MAGNETIC CIRCUITS

Overview

1. Understand magnetic fields and their interactions with moving charges.
2. Use the right-hand rule to determine the direction of the magnetic field around a current-carrying wire or coil.
3. Calculate forces on moving charges and current carrying wires due to magnetic fields.
4. Calculate the voltage induced in a coil by a changing magnetic flux or in a conductor cutting through a magnetic field.
5. Use Lenz's law to determine the polarities of induced voltages.

Overview

6. Apply magnetic-circuit concepts to determine the magnetic fields in practical devices.
7. Determine the inductance and mutual inductance of coils given their physical parameters.
8. Understand hysteresis, saturation, core loss, and eddy currents in cores composed of magnetic materials such as iron.
9. Understand ideal transformers and solve circuits that include transformers.
10. Use the equivalent circuits of real transformers to determine their regulations and power efficiencies.

Magnetic Field

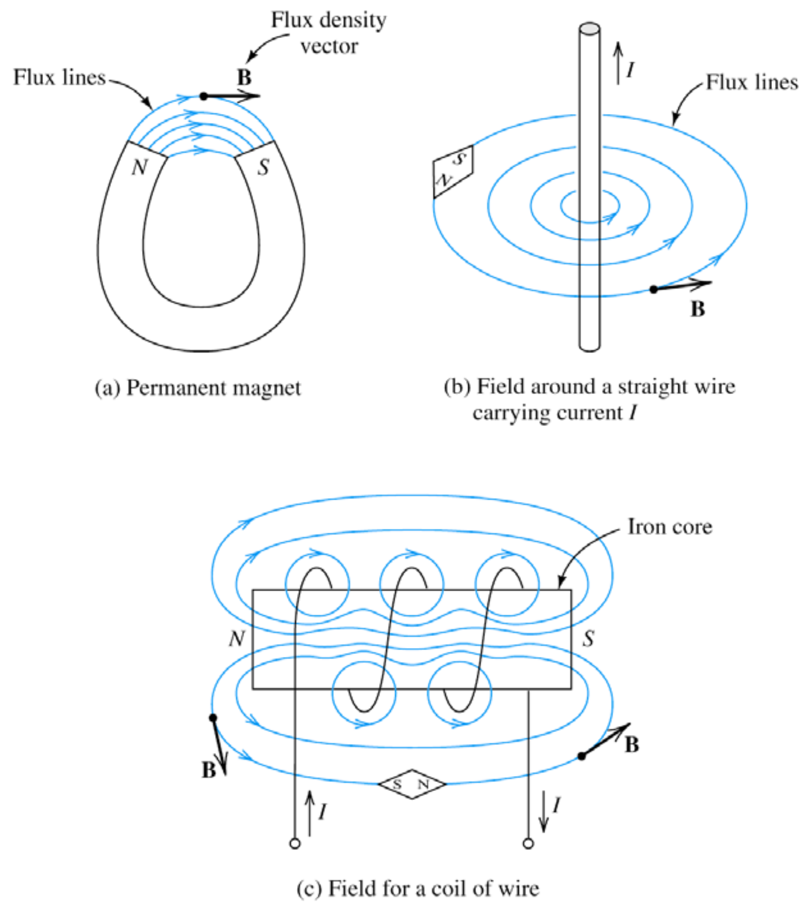


Figure 15.1 Magnetic fields can be visualized as lines of flux that form closed paths. Using a compass, we can determine the direction of the flux lines at any point. Note that the flux density vector **B** is tangent to the lines of flux.

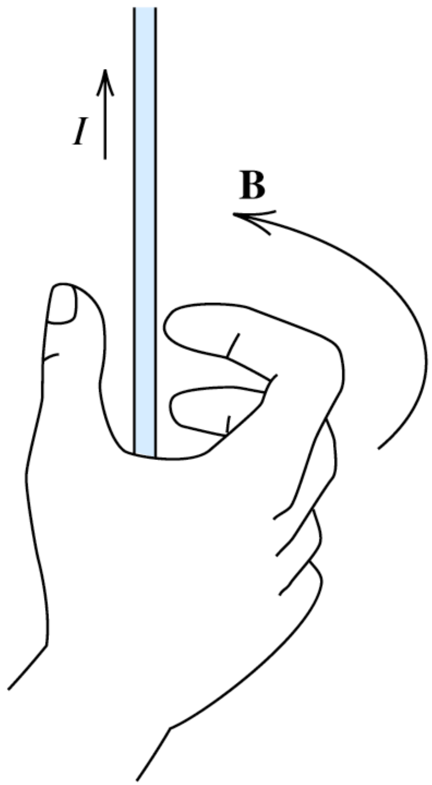
MAGNETIC FIELDS

Magnetic flux lines form closed paths that are close together where the field is strong and farther apart where the field is weak.

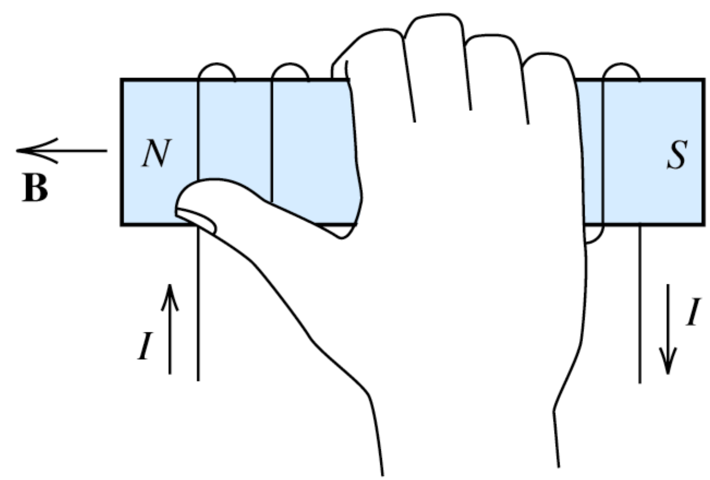
Flux lines leave the north-seeking end of a magnet and enter the south-seeking end.

When placed in a magnetic field, a compass indicates north in the direction of the flux lines.

Right-Hand Rule

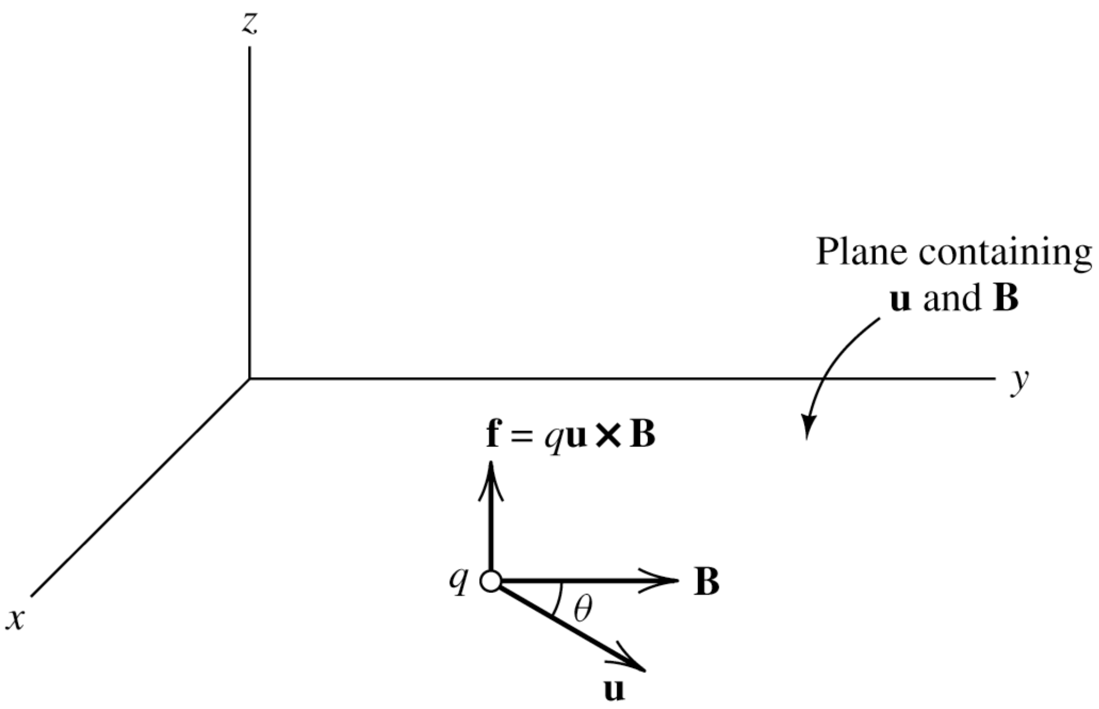


(a) If a wire is grasped with the thumb pointing in the current direction, the fingers encircle the wire in the direction of the magnetic field



(b) If a coil is grasped with the fingers pointing in the current direction, the thumb points in the direction of the magnetic field inside the coil

Forces on Charges Moving in Magnetic Fields



$$\mathbf{f} = q\mathbf{u} \times \mathbf{B}$$

$$f = quB\sin(\theta)$$

Forces on Current-Carrying Wires

$$d\mathbf{f} = i d\mathbf{l} \times \mathbf{B}$$

$$f = ilB\sin(\theta)$$

Flux Linkages and Faraday's Law

$$\phi = \int_A \mathbf{B} \cdot d\mathbf{A}$$

$$\lambda = N\phi$$

Faraday's law of magnetic induction:

$$e = \frac{d\lambda}{dt}$$

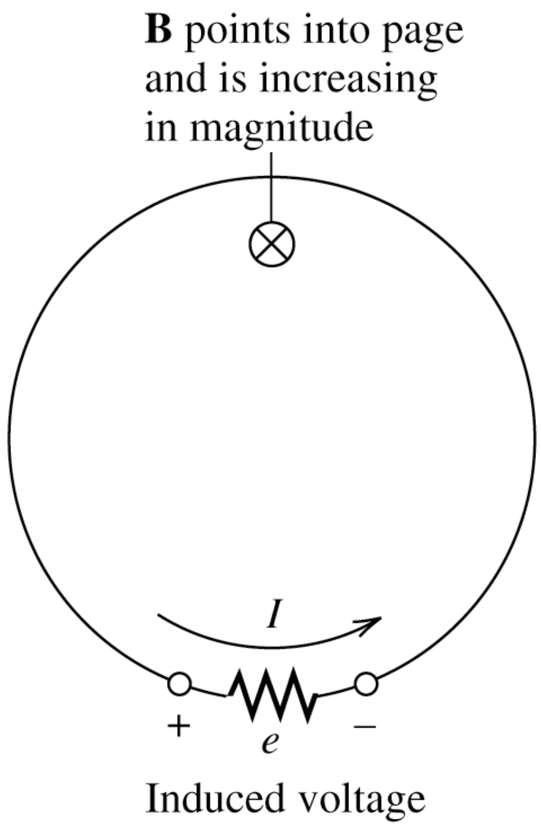
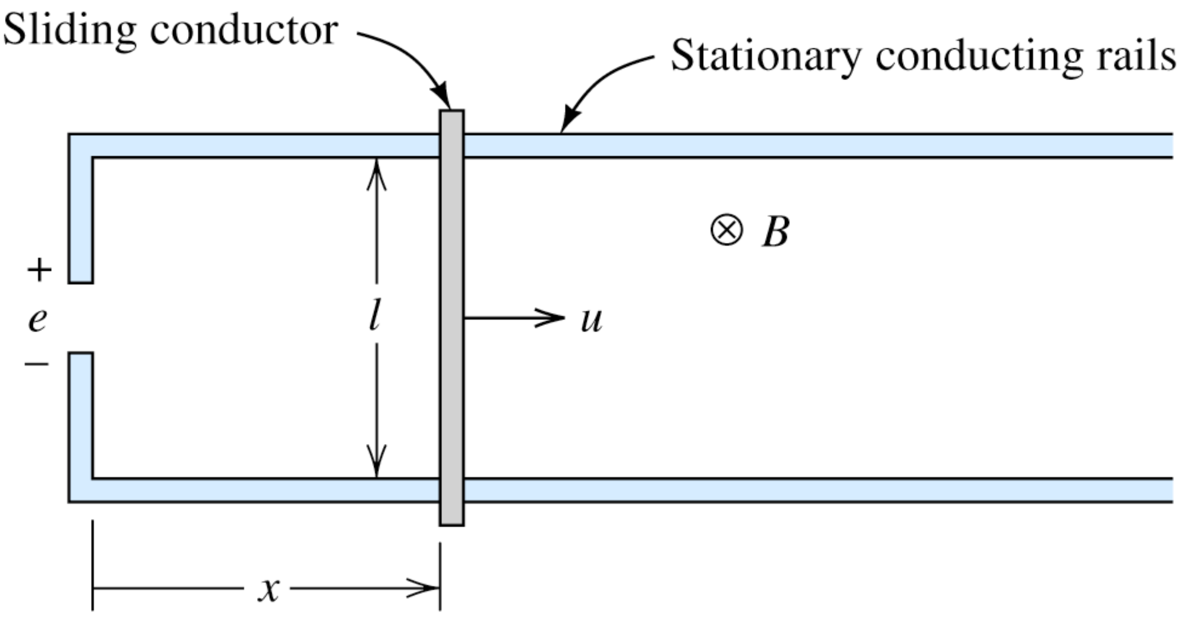


Figure 15.4 When the flux linking a coil changes, a voltage is induced in the coil. The polarity of the voltage is such that if a circuit is formed by placing a resistance across the coil terminals, the resulting current produces a field that tends to oppose the original change in the field.

Lenz's Law

Lenz's law states that the polarity of the induced voltage is such that the voltage would produce a current (through an external resistance) that opposes the original change in flux linkages.

Voltages Induced in Field-Cutting Conductors



$$e = Blu$$

Magnetic Field Intensity and Ampère's Law

$$\mathbf{B} = \mu\mathbf{H}$$

$$\mu_0 = 4\pi \times 10^{-7}\ \text{Wb/Am}$$

$$\mu_r = \frac{\mu}{\mu_0}$$

Ampère's Law:

$$\oint \mathbf{H} \cdot d\mathbf{l} = \sum i$$

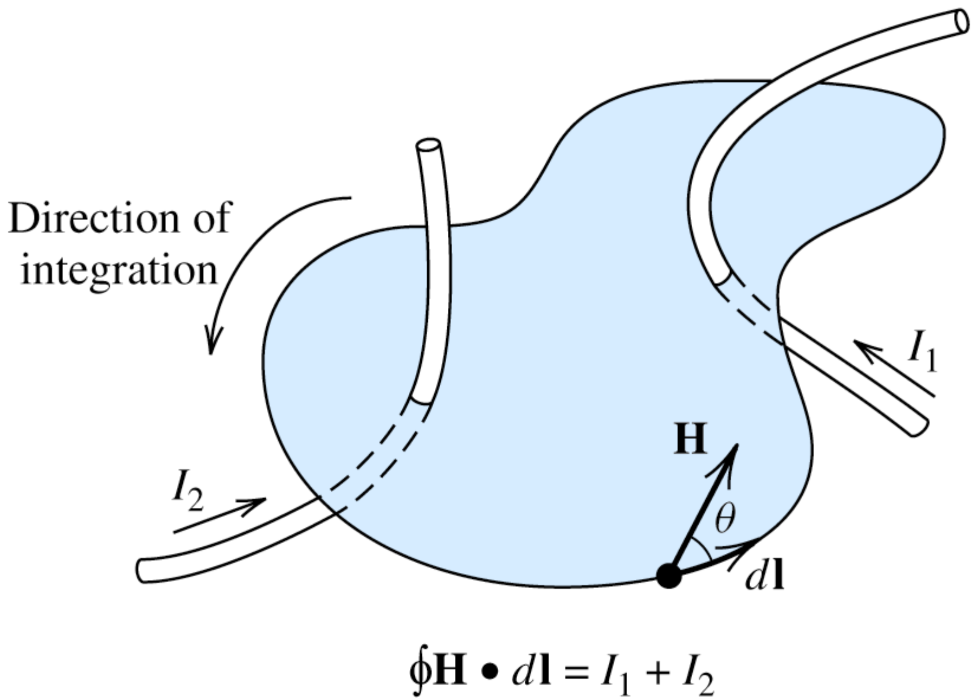
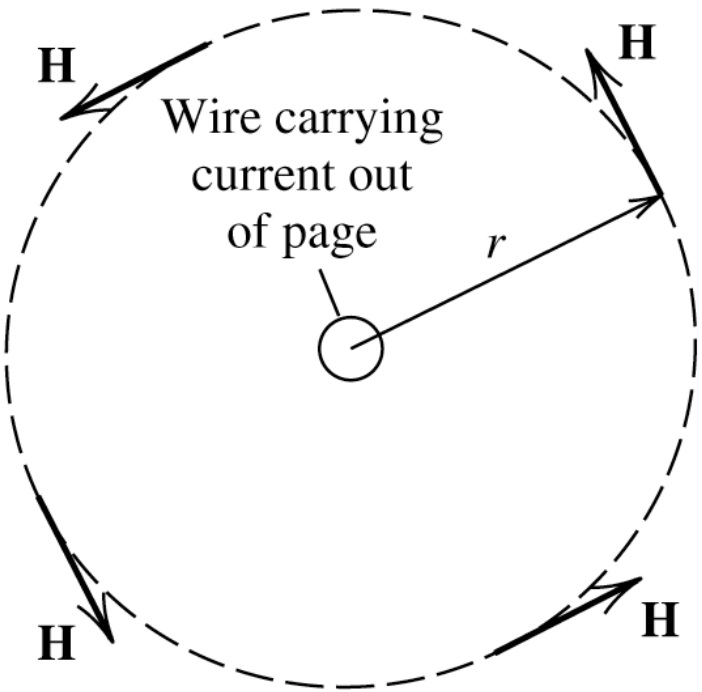


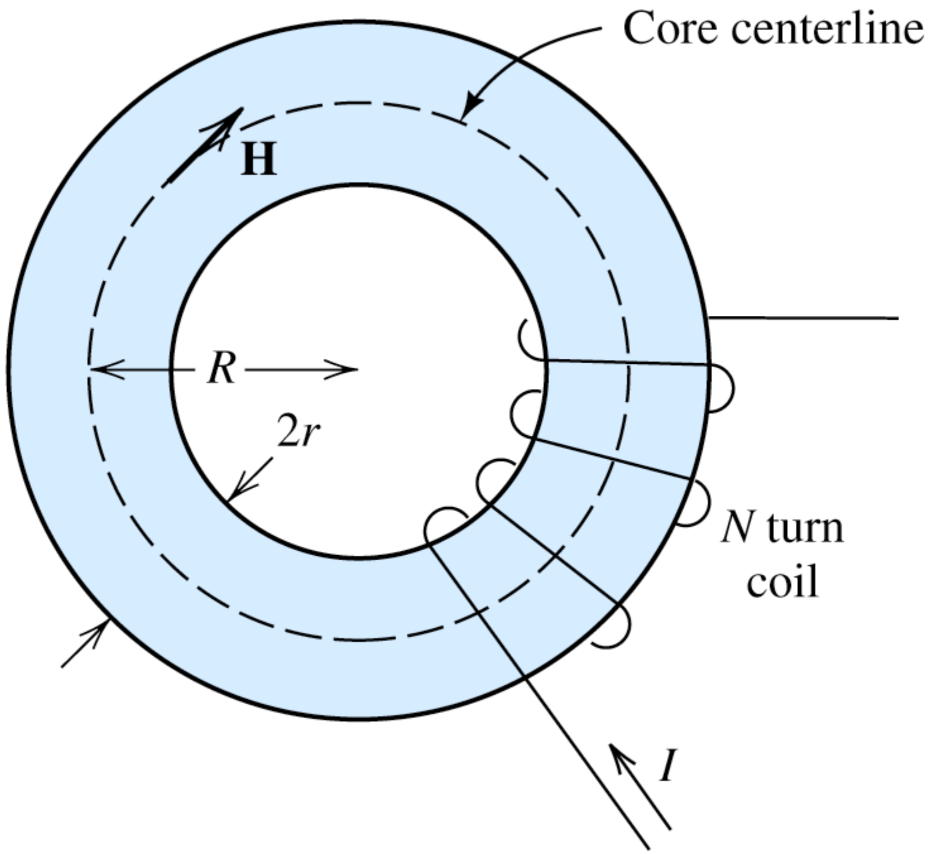
Figure 15.6 Ampère's law states that the line integral of magnetic field intensity around a closed path is equal to the sum of the currents flowing through the surface bounded by the path.

Magnetic Field Around a Long Straight Wire



$$B = \mu H = \frac{\mu I}{2\pi r}$$

Flux Density in a Toroidal Core



$$B = \frac{\mu N I}{2\pi R}$$

$$\phi = \frac{\mu N I r^2}{2R}$$

$$\lambda = N\phi = \frac{\mu N^2 I r^2}{2R}$$

Example Exercise

- Toroidal core with mu_r=5000, R=1cm, r=2cm, N=100. Current i(t)=2sin(200*pi*t). Compute flux, linkages. Use Faraday's law to determine voltage induced in the coil.

MAGNETIC CIRCUITS

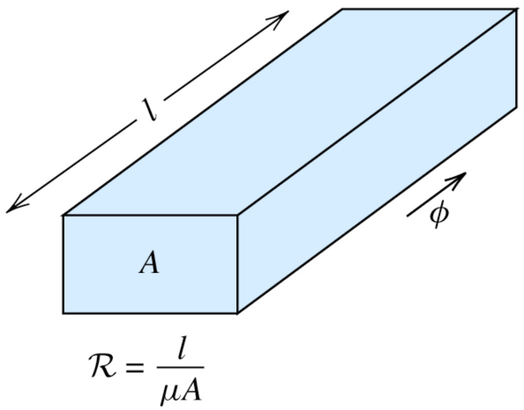
In many engineering applications, we need to compute the magnetic fields for structures that lack sufficient symmetry for straight-forward application of Ampère’s law. Then, we use an approximate method known as magnetic circuit analysis.

Circuit Analysis

magnetomotive force (mmf) of an N–turn current-carrying coil

$$\mathfrak{I} = N\,I$$

reluctance of a path for magnetic flux



$$\Re = \frac{\ell}{\mu A}$$

$$\mathfrak{I} = \Re\,\phi$$

Toroidal Coil as a Magnetic Circuit

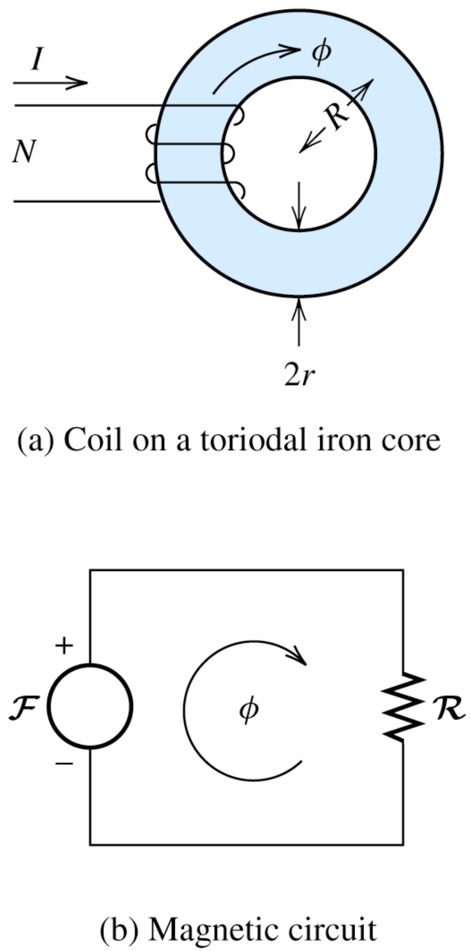


Figure 15.11 The magnetic circuit for the toriodal coil.

$$l = 2\pi R$$

$$\Re = \frac{\ell}{\mu A} = \frac{2R}{\mu r^2}$$

$$\Im = N\ I$$

$$\phi = \frac{\Im}{R}$$

Advantage of the Magnetic-Circuit Approach

The advantage of the magnetic-circuit approach is that it can be applied to unsymmetrical magnetic cores with multiple coils.

Magnetic Circuit with Air Gap

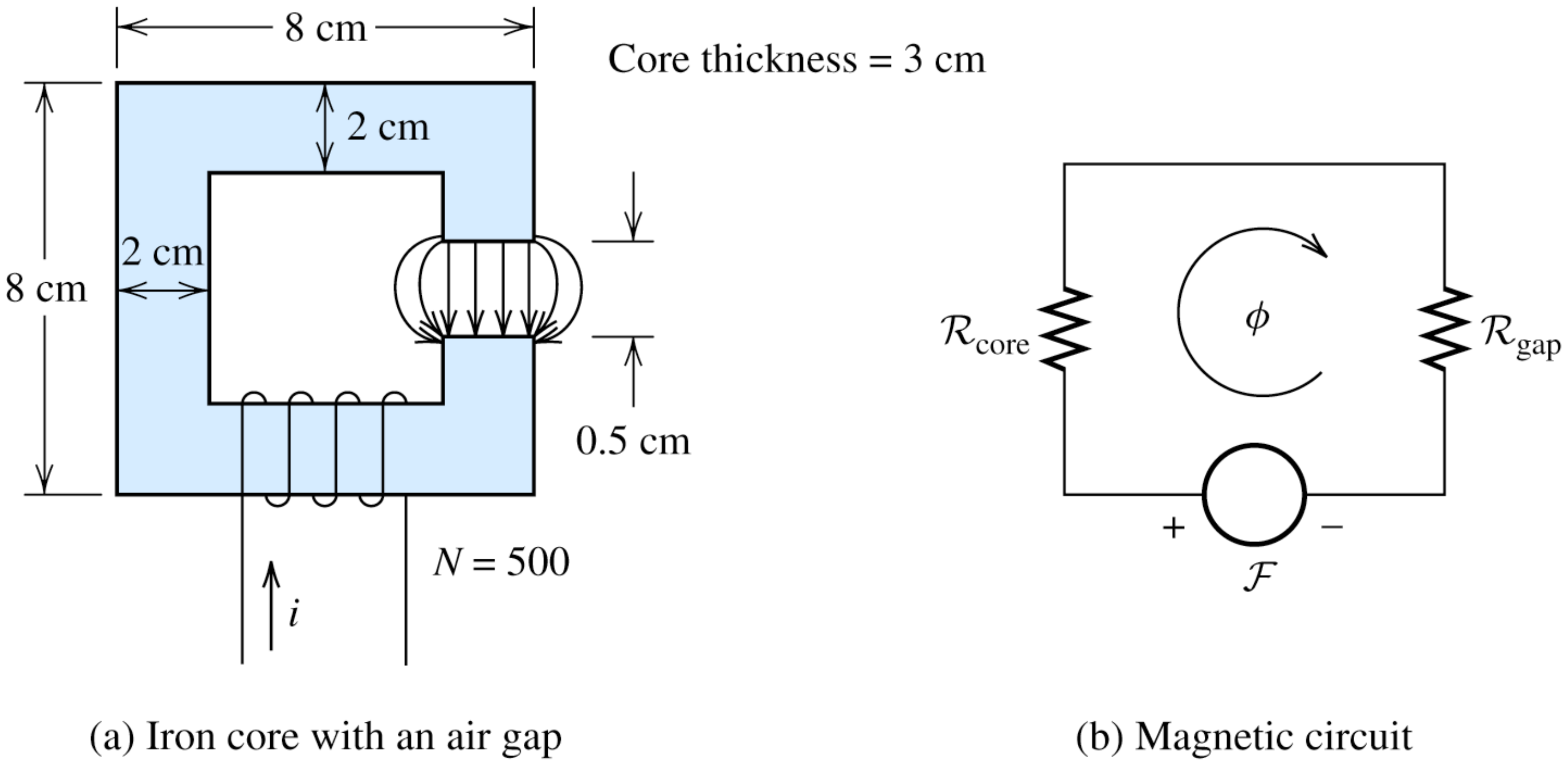
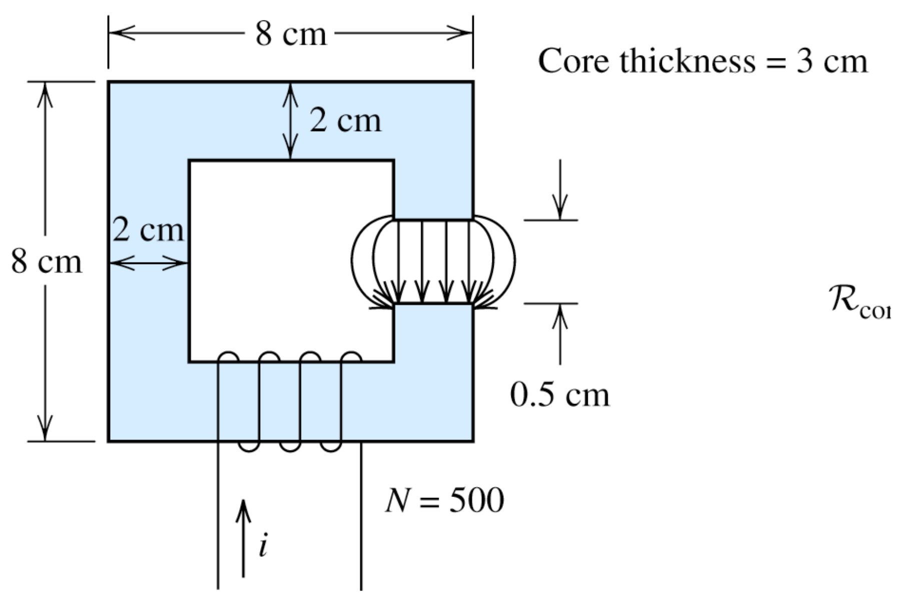


Figure 15.12 Magnetic circuit of Example 15.5.

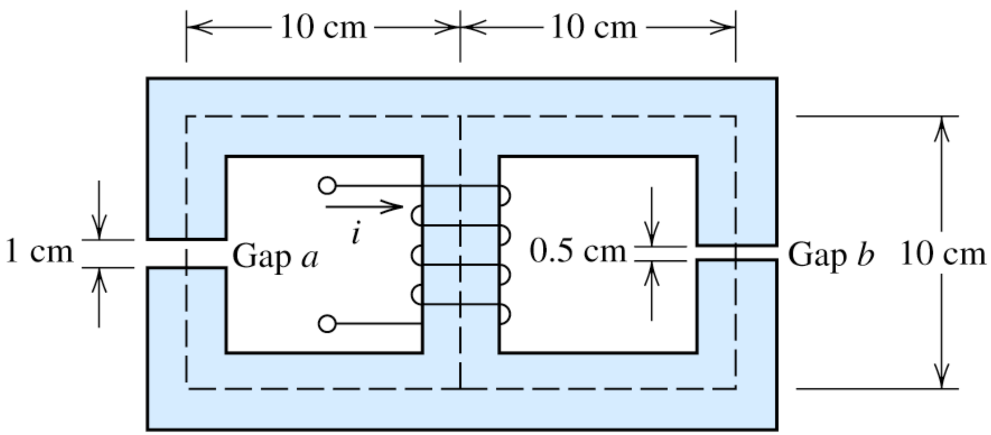
Fringing

We approximately account for fringing by adding the length of the gap to the depth and width in computing effective gap area.

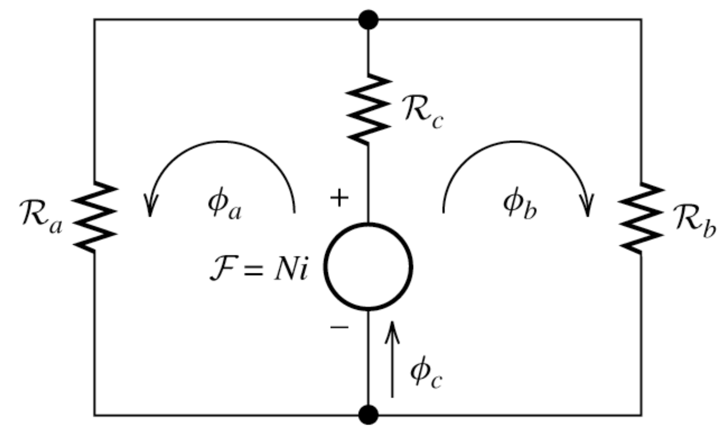


(a) Iron core with an air gap

A Magnetic Circuit with Reluctances in Series and Parallel



(a) Core



(b) Magnetic circuit

INDUCTANCE AND MUTUAL INDUCTANCE

$$L = \frac{\lambda}{i} \qquad L = \frac{N^2}{\Re}$$

$$e = L\frac{di}{dt}$$

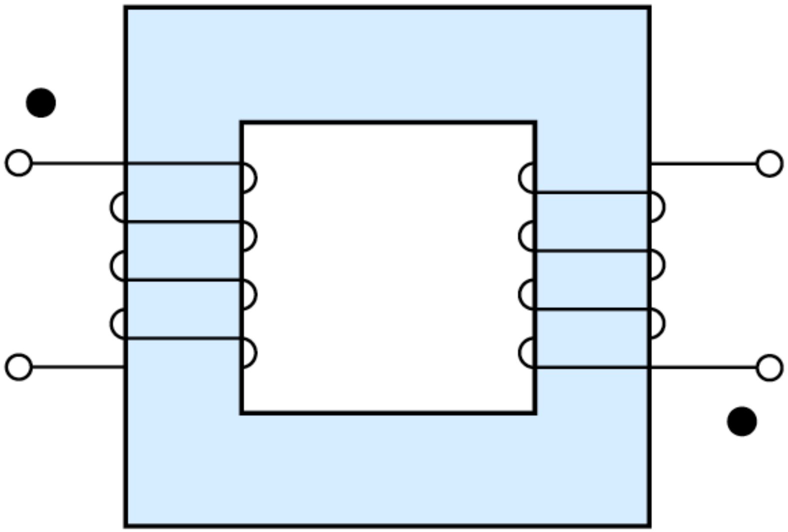
Mutual Inductance

$$L_1 = \frac{\lambda_{11}}{i_1} \qquad L_2 = \frac{\lambda_{22}}{i_2}$$

$$M = \frac{\lambda_{21}}{i_1} = \frac{\lambda_{12}}{i_2}$$

Dot Convention

Aiding fluxes are produced by currents entering like marked terminals.



Circuit Equations for Mutual Inductance

$$\lambda_1 = L_1 i_1 \pm M i_2$$

$$\lambda_2 = \pm M i_1 + L_2 i_2$$

$$e_1 = \frac{d\lambda_1}{dt} = L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt}$$

$$e_2 = \frac{d\lambda_2}{dt} = \pm M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

Example Exercise

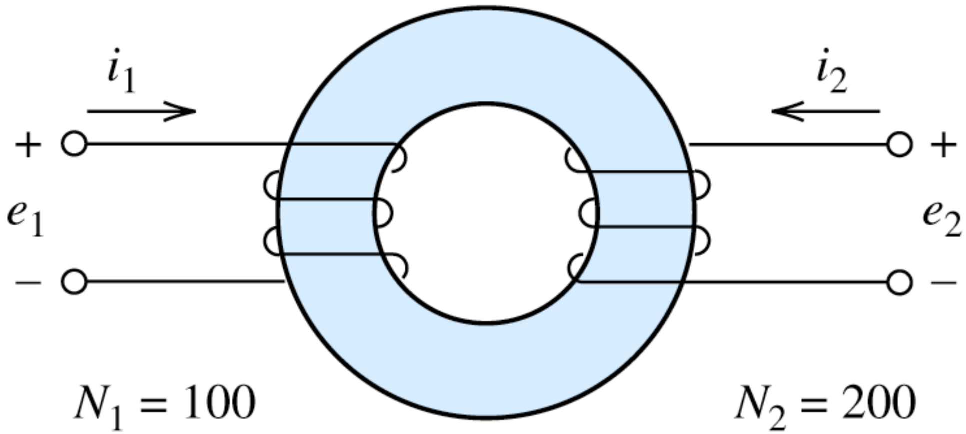


Figure 15.16 Coils of Example 15.8.

Example Exercise

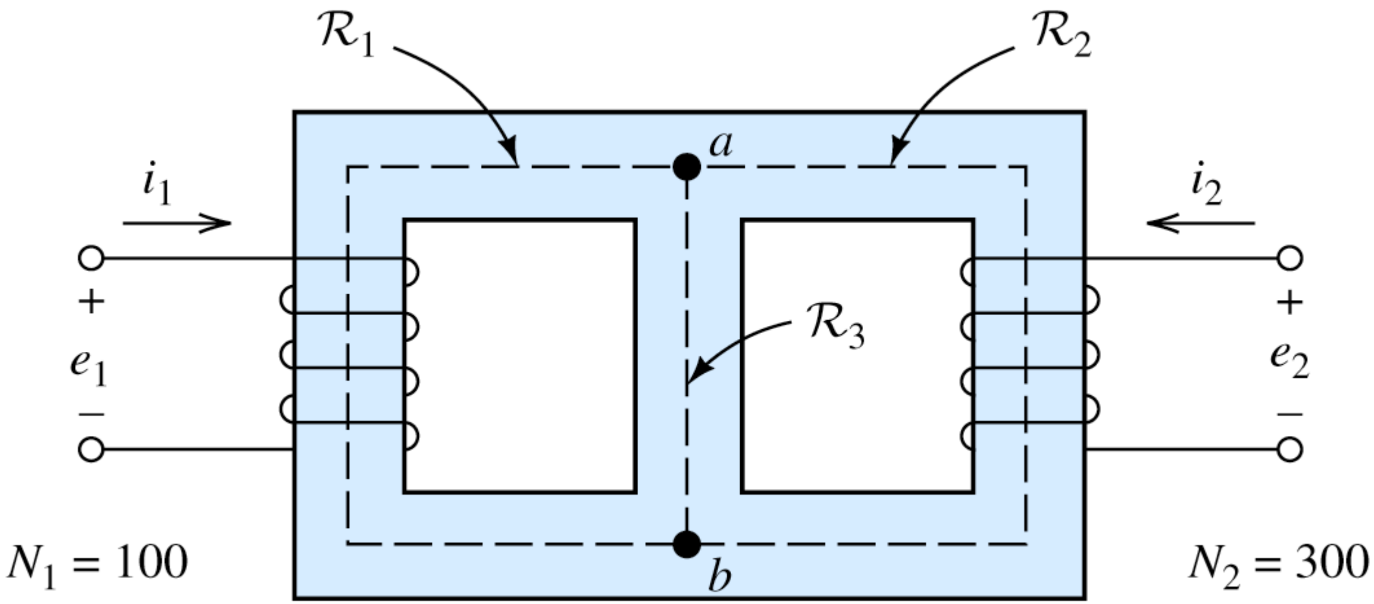
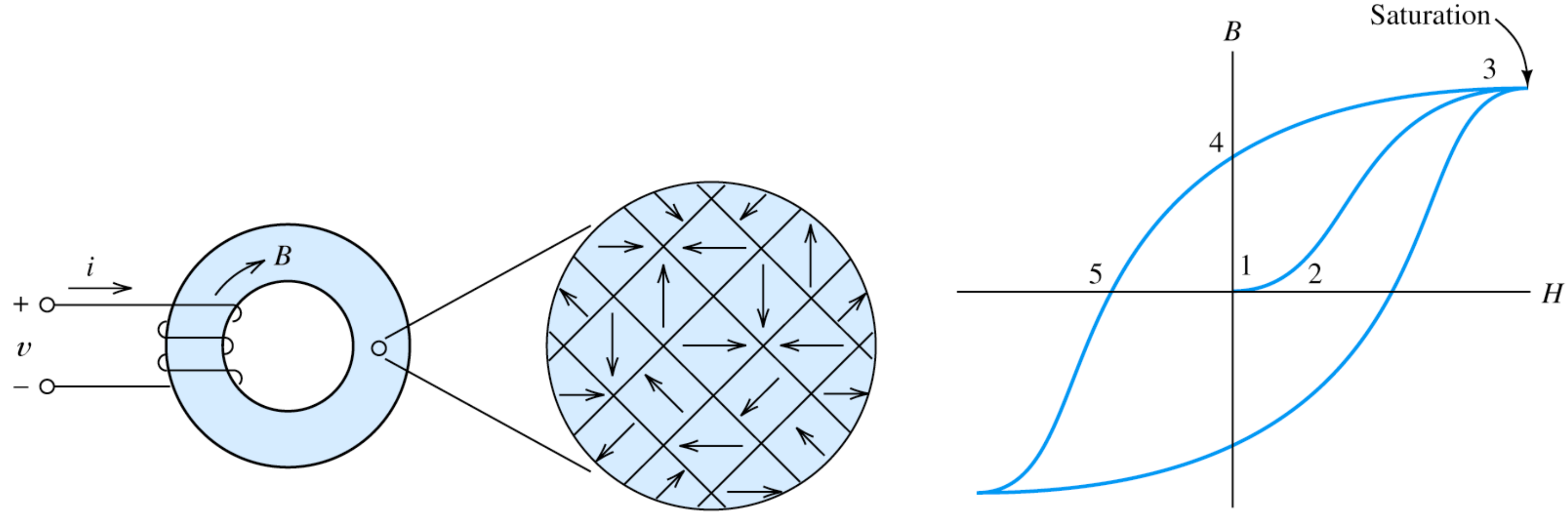


Figure 15.17 Magnetic circuit of Exercise 15.13.

Magnetic Materials

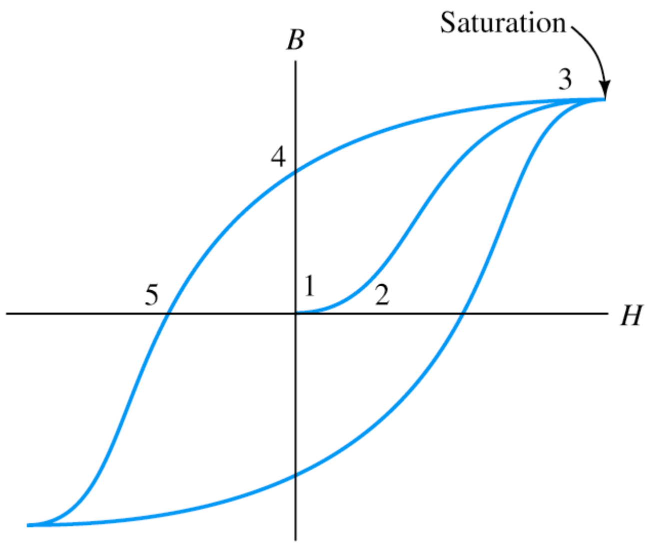


(a) Sample and coil for applying H (b) Magnetic domains (c) Hysteresis loop in the $B-H$ plane

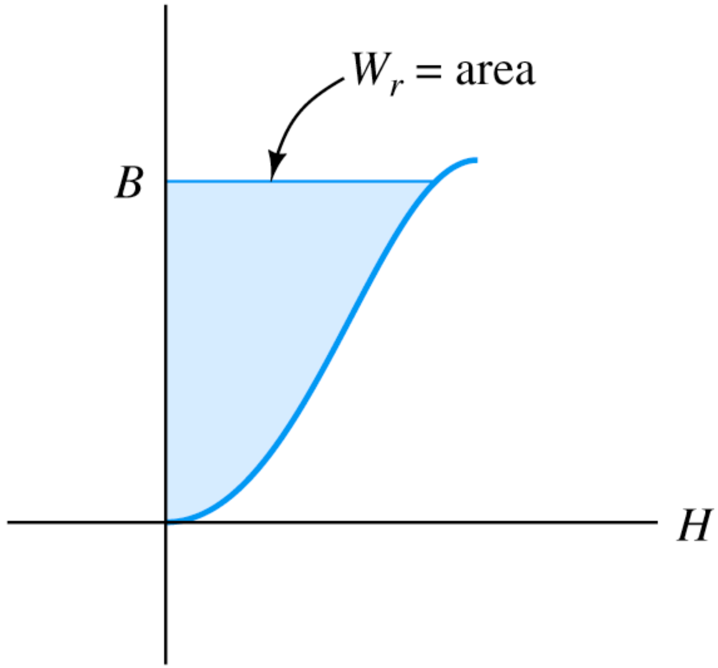
Figure 15.18 Materials such as iron display a $B–H$ relationship with hysteresis and saturation.

MAGNETIC MATERIALS

The relationship between B and H is not linear for the types of iron used in motors and transformers.



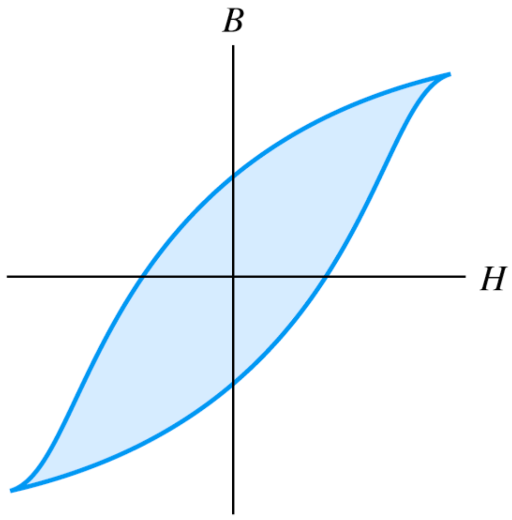
Energy Considerations



$$W_v = \frac{W}{Al} = \int_0^B H\,dB$$

Core Loss

Power loss due to hysteresis is proportional to frequency, assuming constant peak flux.



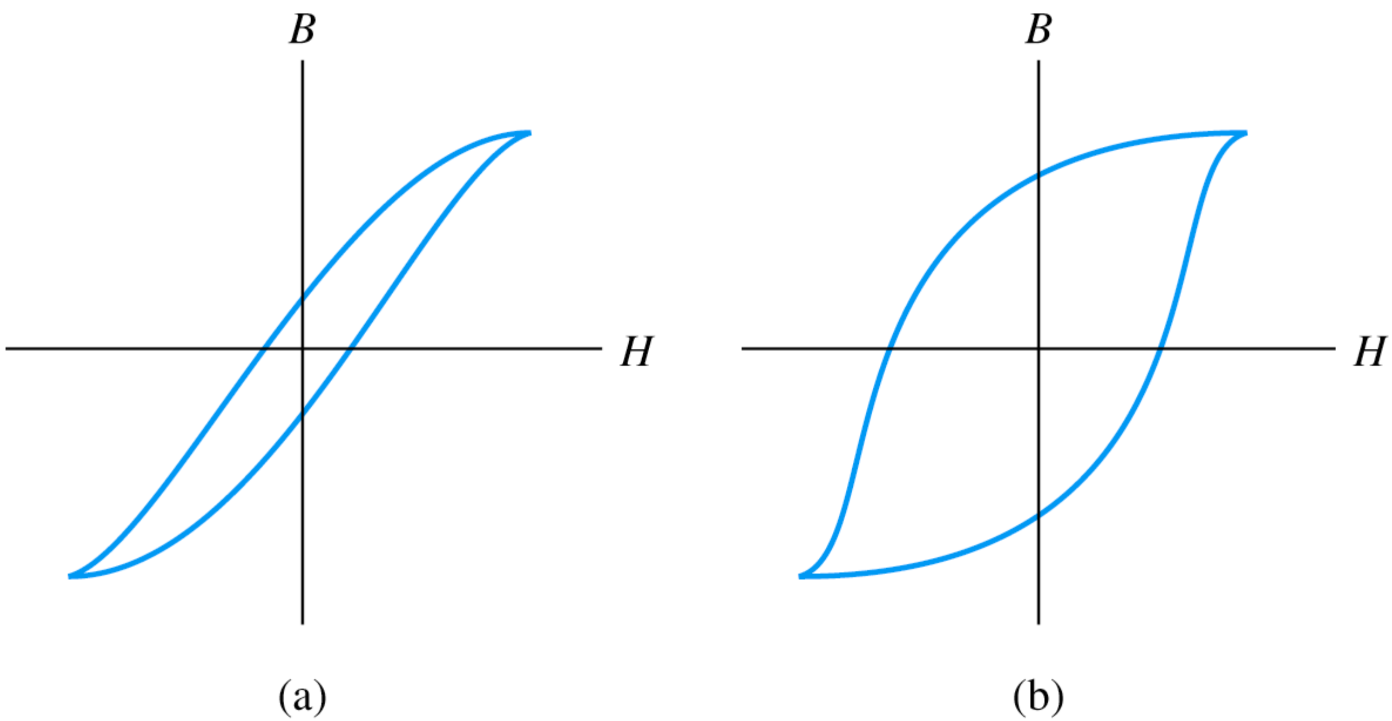


Figure 15.21 When we want to minimize core loss (as in a transformer or motor), we choose a material having a thin hysteresis loop. On the other hand, for a permanent magnet, we should choose a material with a wide loop.

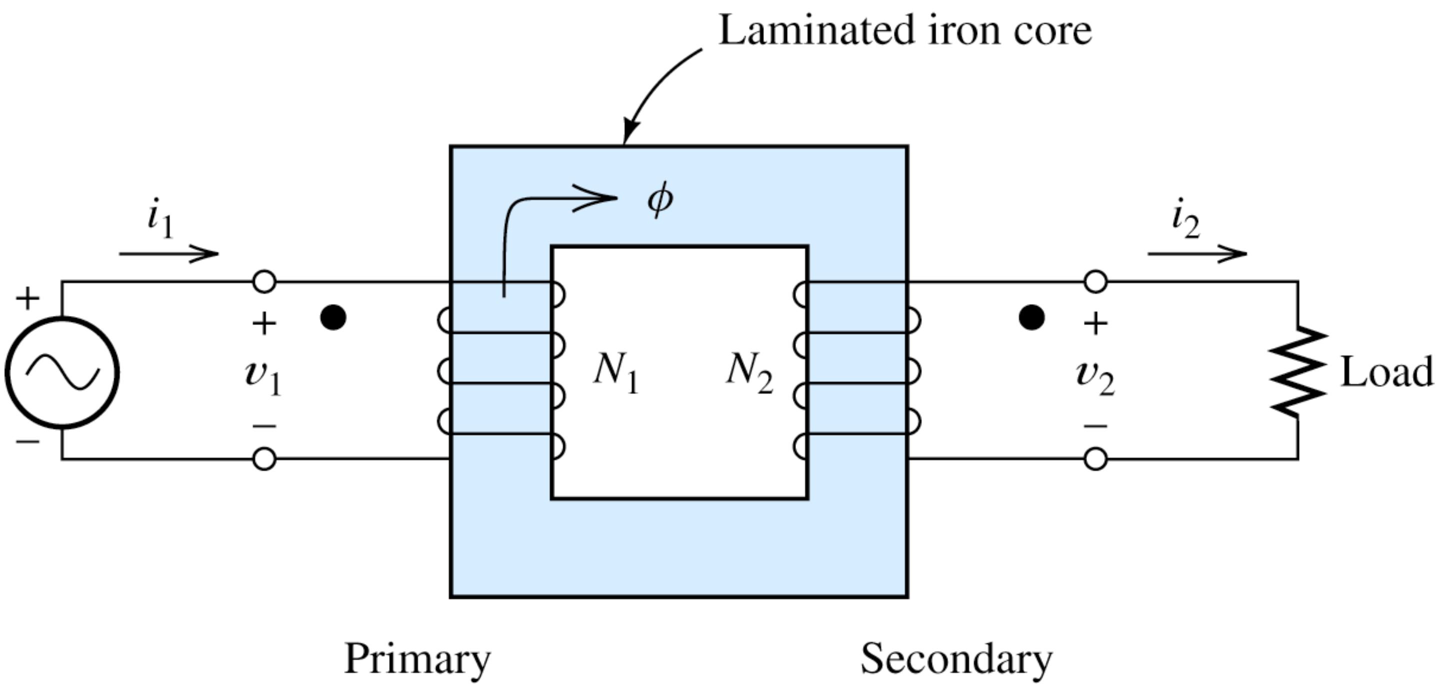
Eddy-Current Loss

Power loss due to eddy currents is proportional to the square of frequency, assuming constant peak flux.

Energy Stored in the Magnetic Field

$$W_v = \int_0^B \frac{B}{\mu} dB = \frac{B^2}{2\mu}$$

IDEAL TRANSFORMERS



$$v_2(t) = \frac{N_2}{N_1} v_1(t)$$

$$I_{2\mathrm{rms}} = \frac{N_1}{N_2} I_{1\mathrm{rms}}$$

$$p_2(t) = p_1(t)$$

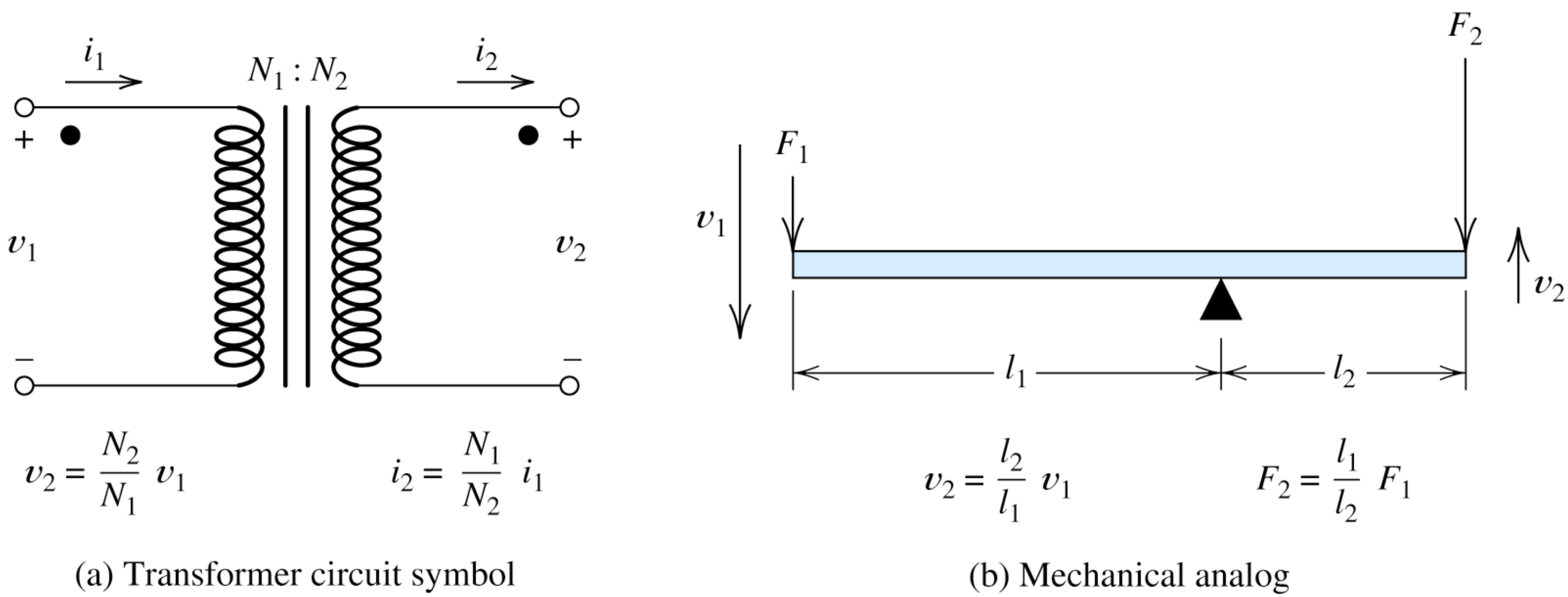


Figure 15.23 The circuit symbol for a transformer and its mechanical analog.

Transformer Summary

1. We assumed that all of the flux links all of the windings of both coils and that the resistance of the coils is zero. Thus, the voltage across each coil is proportional to the number of turns on the coil.

$$v_2(t) = \frac{N_2}{N_1} v_1(t)$$

2. We assumed that the reluctance of the core is negligible, so the total mmf of both coils is zero.

$$i_2(t) = \frac{N_1}{N_2} i_2(t)$$

3. A consequence of the voltage and current relationships is that all of the power delivered to an ideal transformer by the source is transferred to the load.

Example Exercise

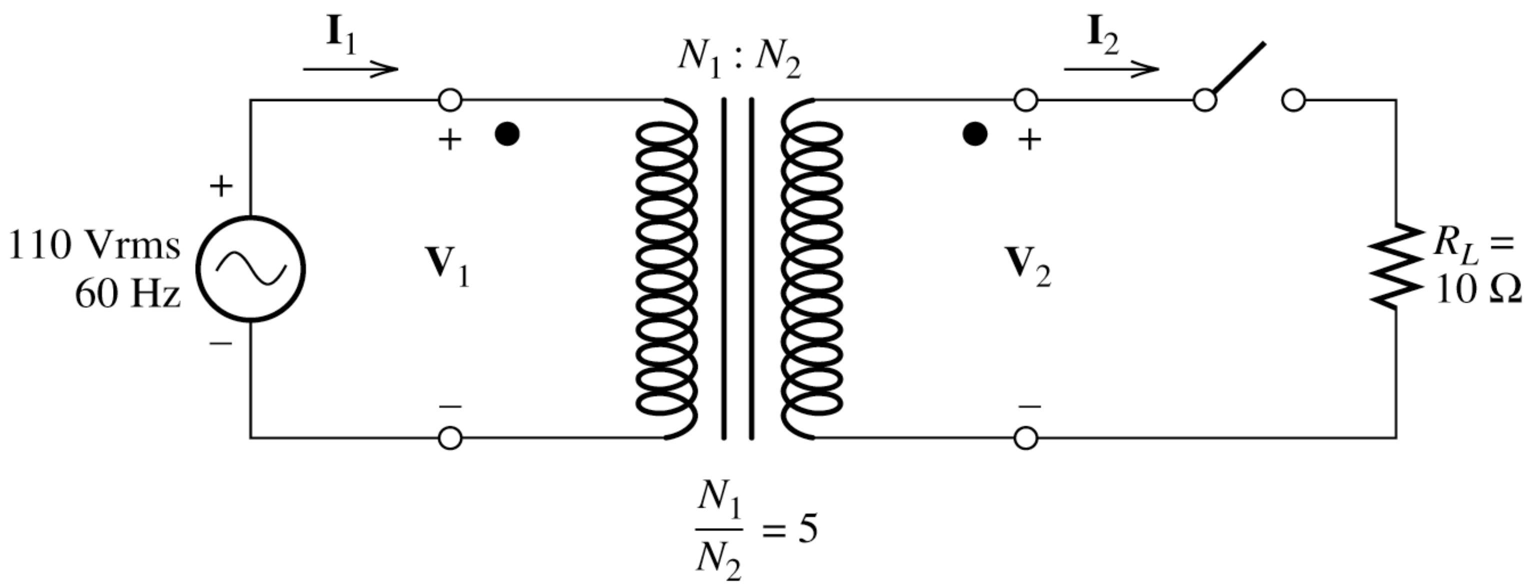
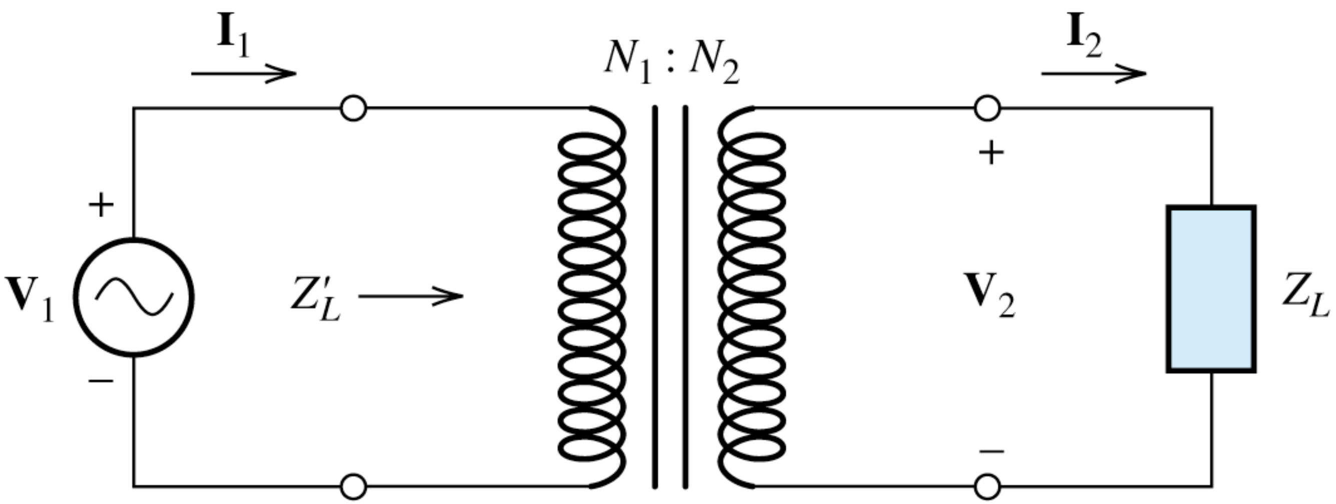


Figure 15.24 Circuit of Example 15.10.

Impedance Transformations



$$Z'_L = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \left(\frac{N_1}{N_2}\right)^2 Z_L$$

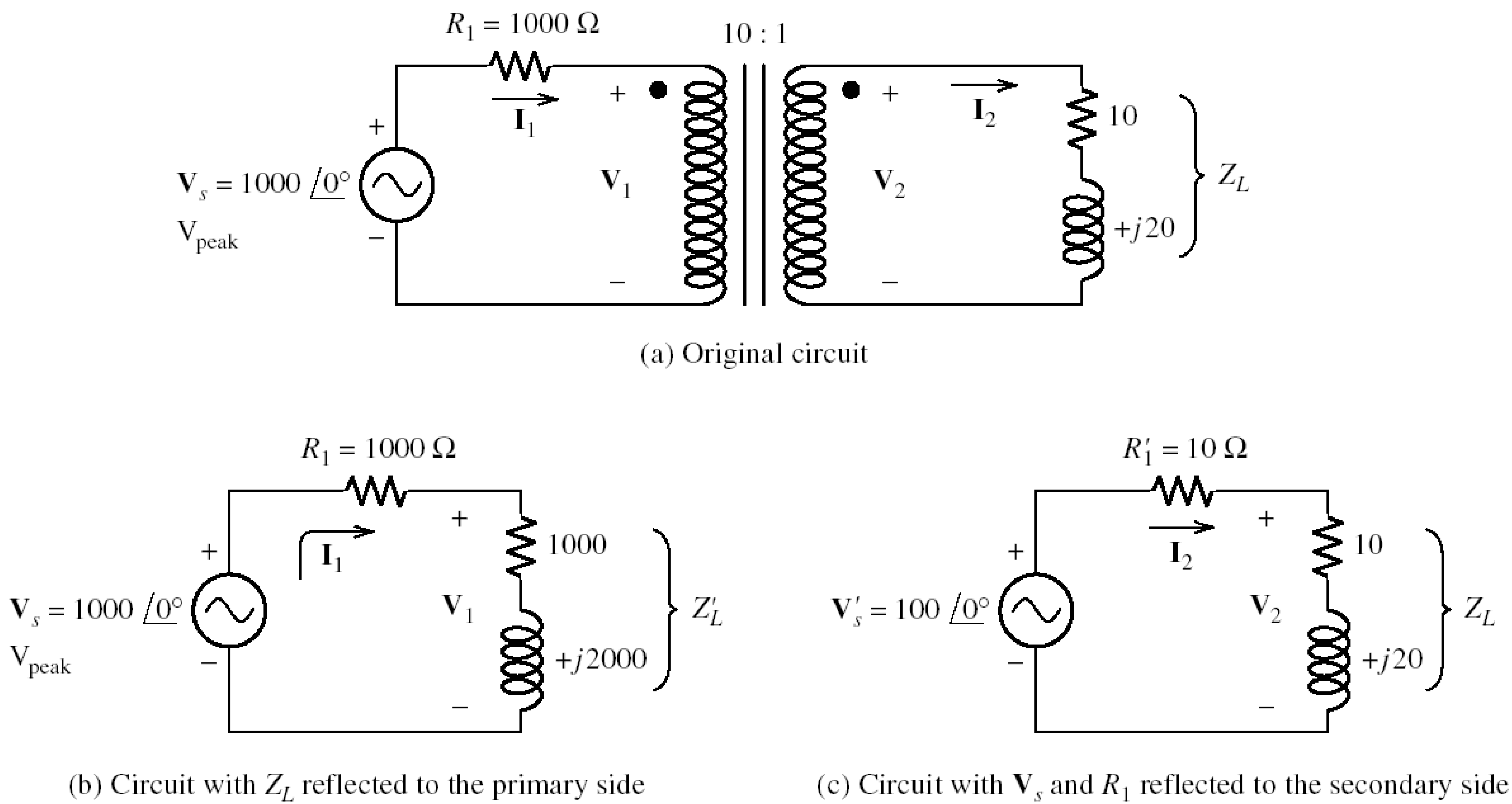
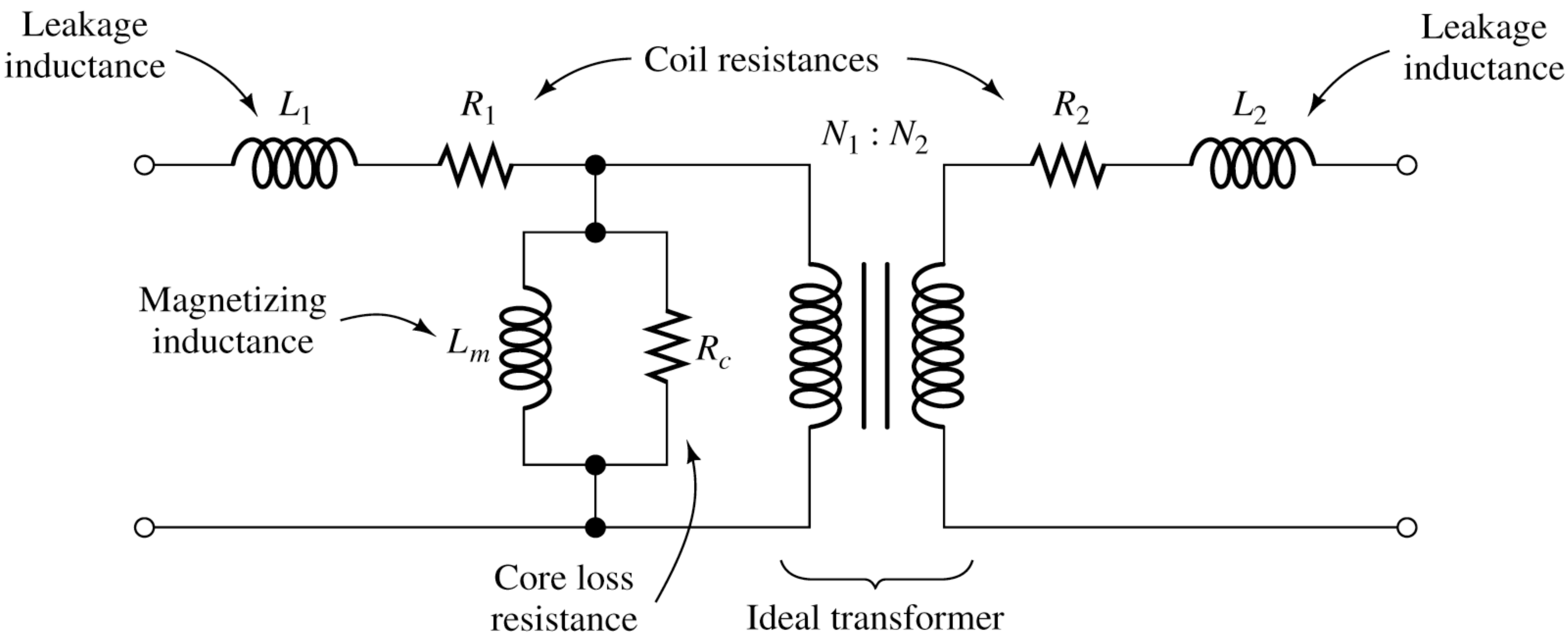
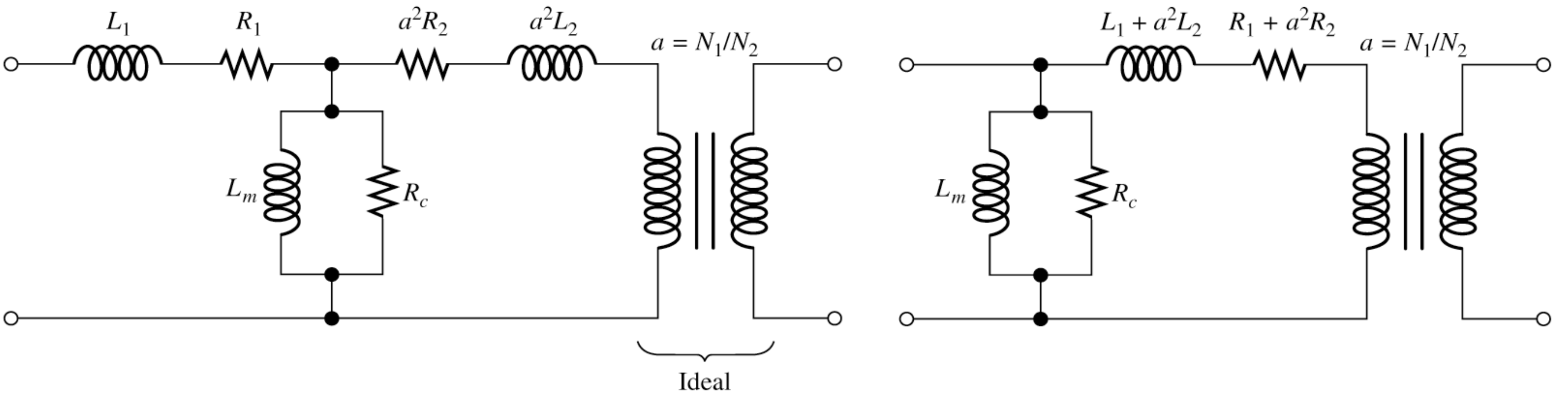


Figure 15.26 The circuit of Examples 15.11 and 15.12.

REAL TRANSFORMERS



Variations of the Transformer Model



(a) All elements referred to the primary side

(b) Approximate equivalent circuit that is sometimes more convenient to use than that of part (a)

Table 15.1. Circuit Values of a 60-Hz 20-kVA 2400/240-V Transformer Compared to Those of an Ideal Transformer

Element Name	*Symbol*	*Ideal*	*Real*
Primary resistance	R_1	0	3.0 Ω
Secondary resistance	R_2	0	0.03 Ω
Primary leakage reactance	$X_1 = \omega L_1$	0	6.5 Ω
Secondary leakage reactance	$X_2 = \omega L_2$	0	0.07 Ω
Magnetizing reactance	$X_m = \omega L_m$	∞	15 kΩ
Core-loss resistance	R_c	∞	100 kΩ

Regulation and Efficiency

$$\text{percent regulation} = \frac{V_{\text{no-load}} - V_{\text{load}}}{V_{\text{load}}} \times 100\%$$

$$\text{power efficiency} = \frac{P_{\text{load}}}{P_{\text{in}}} \times 100\% = \left(1 - \frac{P_{\text{loss}}}{P_{\text{in}}}\right) \times 100\%$$

Example Exercise

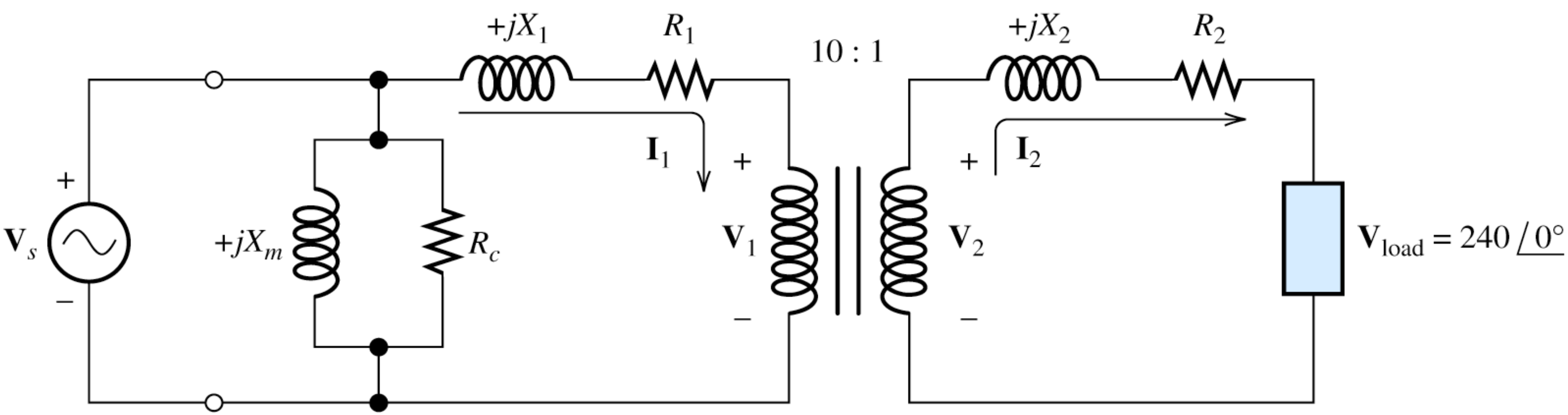


Figure 15.30 Circuit of Example 15.13.