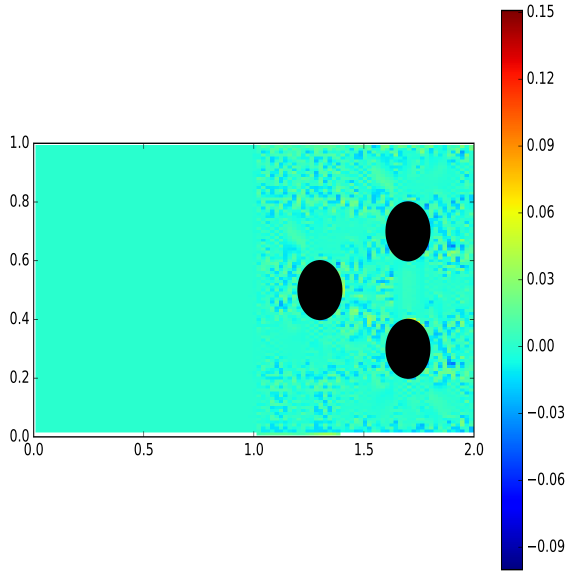

Figure 45: Mesh4: Contour plot for divergence

is added to the above divergenceless field, which gives the starred velocity fields (shown in figures , , and). These starred velocities are corrected through a pressure-correction step to give the the corrected velocities (shown in blue in figures 46, 48, 50, 52). The ranks of A are 1381, 2615, 4593 and 7796 which is one less than the corresponding no. of CVs

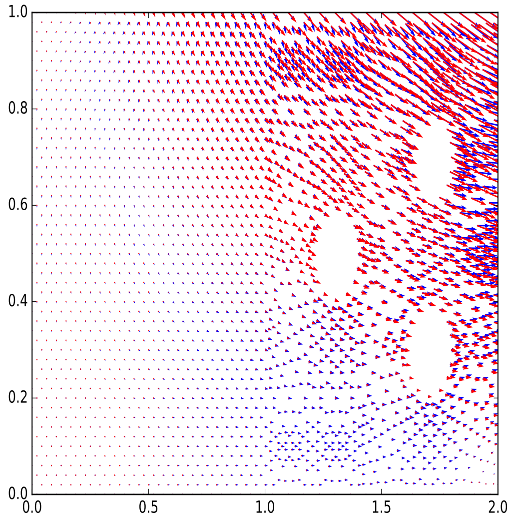

Figure 46: Mesh1: Computed and original velocity fields

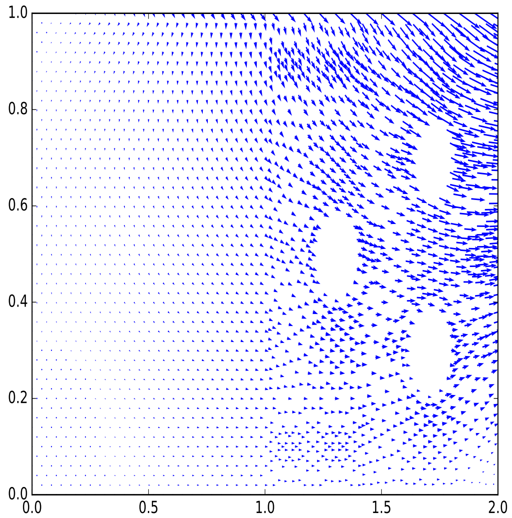

Figure 47: Mesh1: Starred velocity fields

The following figures show the log of the absolute value of the divergence of the perturbed (figures 54, 56, 58 and 60) and corrected (figures 55, 57, 59 and 61) fields:

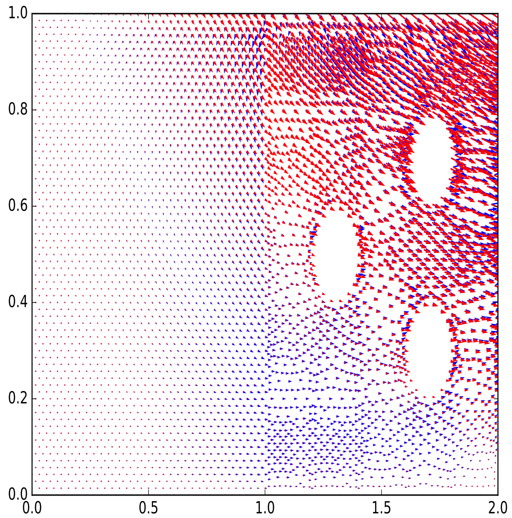

Figure 48: Mesh2: Computed and original velocity fields

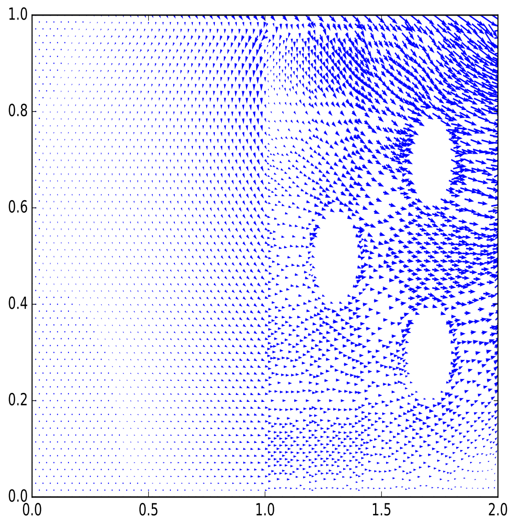

Figure 49: Mesh2: Starred velocity fields

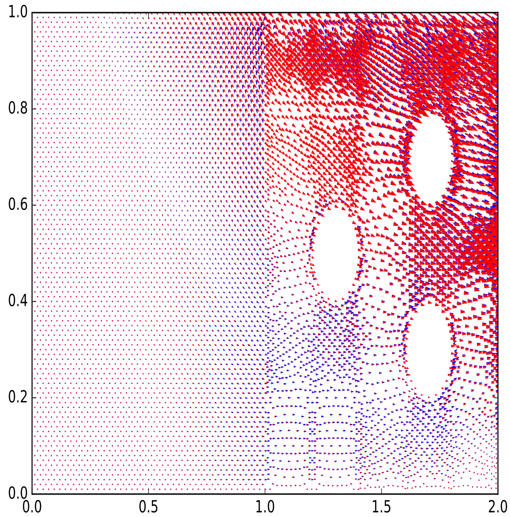

Figure 50: Mesh3: Computed and original velocity fields

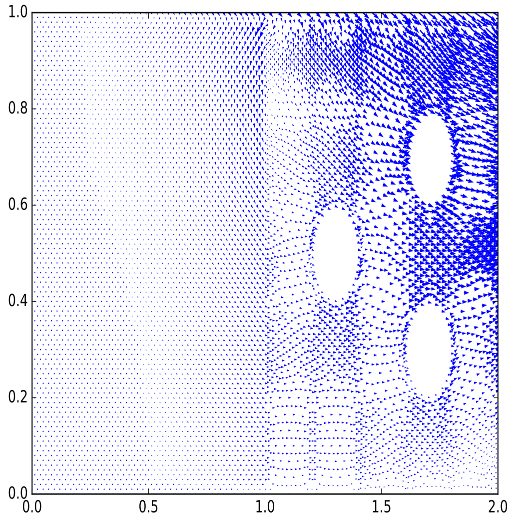

Figure 51: Mesh3: Starred velocity fields

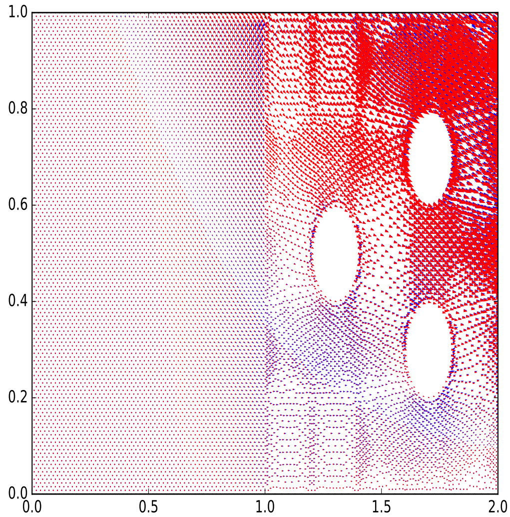

Figure 52: Mesh4: Computed and original velocity fields

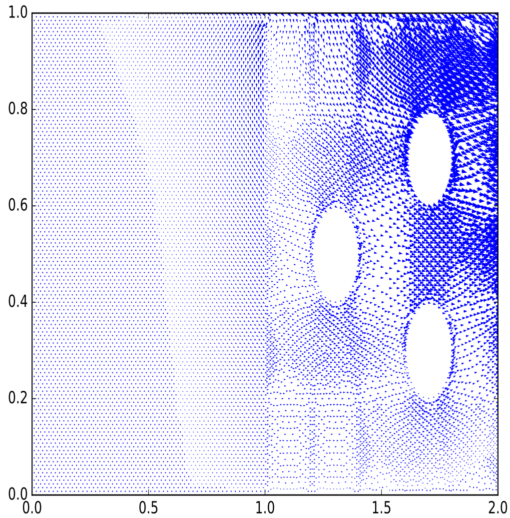

Figure 53: Mesh4: Starred velocity fields

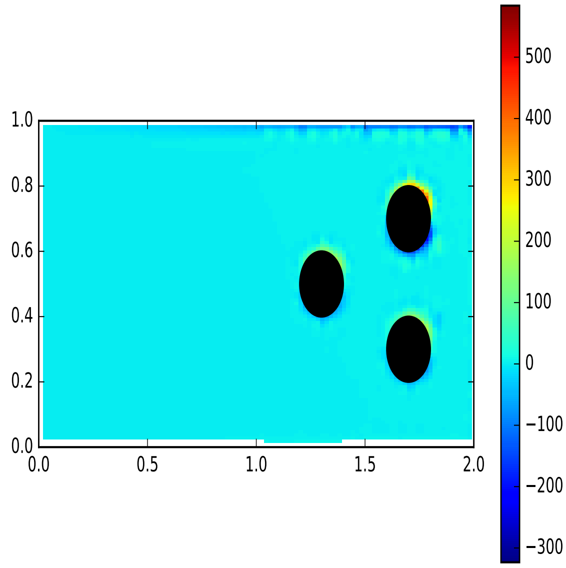

Figure 54: Mesh1:Log of absolute value of divergence of perturbed (starred) velocity field

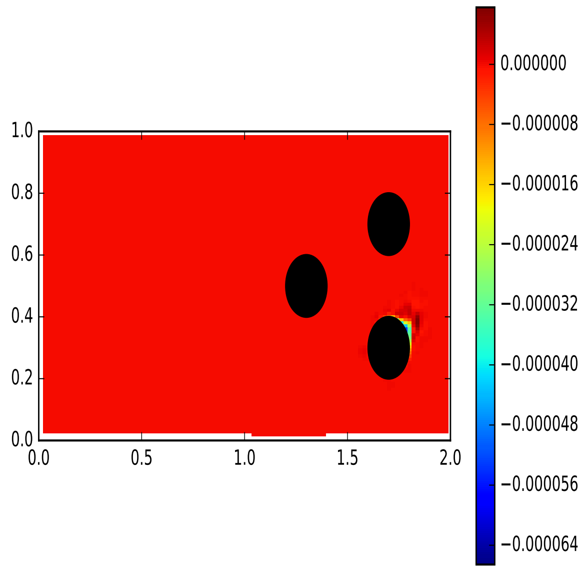

Figure 55: Mesh1:Log of absolute value of divergence of corrected velocity field

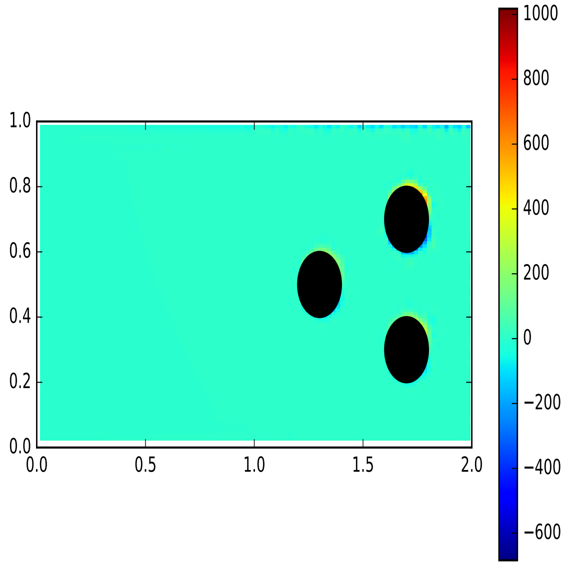

Figure 56: Mesh2:Log of absolute value of divergence of perturbed (starred) velocity field

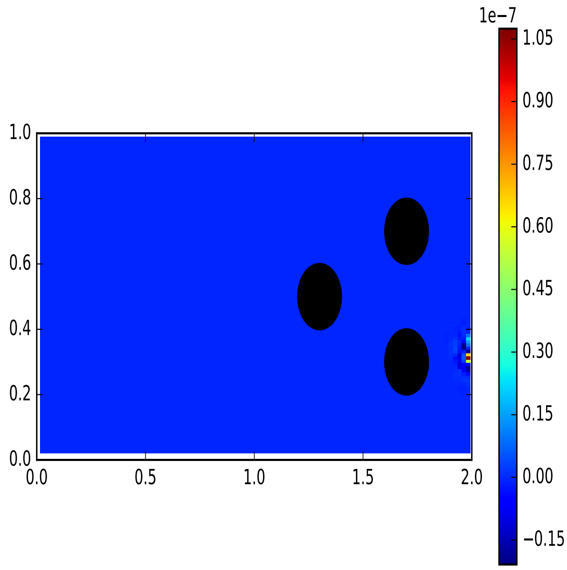

Figure 57: Mesh2:Log of absolute value of divergence of corrected velocity field

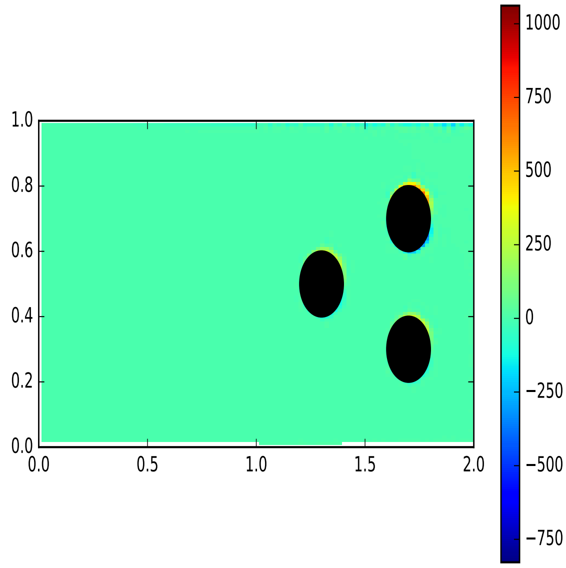

Figure 58: Mesh3:Log of absolute value of divergence of perturbed (starred) velocity field

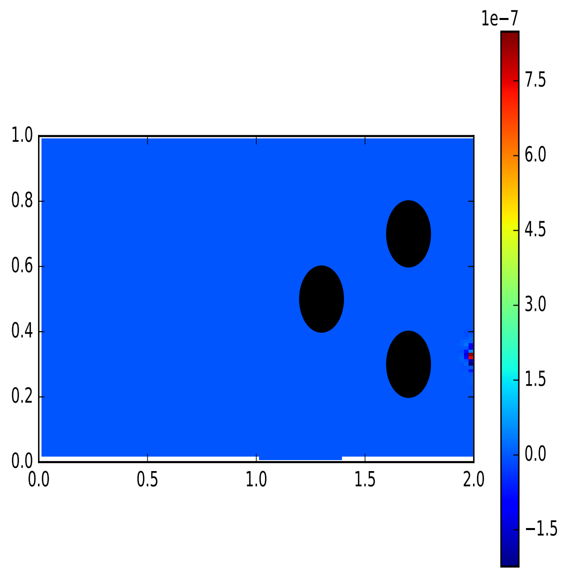

Figure 59: Mesh3:Log of absolute value of divergence of corrected velocity field

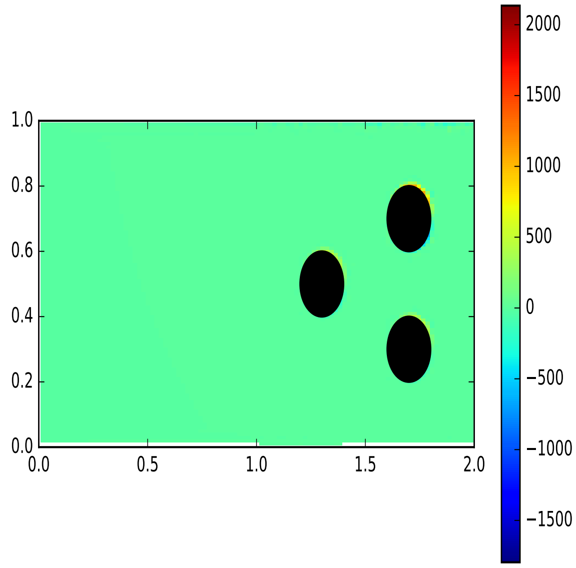

Figure 60: Mesh4:Log of absolute value of divergence of perturbed (starred) velocity field

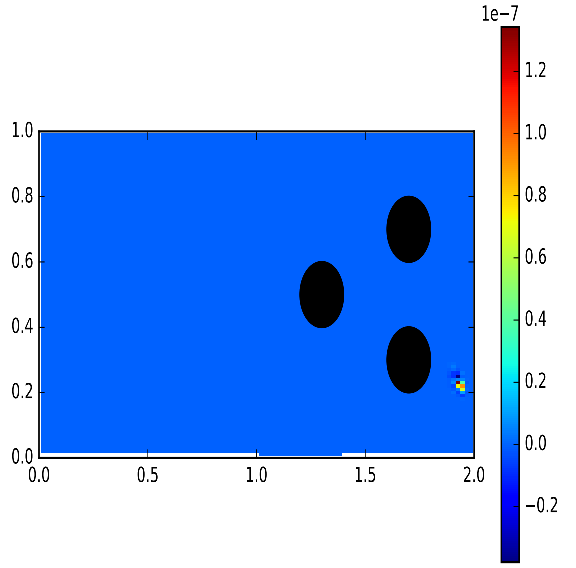

Figure 61: Mesh4:Log of absolute value of divergence of corrected velocity field