

Week 3 HOMEWORK

1.) Given, virtual address space = 36 bit
page size = 8K
per page table entry = 4B.

a.) No. of pages = $\frac{\text{Virtual memory size}}{\text{page size}}$

$$= \frac{2^{36} \text{ B.}}{2^{13} \text{ B.}}$$
$$= 2^{23} \text{ pages.}$$

b.) Per page table entry is 4 bytes i.e. 32 bits, so,
it can reference 2^{32} pages.

Since, the size of each page is 2^{13} B,
the maximum possible physical memory size is

$$2^{32} \times 2^{13} = 2^{45} \text{ B. i.e. } 32 \text{ TB.}$$

c.) The virtual memory size is 2^{36} B, so, no. of pages
that can be referenced by the page table is
 $\frac{2^{36} \text{ B.}}{2^{13}}$ i.e 2^{23} pages.

Each page entry size is 4 bytes.
So total size of the page table is $2^{23} \times 4 = 2^{25}$ B
i.e. 32 MB.

Now, given that process size is 8GB.

32 MB size page table will occupy a lot of process
memory space. (hence costly).

∴ 1st level page table will not be good.

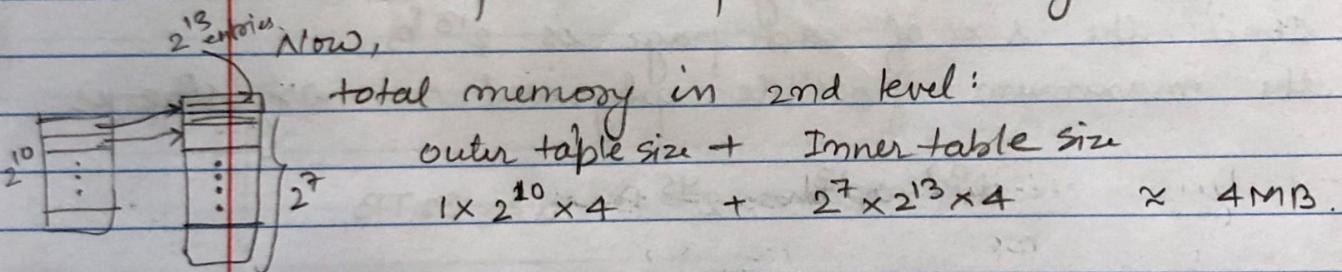
2nd level - The virtual addresses can be divided up as

P ₁	P ₂	
10	13	13

Now, since the process size is $2^{33} B$: No of pages to be referenced are 2^{20} pages.

Inner table P₂ can reference 2^{20} pages and since each entry in inner table can be 13 bits i.e. 2^{13} entries, we need $\frac{2^{20}}{2^{13}} = 2^7$ such entries in inner table.

Outer table can have 2^{10} entries (outer table is page directory). So, size of outer table which is stored in physical memory is $2^{10} \times 4 \text{ bytes} = 4 \text{ kB}$. 4 kB occupies less space so it is good.



P₁ P₂ P₃
3rd level paging - Suppose we divide virtual address as,

P ₁	P ₂	P ₃	
9	7	7	13

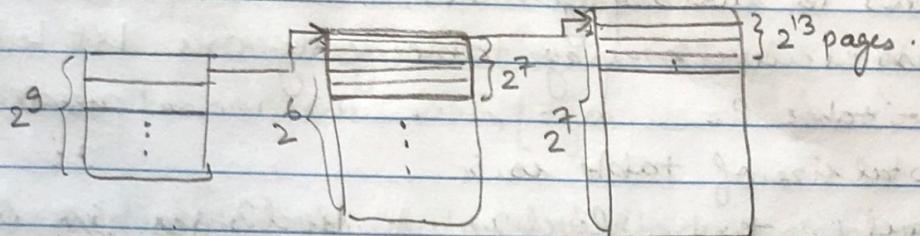
Similarly as above, to address 2^{20} pages in inner table P₃, we need $\frac{2^{20}}{2^7} = 2^{13}$ inner tables (because

each inner tables can have 2^7 entries i.e. 2^7 pages)
 Each entry in 2nd level table P₂, points to 2^{13} 3rd level page table entries. So we need $\frac{2^{13}}{2^7} = 2^6$ such level 2 tables.

And we one level 1 table to point to 2^6 level 2 page tables.

So, total size of table

$$\text{level 1} + \text{level 2} + \text{level 3}$$
$$1 \times 2^9 \times 4B + 2^6 \times 2^7 \times 4B + 2^{13} \times 2^7 \times 4 \approx 4MB$$



Here we see that for both level 2 and level 3 paging, the total memory occupied is 4MB. But level 2 is simple. So level-2 is better choice.

d.) The average size of the table as calculated above is 4MB.

2.) Given, virtual address = 32-bit i.e. $2^{32}B$.

divided as, $P_1 \quad P_2 \quad P_3$

10	18	16	8
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Clearly, level 3 paging is used.

a.) We see that the last 8-bits are the offset of page
∴ the page size is 2^8 bytes i.e. 256 bytes.

b.) Now given, process size = 256K i.e. $2^{18}B$.
i.e. the pages required to access are

$$\frac{2^{18}}{2^8} = 2^{10} \text{ pages.}$$

Now, here we see that,

level 1 table P_1 points to 2^{10} entries (1024) of P_2

level 2 table P_2 points to 2^8 entries (256) of P_3

level 3 table P_3 has 64 entries.

We have 2^{10} pages to be addressed.
So, we need $\frac{2^{10}}{2^4} = 16$ level 3-page tables $\rightarrow P_3$.

To access 16 level 3-page tables, we need 1 2nd level table
To access 1 level 2-page table, only one 1st level is needed.

∴ Total size of table is :

$$\begin{aligned} & \text{level 1} + \text{level 2} + \text{level 3} \\ & 1 \times 1024 \times 4 + 1 \times 256 \times 4 + 16 \times 64 \times 4 = \\ & (\text{Assume 4 bytes per entry}) \\ & = 4096 + 1024 + 4096 \\ & = 9216 \text{ bytes} \end{aligned}$$

c.) Given, in a process,

code segment - 48K

data segment - 600K

stack segment - 64K.

As we know, page size is 2^8 B and it is assumed the page entry size is 4B.

∴ No. of pages for code segment = $\frac{48K}{2^8}$

$$\begin{aligned} & = 48 \times 2^2 \\ & = 192 \text{ pages.} \end{aligned}$$

No. of pages for data segment = $\frac{600K}{2^8} \Rightarrow 2400 \text{ pages.}$

No. of pages of stack segment = $\frac{64K}{2^8} = 256 \text{ pages.}$

For 48K code segment, we need $\frac{192}{64} = 3$, 3rd level tables to address 192 pages.

For 600K data segment, we need $\frac{2400}{64} = 37.5 \approx 38$ 3rd level tables to address 2400 pages.

For 64K stack segment, we need $\frac{256}{64} = 4$ 3rd level tables to address 256 pages.

To address code, data and stack we need 1, 1st level page table in which each entry has address to code data & stack having 3 page table entries. i.e. 1st level table points to 3 2nd level table.

So, size of table,

$$\begin{aligned} & \text{level 1} + \text{level 2} + \text{level 3} \\ & 1 \times 1024 \times 4 + 3 \times 256 \times 4 + (3+38+4) \times 64 \times 4 \\ & = 4096 + 3072 + 11520 \\ & = 18688 \text{ bytes.} \end{aligned}$$

3) Given, a virtual machine 32 bits divided as.

8	4	8	12
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Physical address = 44 bits

protection bits per page = 4 bits.

a) The last 12-bits are page offset. So the size of the page is 2^{12} i.e. 4KB.

b.)

	P ₁	P ₂	P ₃	
	8	4	8	12

Using above division, we see that

1st level has $2^8 = 256$ entries each pointing to 2nd level
 2nd level has $2^4 = 16$ entries, each pointing to
 3rd level table

3rd level has $2^8 = 256$ entries each pointing to a page.

We have a process size 64k,

So it contains $\frac{2^{16}}{2^{12}}$ pages i.e. $2^4 = 16$ pages.

To address 16 pages, we need 1 3rd level page table.
 ∴ 1 entry in 2nd level and one entry in 1st level page.

Now, physical address is 44 bits and page size is 4k ∴ 32 bits for frame no.

32	12
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Given that table entry has 4 bit protection bit $\leftarrow 44 \rightarrow$
 ∴ the entry would be $32+4=36$ bits. Round up to entire word aligned, it is 64 bits or 8 bytes.

∴ The page table size of 3rd level is

$$256 \times 8 = 2048 \text{ bytes.}$$

Now, for level 1 page table, we should not assume that in 2nd level page table pages are aligned. So, full address should be stored.

each page entry is at least 44 bits. Rounding up to 64 bits (8 bytes)

$$\therefore \text{1st level page table size} \\ = 256 \times 8 = 2048$$

Similarly, 2nd page table also cannot assume page alignment. Hence 44 bits size page entry.

$$\text{2nd level page table size} = 16 \times 8 = 128 \text{ Bytes.}$$

$$\begin{aligned}\therefore \text{Total size of page table} &= \text{1st level} + \text{2nd level} + \text{3rd level} \\ &= 2048 + 128 + 2048 \\ &= 4224 \text{ bytes.}\end{aligned}$$

The total size of pagetable is 4224 bytes and each frame is 4kB i.e. 4096 bytes, 4224 bytes would need 2 such frame to be stored. $2 \text{ pages} = 4096 \times 2 = 8192$. 2nd page will not be fully utilized (only 128 out of 4096 utilized). So there will be internal fragmentation & $4096 - 128 = 3968$ bytes will be wasted.

c.) We know that,

1st level page table has 256 entries

2nd level page table has 16 entries

3rd level page table has 256 entries.

for code segment, we have $\frac{48K}{4K} = 12 \text{ page}$. To access

12 pages we need 1 3rd level page table.

for data segment, we have $\frac{600K}{4K} = 150 \text{ pages}$ and

to access 150 pages we need 1 3rd level page table

for stack segment we have $\frac{64K}{4K} = 16 \text{ pages}$ and

to access 16 page we need

1 3rd level page table.

we have 1 entry in 2nd level page table.
and 3 entries in 1st level page table to point
code, data and stack.

∴ We need 3 2nd level page table & 1 1st level
page table.

To calculate size of page table we consider
protection bit in the page entry too.

$$\therefore \text{3rd level page table size} = (1+1+1) \times 256 \times 8 \\ = 6144 \text{ bytes}$$

(8 byte page entry because total bits are $32+4=36$)

To round up to next word, we use 64 bits i.e. 8 bytes)

$$\text{2nd level page table size} = 3 \times 16 \times 8 \\ = 384 \text{ bytes}$$

$$\text{1st level page table size} = 1 \times 256 \times 8 \\ = 2048 \text{ bytes}$$

$$\therefore \text{Total size} = 6144 + 384 + 2048 \\ = 8576 \text{ bytes.}$$

Here, the page table size is 8576 bytes. each frame size
is 4096 bytes. So, it would need 3 frames to
get stored. 2 frames are fully utilized by 3rd
is not.

$$8576 - 8192 = 384 \text{ bytes are stored in}\\ \text{3rd frame.}$$

$$\therefore 4096 - 384 = 3712 \text{ bytes gets wasted in}\\ \text{the 3rd frame.}$$

So, internal fragmentation occurs and
3712 bytes get wasted.