Indian Institute of Technology Mandi
भारतीय प्रौद्योगिकी संस्थान मण्डी

IC-111: Linear Algebra **End Sem Exam** **Even Semester: 2012-13**

Duration: 3.00 Hours **Total Marks: 45**

There are 15 questions in this question paper.
Answer only 11 questions.
Question No. 1 and 2 are compulsory.

1. Consider the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ and an ordered basis B given by $T(x,y) = \{2x - 7y, 4x + 3y\}$ and $B = \{(1,3),(2,5)\}$. Then,

   (a) Find coordinate vectors $[u]_B$ of a general element $u = (x,y) \in \mathbb{R}^2$ with respect to the ordered basis B.

   (b) Find matrix representation $[T]_B$ with respect to basis B

   (c) Using $[T]_B$ and $[u]_B$ find $[T(u)]_B$.

   (d) If $B' = \{(1,0),(0,1)\}$ then find the transition matrix of $P = [Id]^B_{B'}$. **[7]**

2. Let $V = \{p(x) \mid p(x)\ \textit{is a polynomial of degree at most}\ 3\}$ be a vector space of all polynomial of degree at most degree 3 over a field $(\mathbb{R}, \boldsymbol{\oplus}, \boldsymbol{\odot})$ where vector addition and scalar multiplication are defined in usual manner. Suppose $U = \{p(x) \in V \mid p(1) = 0\}$ and $W = \{p(x) \in V \mid p'(1) = 0\}$ then find dimension and basis of the subspaces $U, W, U \cap W$ and $U \oplus W$. $\left[\mathbf{6}\frac{1}{2}\right]$

3. Let $(V, \oplus, \odot)$ be a vector space over a field $(\mathbb{F}, \boldsymbol{\oplus}, \boldsymbol{\odot})$ and $(U_i, \oplus, \odot)$ $i = 1, 2, \ldots, n$ be n subspaces of $(V, \oplus, \odot)$ over $(\mathbb{F}, \boldsymbol{\oplus}, \boldsymbol{\odot})$, then show that $\bigcap_{i=1}^n U_i$ is subspace of V. $\left[\mathbf{3}\frac{1}{2}\right]$

4. Let U and W be two subspaces of a vector space $(V, \oplus, \odot)$ over the field $(\mathbb{F}, \boldsymbol{\oplus}, \boldsymbol{\odot})$ then show that $U \oplus W = [U \cup W]$. $\left[\mathbf{3}\frac{1}{2}\right]$

5. Using Wronskian technique find a LI subset of A of S such that $[A] = [S]$, where $S = \{x^2, x^2 + 2x, x^2 + 2, 1 - x\}$. Find coordinate vectors of $3x^2 + x + 5$ corresponding to the A. $\left[\mathbf{3}\frac{1}{2}\right]$

6. Let $(V, \oplus, \odot)$ be a vector space over a field $(\mathbb{F}, \boldsymbol{\oplus}, \boldsymbol{\odot})$. Suppose $S = \{v_1, v_2, \ldots, v_n\}$ is an ordered set of vectors with $v_1 \neq 0$. Then show that S is LD **iff** one of the vectors $v_2, v_3, \ldots, v_n$, say v_k belongs to span of $v_1, v_2, \ldots v_{k-1}$, i.e. $v_k \in [v_1, v_2, \ldots v_{k-1}]$ for some $k = 2, 3, \ldots n$. $\left[\mathbf{3}\frac{1}{2}\right]$

7. Let $(V, \oplus, \odot)$ be a n-dimensional vector space over a field $(\mathbb{F}, \boldsymbol{\oplus}, \boldsymbol{\odot})$. Then show that any LI set $B = \{v_1, v_2, \ldots v_n\}$ of n vectors is a basis of V. $\left[\mathbf{3}\frac{1}{2}\right]$