Number System

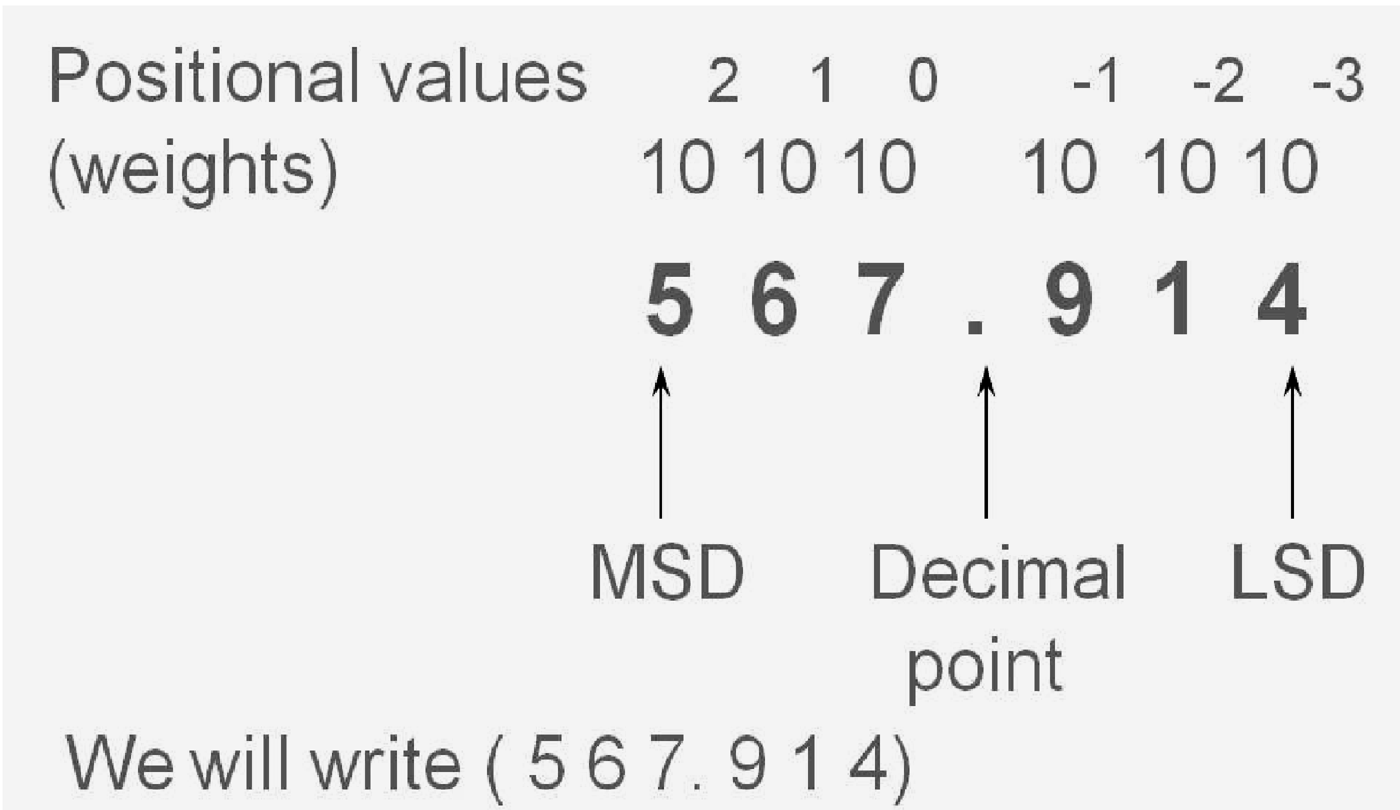
Number Systems

- To talk about binary data, we must first talk about number systems
- The decimal number system (base 10) you should be familiar with!
  - A digit in base 10 ranges from 0 to 9.
  - A digit in base 2 ranges from 0 to 1 (binary number system). A digit in base 2 is also called a 'bit'.
  - A digit in base R can range from 0 to R-1
  - A digit in Base 16 can range from 0 to 16-1 (0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F).

  Use letters A-F to represent values 10 to 15. Base 16 is also called Hexadecimal or just 'Hex'.

Positional-Value System

- The value of a digit (“digit” from Latin word for finger) depends on its position



Positional Number Systems

- The traditional number system is called a positional number system.

$$6354 = 6*1000 + 3*100 + 5*10 + 4$$

- A number is represented as a string of digits.

$$D = \sum_{i=0}^{p-1} d_i 10^i$$

- Each digit position has a *weight* assoc. with it.
- Number's value = a *weighted sum* of the digits

Decimal Numbers

- Decimal Numbers: each of ten digits (0-9)

$$D_n \quad D_3D_2D_1D_0.d_1d_2d_3$$

D_n*10^n

D_3*10^3 d_3*10^{-3}

D_2*10^2 d_2*10^{-2}

D_1*10^1 d_1*10^{-1}

D_0*10^0

Cont….

Base : **The number of characters in the character set of a positional number system**.

a) Decimal number base 10
b) Binary number base 2

Binary coded Decimal (BCD): **A special binary code used for directly represent the decimal character**.

a) Each four bit value in BCD represent a single decimal character.

Code: A unique way to represent a value or a character in the alphabet. Digital codes use bits configured in a given way to present the number of alphabet character.

Complement :The complement of a single character represented in a positional number system is the difference between the total number of characters in the character set and the value of the character for which a complement is being sought.

E. g: 10s complement of 4 is 6 , because 10 – 4 = 6.

Hexadecimal : **A base 16 number system, and four bit used to represent a single hexadecimal number.**

Logic : **It is process of determine the truth or false.**

Binary Representation

- The basis of all digital data is binary representation.
- Binary - means ‘two’
  - 1, 0
  - True, False
  - Hot, Cold
  - On, Off
- We must be able to handle more than just values for real world problems
  - 1, 0, 56
  - True, False, Maybe
  - Hot, Cold, Warm, Cool
  - On, Off, Leaky

Binary: Base-2 Number System

$2^{5}\ 2^{4}\ 2^{3}\ 2^{2}\ 2^{1}\ 2^{0}\quad 2^{-1}\ 2^{-2}\ 2^{-3}$

1 0 1 1 1 1 . 0 0 1

base point or radix

We write: $(1\,0\,1\,1\,1\,1\,.\,0\,0\,1)_2$

Digits are called bits

Binary Numbers

- Binary Numbers: each of 2 digits (0,1) called bits
- Largest decimal number = 2^n-1; n is # of bits
- Binary-to-Decimal Conversion

$$B_n \qquad B_3B_2B_1B_0.b_1b_2b_3$$

(B_n*2^{n}) +

(B_3*2^{3}) +

(B_2*2^{2}) +

(B_1*2^{1}) +

(B_0*2^{0}) +

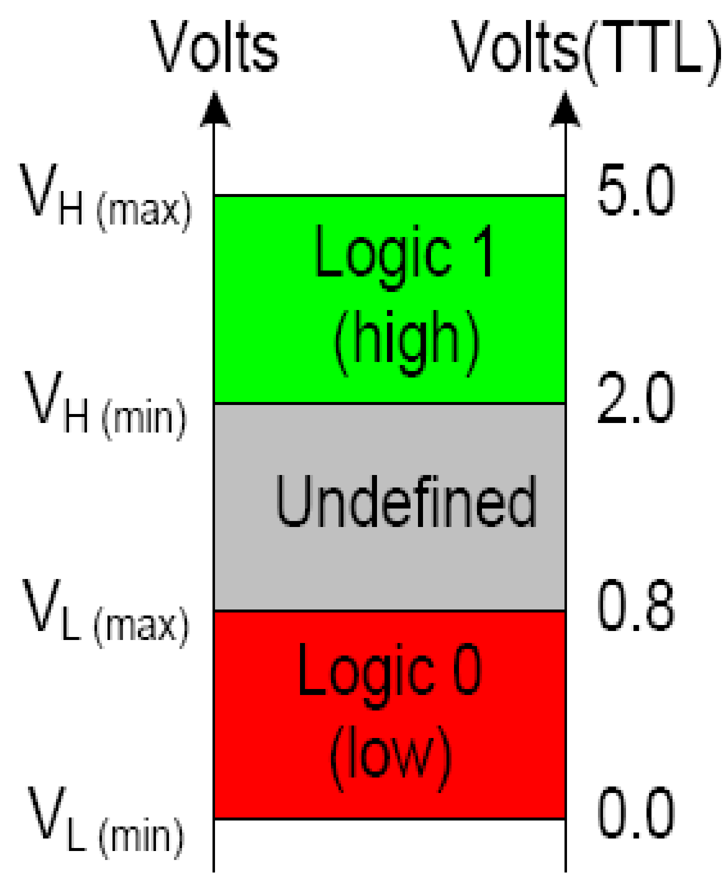
(b_1*2^{-1}) +

(b_2*2^{-2}) +

b_3*2^{-3}

Binary Digits, Logic Levels, and Digital Waveforms

- Binary Digits: 0 → Low,
  1 → High
- Code:
  Group of Bits
- Logic Level:
  represented by voltages



Base 10, Base 2, Base 16

The text book uses subscripts to represent different bases (ie. $A2F_{16}$, 953.78_{10}, 1011.11_{2})

We will use special symbols to represent the different bases.

- **The default base will be decimal, no special symbol for base 10.**

- **The '$' will be used for base 16 ($A2F)**

- **The '%' will be used for base 2 (%10101111)**

If ALL numbers on a page are the same base (ie, all in base 16 or base 2 or whatever) then no symbols will be used and a statement will be present that will state the base (ie, all numbers on this page are in base 16).

we were talking about Binary DATA!!!

How many binary DIGITS does it take to represent our data??

Binary Codes

One Binary Digit (one bit) can take on values 0, 1. We can represent TWO values:

(0 = hot, 1 = cold), (1 = True, 0 = False),
(1 = on, 0 = off).

Two Binary digits (two bits) can take on values of
00, 01, 10, 11. We can represent FOUR values:

(00 = hot, 01 = warm, 10 = cool, 11 = cold).

Three Binary digits (three bits) can take on values of 000, 001, 010, 011, 100, 101, 110, 111.

We can represent 8 values
000 = Black, 001 = Red, 010 = Pink, 011 = Yellow, 100 = Brown,
101 = Blue, 110 = Green , 111 = White.

Binary Codes (cont.)

- **N bits (or N binary Digits) can represent 2^N different values.**

  (for example, 4 bits can represent 2^4 or 16 different values)

- **N bits can take on unsigned decimal values from 0 to 2^N-1.**

Codes usually given in tabular form.

000	black
001	red
010	pink
011	yellow
100	brown
101	blue
110	green
111	white

2^{10} = 1024 = 1 K

2^{20} = 1048576 = 1 M (1 Megabits) = 1024 K = 2^{10} x 2^{10}

2^{30} = 1073741824 = 1 G (1 Gigabits)

Octal and Hexadecimal ("Hex") Numbers

- Octal = base 8
- Hexadecimal = base 16
  - Use A – F to represent the values 10 through 16 in each position.

Useful for representing multi-bit binary numbers because their radices are integer multiples of 2.

$$10\ 0101\ 1010\ 1111\ .\ 1011\ 111_2 = 2\ 5\ A\ F\ .\ B\ E_{16}$$

Comparative Counting in Different Number System

Decimal	Binary	Octal	Hex
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Octal Numbers

- The octal system is composed of eight digits; 0-7
- One octal bit: 0-7

Hexadecimal Numbers

- The Hexadecimal system has base sixteen; 0-9 & A-F
- One hex bit: 0-F

Comparison of binary, decimal, octal and hexadecimal numbers

Binary	Decimal	Octal	3-Bit String	Hexadecimal	4-Bit String
0	0	0	000	0	0000
1	1	1	001	1	0001
10	2	2	010	2	0010
11	3	3	011	3	0011
100	4	4	100	4	0100
101	5	5	101	5	0101
110	6	6	110	6	0110
111	7	7	111	7	0111
1000	8	10	—	8	1000
1001	9	11	—	9	1001
1010	10	12	—	A	1010
1011	11	13	—	B	1011
1100	12	14	—	C	1100
1101	13	15	—	D	1101
1110	14	16	—	E	1110
1111	15	17	—	F	1111

Table

Binary, decimal, octal, and hexadecimal numbers.

Positional Notation Number System Examples

Value of number is determined by multiplying each digit by a weight and then summing. The weight of each digit is a POWER of the BASE and is determined by position.

$$953.78 = 9 \times 10^2 + 5 \times 10^1 + 3 \times 10^0 + 7 \times 10^{-1} + 8 \times 10^{-2}$$
$$= 900 + 50 + 3 + .7 + .08 = 953.78$$

decimal

$$\%\ 1011.11 = 1x2^3 + 0x2^2 + 1x2^1 + 1x2^0 + 1x2^{-1} + 1x2^{-2}$$
$$= 8 + 0 + 2 + 1 + 0.5 + 0.25$$
$$= 11.75$$

binary

$$\$\ A2F = 10x16^2 + 2x16^1 + 15x16^0$$
$$= 10 \times 256 + 2 \times 16 + 15 \times 1$$
$$= 2560 + 32 + 15 = 2607$$

hex