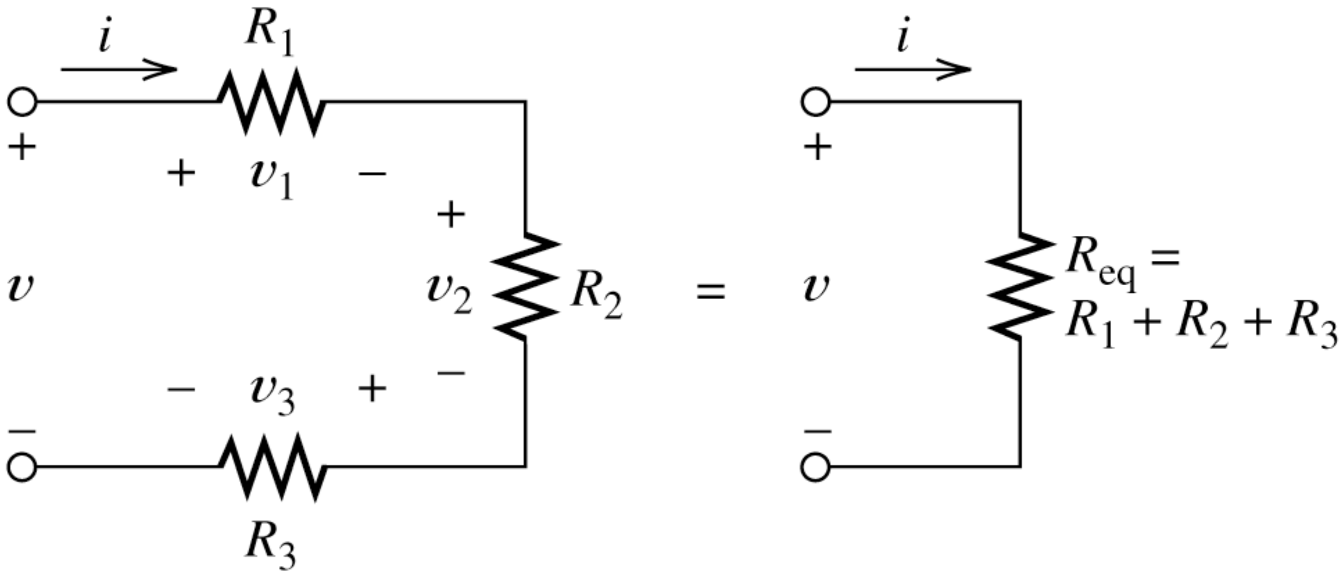
RESISTIVE CIRCUITS

Overview

1. Solve circuits (i.e., find currents and voltages of interest) by combining resistances in series and parallel.
2. Apply the voltage-division and current-division principles.
3. Solve circuits by the node-voltage technique.
4. Solve circuits by the mesh-current technique.
5. Find Thévenin and Norton equivalents and apply source transformations.
6. Apply the superposition principle.
7. Draw the circuit diagram and state the principles of operation for the Wheatstone bridge.

Series Resistance

- A series combination of resistances has an equivalent resistance equal to the sum of the original resistances



(a) Three resistances in series

(b) Equivalent resistance

Why?

- By Ohm's law

$$v_1 = R_1 i$$

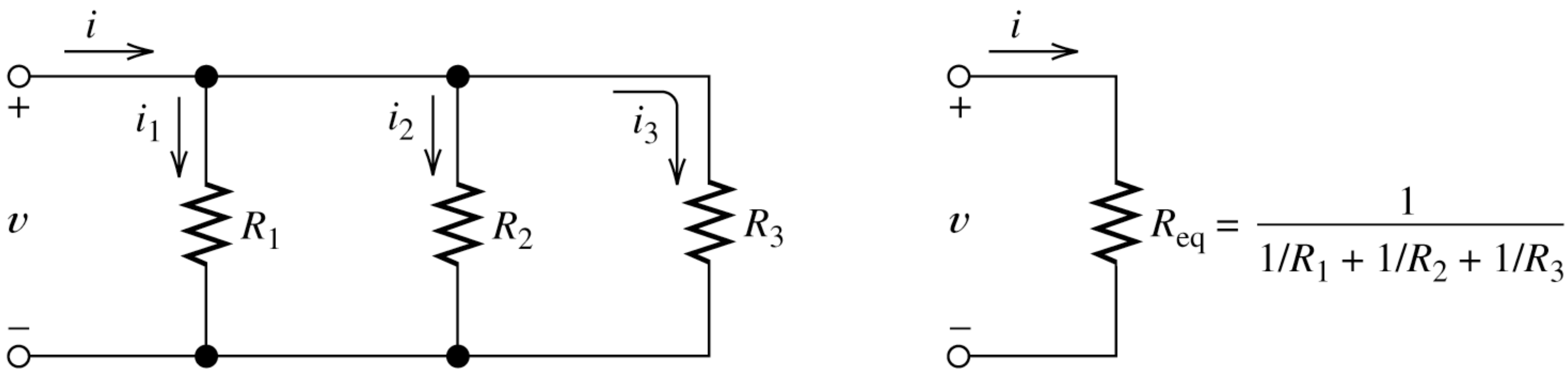
$$v_2 = R_2 i$$

$$v_3 = R_3 i$$

- By KVL

$$\begin{aligned} v &= v_1 + v_2 + v_3 \\ &= R_1 i + R_2 i + R_3 i \\ &= (R_1 + R_2 + R_3) i \\ &= R_{eq} i \end{aligned}$$

Parallel Resistance



(a) Three resistances in parallel

(b) Equivalent resistance

Why?

- By Ohm's law

$$i_1 = v / R_1$$

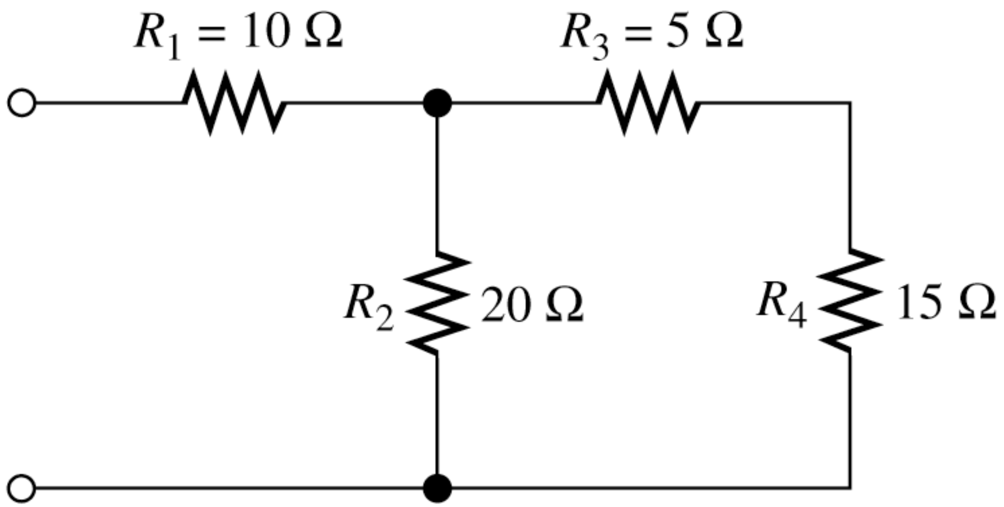
$$i_2 = v / R_2$$

$$i_3 = v / R_3$$

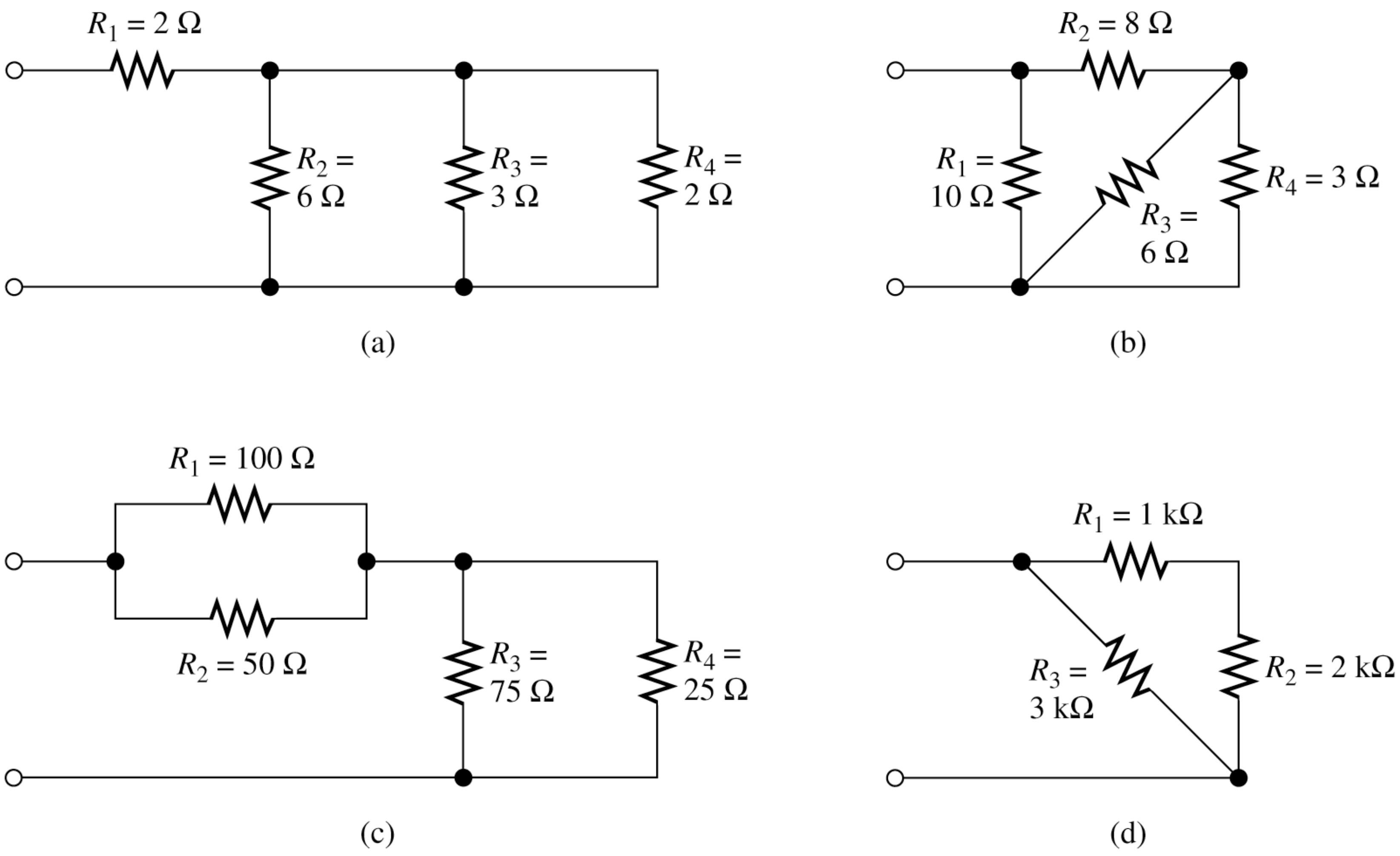
- By KCL

$$\begin{aligned} i &= i_1 + i_2 + i_3 \\ &= v / R_1 + v / R_2 + v / R_3 \\ &= (1 / R_1 + 1 / R_2 + 1 / R_3) v \\ &= v / R_{eq} \end{aligned}$$

Example Exercise



Example Exercise



Circuits

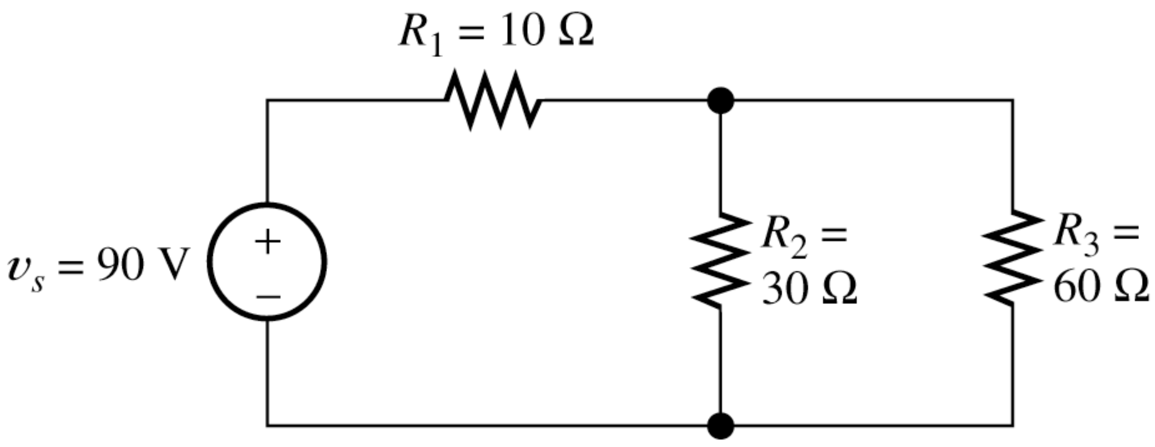
- Conductance is inverse of resistance
  - Series conductance combine as resistance in parallel
- In reality, loads connected in parallel
  - Switch in series

Circuit Analysis using Series/Parallel Equivalents

1. Begin by locating a combination of resistances that are in series or parallel. Often the place to start is farthest from the source.
2. Redraw the circuit with the equivalent resistance for the combination found in step 1.
3. Repeat steps 1 and 2 until the circuit is reduced as far as possible. Often (but not always) we end up with a single source and a single resistance.
4. Solve for the currents and voltages in the final equivalent circuit.

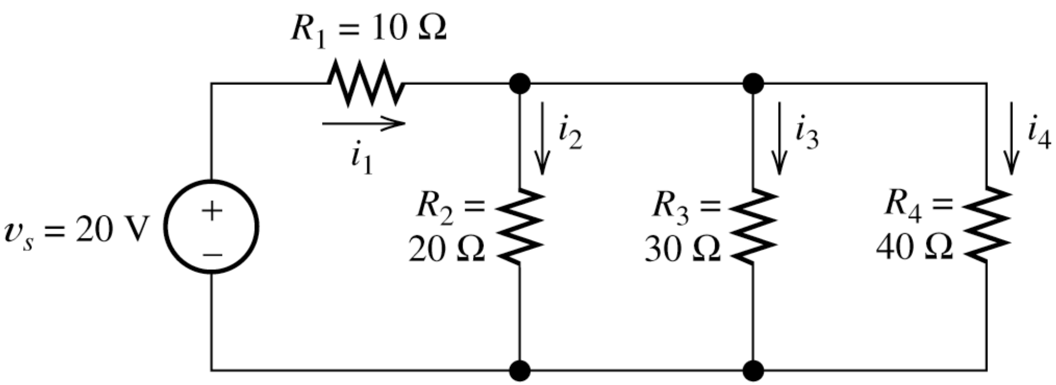
Example Exercise

- Find currents and voltages

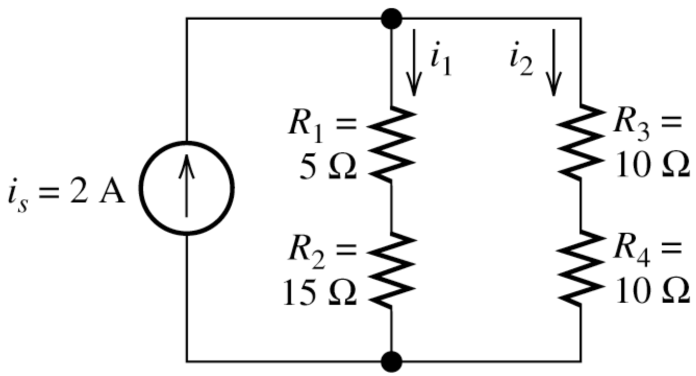


(a) Original circuit

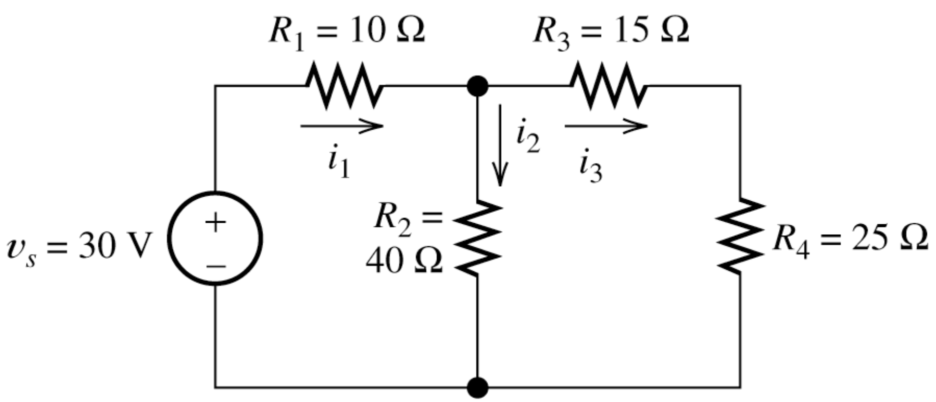
Example Exercise



(a)



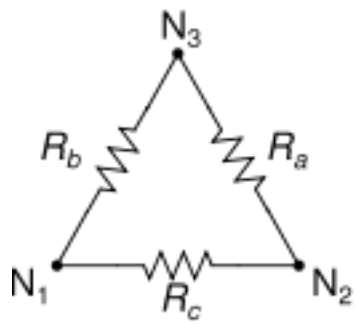
(b)

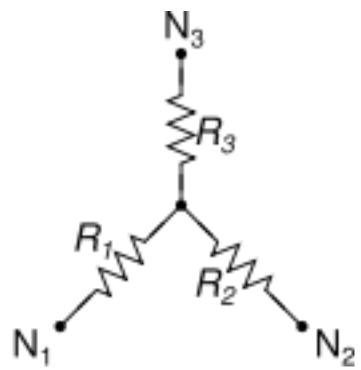


(c)

Wye-Delta (Star-Delta) Conversion

- Node Reduction





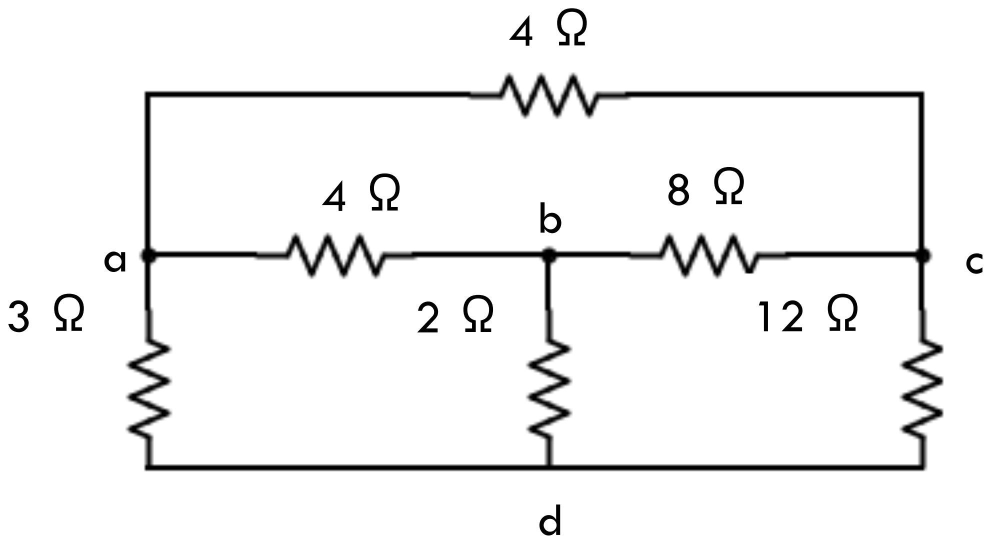
$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}, \qquad R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1},$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}, \qquad R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2},$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}. \qquad R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}.$$

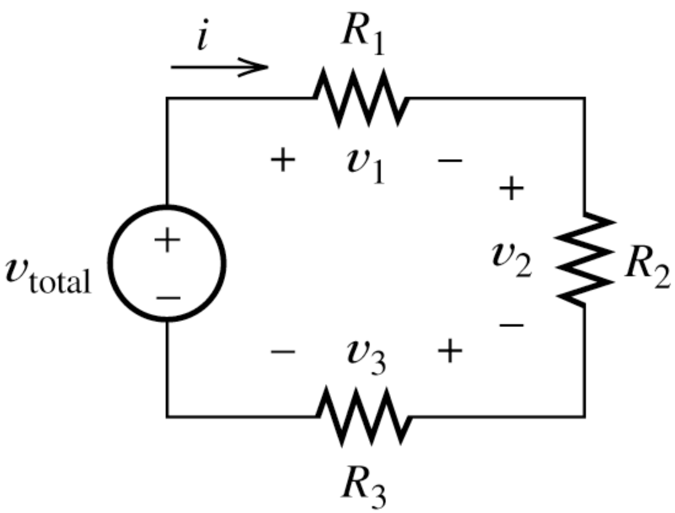
Example Exercise

- Find resistance between a and d.



Voltage Division Principle

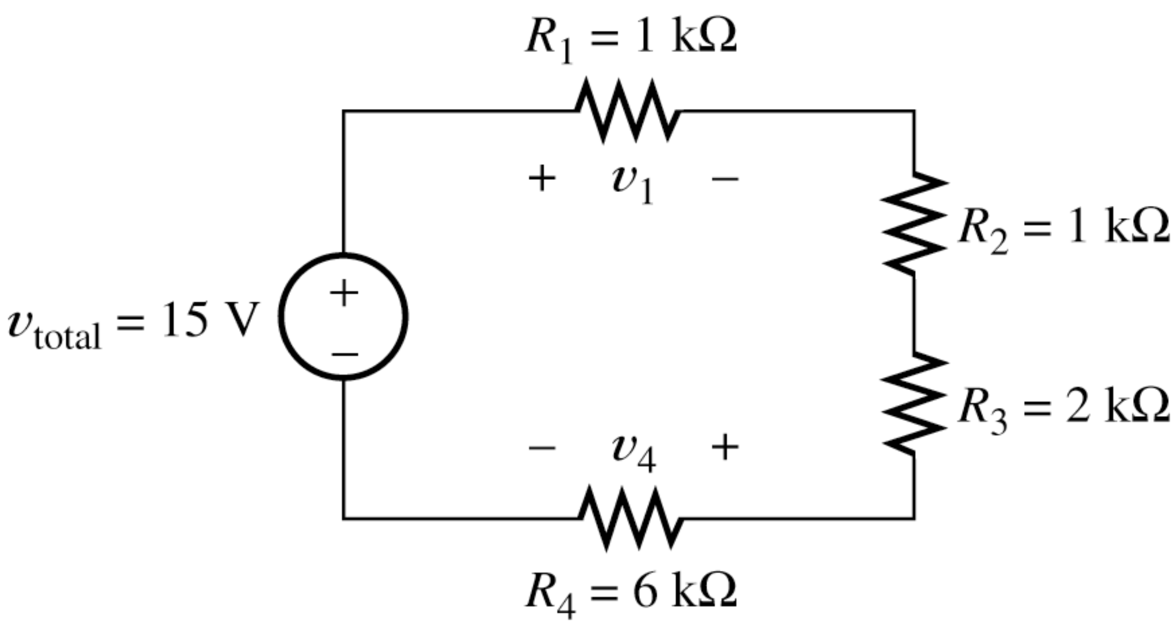
- Of the total voltage, the fraction that appears across a given resistance in a series circuit is the ratio of the given resistance to the total series resistance.



$$v_1 = R_1 i = \frac{R_1}{R_1 + R_2 + R_3} v_{\text{total}}$$

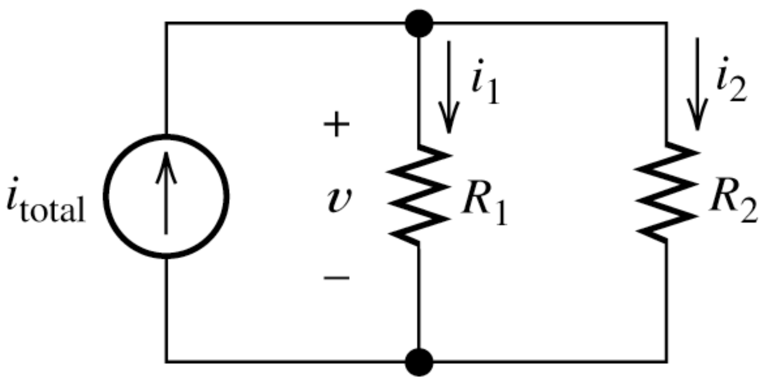
$$v_2 = R_2 i = \frac{R_2}{R_1 + R_2 + R_3} v_{\text{total}}$$

Example Exercise



Current Division Principle

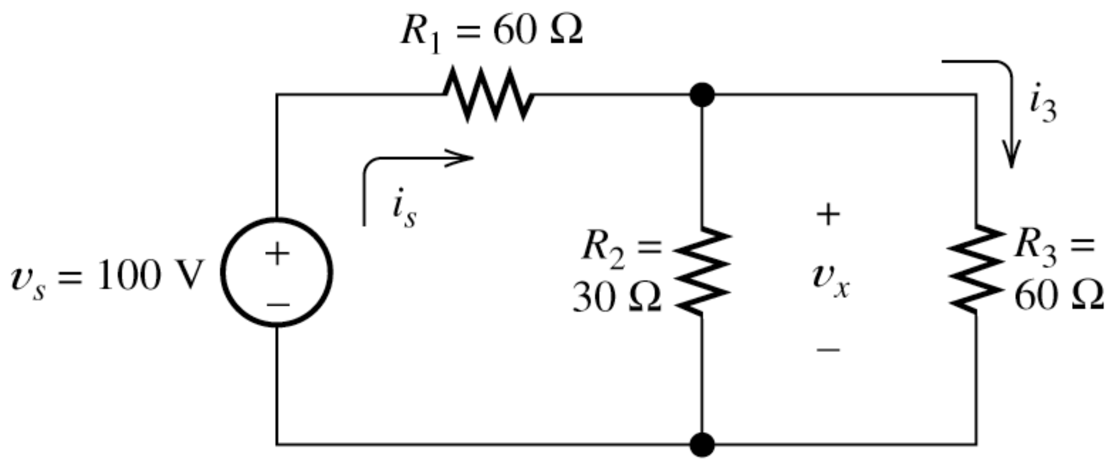
- For two resistances in parallel, the fraction of the total current flowing in a resistance is the ratio of the other resistance to the sum of the two resistances.



$$i_1 = \frac{v}{R_1} = \frac{R_2}{R_1 + R_2} i_{total}$$

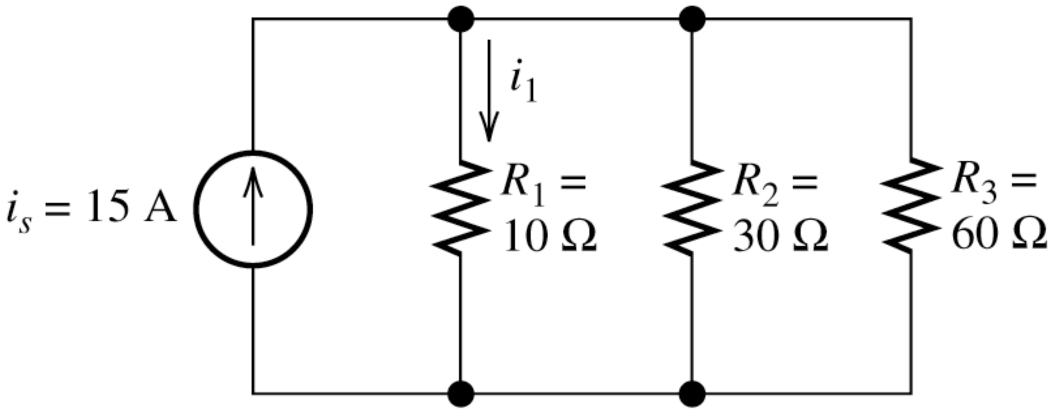
$$i_2 = \frac{v}{R_2} = \frac{R_1}{R_1 + R_2} i_{total}$$

Example Exercise



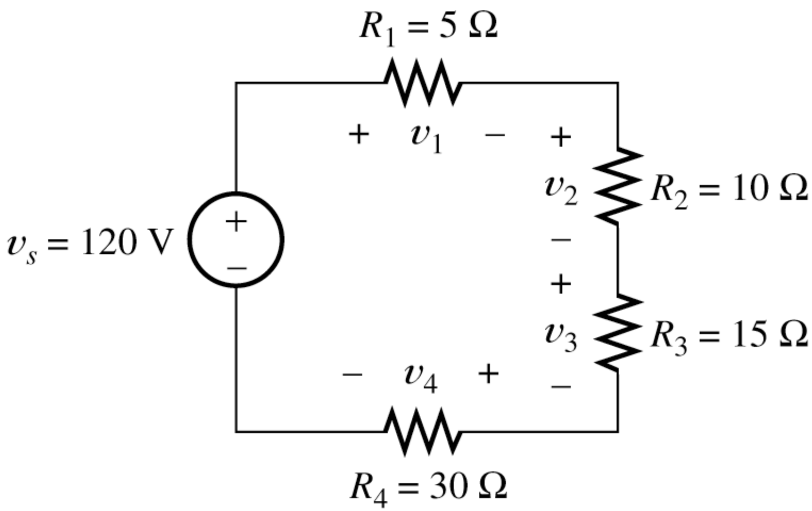
(a) Original circuit

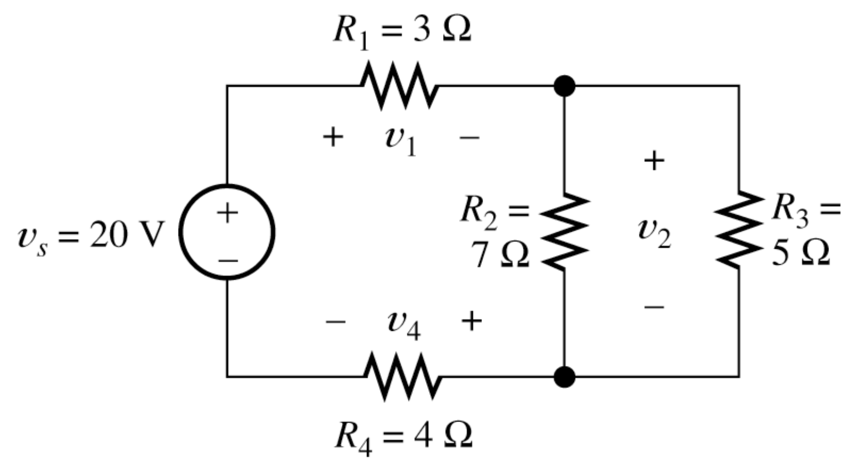
Example Exercise

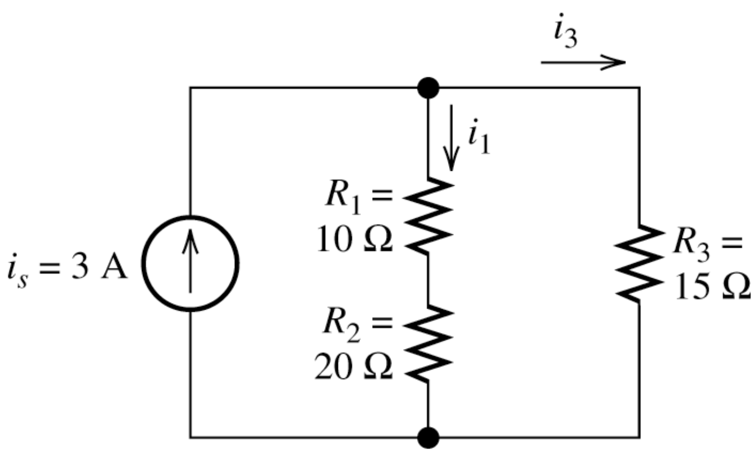


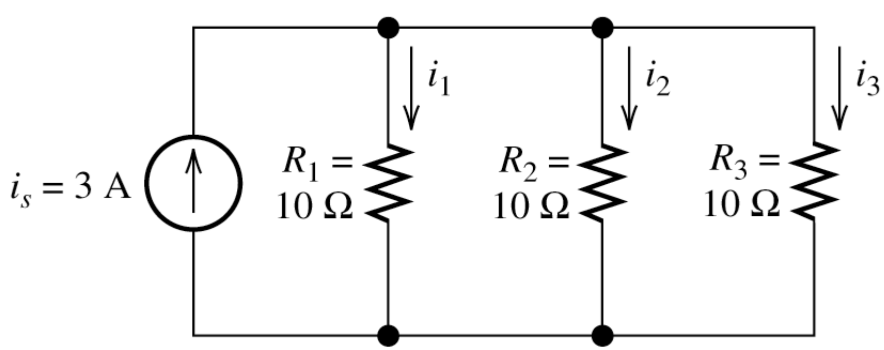
(a) Original circuit

Example Exercise









Circuit Analysis

Although they are very important concepts, series/parallel equivalents and the current/voltage division principles are not sufficient to solve all circuits.

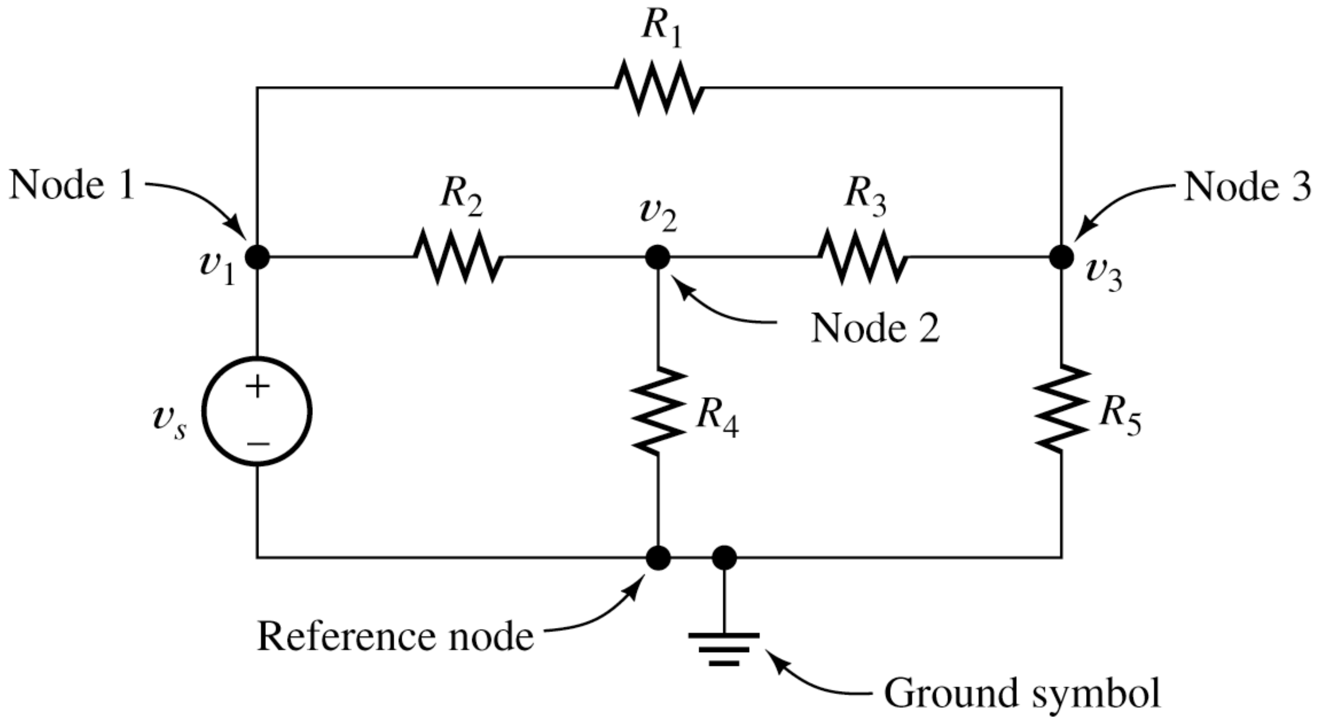
1. Node Voltage Analysis
2. Mesh Current Analysis

Node Voltage Analysis

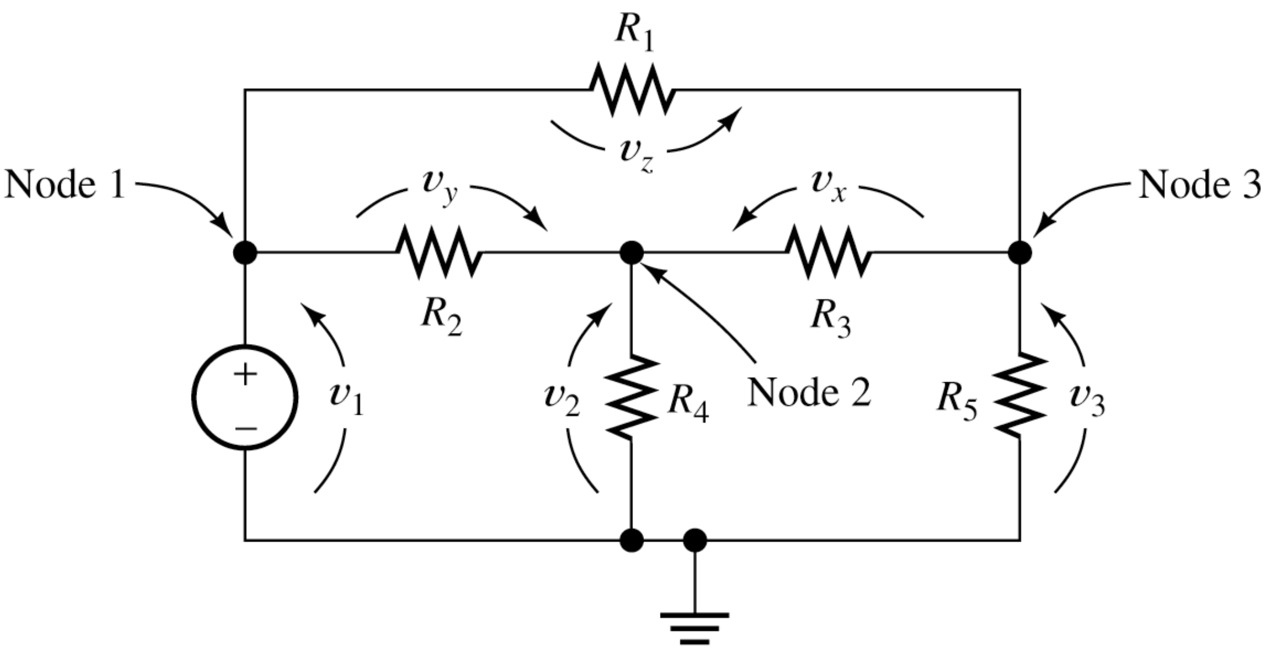
1. Identify all nodes in the circuit
2. Assign a reference node
3. Assign node voltages
4. Identify unknown node voltages
5. Write KCL equations
6. Solve for unknown node voltages
7. Calculate currents

Node Voltage Analysis

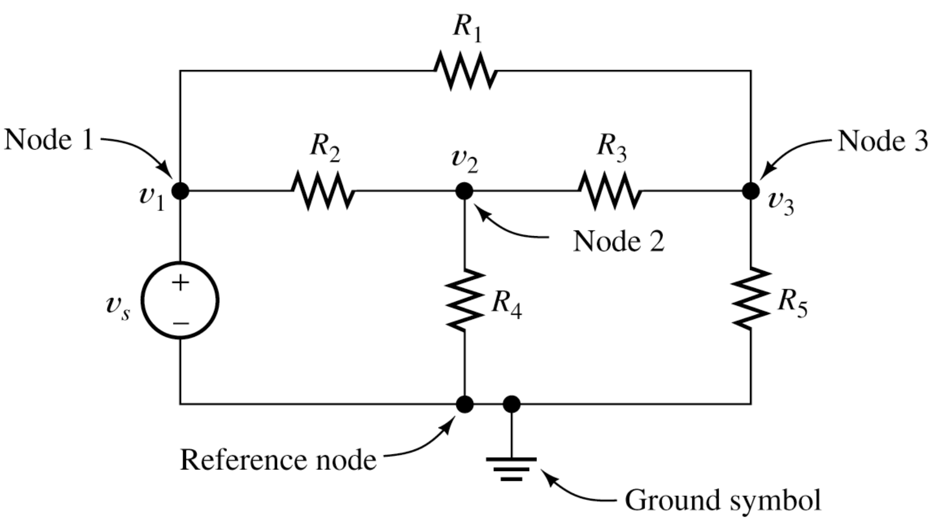
- Step 1: Select reference node and label voltages at other nodes
  - Typically one end of a voltage source



Element Voltages



KCL Equations in Node Voltages

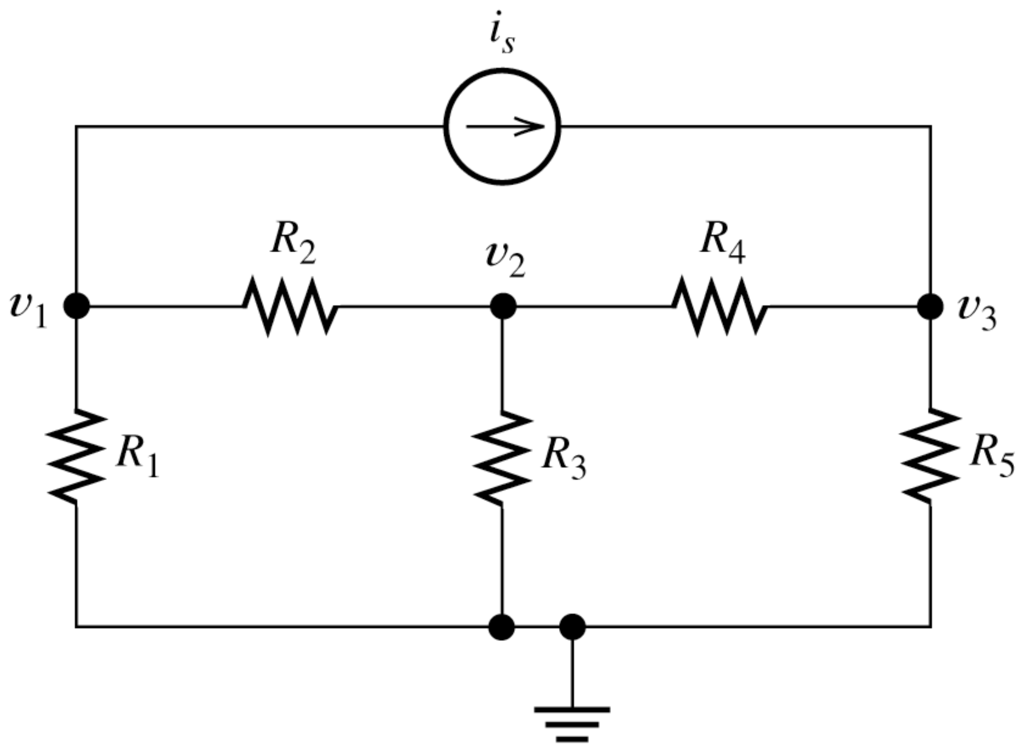


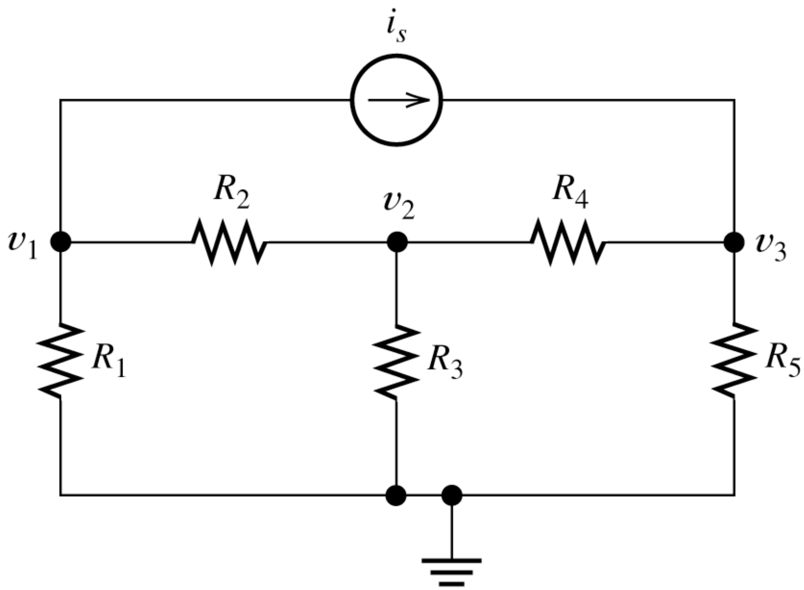
$$v_1 = v_s$$

$$\frac{v_2 - v_1}{R_2} + \frac{v_2}{R_4} + \frac{v_2 - v_3}{R_3} = 0$$

$$\frac{v_3 - v_1}{R_1} + \frac{v_3}{R_5} + \frac{v_3 - v_2}{R_3} = 0$$

Example Exercise



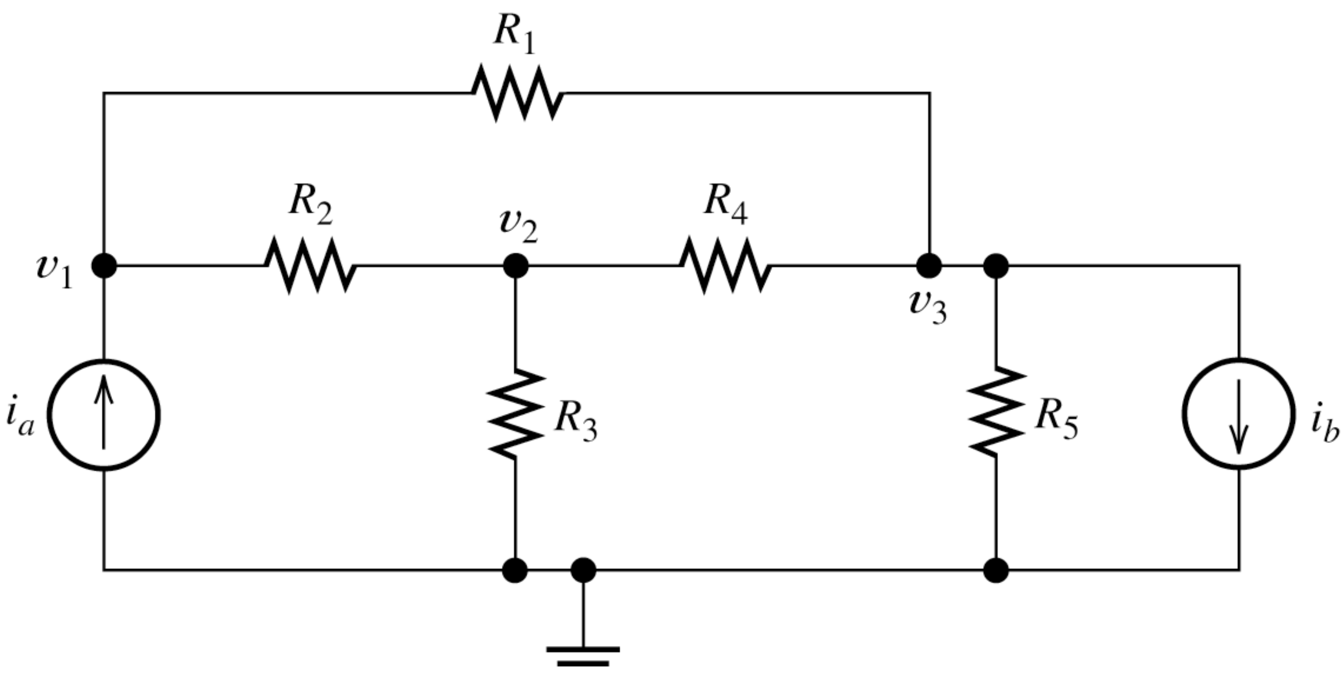


$$\frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} + i_s = 0$$

$$\frac{v_2 - v_1}{R_2} + \frac{v_2}{R_3} + \frac{v_2 - v_3}{R_4} = 0$$

$$\frac{v_3}{R_5} + \frac{v_3 - v_2}{R_4} = i_s$$

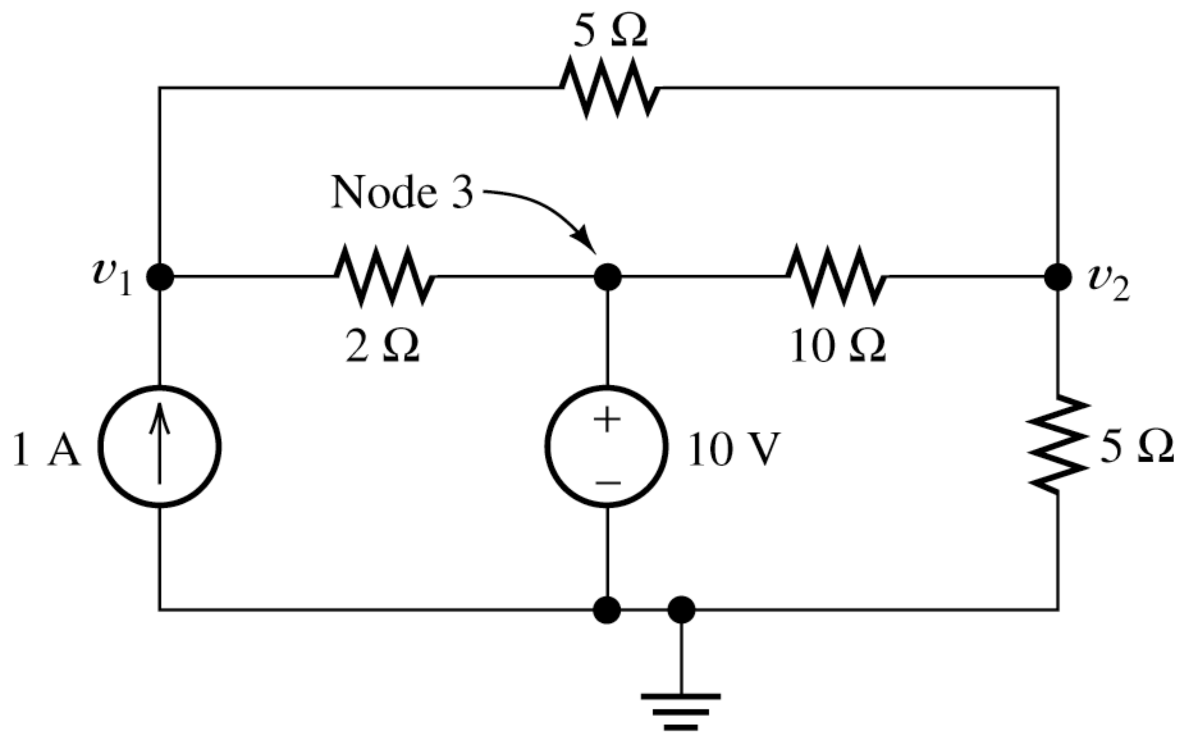
Example Exercise



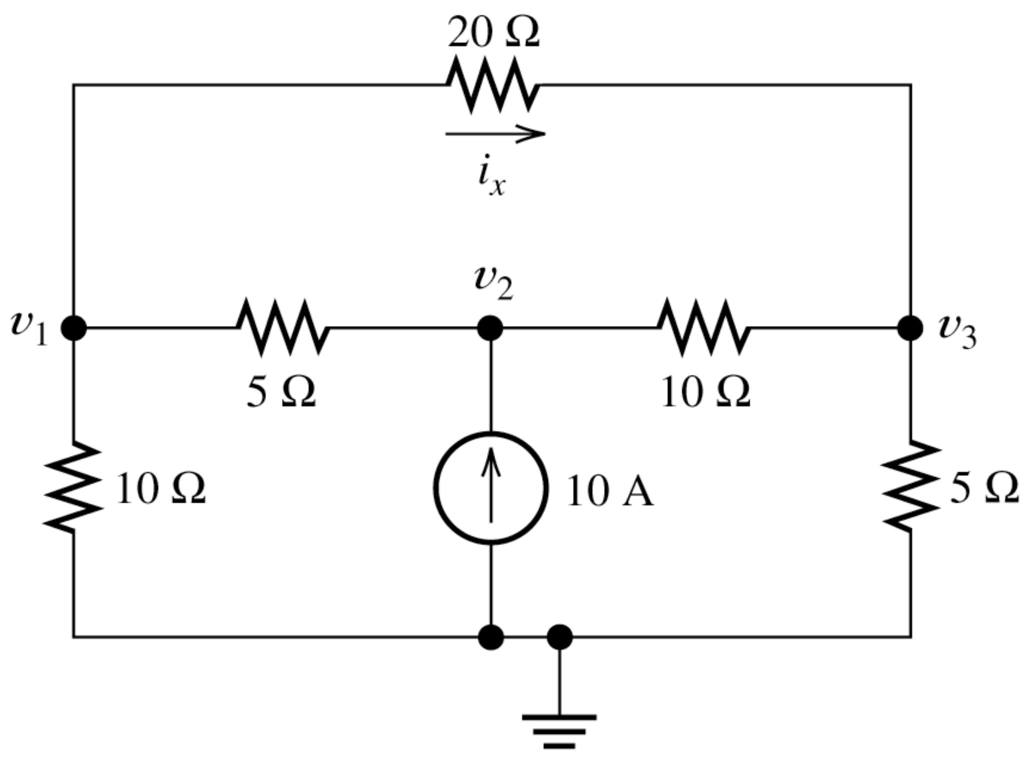
Solve Equations

- Calculators or notebooks
- Matrix inversion
  - Matlab
  - LabVIEW

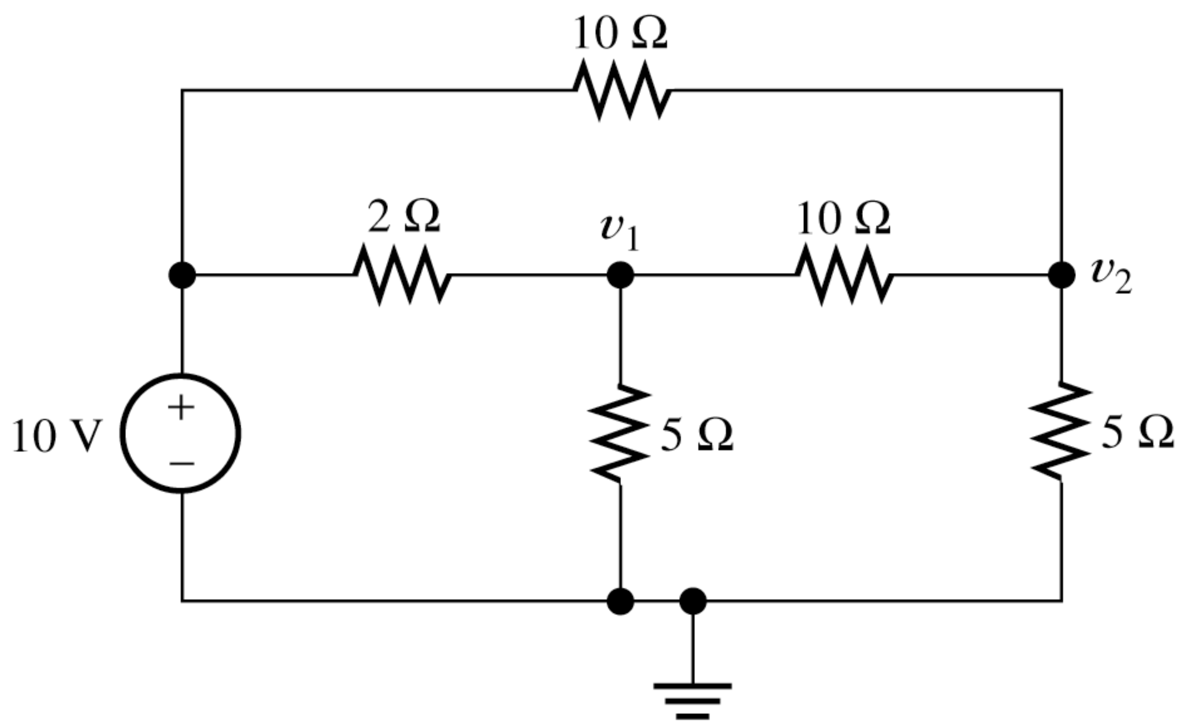
Example Exercise



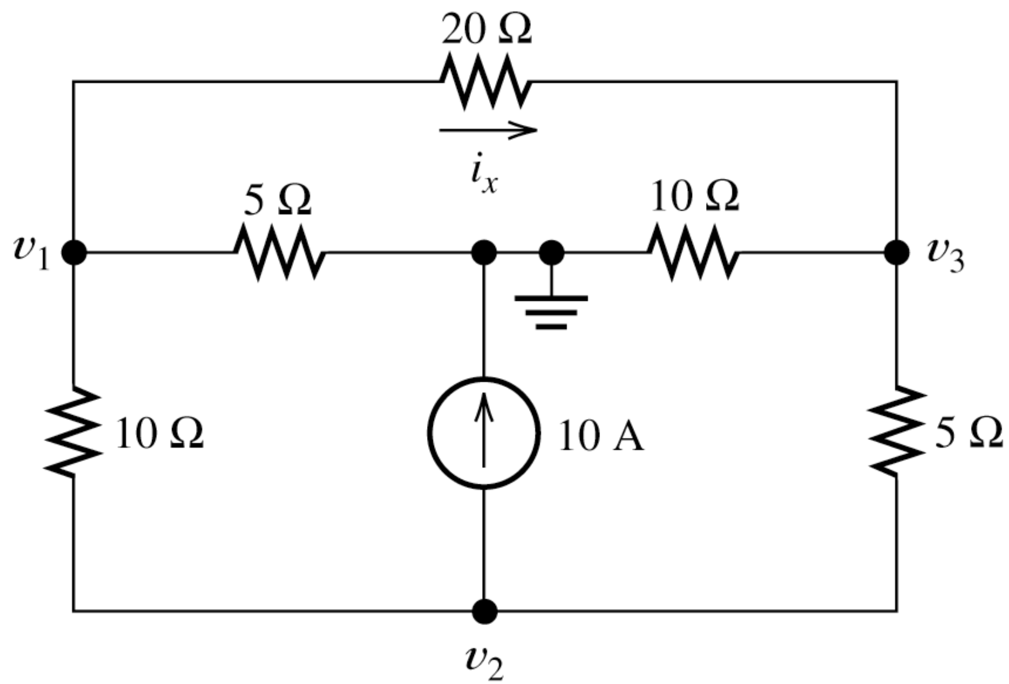
Example Exercise



Example Exercise

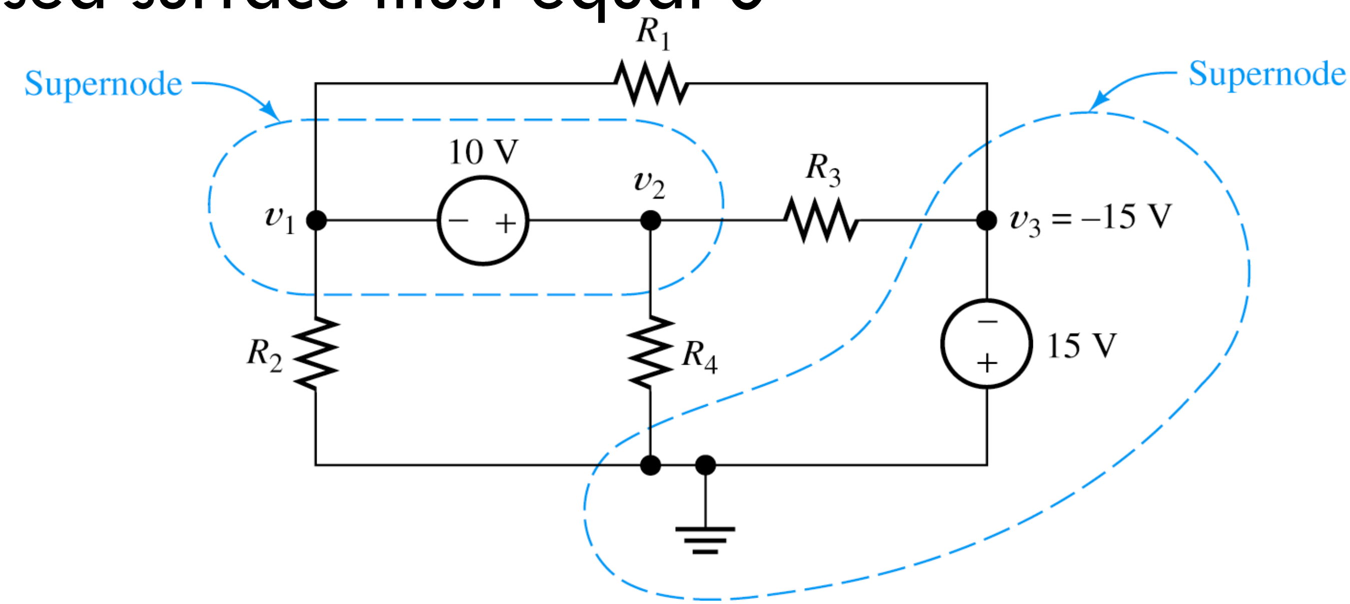


Example Exercise (Different Reference)

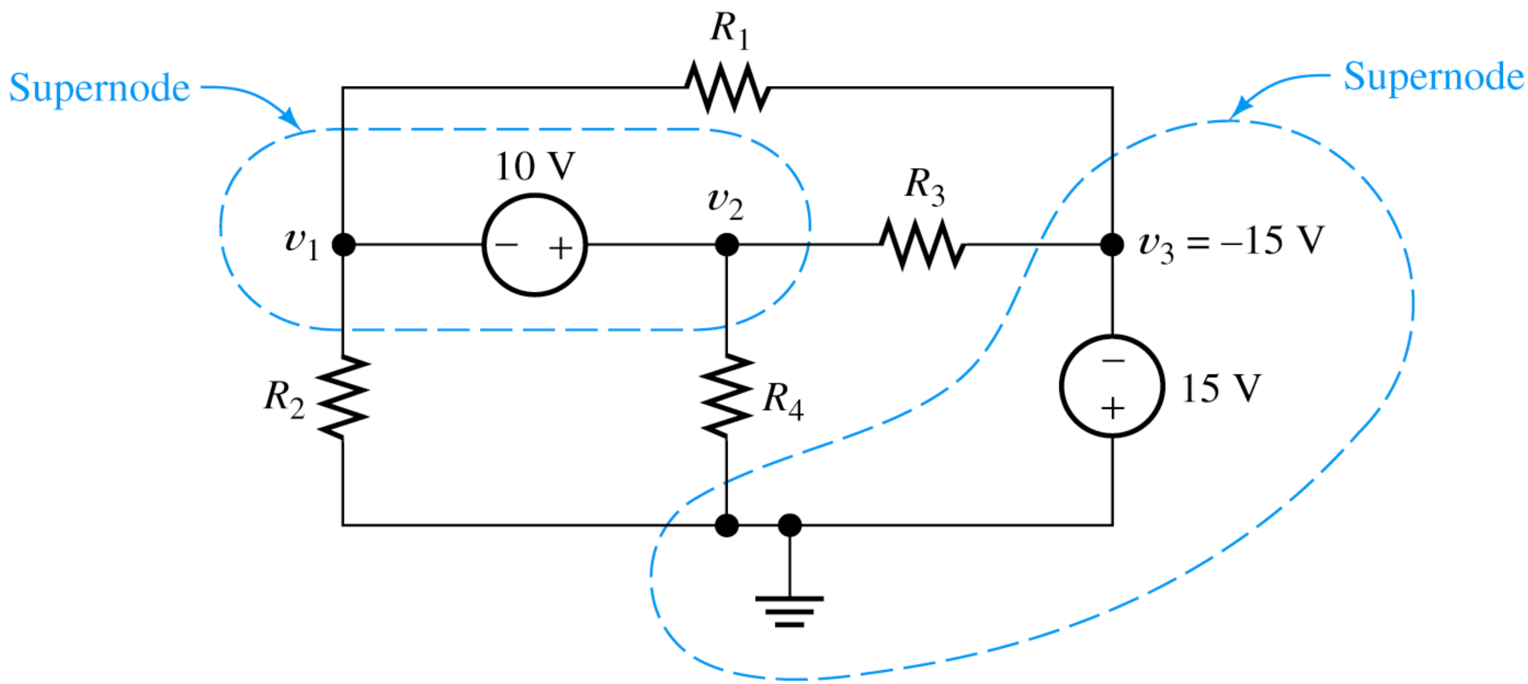


Supernode

- If one end of voltage source not a reference, need a current variable for the two nodes => higher order system of equations
- Generalized KCL: Net current flowing through any closed surface must equal 0



$$\frac{v_1}{R_2} + \frac{v_1 - (-15)}{R_1} + \frac{v_2}{R_4} + \frac{v_2 - (-15)}{R_3} = 0$$



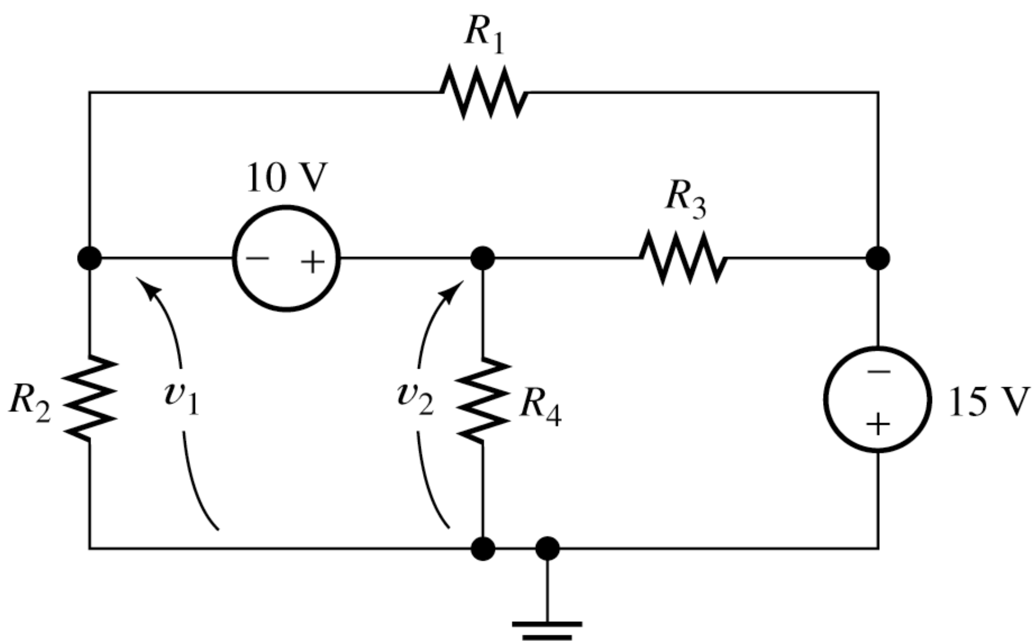
Circuits with Voltage Sources

We obtain dependent equations if we use all of the nodes in a network to write KCL equations.

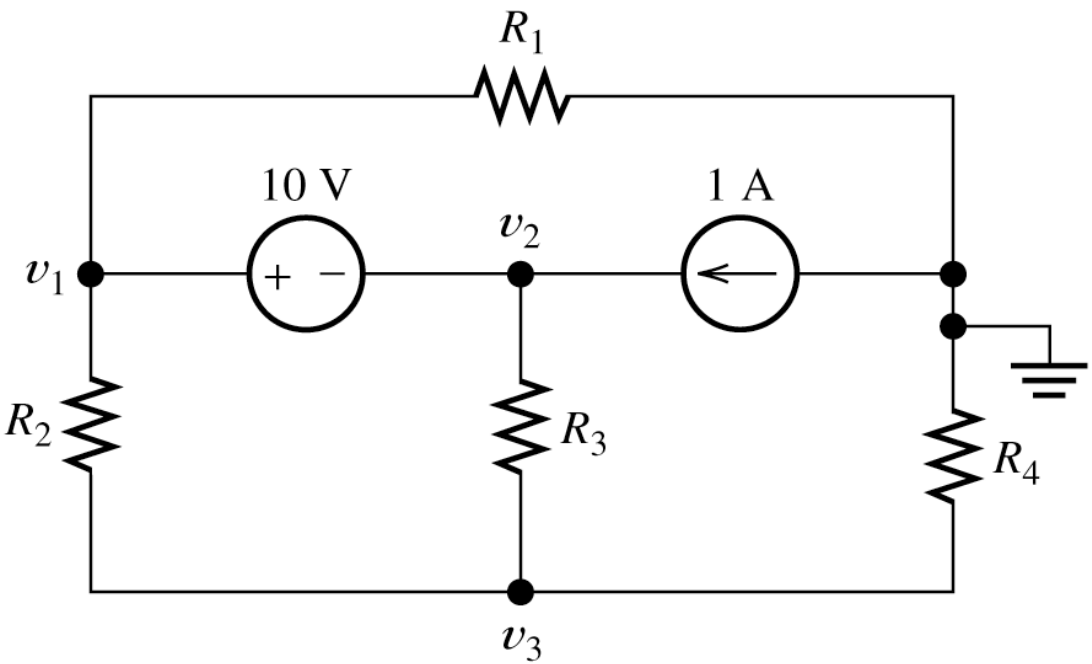
KVL

- Another independent equation from the loop with v_1, v_2 and 10V source.

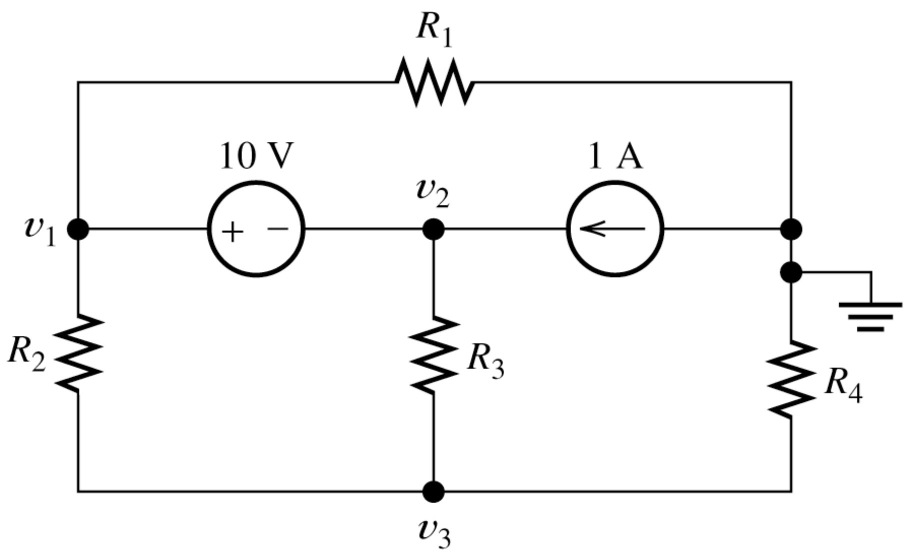
$$-v_1 - 10 + v_2 = 0$$



Example Exercise



$$-v_1 + 10 + v_2 = 0$$



$$\frac{v_1}{R_1} + \frac{v_1 - v_3}{R_2} + \frac{v_2 - v_3}{R_3} = 1$$

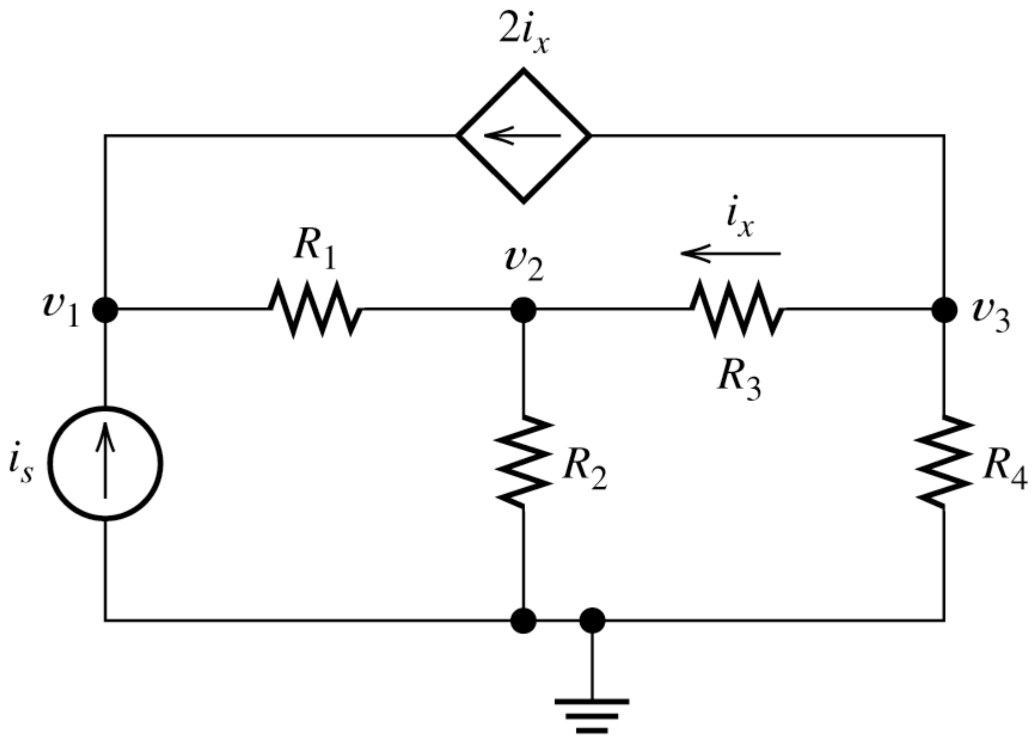
$$\frac{v_3 - v_1}{R_2} + \frac{v_3 - v_2}{R_3} + \frac{v_3}{R_4} = 0$$

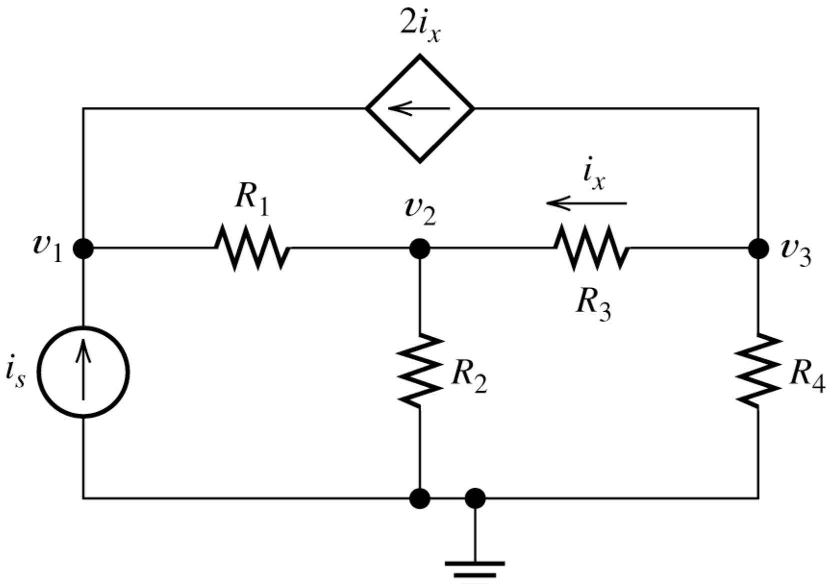
$$\frac{v_1}{R_1} + \frac{v_3}{R_4} = 1$$

Circuits with Controlled Source

- First, we write KCL equations at each node, including the current of the controlled source just as if it were an ordinary independent source.
- Then, express the controlling variable in terms of the node voltage variables and substitute into network equations.

Example Exercise





$$\frac{v_1 - v_2}{R_1} = i_s + 2i_x$$

$$\frac{v_2 - v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_2 - v_3}{R_3} = 0$$

$$\frac{v_3 - v_2}{R_3} + \frac{v_3}{R_4} + 2i_x = 0$$

Next, we find an expression for the controlling variable i_x in terms of the node voltages.

$$i_x = \frac{v_3 - v_2}{R_3}$$

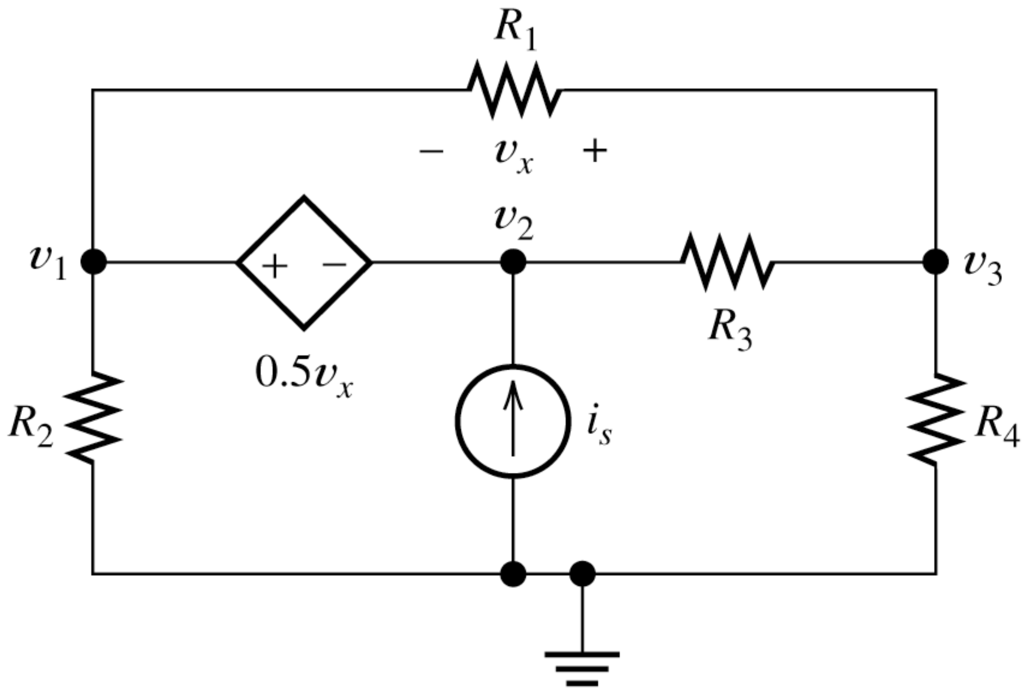
Substitution yields

$$\frac{v_1 - v_2}{R_1} = i_s + 2\frac{v_3 - v_2}{R_3}$$

$$\frac{v_2 - v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_2 - v_3}{R_3} = 0$$

$$\frac{v_3 - v_2}{R_3} + \frac{v_3}{R_4} + 2\frac{v_3 - v_2}{R_3} = 0$$

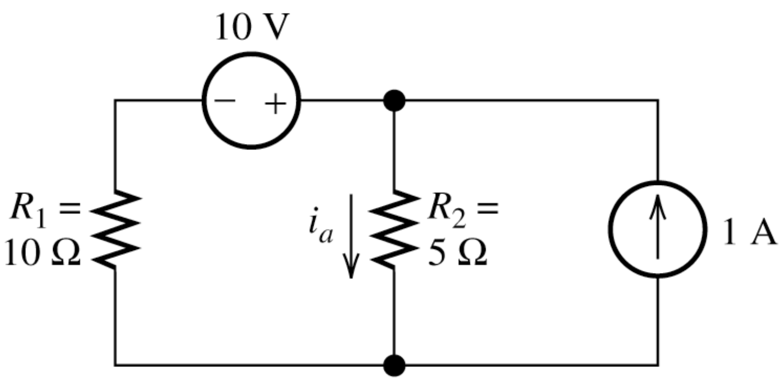
Example Exercise



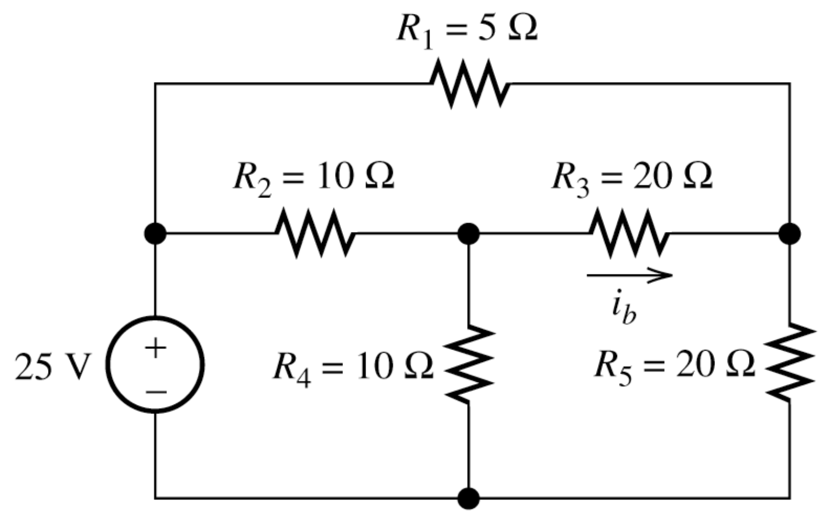
Node-Voltage Analysis

- Select a reference node and assign variables for the unknown node voltages. If the reference node is chosen at one end of an independent voltage source, one node voltage is known at the start, and fewer need to be computed.
- Write network equations. First, use KCL to write current equations for nodes and supernodes. Write as many current equations as you can without using all of the nodes. Then if you do not have enough equations because of voltage sources connected between nodes, use KVL to write additional equations.
- If the circuit contains dependent sources, find expressions for the controlling variables in terms of the node voltages. Substitute into the network equations, and obtain equations having only the node voltages as unknowns.
- Put the equations into standard form and solve for the node voltages.
- Use the values found for the node voltages to calculate any other currents or voltages of interest.

Example Exercise

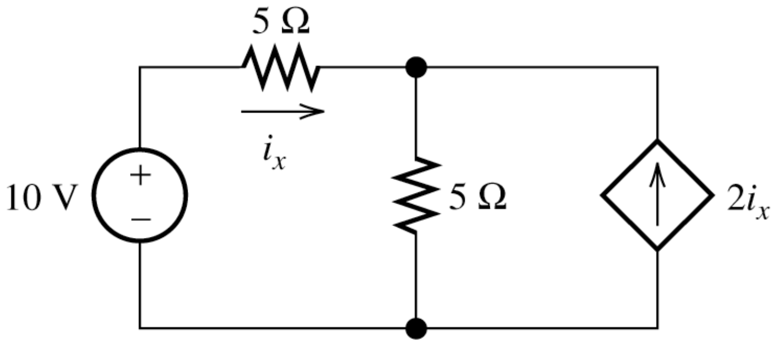


(a)

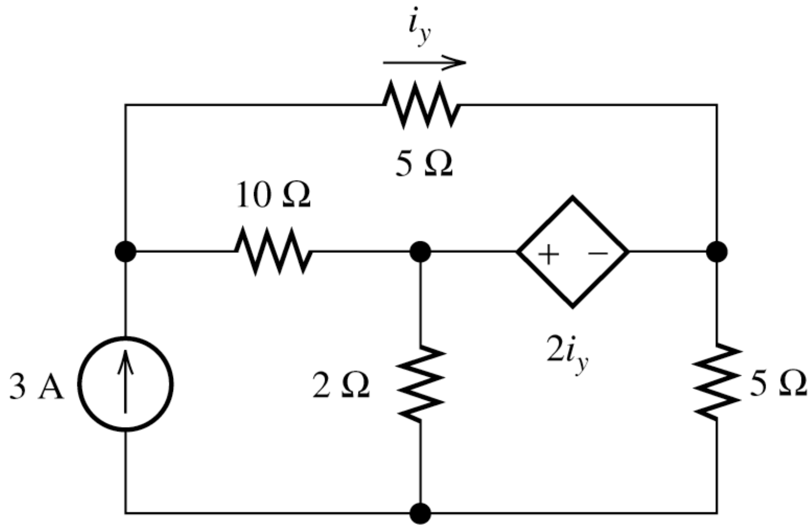


(b)

Example Exercise



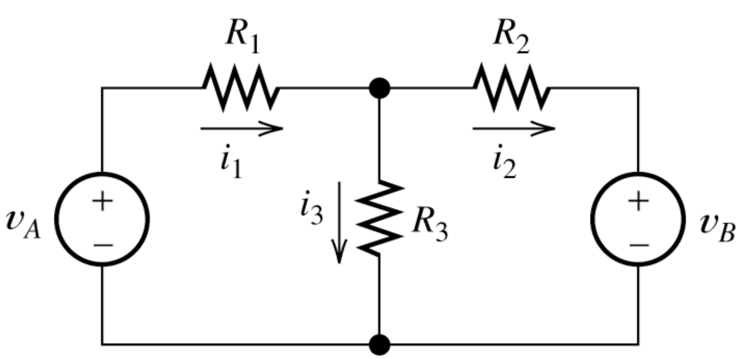
(a)



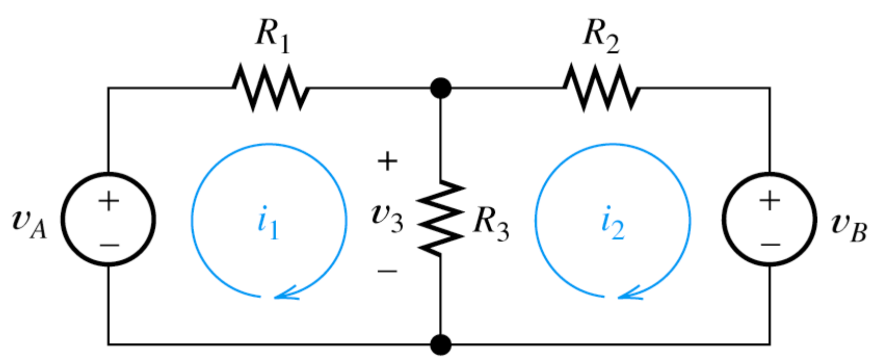
(b)

Mesh Current Analysis

- For planar networks
- Branch currents vs. mesh currents



(a) Circuit with branch currents

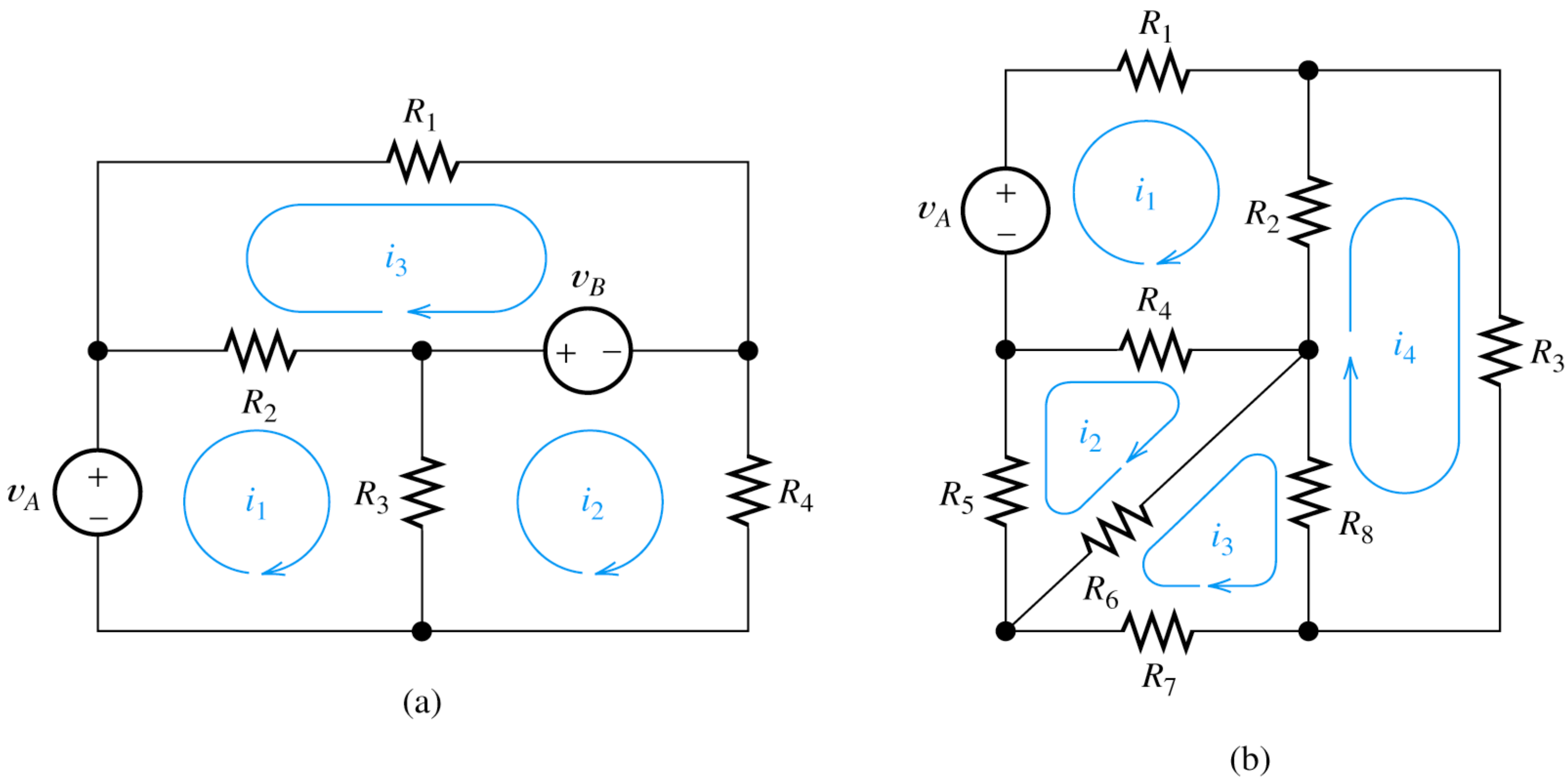


(b) Circuit with mesh currents

Choosing the Mesh Currents

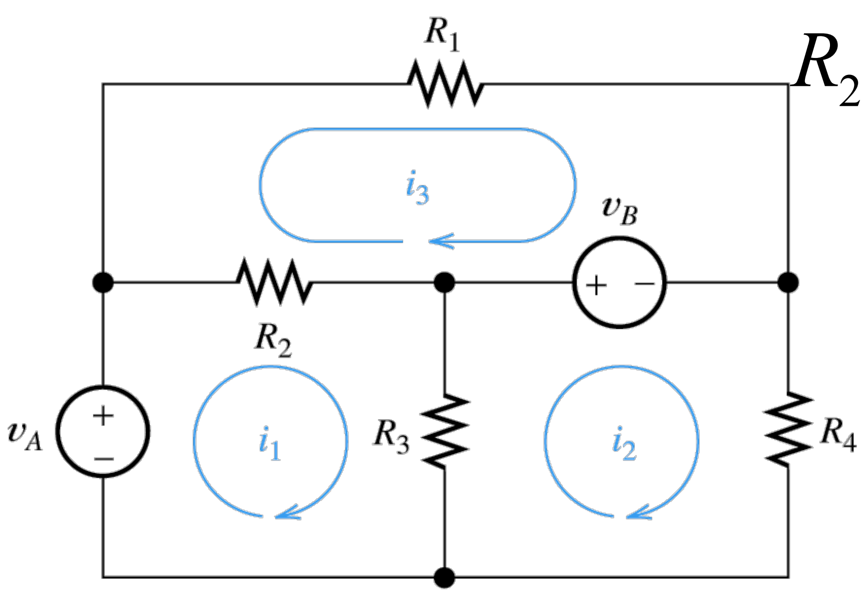
- When several mesh currents flow through one element, we consider the current in that element to be the algebraic sum of the mesh currents.
- Sometimes it is said that the mesh currents are defined by "soaping the window panes."

Mesh Currents



Writing Equations to Solve for Mesh Currents

- If a network contains only resistances and independent voltage sources, we can write the required equations by following each current around its mesh and applying KVL.

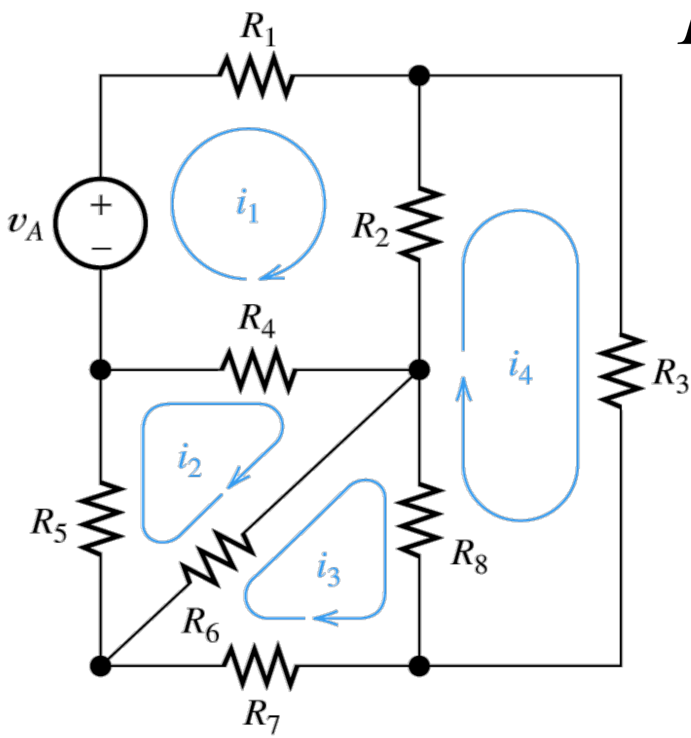


(a)

$$R_2\left(i_1 - i_s\right) + R_3\left(i_1 - i_2\right) - v_A = 0$$

$$R_3\left(i_2 - i_1\right) + R_4 i_2 + v_B = 0$$

$$R_2\left(i_3 - i_1\right) + R_1 i_3 - v_B = 0$$



(b)

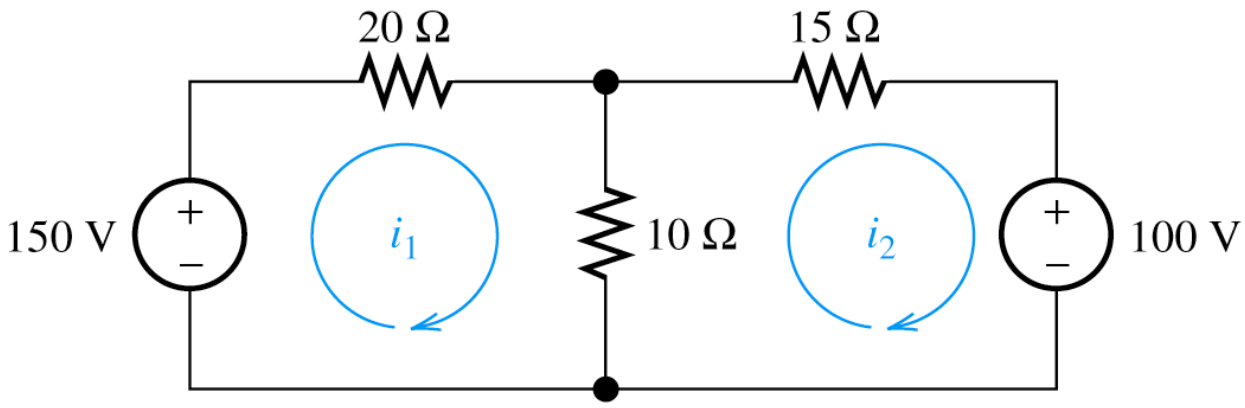
$$R_1 i_1 + R_2(i_1 - i_4) + R_4(i_1 - i_2) - v_A = 0$$

$$R_5 i_2 + R_4(i_2 - i_1) + R_6(i_2 - i_3) = 0$$

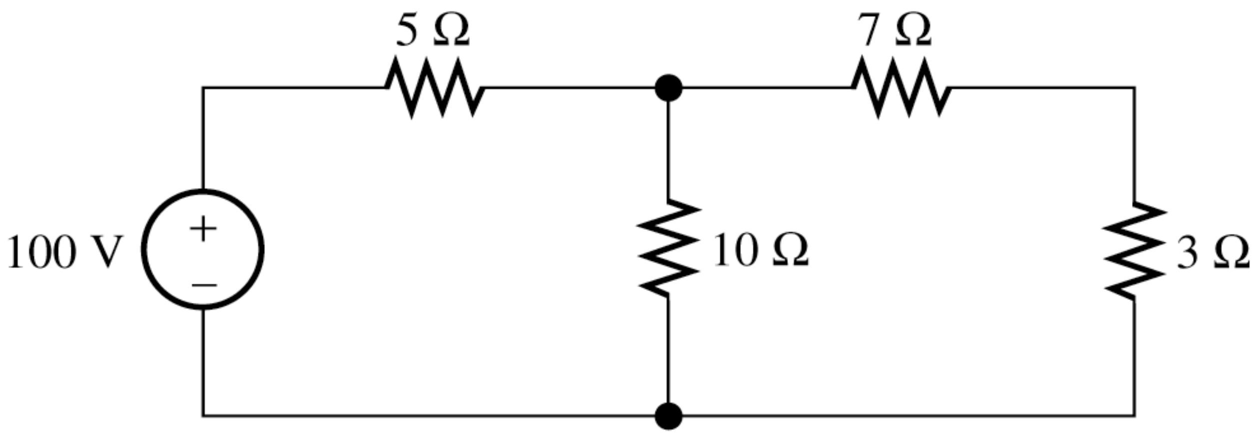
$$R_7 i_3 + R_6(i_3 - i_2) + R_8(i_3 - i_4) = 0$$

$$R_3 i_4 + R_2(i_4 - i_1) + R_8(i_4 - i_3) = 0$$

Example Exercise

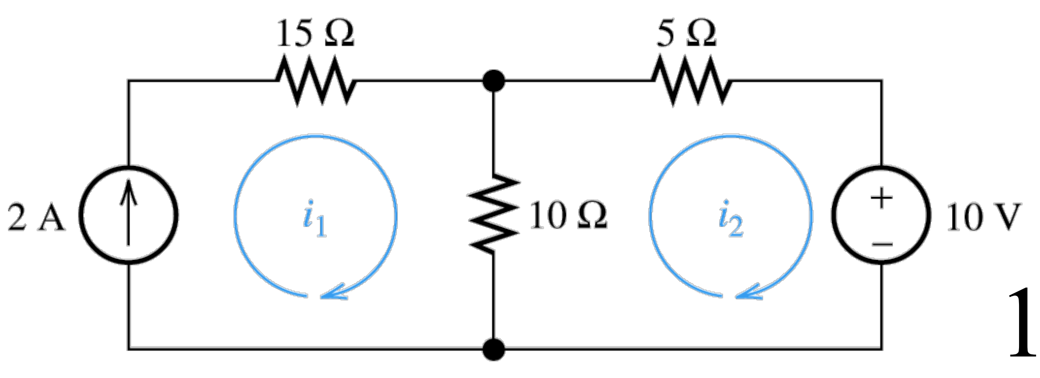


Example Exercise



Circuits Containing Current Sources

A common mistake made by beginning students is to assume that the voltages across current sources are zero.

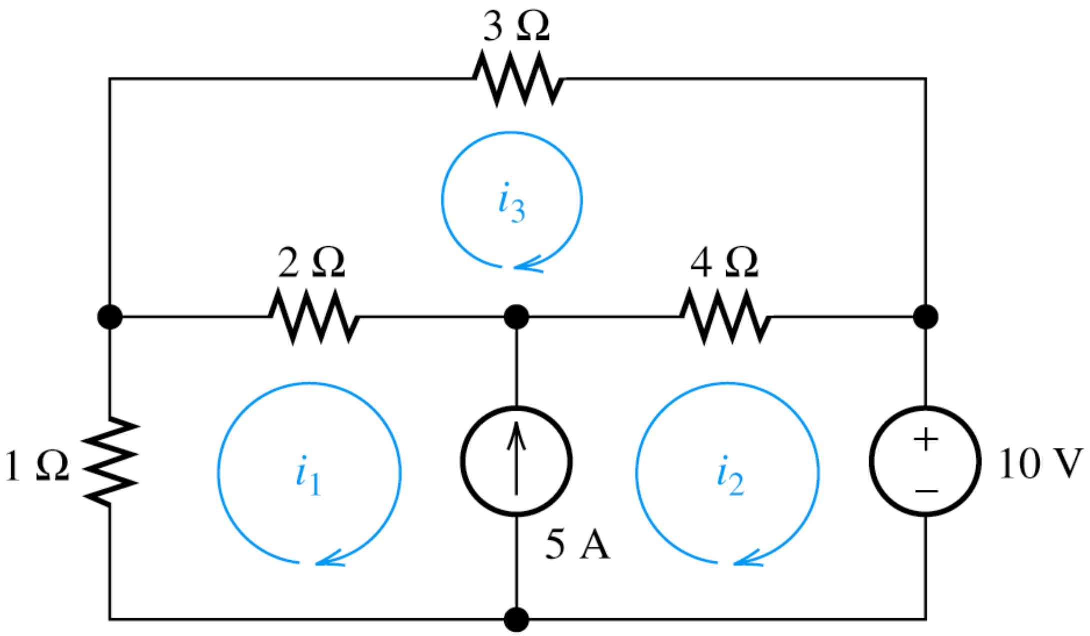


$$i_1 = 2\text{A}$$

$$10(i_2 - i_1) + 5i_2 + 10 = 0$$

KVL in loop 1?

Example Exercise



Supermesh

- Combine meshes 1 and 2 into a **supermesh**. In other words, we write a KVL equation around the periphery of meshes 1 and 2 combined.

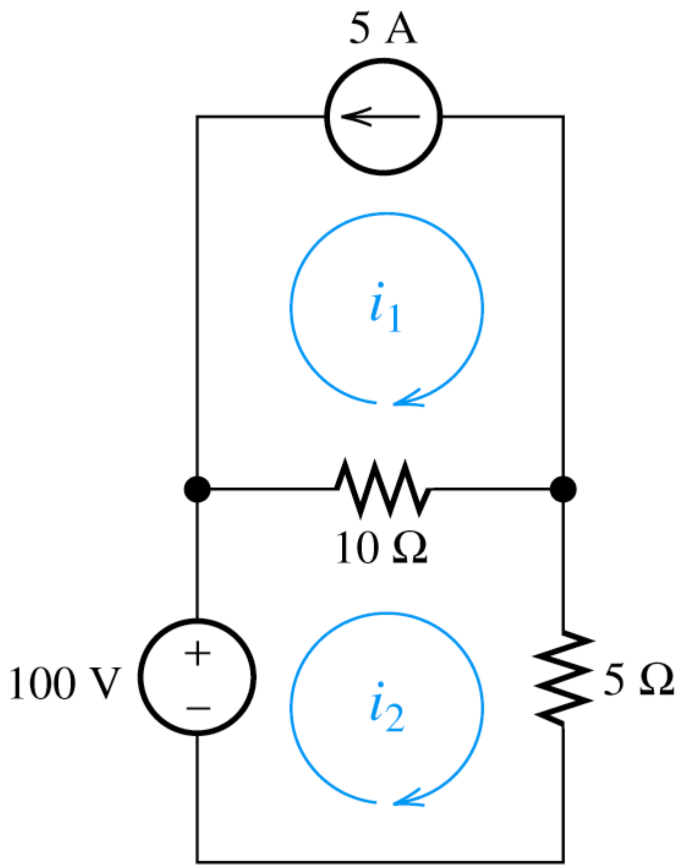
$$i_1 + 2(i_1 - i_3) + 4(i_2 - i_3) + 10 = 0$$

Mesh 3:

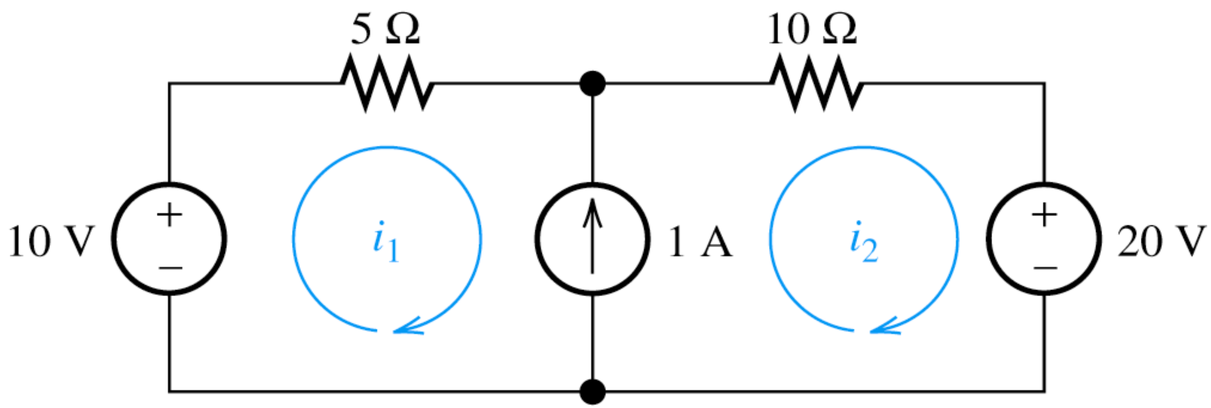
$$3i_3 + 4(i_3 - i_2) + 2(i_3 - i_1) = 0$$

$$i_2 - i_1 = 5$$

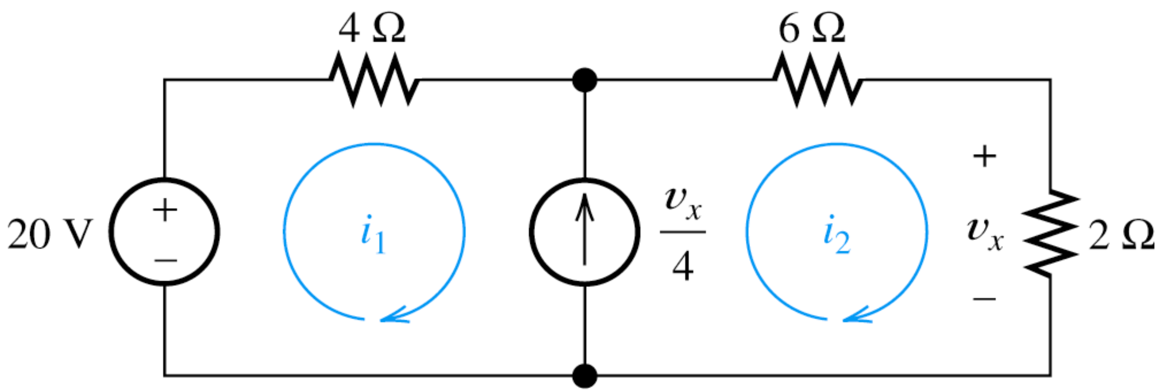
Example Exercise



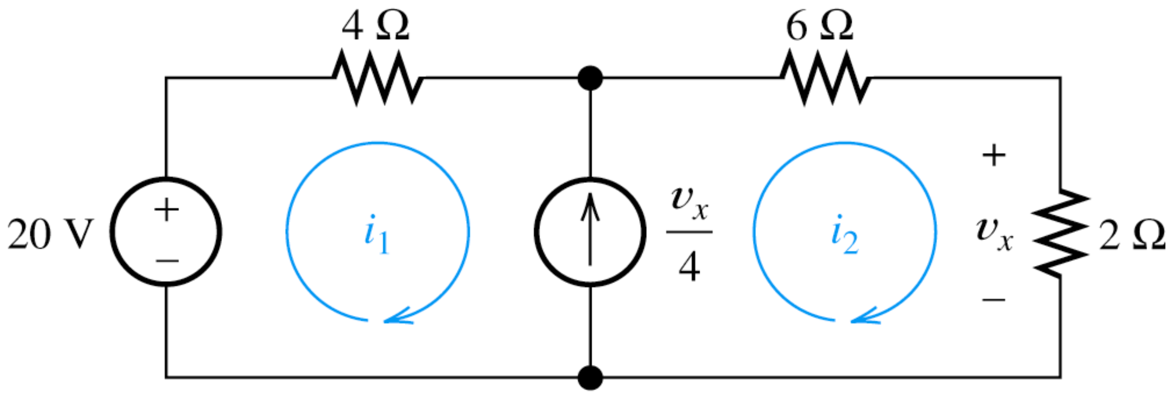
Example Exercise



Example Exercise



Example Exercise



$$-4i_1 + 6i_2 + 2i_2 = 0$$

$$\frac{v_x}{4} = i_2 - i_1$$

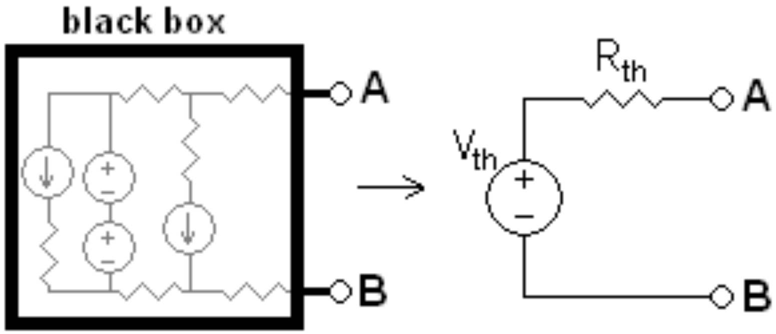
$$v_x = 2i_2$$

Mesh-Current Analysis

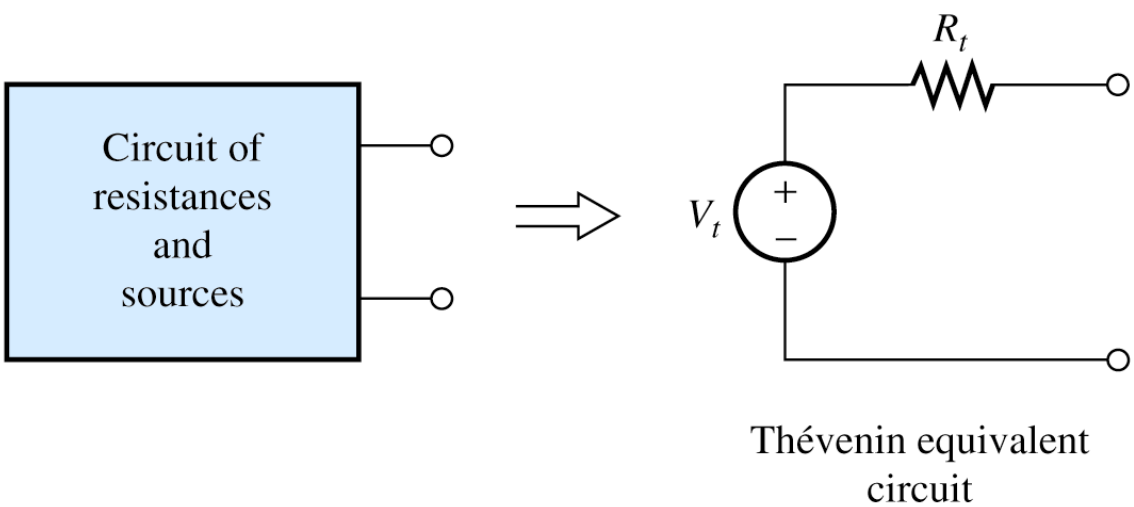
1. If necessary, redraw the network without crossing conductors or elements. Then define the mesh currents flowing around each of the open areas defined by the network. For consistency, we usually select a clockwise direction for each of the mesh currents, but this is not a requirement.
2. Write network equations, stopping after the number of equations is equal to the number of mesh currents. First, use KVL to write voltage equations for meshes that do not contain current sources. Next, if any current sources are present, write expressions for their currents in terms of the mesh currents. Finally, if a current source is common to two meshes, write a KVL equation for the supermesh.
3. If the circuit contains dependent sources, find expressions for the controlling variables in terms of the mesh currents. Substitute into the network equations, and obtain equations having only the mesh currents as unknowns.
4. Put the equations into standard form. Solve for the mesh currents by use of determinants or other means.
5. Use the values found for the mesh currents to calculate any other currents or voltages of interest.

Thévenin Theorem

- Any linear electrical network with voltage and current sources and resistances can be replaced at terminals A-B by an equivalent voltage source V_{th} in series connection with an equivalent resistance R_{th}.

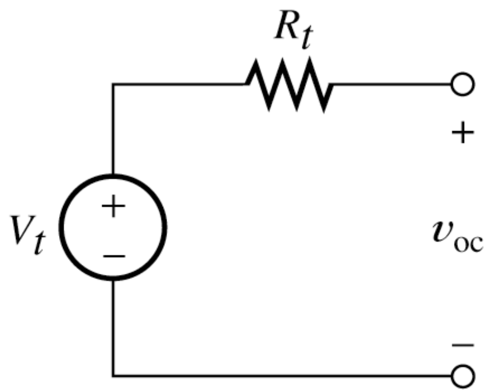


Thévenin Equivalent Circuits



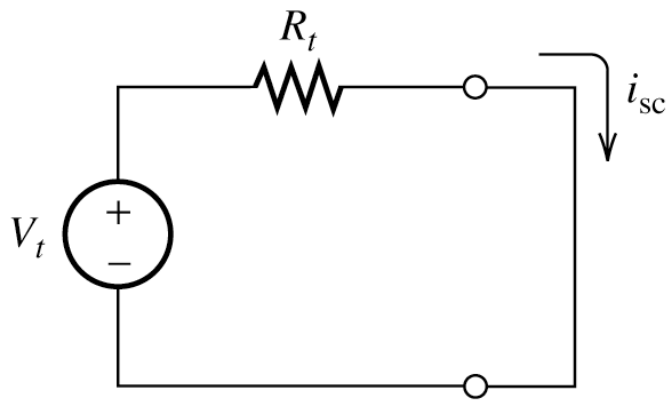
Thévenin Equivalent Circuit (Contd.)

- With open circuited terminals, the open circuit voltage v_{oc} is equal to the Thévenin voltage V_t.

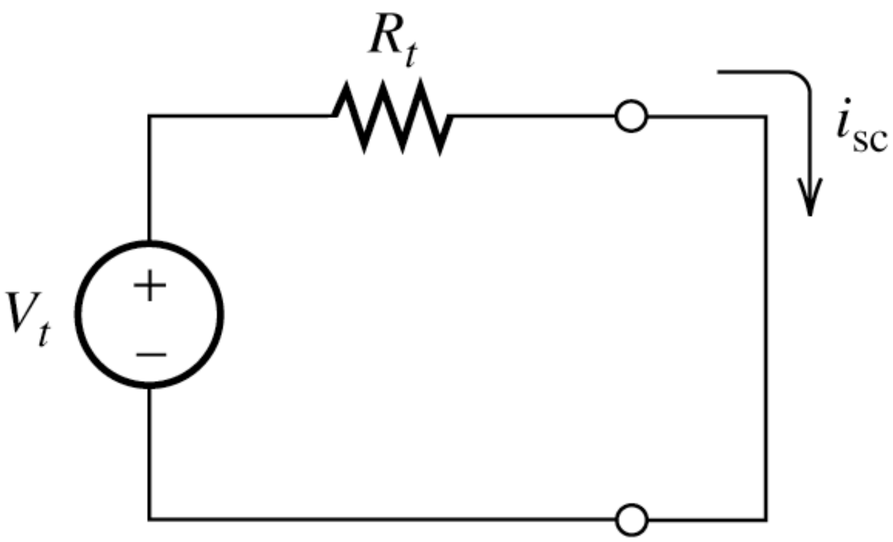


Thévenin Equivalent Circuit (Contd.)

- With short circuited terminals, the short circuit current i_{sc} is equal to V_t/R_t.



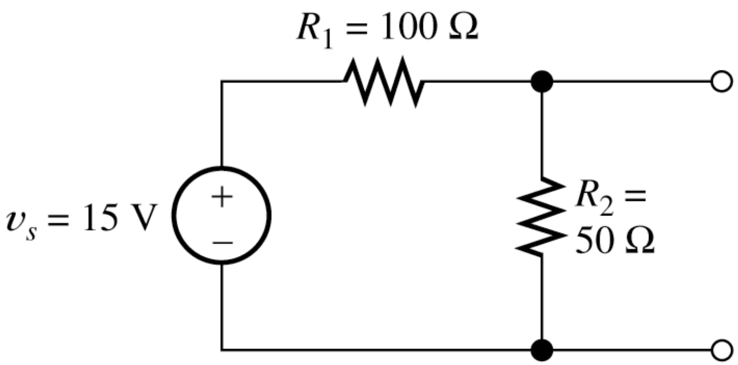
Thévenin Equivalent Circuits



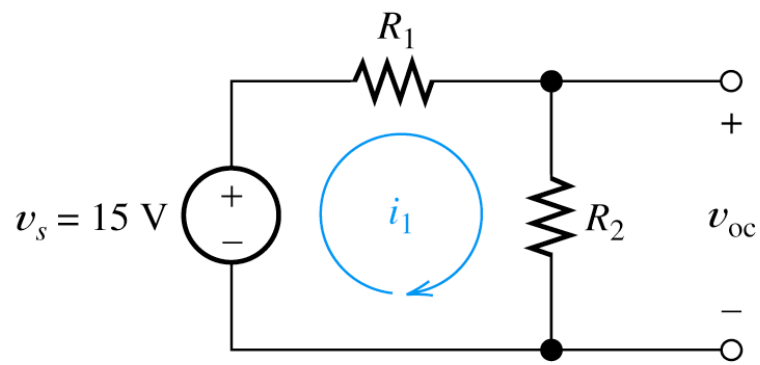
$$V_t = v_{oc}$$

$$R_t = \frac{v_{oc}}{i_{sc}}$$

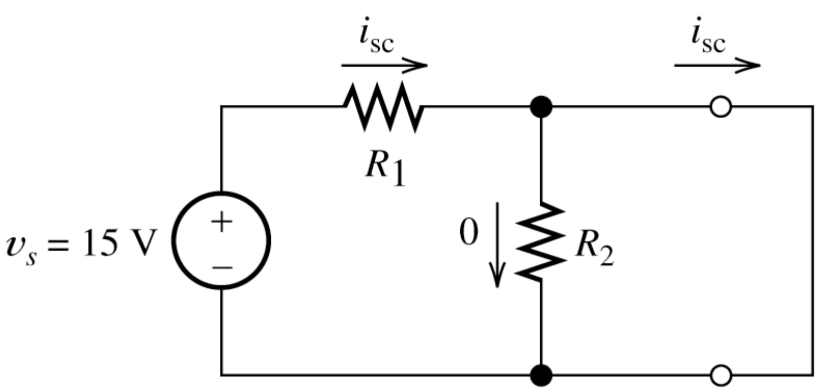
Example Exercise



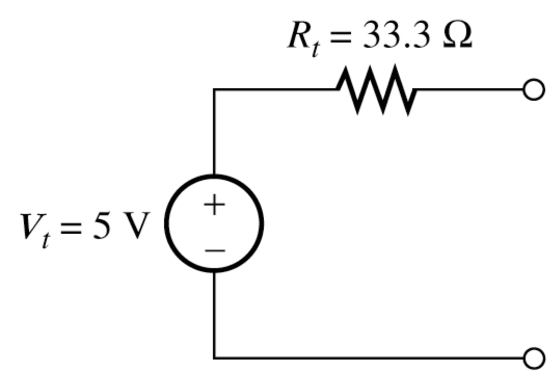
(a) Original circuit



(b) Analysis with an open circuit

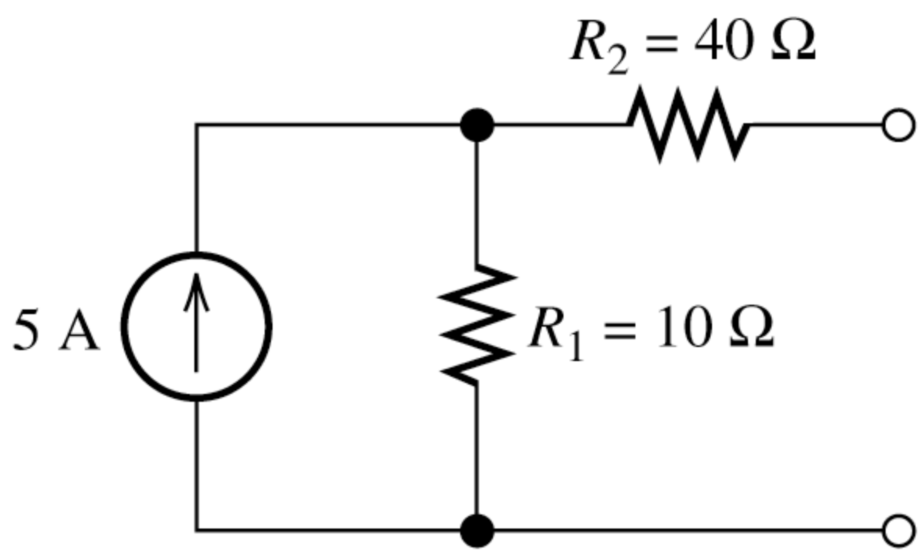


(c) Analysis with a short circuit



(d) Thévenin equivalent

Example Exercise

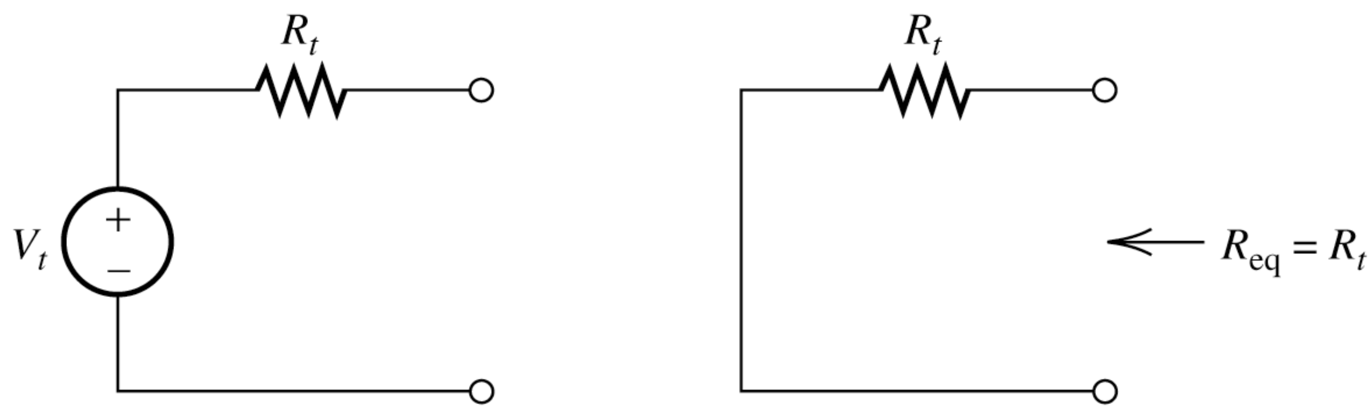


Finding the Thévenin Resistance Directly

- We can find the Thévenin resistance by zeroing the sources in the original network and then computing the resistance between the terminals.

- Note: When zeroing a voltage source, it becomes a short circuit. When zeroing a current source, it becomes an open circuit.

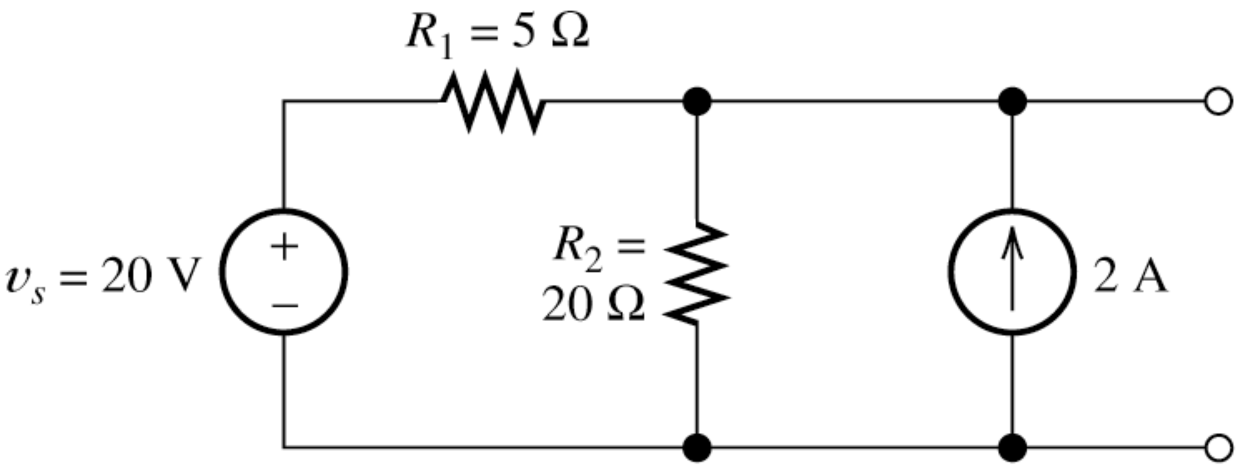
Thévenin Resistance



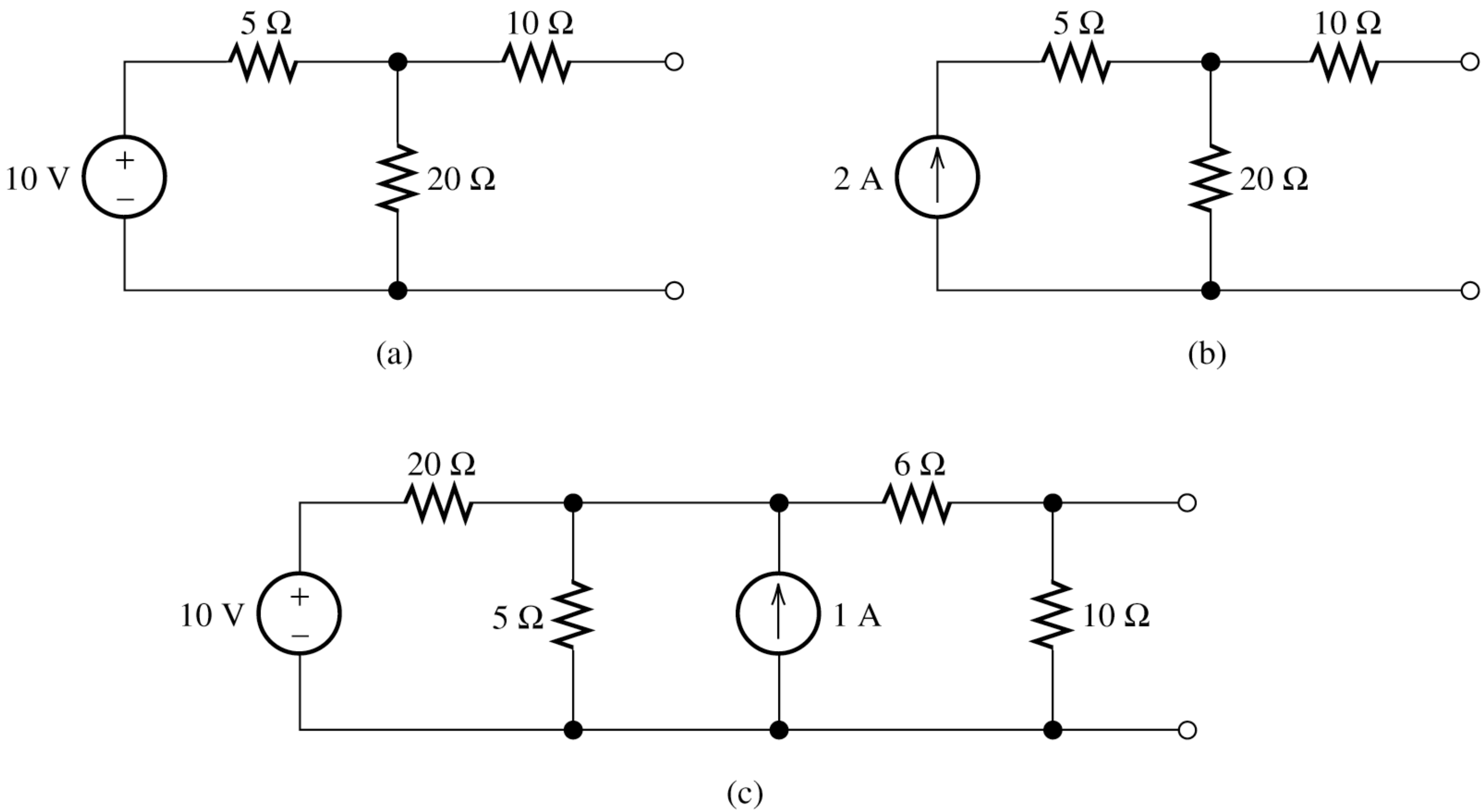
(a) Thévenin equivalent

(b) Thévenin equivalent with its source zeroed

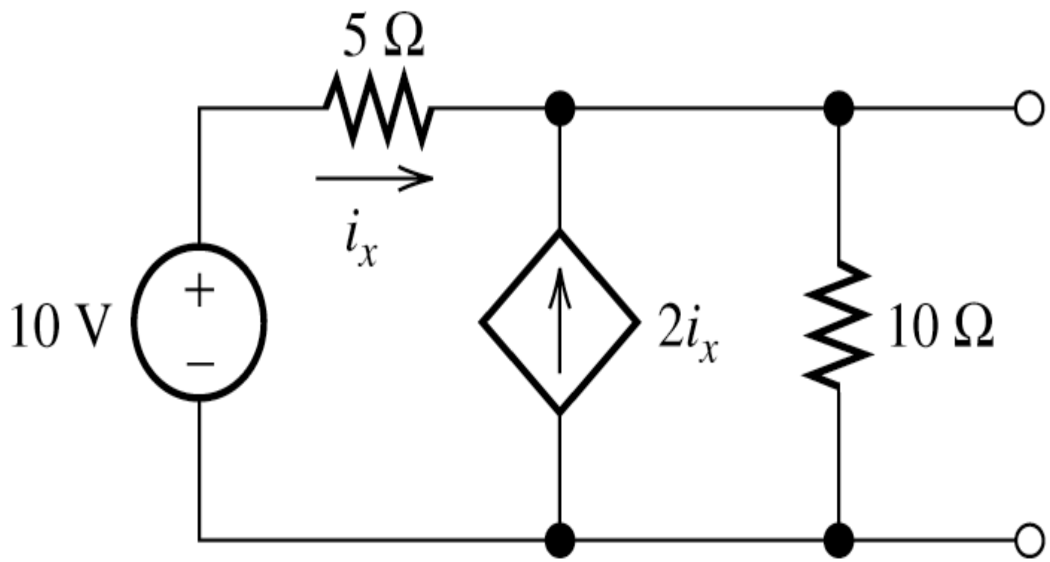
Thévenin Resistance



Example Exercise

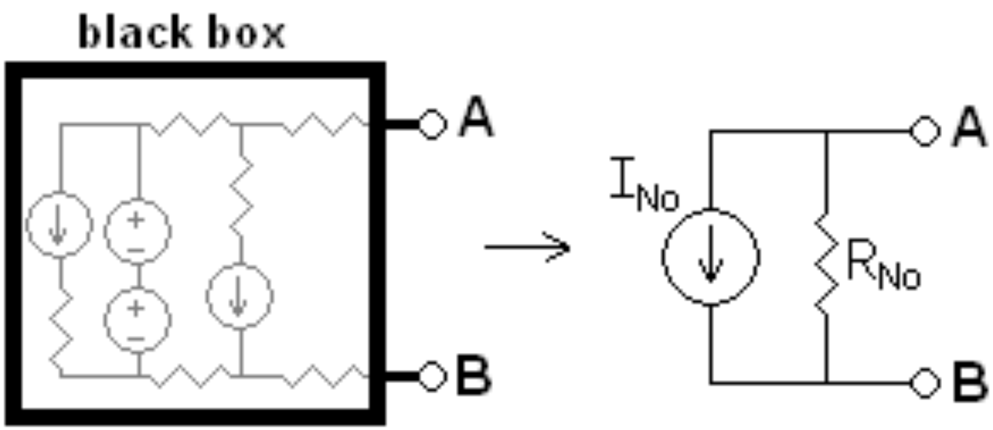


Example Exercise

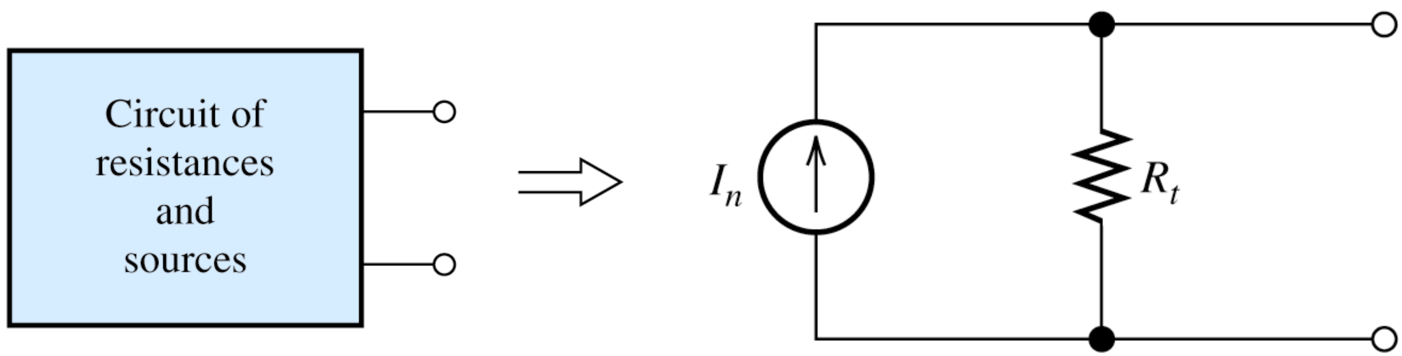


Norton Theorem

- Any linear electrical network with voltage and current sources and only resistances can be replaced at terminals A-B by an equivalent current source I_{NO} in parallel connection with an equivalent resistance R_{NO}.

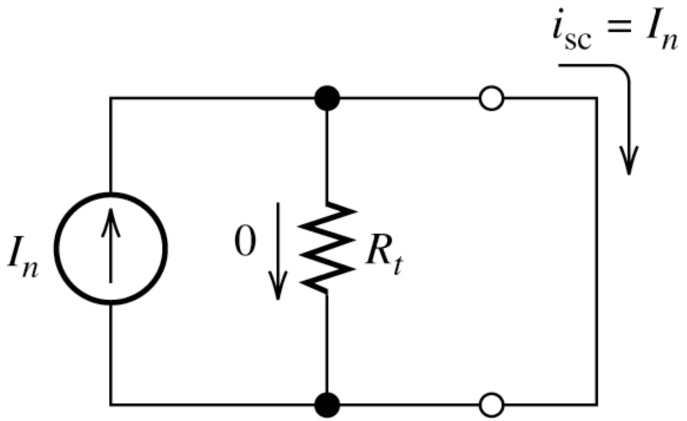


Norton Equivalent Circuit



Is R_t same as Thévenin resistance? If yes, why? If no, what is its value?

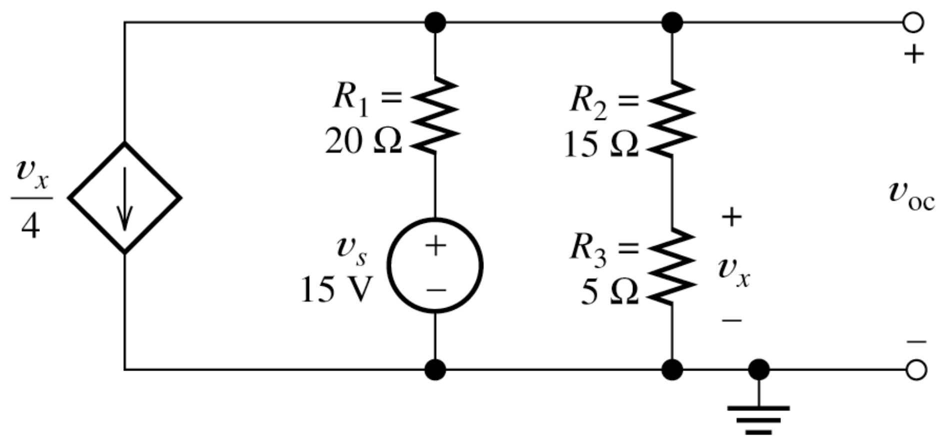
Norton Equivalent Circuit



Step-by-step Thévenin/Norton-Equivalent-Circuit Analysis

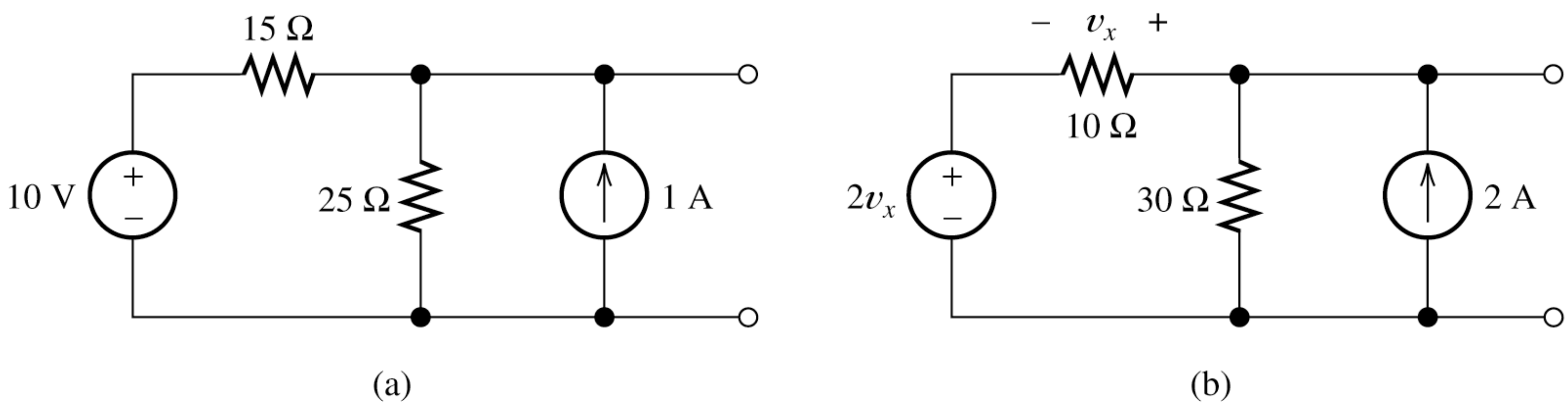
1. Perform two of these:
   1. Determine the open-circuit voltage $V_t = v_{oc}$.
   2. Determine the short-circuit current $I_n = i_{sc}$.
   3. Zero the sources and find the Thévenin resistance R_t looking back into the terminals.
2. Use the equation $V_t = R_t I_n$ to compute the remaining value.
3. The Thévenin equivalent consists of a voltage source V_t in series with R_t .
4. The Norton equivalent consists of a current source I_n in parallel with R_t .

Example Exercise



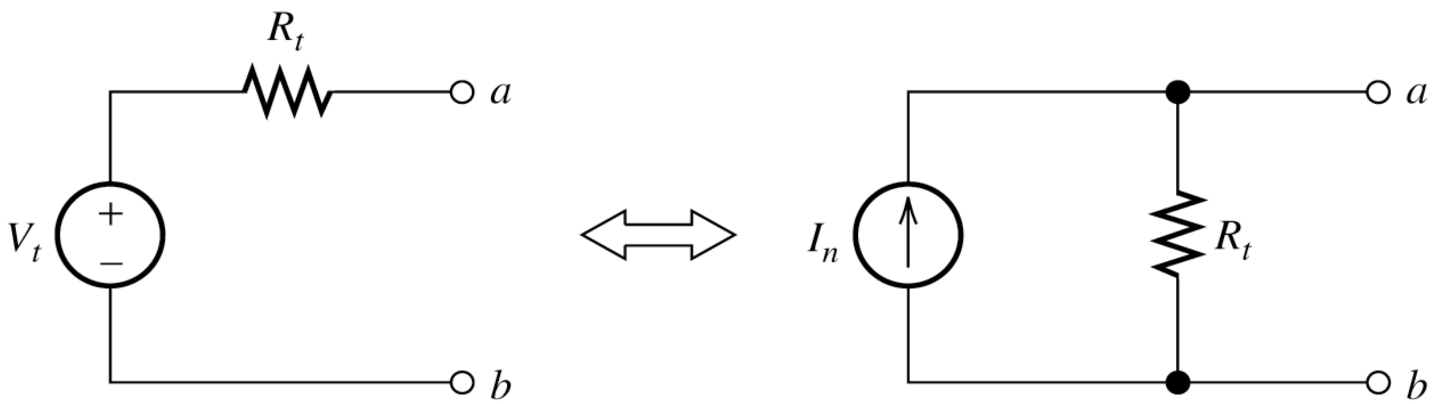
(a) Original circuit under open-circuit conditions

Example Exercise

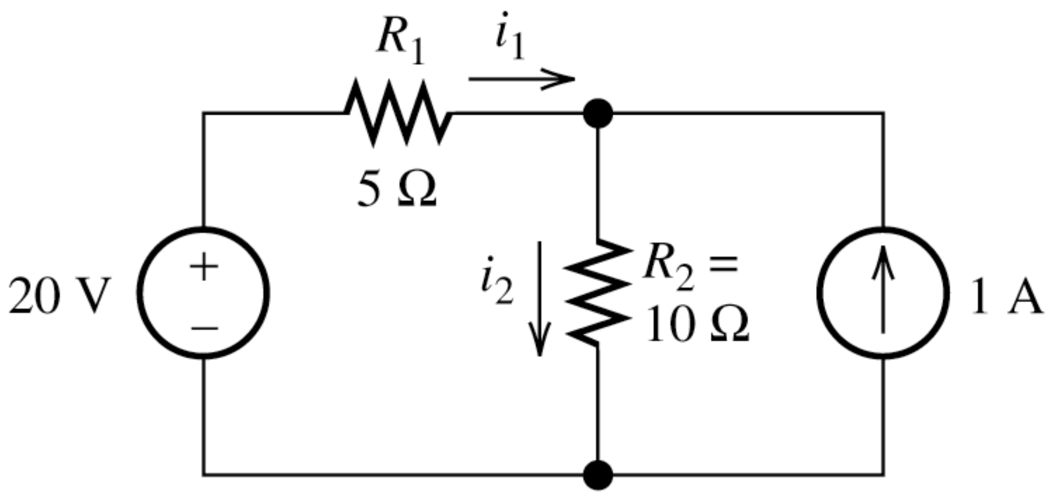


Source Transformations

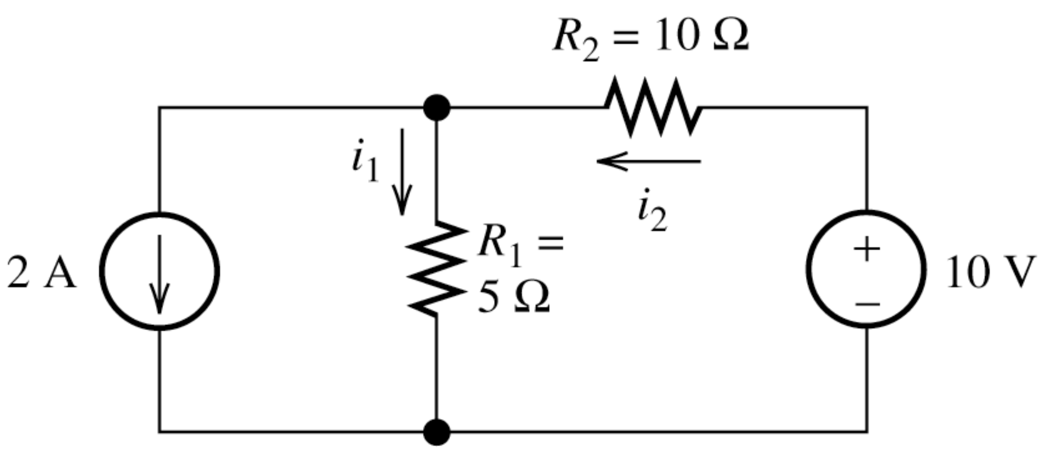
- Identical in terms of external behavior



Example Exercise



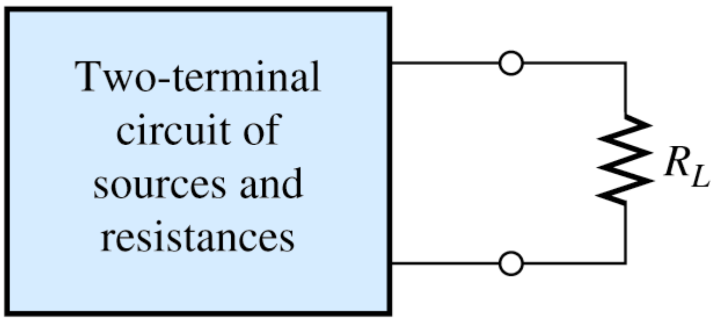
Example Exercise



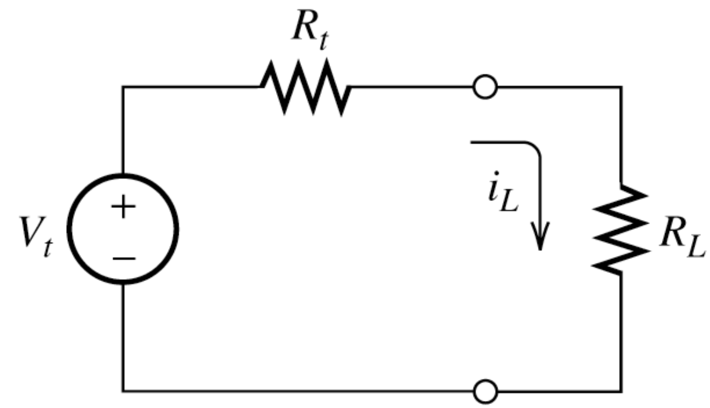
Maximum Power Transfer

The load resistance that absorbs the maximum power from a two-terminal circuit is equal to the Thévenin resistance.

Maximum Power Transfer



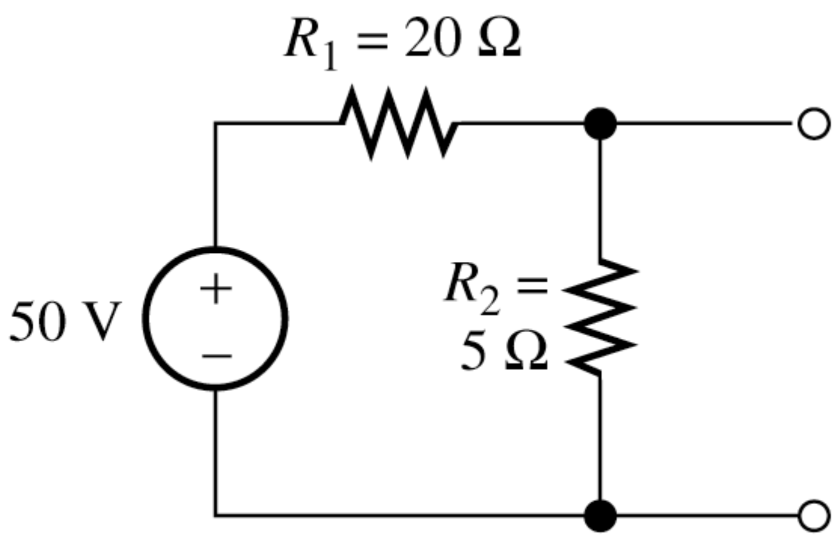
(a) Original circuit with load



(b) Thévenin equivalent circuit with load

Why $R_L = R_t$ for maximum power transfer?

Example Exercise



Practical Application

- High efficiency system not usually designed for maximum power transfer
  - Consider EV. Minimize power loss in battery and wiring
- For small power systems, maximum power transfer is used
  - Consider radio receiver. Extract maximum signal power from receiving antenna.

Superposition Principle

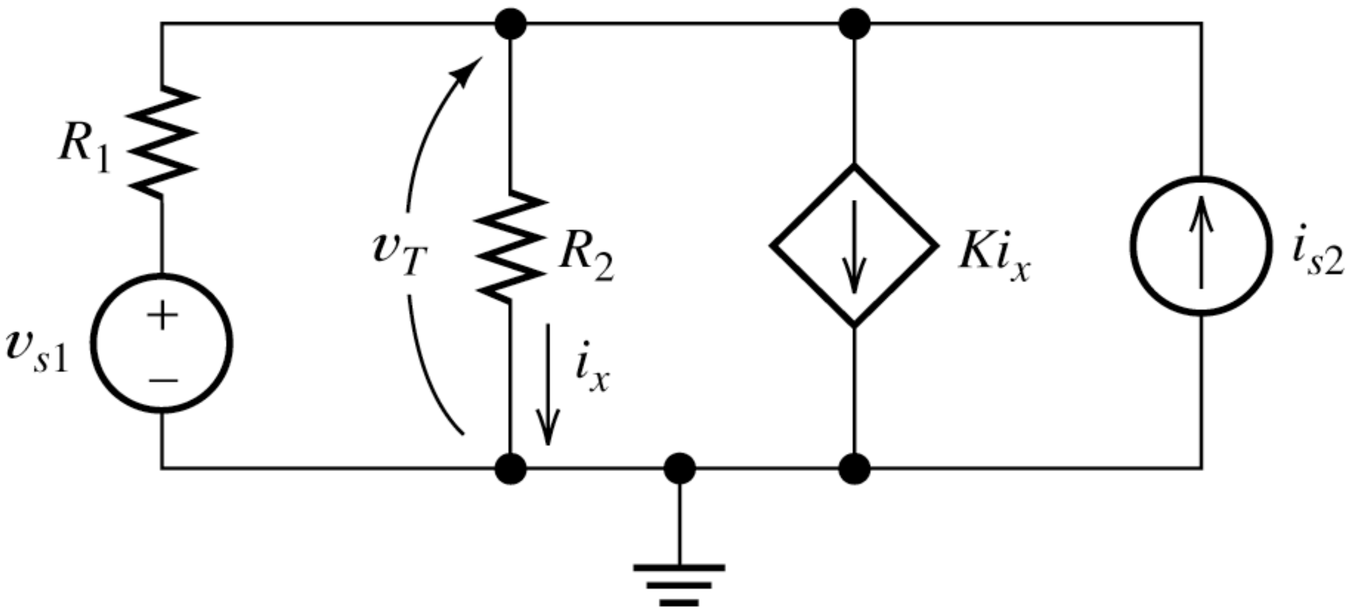
- Applicable to circuits with resistances, linear dependent sources and independent sources.
- Response: Current flowing through or voltage across an element

Superposition Principle

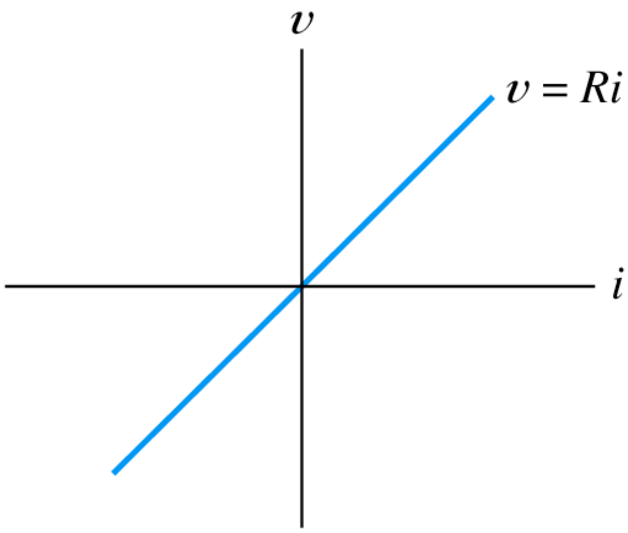
The **superposition principle** states that the total response is the sum of the responses to each of the independent sources acting individually. In equation form, this is

$$r_T = r_1 + r_2 + \cdots + r_n$$

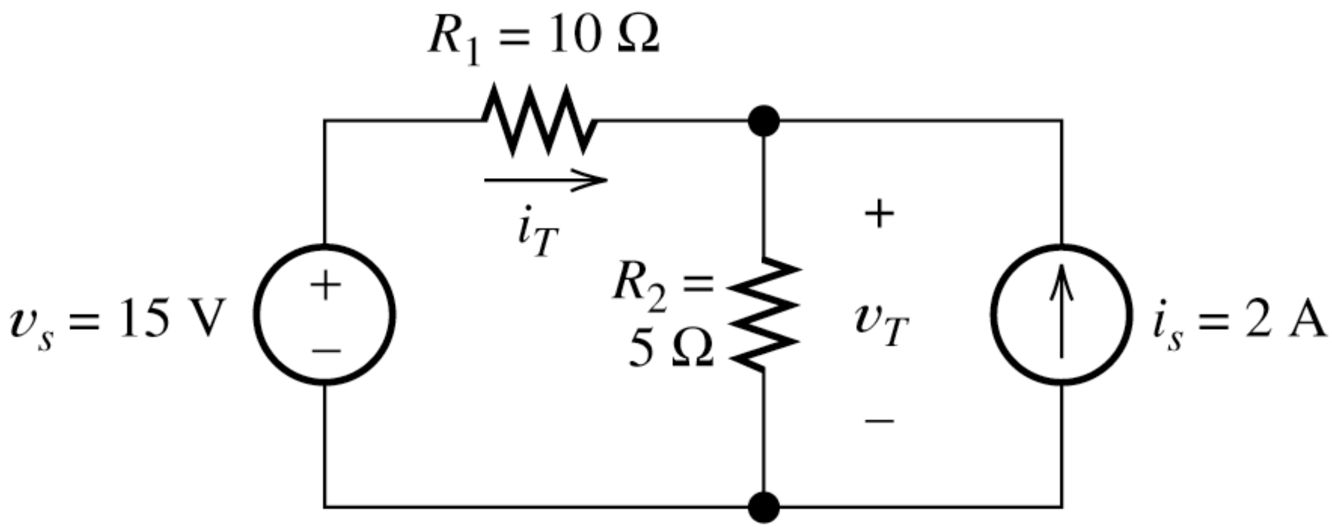
Superposition Principle (Contd.)



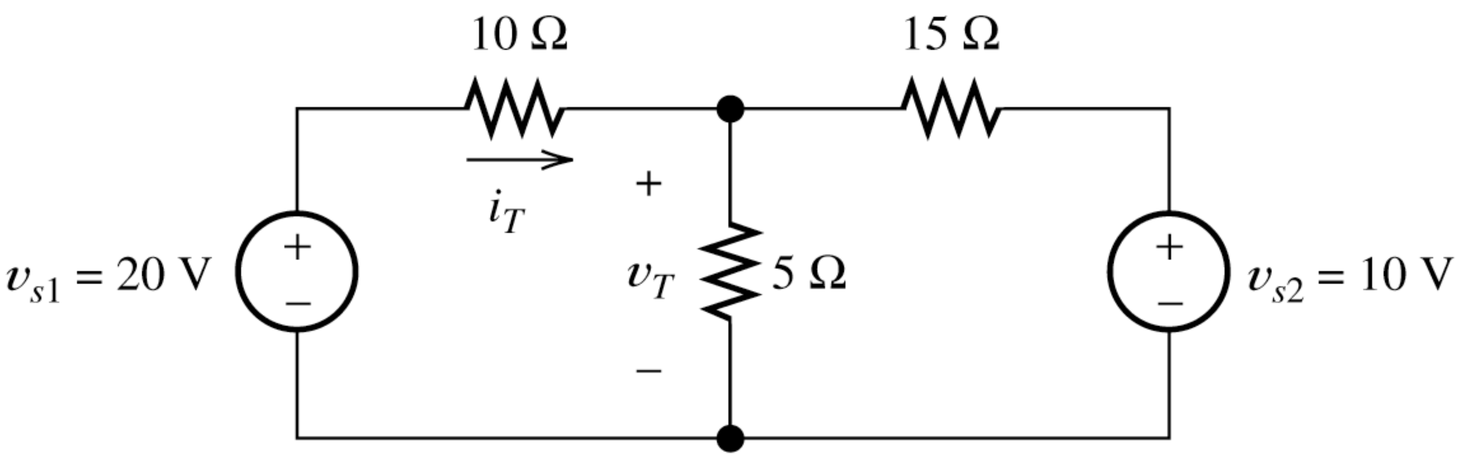
Linear Elements



Example Exercise



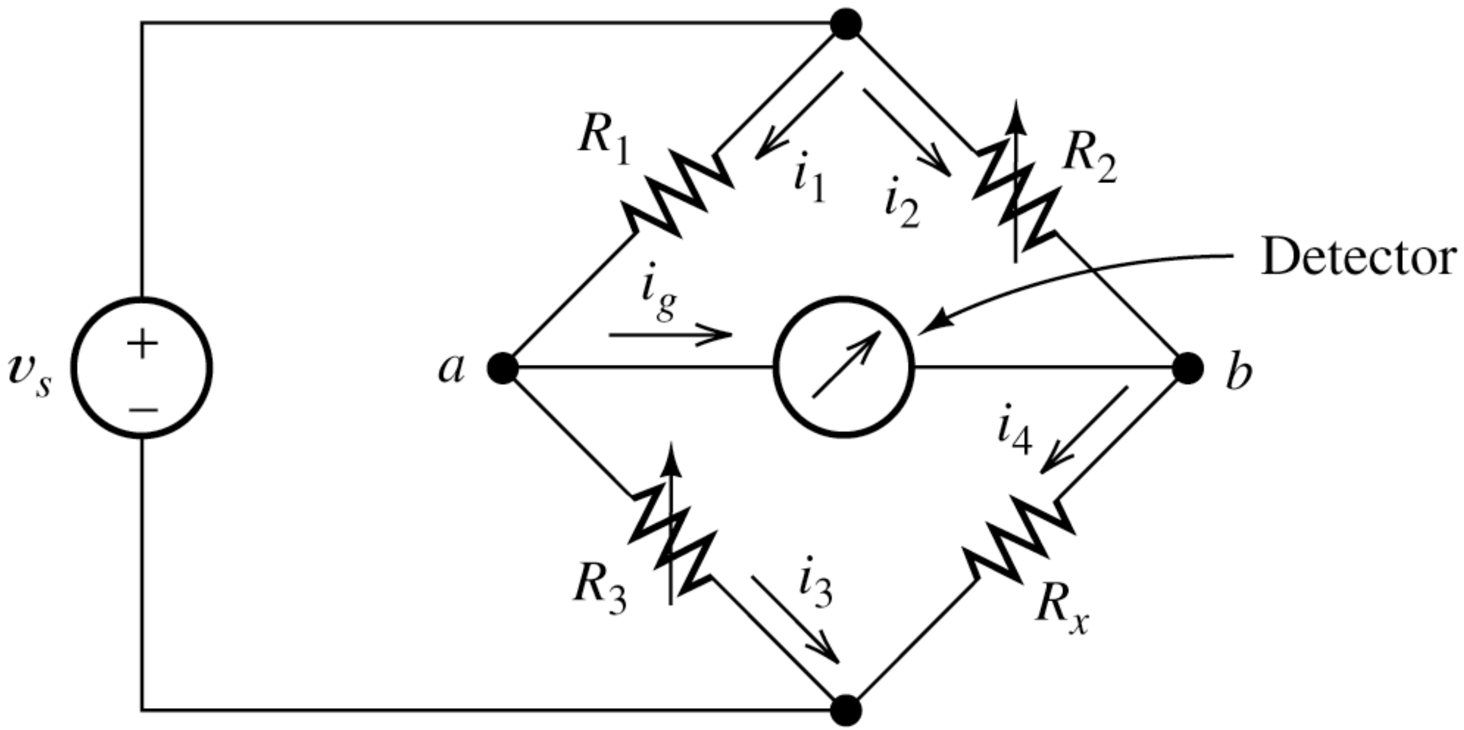
Example Exercise



Wheatstone Bridge

- Circuit used to measure unknown resistances
- Used by mechanical and civil engineers to measure the resistances of strain gauges in experimental stress studies of machines and buildings.

Wheatstone Bridge



Wheatstone Bridge (Contd.)

- Bridge balanced when no current through detector i.e. $i_g=0$ and $v_{ab}=0$
- R_2 and R_3 adjusted until balanced

$$R_x = \frac{R_2}{R_1} R_3$$