

The theory of state space search uses graph theory to model and solve problems by representing them as graphs with nodes (states) and arcs (transitions).

Each node represents a specific state in the problem-solving process, such as game positions or logical conclusions, while arcs represent actions or inferences that move between states.

This approach is especially useful in **expert systems** where **rules generate new knowledge**.

Graph theory, originally developed by Leonhard Euler to solve the “bridges of Königsberg” problem, remains a powerful tool for analysing the structure and relationships within such problems. The city of Königsberg occupied both banks and two islands of a river. The islands and the riverbanks were connected by seven bridges, as indicated in Figure 3.1.

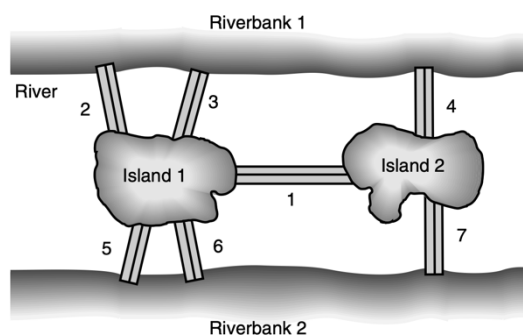


Figure 3.1 The city of Königsberg.

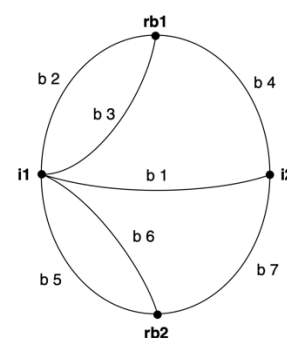


Figure 3.2 Graph of the Königsberg bridge system.

Structures refer to the way we organize and represent the *state space*. This is typically done using graphs or trees, where **nodes represent possible states of the problem** and **edges (or arcs) represent actions or transitions between these states**. The starting point (**initial state**), the desired outcome (**goal state**).

Strategies are the methods or algorithms used to explore the state space in search of a solution. **These strategies determine the order in which nodes are visited and include uninformed search strategies (such as breadth-first search and depth-first search) and informed search strategies (such as best-first search and A*)**. Each strategy has its own strengths, weaknesses, and suitability depending on the nature of the problem, such as its size, complexity, and whether or not heuristic information is available.

Definition of a State Space Search

A state space is represented by a four-tuple $[N, A, S, GD]$, where:

- N is the set of nodes or states of the graph. These correspond to the states in a problem-solving process.
- A is the set of arcs between nodes. These correspond to the steps in a problem-solving process.
- S , a nonempty subset of N , contains the start state(s) of the problem.
- GD , a nonempty subset of N , contains the goal state(s) of the problem.

The states in GD are described using either:

- A measurable property of the states encountered in the search.
- A measurable property of the path developed in the search, for example, the sum of the transition costs for the arcs of the path.

A solution path is a path through this graph from a node in S to a node in GD

EXAMPLE 3.1.3: THE TRAVELING SALESPERSON

Suppose a salesperson has five cities to visit and then must return home. The goal of the problem is to find the shortest path for the salesperson to travel, visiting each city, and then returning to the starting city.

Figure 3.9 gives an instance of this problem. The nodes of the graph represent cities, and each arc is labelled with a weight indicating the cost of traveling that arc. **Assume the salesperson lives in city A and will return there.**

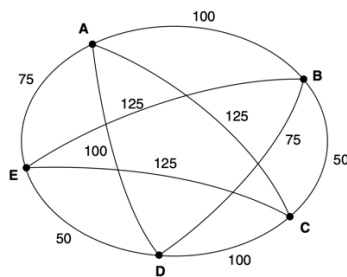


Figure 3.9 An instance of the traveling salesperson problem.

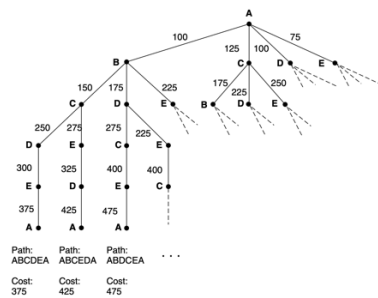


Figure 3.10 Search of the traveling salesperson problem. Each arc is marked with the total weight of all paths from the start node (A) to its endpoint.

Nearest neighbour approach:

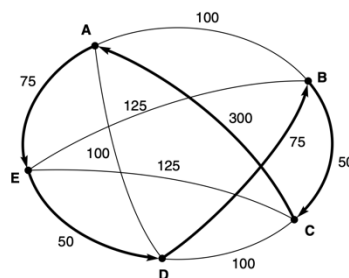


Figure 3.11 An instance of the traveling salesperson problem with the nearest neighbor path in bold. Note that this path (A, E, D, B, C, A), at a cost of 550, is not the shortest path. The comparatively high cost of arc (C, A) defeated the heuristic.