

# Projections on Auxiliary Planes

## Indian Institute of Technology Mandi

**Graphics for Design** 



## **Need for Auxiliary Planes**

- Sometimes none of the three principal orthographic views of an object show the different edges and faces of an object in their true sizes, since these edges and faces, are not parallel to any one of the three principal planes of projection.
- In order to show such edges and faces in their true sizes, it becomes necessary to set up *additional planes of projection* other than the three principal planes of projection in the positions which will show them in true sizes.

- If an edge or a face is to be shown in true size, it should be parallel to the plane of projection.
- Hence the additional planes are set up so as to be parallel to the edges and faces which should be shown in true sizes.
- These additional planes of projection which are set up to obtain the true sizes are called Auxiliary Planes.
- The views projected on these auxiliary planes are called Auxiliary Views.

The auxiliary view method may be applied

- To find the true length of a line.
- To project a line which is inclined to both HP and VP as a point.
- To project a plane surface or a lamina as a line.

### Types of auxiliary planes

Usually the auxiliary planes are set up such that they are *parallel* to the edge or face which is to be shown in true size and *perpendicular* to any one of the three principal planes of projection

Therefore, the selection of the auxiliary plane as to which of the principal planes of projection it should be perpendicular, obviously depends on the shape of the object whose edge or face that is to be shown in true size.

If the auxiliary plane selected is **perpendicular** to **HP** and **inclined** to **VP**, the views of the object projected on the **auxiliary plane** is called **auxiliary front view** and the auxiliary plane is called **auxiliary vertical plane** and denoted as **AVP**.

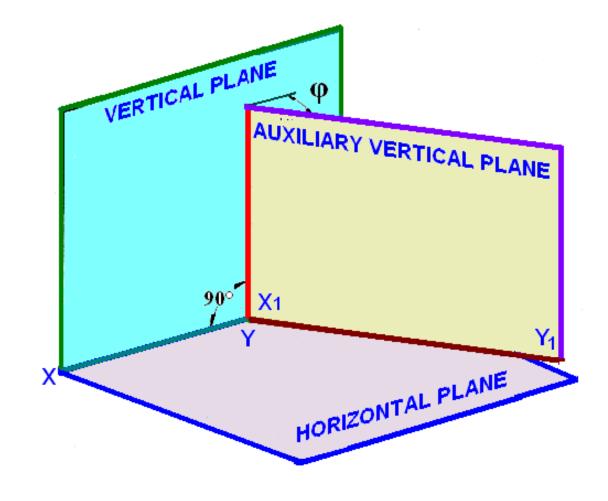
If the auxiliary plane is **perpendicular** to **VP** and **inclined** to **HP**, the view of the object projected on the auxiliary plane is called **auxiliary top view** and the **auxiliary plane** is called **auxiliary inclined plane** and denoted as **AIP**.

#### **Auxiliary Vertical Plane**

**AVP** is placed in the first quadrant with its surface perpendicular to **HP** and inclined at  $\phi$  to **VP**.

The object is to be placed in the space in between HP, VP and AVP. The AVP intersects HP along the X<sub>1</sub>Y<sub>1</sub> line.

The direction of sight to project the auxiliary **front** view will be normal to AVP.



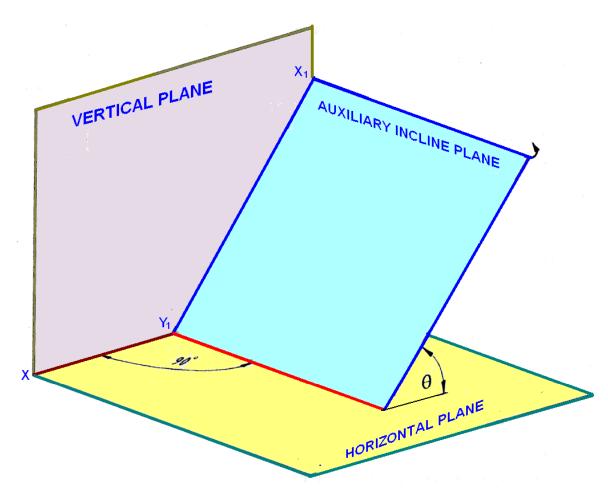
After obtaining the top view, front view and auxiliary front view on **HP**, **VP** and **AVP**, the **HP**, with the **AVP** being held perpendicular to it, is rotated so as to be in-plane with that of **VP**, and then the **AVP** is rotated about the  $X_1Y_1$  line so as to be in-plane with that of already rotated **HP** 

#### **Auxiliary Inclined Plane**

**AIP** is placed in the first quadrant with its surface perpendicular to **VP** and inclined at  $\theta$  to **HP**.

The object is to be placed in the space between **HP**, **VP** and **AIP**.

The AIP intersects the VP along the  $X_1Y_1$  line.



The direction of sight to project the auxiliary top view will be normal to the AIP.

After obtaining the top view, front view and auxiliary top view on *HP*, *VP* and *AIP*, *HP* is rotated about the **XY** line independently (detaching the AIP from HP).

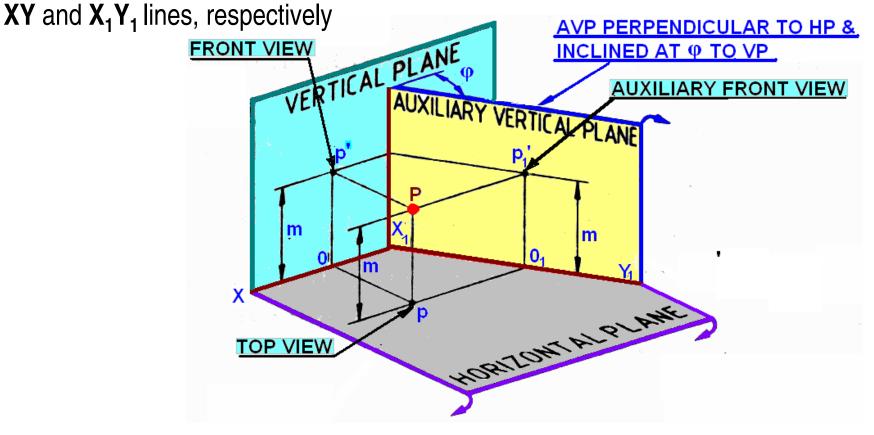
The AIP is then rotated about  $X_1Y_1$  line independently so as to be in-plane with that of **VP**.

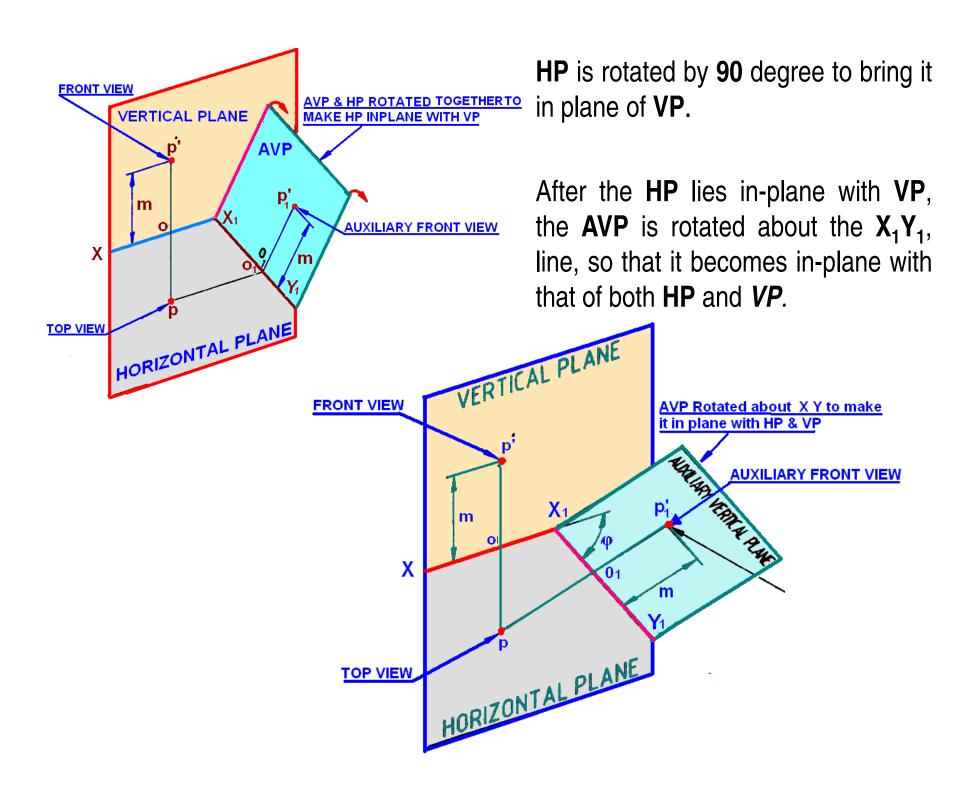
## Projection of Points on Auxiliary Planes Projection on AVP

Point **P** is situated in the first quadrant at a height **m** above **HP**. An auxiliary vertical plane AVP is set up perpendicular to **HP** and inclined at  $\phi$  to VP. The point **P** is projected on VP, HP and AVP.

p' is the projection on VP, p is the projection on HP and  $P_1'$  is the projection on AVP.

Since point is at a height  $\mathbf{m}$  above  $\mathbf{HP}$ , both  $\mathbf{p'}$  and  $\mathbf{p_1'}$  are at a height  $\mathbf{m}$  above the

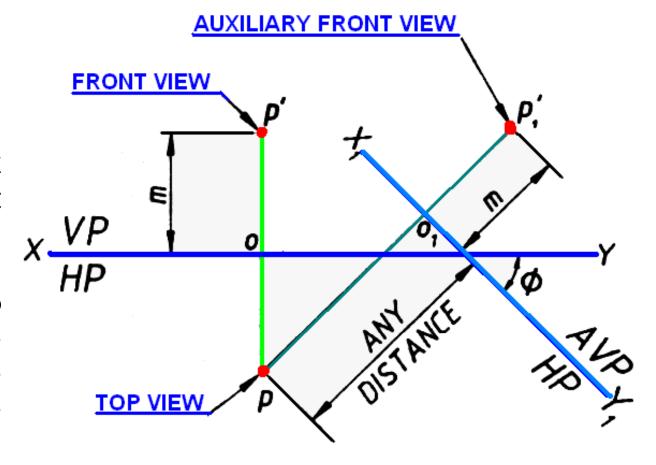




## Orthographic projections

**p** and **p**', the top and front views of the point **P**.

Since **AVP** is inclined at  $\phi$  to **VP**, draw the  $X_1Y_1$  line inclined at  $\phi$  to the **XY** line at any convenient distance from **p**.



Since point **P** is at a height **m** above **HP**, the auxiliary front view  $\mathbf{p_1}'$  will also be at a height **m** above the  $\mathbf{X_1Y_1}$  line.

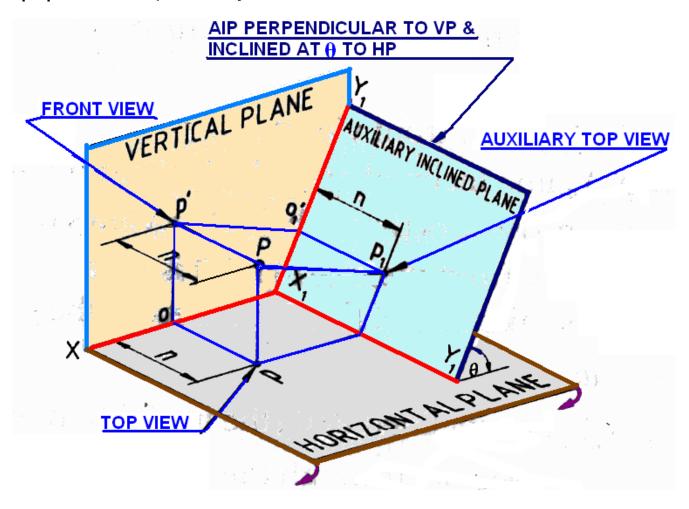
Therefore, mark  $P_1$  by measuring  $o_1p_1$ '=op' = m on the projector drawn from p perpendicular to the  $X_1Y_1$  line.

#### **Projection on AIP**

Point **P** is situated in first quadrant at a distance **n** from **VP**. An auxiliary plane AIP is set up perpendicular to **VP** and inclined at  $\theta$  to HP. The point **P** is projected on VP, HP and AIP.

p' is the projection on VP, p is the projection on HP and  $P_1$  is the projection on AIP.

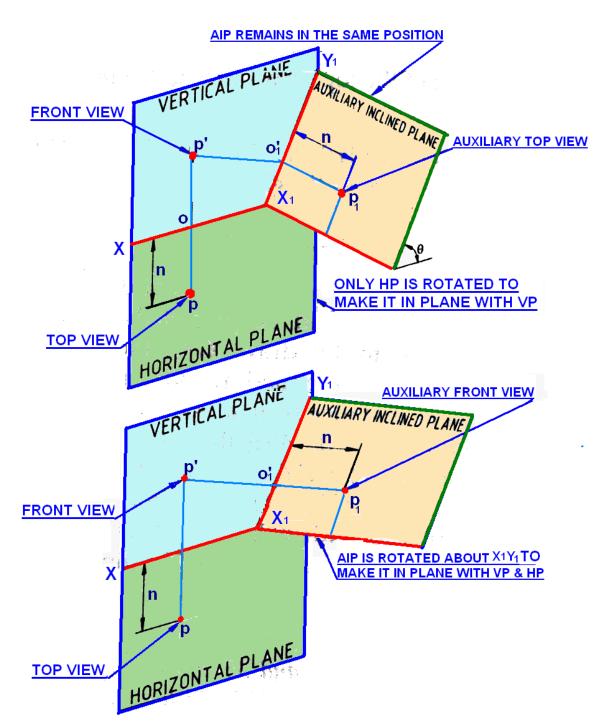
Since the point is at a distance  $\mathbf{n}$  from  $\mathbf{VP}$ , both  $\mathbf{p}$  and  $\mathbf{p_1}$  are at a distance  $\mathbf{n}$  above the  $\mathbf{XY}$  and  $\mathbf{X_1Y_1}$  lines, respectively



**HP** is rotated by 90 degree about **XY** line to bring it in plane with **VP**.

After the **HP** lies in-plane with **VP**, the AIP is rotated about the  $X_1Y_1$ , line, so that it becomes in-plane with that of both **HP** and **VP**.

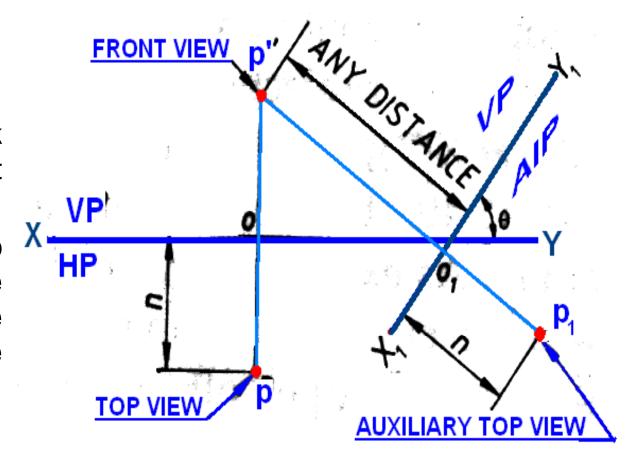
**p** and **p'** lie on a vertical projector perpendicular to the **XY** line, and **p'** and **p**<sub>1</sub> lie on a projector perpendicular to the  $X_1Y_1$  line which it self is inclined at  $\theta$  to **XY** line.



## Orthographic projections

**p** and **p**', the top and front views of the point **P**.

Since **AIP** is inclined at  $\theta$  to **HP**, draw the  $X_1Y_1$  line inclined at  $\theta$  to the **XY** line at any convenient distance from **p**'.



Since point **P** is at a distance **n** infront of **VP**, the auxiliary top view  $\mathbf{p}_1$  will also be at a distance **n** from the  $\mathbf{X}_1\mathbf{Y}_1$  line.

Therefore, mark  $P_1$  by measuring  $o_1p_1=op=n$  on the projector drawn from p' perpendicular to the  $X_1Y_1$  line.

## Step by step procedure to draw auxiliary views

Auxiliary front view	Auxiliary top view
Draw the top and front views.	Draw the top and front views
<ul> <li>Draw X<sub>1</sub>Y<sub>1</sub> line inclined at φ (the inclination of AVP with VP) to the XY line.</li> </ul>	<ul> <li>Draw X<sub>1</sub>Y<sub>1</sub> line inclined at θ     (the inclination of AIP with HP)     to XY line.</li> </ul>
<ul> <li>Draw the projectors through the top views of the points perpendicular to the X<sub>1</sub>Y<sub>1</sub> line.</li> </ul>	<ul> <li>Draw the projectors through the front views of the points perpendicular to the X<sub>1</sub>Y<sub>1</sub> line.</li> </ul>
<ul> <li>The auxiliary front view of a point is obtained by stepping off a distance from the X<sub>1</sub>Y<sub>1</sub> line equal to the distance of the front view of the given point from the XY line.</li> </ul>	<ul> <li>The auxiliary top view of a point is obtained by stepping off a distance from X<sub>1</sub>Y<sub>1</sub> line equal to to the distance of the top view of the given point from the XY line</li> </ul>

## Projection of lines on auxiliary planes

The problems on projection of lines inclined to both the planes may also be solved by the auxiliary plane methods.

In this method, the line is always placed parallel to both **HP** and **VP**, and then two auxiliary planes are set up — one auxiliary plane will be perpendicular to **VP** and inclined at  $\theta$  to **HP**, i.e., **AIP**, and the other will be perpendicular to **HP** and inclined at  $\phi$  (true inclination) or  $\beta$  (apparent inclination) to **VP**.

#### Problem 1:

Draw the projections of a line **80 mm** long inclined at **30°** to **HP** and its top view appears to be inclined at **60°** to **VP**. One of the ends of the line is **45** mm above **HP** and **60** mm infront of **VP**. Draw its projections by

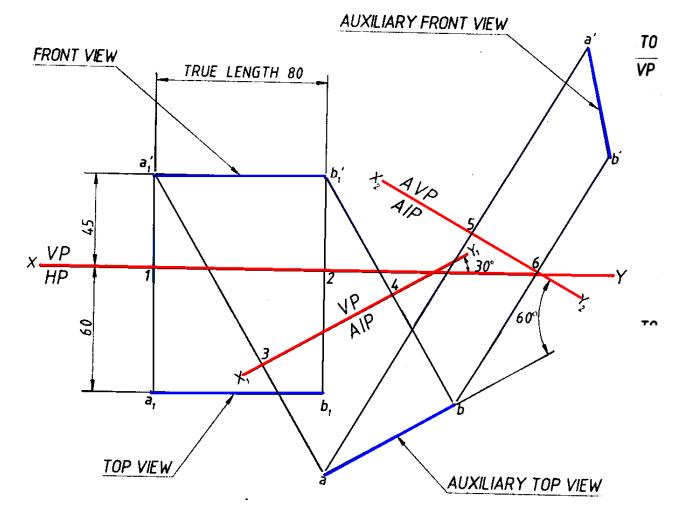
auxiliary plane method

#### Solution

Draw the top and front views of one of the ends, say A, 45 mm above HP and 60 mm infront of VP.

Assume that the line is parallel to both HP and VP and draw its top and front views.

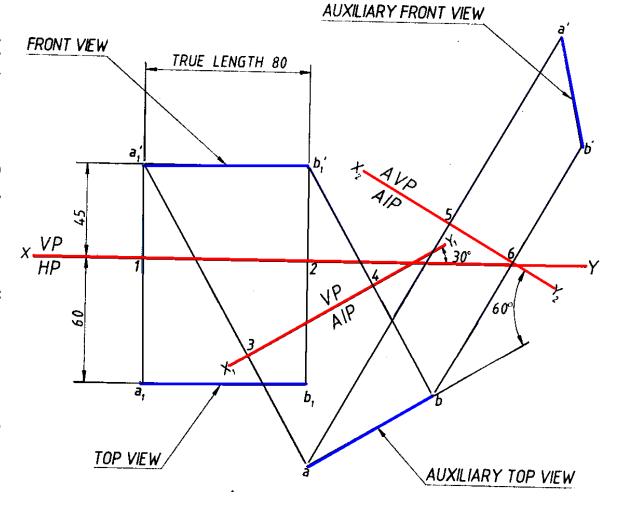
Since the line is to be inclined at 30° to HP, set up an AIP inclined at 30° to HP and perpendicular to VP.



Draw  $X_1Y_1$  line inclined at  $30^0$  to XY line at any convenient distance from it.

To project an auxiliary top view on AIP, draw projections from  $a_1$ ' and  $b_1$ ' perpendicular to  $X_1Y_1$  line, and on them step off  $1a_1=3a$  and  $2b_1=4b$  from the  $X_1Y_1$  line.

Connect **ab** which will be the auxiliary top view.



Since the top view of the line appears inclined to VP at  $60^{\circ}$ , draw the  $X_2Y_2$  line inclined at  $60^{\circ}$  to the auxiliary top view **ab** at any convenient distance from it. Draw the projections from **a** and **b** perpendicular to  $X_2Y_2$  and on them step off  $5a' = 3a_1'$  and  $6b' = 4b_1'$ . Connect **a'b'** which will be the auxiliary front view.

#### **Problem 2:**

A line AB 60 mm long has one of its extremities 60 mm infront of VP and 45 mm above HP. The line is inclined at 30° to HP and 45° to VP. Draw the projections of the line by the auxiliary plane method.

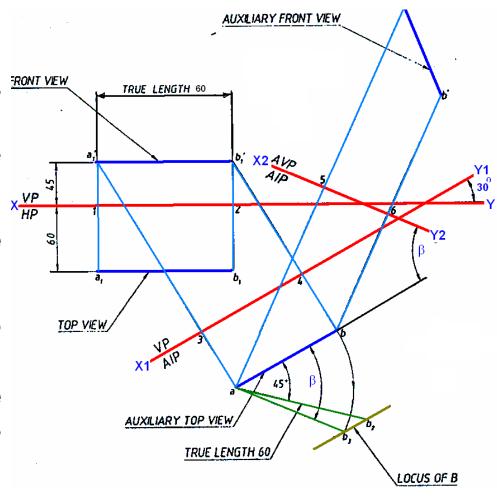
### **Solution**

Let A be one of the extremities of the line AB at distance 60 mm infront of VP and 45 mm above HP.

Mark  $a_1$  and  $a_1$ ' the top and the front views of the extremity A.

Initially the line is assumed to be parallel to HP and VP.

 $a_1b_1$  and  $a_1'b_1'$  are the projections of the line in this position.

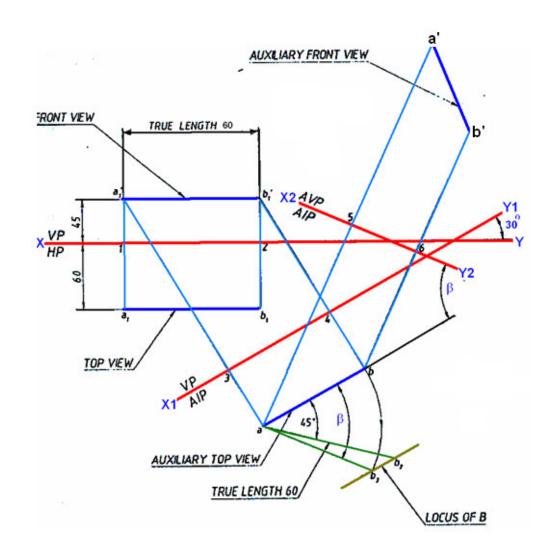


Then instead of rotating the line so as to make it inclined to both the planes, an AIP is set up at an angle  $\theta$ , which the line is supposed to make with HP and the auxiliary top view is projected on it.

To draw the Auxiliary Top View on AIP

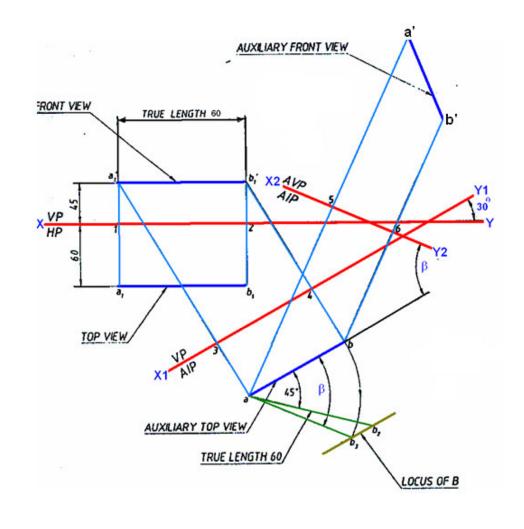
Draw  $X_1Y_1$  line inclined at  $\theta = 30^{\circ}$  to the XY line. Mark AIP and VP. Project the auxiliary top view ab The projections ab on the AIP and  $a_1'b_1'$  on VP are the auxiliary view and the front view of the line when it is inclined at  $\theta$  to HP and parallel to VP.

Since the line is inclined at true inclination  $\phi$  to VP, to project the auxiliary front view an AVP inclined at  $\phi$  to VP should be setup.



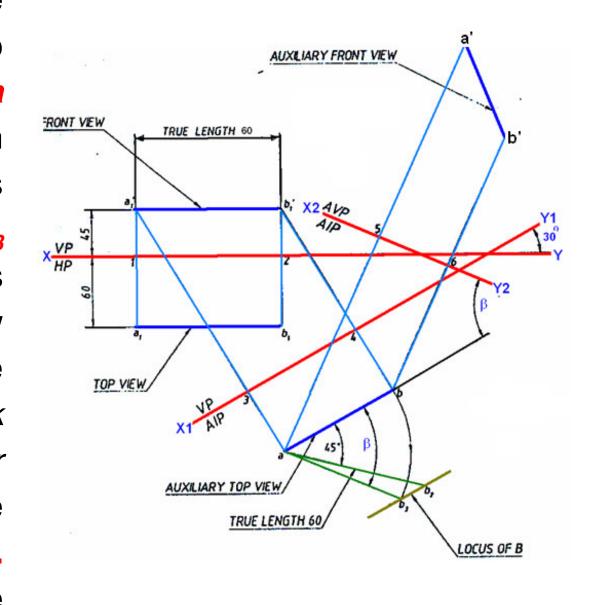
#### To draw the Auxiliary F.V. on AVP

Already the line is inclined at  $\theta$  to AIP and parallel to VP. If the line is to be inclined at  $\phi$  to VP, an AVP inclined at  $\phi$  to the given line should be setup. But we know that when a line is inclined to both the planes, they will not be inclined at true inclinations to the XY line, instead they will be at apparent inclinations with the XY line. Therefore  $X_2Y_2$ , the line of intersection of AIP and AVP cannot be drawn directly at  $\phi$  to ab.



The apparent inclination  $\beta$  of ab with the  $X_2Y_2$  line should be found out. To find  $\beta$ , through a draw  $ab_2$  equal to 60 mm, the true length of AB inclined at  $\phi = 45^{\circ}$  to ab.

Through  $b_2$ , draw the locus of **B** parallel to X<sub>1</sub>Y<sub>1</sub> line. With center a and radius ab strike an arc to intersect the locus of **B** at  $b_3$ . Connect  $ab_3$ and its measure inclination  $\beta$  with **ab**. Now draw the  $X_2Y_2$  line inclined at  $\beta$  to ab. Mark AVP and AIP on either side of X<sub>2</sub>Y<sub>2</sub> Project the auxiliary front view a'b'. ab and a'b' are the required projections.



## **Shortest distance between two lines**

Two lines may be *parallel*, or *intersecting*, or *non-parallel* and *non-intersecting*.

When the lines are intersecting, the point of intersection lies on both the lines and hence these lines have no shortest distance between them.

Non-parallel and non-intersecting lines are called **Skew Lines**.

The parallel lines and the skew lines have a *shortest distance* between them.

The shortest distance between the two lines is the shortest perpendicular drawn between the two lines.

## Shortest distance between two parallel lines

The shortest distance between two parallel lines is equal to the length of the perpendicular drawn between them.

If its true length is to be measured, then the two given parallel lines should be shown in their point views.

If the point views of the lines are required, then first they have to be shown in their true lengths in one of the orthographic views.

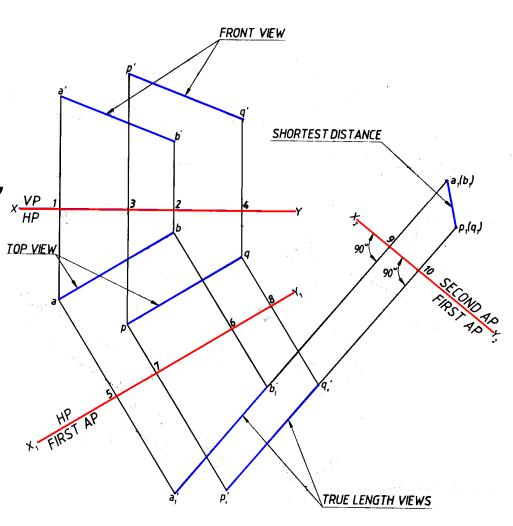
If none of the orthographic views show the given lines in their true lengths, an auxiliary plane parallel to the two given lines should be set up to project them in their true lengths on it.

Even the auxiliary view which shows the lines in their true lengths may not show the perpendicular distance between them in true length. Hence another auxiliary plane perpendicular to the two given lines should be set up. Then the lines appear as points on this auxiliary plane and the distance between these point views will be the shortest distance between them.

## Shortest distance between two parallel lines

Projections of a pair of parallel lines **AB** and **PQ** are shown. **ab** and **a'b'**are the top and front views of the line **AB**. **pq** and **p'q'** are the top and front views of the line **PQ**.

Since the top and front views of the lines are inclined to the **XY** line, neither the top view nor the front view show the lines in their true lengths.



To show these lines in their true lengths, an auxiliary plane, parallel to the two given lines, should be set up parallel to the projections of the lines either in the top view or front view.

In this case the auxiliary plane is set up so as to be parallel to the two given lines in top view

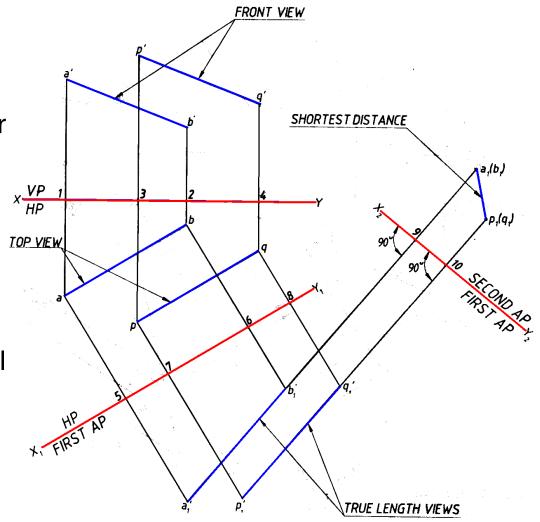
Draw the  $X_1Y_1$  line parallel to **ab** and **pq** at any convenient distance from them.

Through the points  $\mathbf{a}$ ,  $\mathbf{b}$ ,  $\mathbf{p}$  and  $\mathbf{q}$ , draw projector lines perpendicular to  $\mathbf{X}_1\mathbf{Y}_1$  line.

Measure  $5a_1$ '= $1a_1$ ' along the projector drawn through a from the  $X_1Y_1$  line, and  $6b_1$ '=2b' along the projector drawn through b from the  $X_1Y_1$  line.

Connect  $\mathbf{a_1'b_1'}$  which will be equal to the true length of the line  $\mathbf{AB}$ .

Similarly by measuring  $7p_{1'} = 3p'$  and  $8q_1' = 4q'$  obtain  $p_1'q_1'$  the true length view of the line PQ.



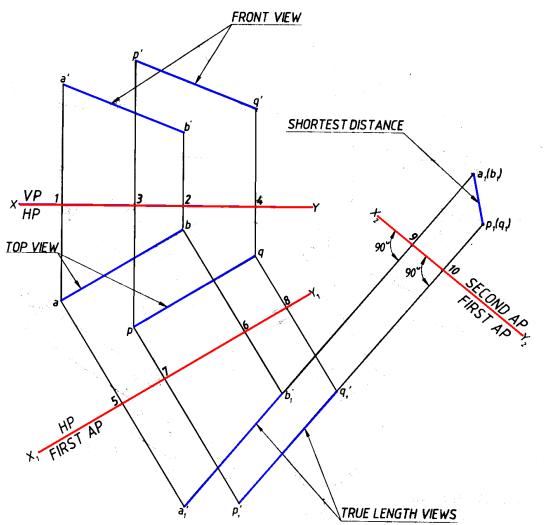
The line **AB** and **PQ** are shown in their true lengths, and now an another auxiliary plane perpendicular to the two given lines should be set up to project their point views on it.

Draw the line  $X_2Y_2$  perpendicular to  $a_1b_1$  and  $p_1q_1$  at any convenient distance from them.

Produce  $a_1'b_1'$  and  $p_1'q_1'$ .

Measure  $a5 = b6 = 9a_1$  along  $a_1'b_1'$  produced from  $X_2Y_2$ . Similarly obtain the point, view p1(q1) by measuring p1(10)=p7 = q8.

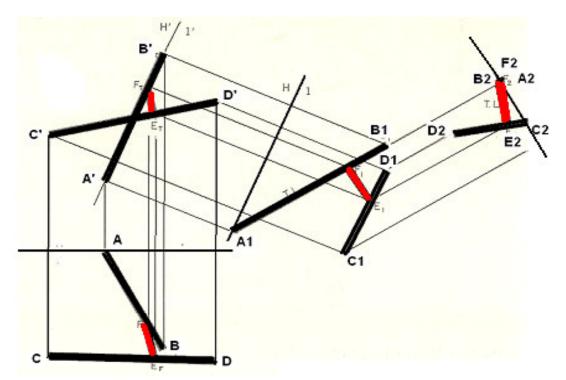
Connect *p1a1* the required shortest distance between the lines **AB** and **PQ** in its true length



### Shortest distance between two skew lines

Projections of two skew lines AB and CD are shown as A'B', C'D' and AB and CD.

Determine the shortest distance EF between the line segments



First an Auxiliary  $A_1B_1$  is made showing the true length of AB. A second auxiliary view showing the point view of AB is projected. For this draw the reference line normal to  $A_1B_1$  and draw the projectors  $C_2$   $D_2$  (of  $C_1$  and  $D_1$ ).

The shortest distance  $F_2E_2$  can be established perpendicular to CD. To project FE back to the Front and Top Views, FE is first projected in first auxiliary plane by first projecting point E, which is on CD, from the second to the first auxiliary view and then back to the front and top views.