IC 110: Engineering Mathematics; Practice sheet

---

1. Show that $\lim_{x\to 4}[x^2+1]$ does not exist, where [.] is the greatest integer function.
2. Show that the function
   $$f(x)=\begin{cases} x^2\cos(1/x), & x\neq 0 \\ 0, & x=0 \end{cases}$$
   is differentiable at x = 0 but $f'(x)$ is not continuous at x = 0.
3. Find the derivative of $f(x)=x|x|$.
4. Show that $\lim_{x\to 0} x[x+1]=0$, where [.] is the greatest integer function.
5. Discuss the continuity of the function $[x]+\sqrt{x-[x]}$ at x = 1.
6. Show that the function $f(x)=\begin{cases} \frac{|x|}{x}, & x\neq 0 \\ 1, & x=0 \end{cases}$ is bounded in $\Re$, though it is not continuous over any interval containing x = 0.
7. Show that the function $f(x)=\cos(x)$ is uniformly continuous on $[0,\infty)$.
8. Show that the function $f(x)=x^3$ is uniformly continuous on [0, 1] but not on $[0,\infty)$.
9. Find the derivative of $\sqrt{x+\sqrt{x+\sqrt{x+.....}}}$ at any point x.
10. Show that the function $f(x)=\begin{cases} (x-1)\tan(\pi x/2), & x\neq 1 \\ -1, & x=1 \end{cases}$ is not differentiable at x = 1.
11. Show that the function $f(x)=\begin{cases} \frac{xe^{1/x}}{1+e^{1/x}}, & x\neq 0 \\ 0, & x=0 \end{cases}$ is not differentiable at x = 0.
12. Find all the value of a and b, so that the function $f(x)=\begin{cases} \tan(x), & x<\pi \\ ax+b, & x\geq \pi \end{cases}$

    and its derivative are continuous at x = 0.
13. Find the approximate value of $(999)^{1/3}$ and $\sin 60^\circ 10'$ using differentials.
14. Using Lagrange mean value theorem, show that
    i) $e^x > 1+x,\ \ x>0$
    ii) $x<\sin^{-1}x<x/\sqrt{1-x^2},\ \ 0<x<1$
15. Obtain the Taylor's polynomial approximation of degree n to the function f(x) about the point x = a. Estimate the error in the given interval
    i) $f(x)=\sqrt{x}$, n=3, a =1, 1≤x≤1.5.
    ii) $f(x)=x^2e^{-x}$, n=4, a =1, 1≤x≤1.5.
16. Show that the following limits
    i) $\lim_{(x,y)\to(0,0)} \frac{x+\sqrt{y}}{x^2+y^2}$
    ii) $\tan^{-1}\frac{y}{x}$

    do not exist.

17. Show that the following function is continuous at the point (0,0)

$$f(x)=\begin{cases}\dfrac{x^2+y^2}{xy}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

18. Discuss the continuity of the following functions at the given points

i) $f(x)=\begin{cases}\dfrac{xy(x-y)}{(x^2+y^2)}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$

ii) $f(x)=\begin{cases}\dfrac{x^4y^4}{(x^2+y^4)^3}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$

iii) $f(x)=\begin{cases}\dfrac{x^2y^2}{(x^3+y^3)}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$

iv) $f(x)=\begin{cases}\dfrac{x^2+y^2}{xy}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$

19. Determine the following limits if they exist

i) $\displaystyle\lim_{(x,y)\to(0,0)}\frac{x}{x^2+y^2}$

ii) $\displaystyle\lim_{(x,y,z)\to(0,0)}\frac{xy+z}{x+y+z^2}$

iii) $\displaystyle\lim_{(x,y,z)\to(0,0)}\frac{x(x+y+z)}{x^2+y^2+z^2}$

20. Show that the function $f(x,y)=\begin{cases}(x+y)\sin(\dfrac{1}{x+y}) & (x+y)\neq 0\\ 0, & (x+y)=0\end{cases}$ is continuous at (0,0) but its partial derivatives f_x and f_y do not exist at (0,0).

21. Show that the function $f(x,y)=\begin{cases}\left(\dfrac{x^2+y^2}{|x|+|y|}\right) & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$ is continuous at (0,0) but its partial derivatives f_x and f_y do not exist at (0,0).

22. If $z=f(x,y), x=u\cos\alpha-v\sin\alpha, y=u\sin\alpha+v\cos\alpha$, where α is a constant, then

$$\left(\frac{\partial f}{\partial u}\right)^2+\left(\frac{\partial f}{\partial v}\right)^2=\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2.$$

23. Obtain the following using implicit differentiation

i) $\dfrac{dy}{dx}$ when $y^x+x^y=\alpha$

ii) $\left(\dfrac{\partial z}{\partial x}\right)_y$ and $\left(\dfrac{\partial z}{\partial y}\right)_x$, when $\cos xy+\cos yz+\cos zx=1$

24. A certain function $z=f(x,y)$ has values $f(2,3)=5, f_x(2,3)=3$ and $f_y(2,3)=7$. Find an approximate value of $f(1.98,3.01)$.