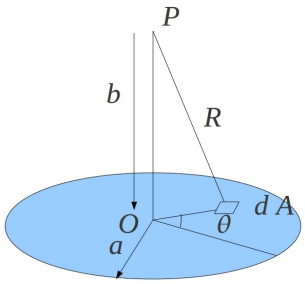
IIT Mandi

School of Basic Sciences

IC-121:Mechanics of particles and waves, Tutorial – 4

1) A particle P of mass m is situated on the axis of uniform disk of mass M and radius a. Find the gravitational force that the disk exerts on the particle. (Hint: Force exerted is given by $F=\frac{2mMG}{a^2}\left[1-\frac{b}{(a^2+b^2)}\right]$)



2) A uniform disc has mass M and radius a and uniform rigid rod has mass M' and length b. The rod is placed along the symmetry axis of the disc with one end in a contact with disk. Find the force required to pull the and the rod apart. Use the result from the previous problem.

3) Show that the gravitational force exerted on a particle inside a hollow symmetric sphere is zero

4) Evaluate the following integrals (a) $\int_6^2 (3x^2-2x-1)\delta(x-3)dx$, (b) $\int_0^5 \cos x\,\delta(x-\pi)dx$ (c) $\int_{-\infty}^{\infty} \ln(x+3)\delta(x+2)dx$, (d) $\int_{-2}^{2} (2x+3)\delta(3x)dx$, (e) $\int_0^2 (x^3+3x+2)\delta(1-x)dx$

5) Show that $\delta(kx)=\frac{1}{|k|}\delta(x)$ where k is any non-zero constant

6) Prove the following results (a) $x\delta(x)=0$ (b) $x\frac{d}{dx}\delta(x)=-\delta(x)$

7) A sphere of radius R centered at the origin carries charge density $\rho(r,\theta)=k\frac{R}{r^2}(R-2r)\sin\theta$ where k is a constant and r,θ are elliptical coordinates. Find the approximate potential for the points on the z axis, far from sphere.

8) For an electrical dipole that consists of two equal and opposite charges $\pm q$ separated by a distance d. Determine the quadrupole and octupole terms in the potential. (Hint: extend the series expansion of dipole to higher order terms).

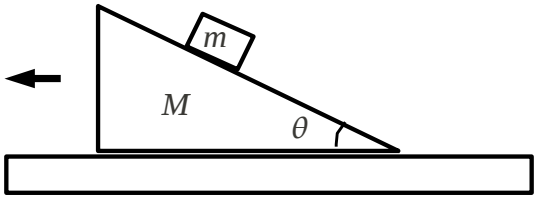
9) Find the equation of motion of the charge using Lagrange's method in an electrical circuit (having no resistance) with components of an inductor of inductuance L is connected to the capacitor of capacitance C. Initially the capacitor has charge Q. Compare the equation

of motion with usual methods in electrodynamics.

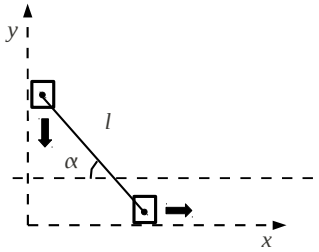
10) Find the Lagrangian and Lagrange's equation of motion for a spring mass system (simple harmonic oscillator)

11) Find the Lagrangian and Lagrange's equation of motion for a simple pendulum,

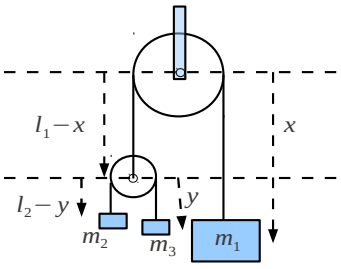
12) A block of mass m is held motionless on a frictionless plane of mass M and the angle of inclination of the plane is θ. The block rests on a frictionless horizontal surface. The block is now released. What is horizontal acceleration of the plane? Find the solution by both Newtonian and Lagrangian methods and compare.



13) Two blocks of equal mass are connected by a rigid bar of length l move with out friction along the path shown in the figure



14) Consider a double pulley system shown in the figure. Find the equation of motion.



15) The point of support of a simple pendulum of length b moves on a massless rim of radius a rotating with constant angular velocity ω. Obtain the expression for velocity and acceleration of the mass m.