TRANSIENTS

Overview

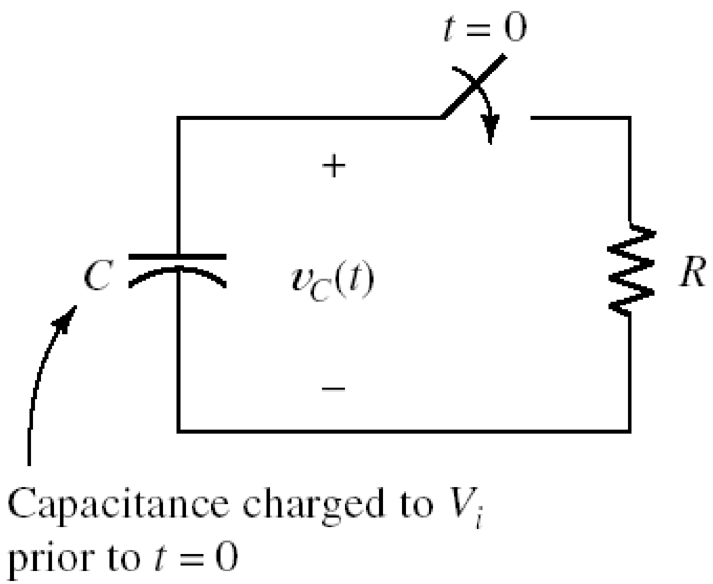
1. Solve first-order *RC* or *RL* circuits.
2. Understand the concepts of transient response and steady-state response.
3. Relate the transient response of first-order circuits to the time constant.
4. Solve *RLC* circuits in dc steady-state conditions.
5. Solve second-order circuits.
6. Relate the step response of a second-order system to its natural frequency and damping ratio.

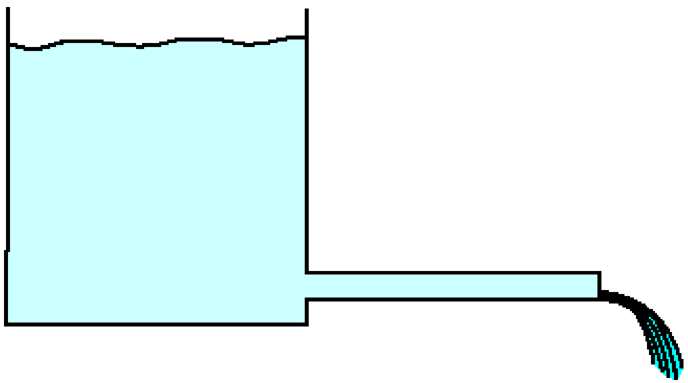
Transients

The time-varying currents and voltages resulting from the sudden application of sources, usually due to switching, are called **transients**. By writing circuit equations, we obtain integrodifferential equations.

First Order RC Circuit

- Capacitor charged to initial voltage V_i.
- At t=0, switch closes and capacitor discharges.





Discharge of a Capacitance through a Resistance

$$C\frac{dv_C(t)}{dt}+\frac{v_C(t)}{R}=0$$

$$v_C(t)=Ke^{st}$$

$$RC\frac{dv_C(t)}{dt}+v_C(t)=0$$

$$RCKse^{st}+Ke^{st}=0$$

Capacitance Discharge (Contd.)

$$s = \frac{-1}{RC}$$

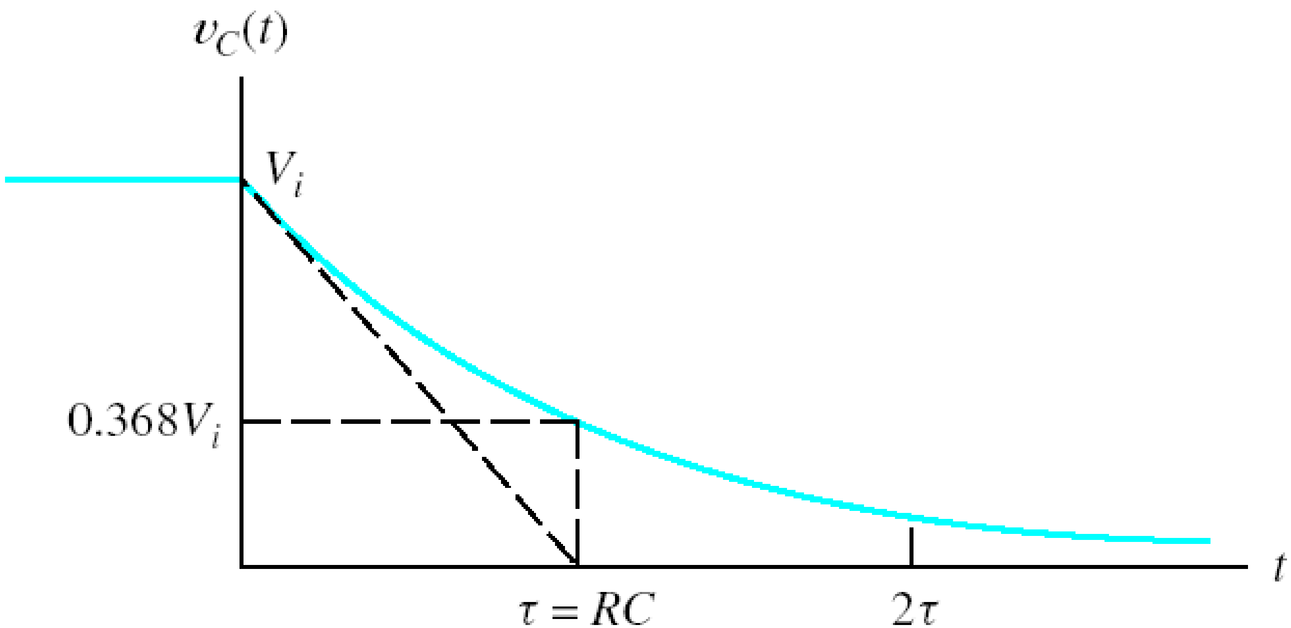
$$v_C(0+) = V_i$$

$$v_C(t) = Ke^{-t/RC}$$

$$v_C(t) = V_i e^{-t/RC}$$

Capacitance Discharge (Contd.)

- Voltage vs. time



Time Constant

- The time interval $\tau = RC$ is called the time constant of the circuit.

$$v_C(t) = V_i e^{-t/\tau}$$

- After 5 time constants, voltage on capacitor is negligible.

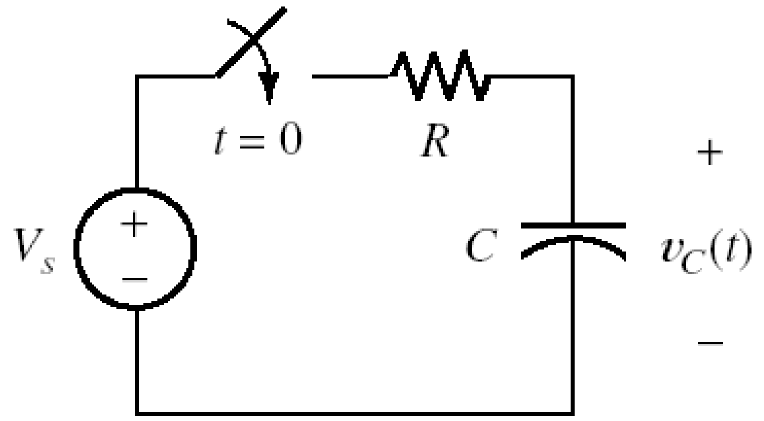
Application

- Garage Door Opener

- Light on for 30 seconds
- Other timing applications

Charging a Capacitance

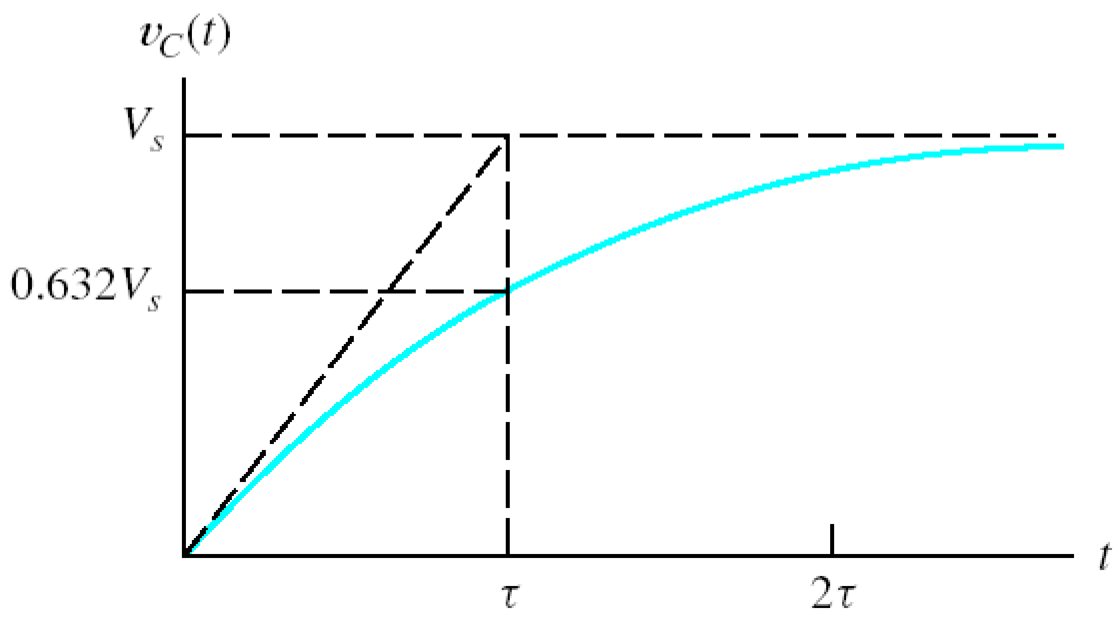
- Switch closes at t=0.
- Steady-state and transient response



$$C\frac{dv_C(t)}{dt}+\frac{v_C(t)-V_s}{R}=0$$

$$v_C(t)=V_s-V_s e^{-t/RC}$$

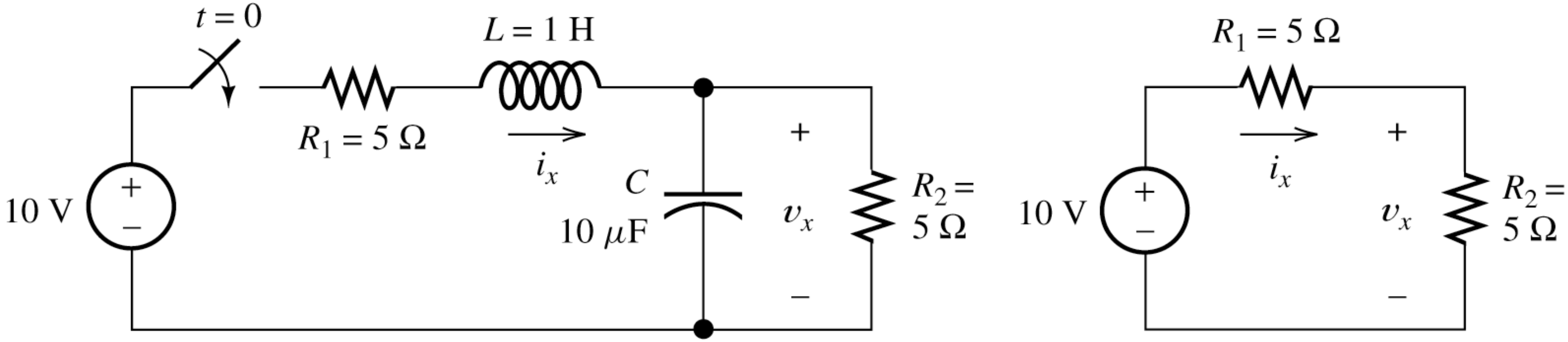
Charging Transient



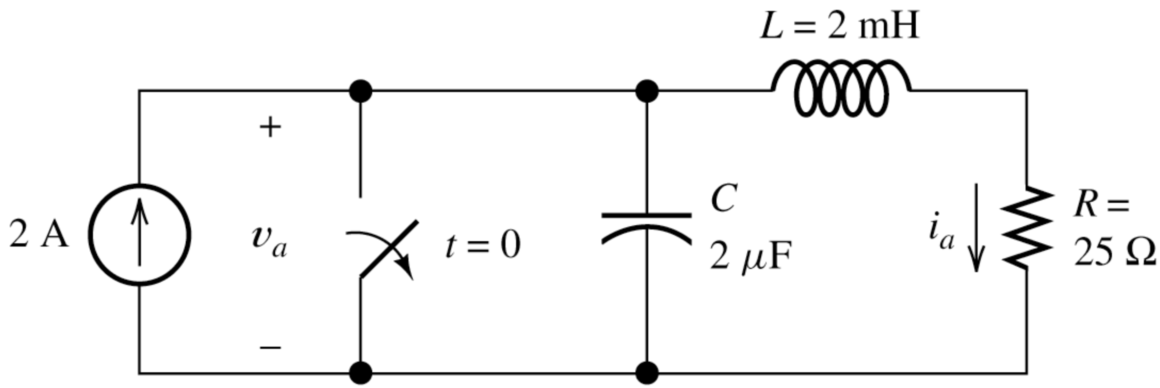
DC Steady State

- The steps in determining the forced response for RLC circuits with DC sources are:
  1. Replace capacitances with open circuits.
  2. Replace inductances with short circuits.
  3. Solve the remaining circuit for steady-state current and voltages.

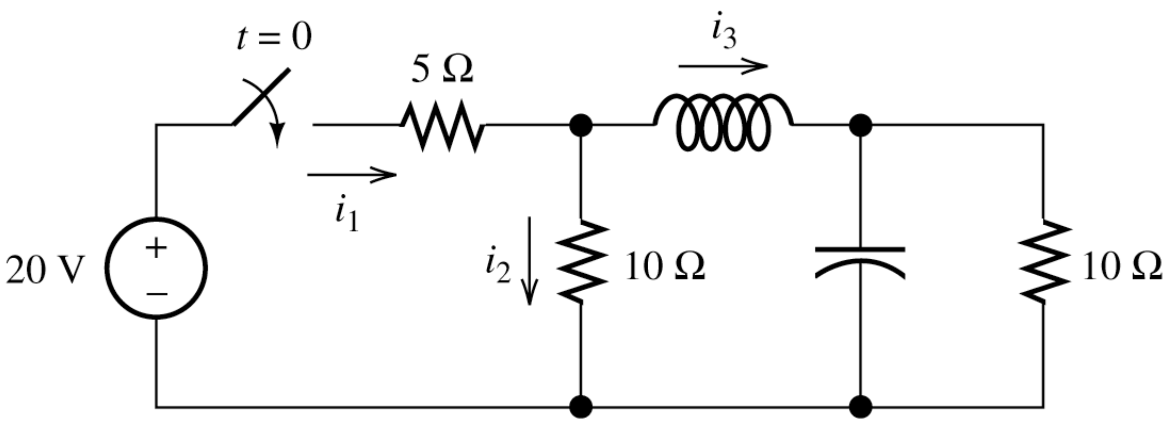
Example Exercise



Example Exercise



(a)

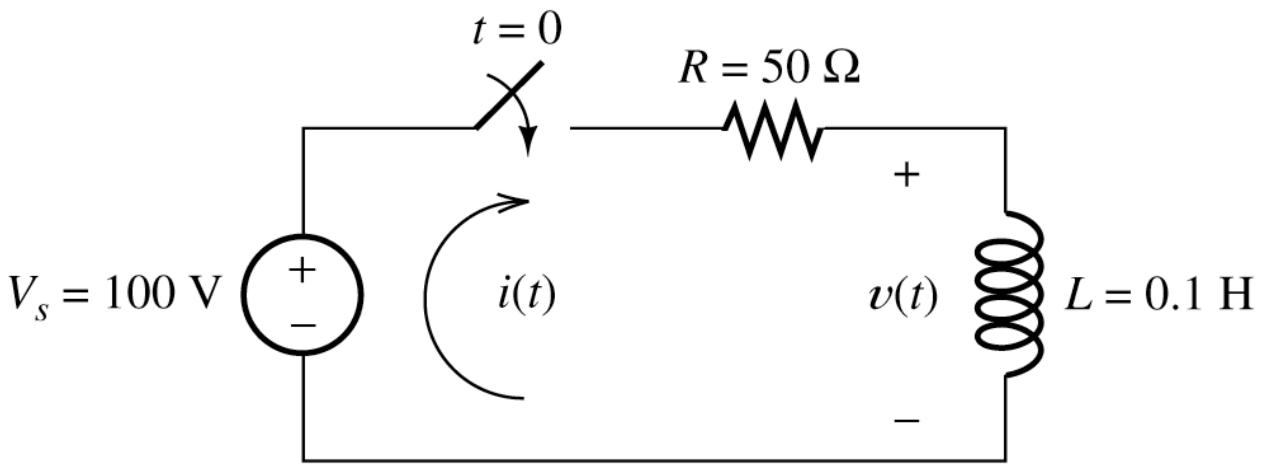


(b)

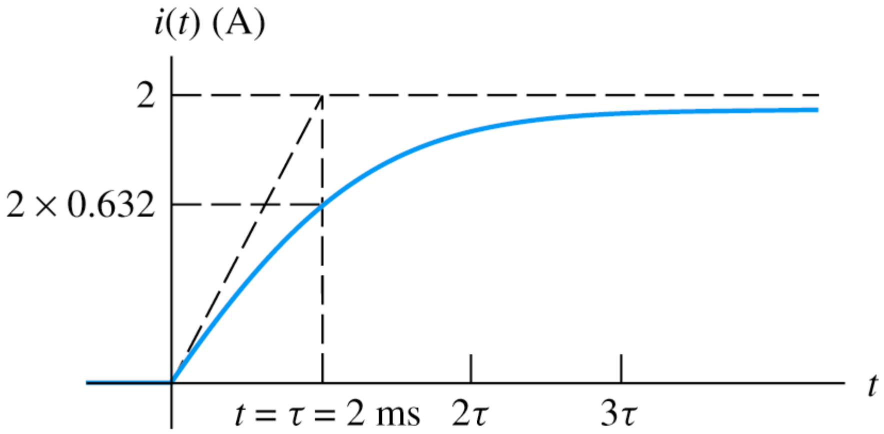
RL Circuits

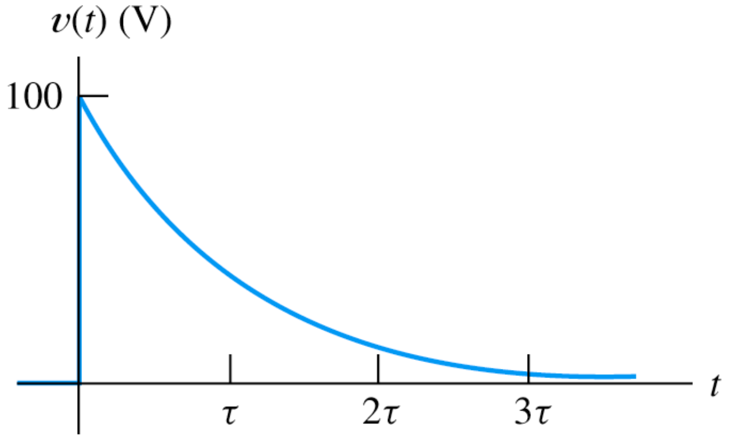
- The steps involved in solving simple circuits containing dc sources, resistances, and one energy-storage element (inductance or capacitance) are:
  1. Apply Kirchhoff's current and voltage laws to write the circuit equation.
  2. If the equation contains integrals, differentiate each term in the equation to produce a pure differential equation.
  3. Assume a solution of the form $K_1 + K_2e^{st}$.
  4. Substitute the solution into the differential equation to determine the values of K_1 and s . (Alternatively, we can determine K_1 by solving the circuit in steady state)
  5. Use the initial conditions to determine the value of K_2.
  6. Write the final solution.

Example Exercise

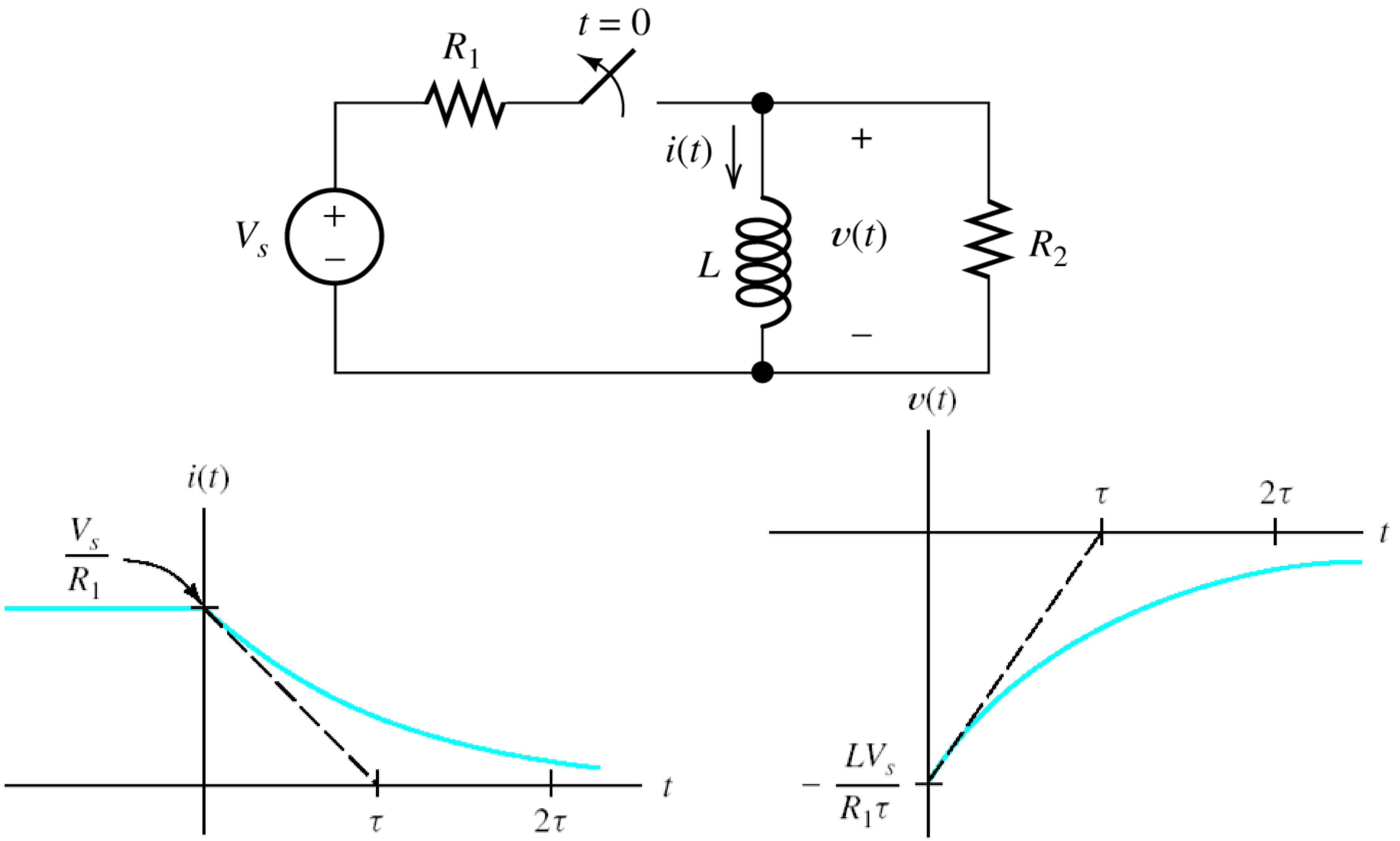


$$i(t) = 2 + K_2 e^{-tR/L}$$

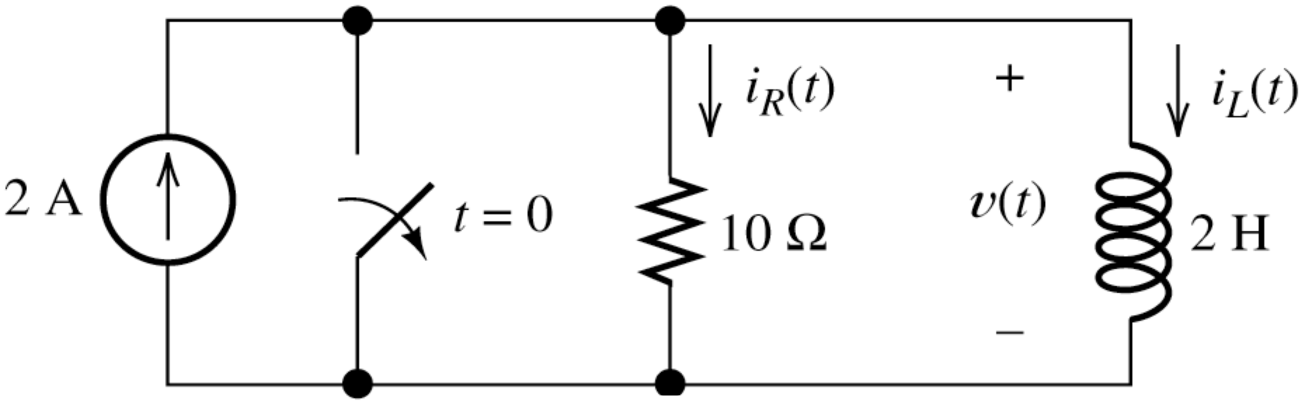


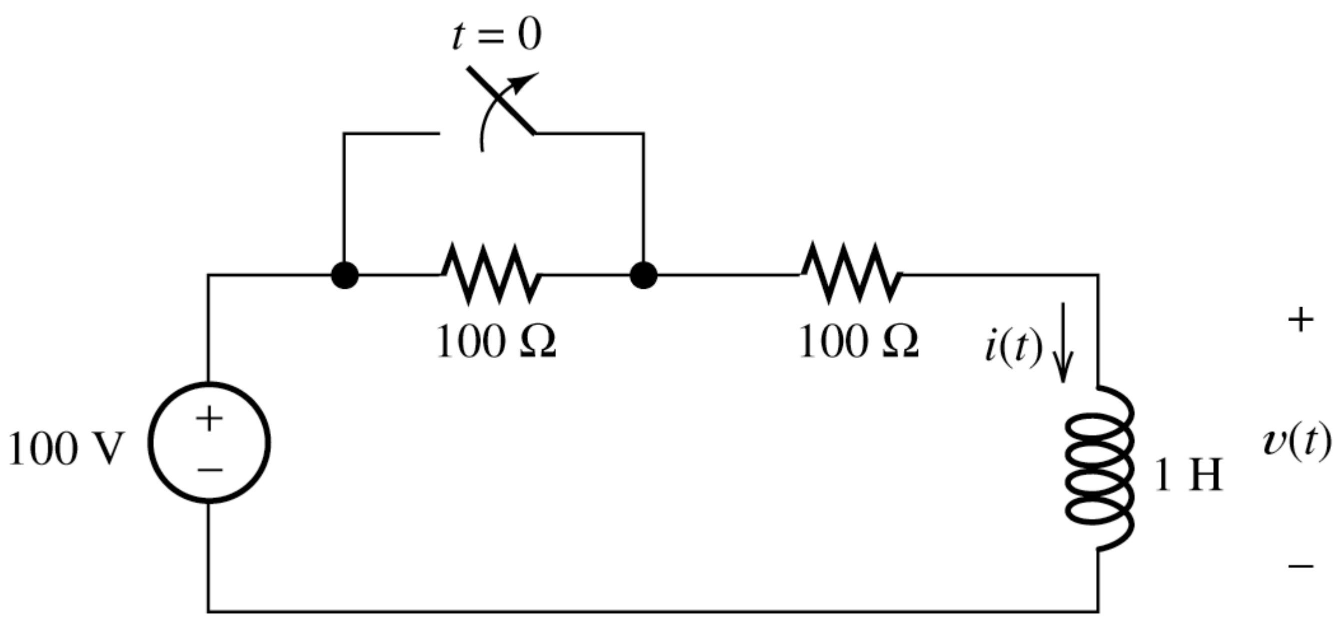


Example Exercise

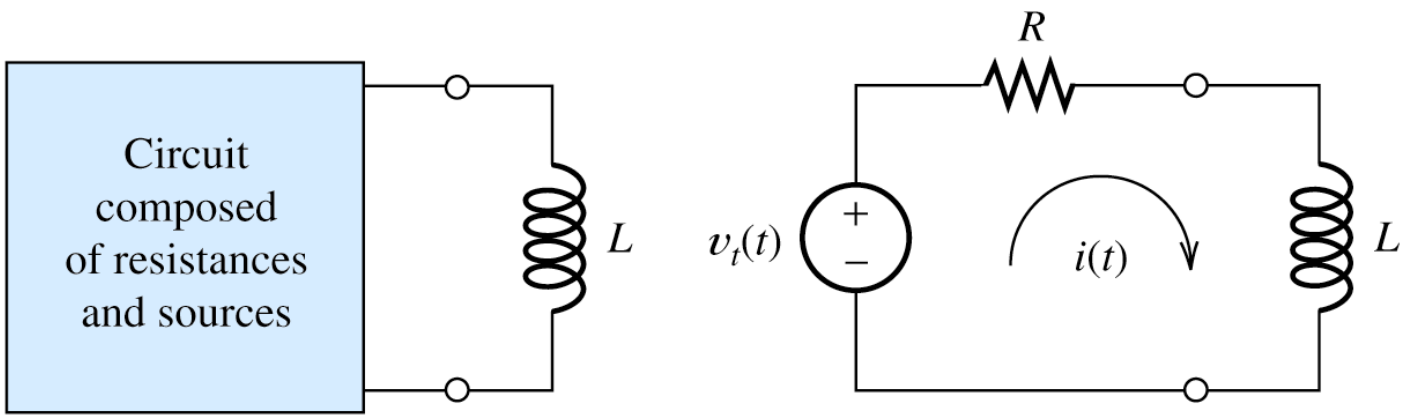


Example Exercises





RC and RL Circuits With General Sources



Example

$$\frac{L}{R}\frac{di(t)}{dt} + i(t) = \frac{v_t(t)}{R}$$

In general

$$\tau\frac{dx(t)}{dt} + x(t) = f(t)$$

where f(t) is the forcing function. Setting the forcing function to zero yields the homogeneous equation.

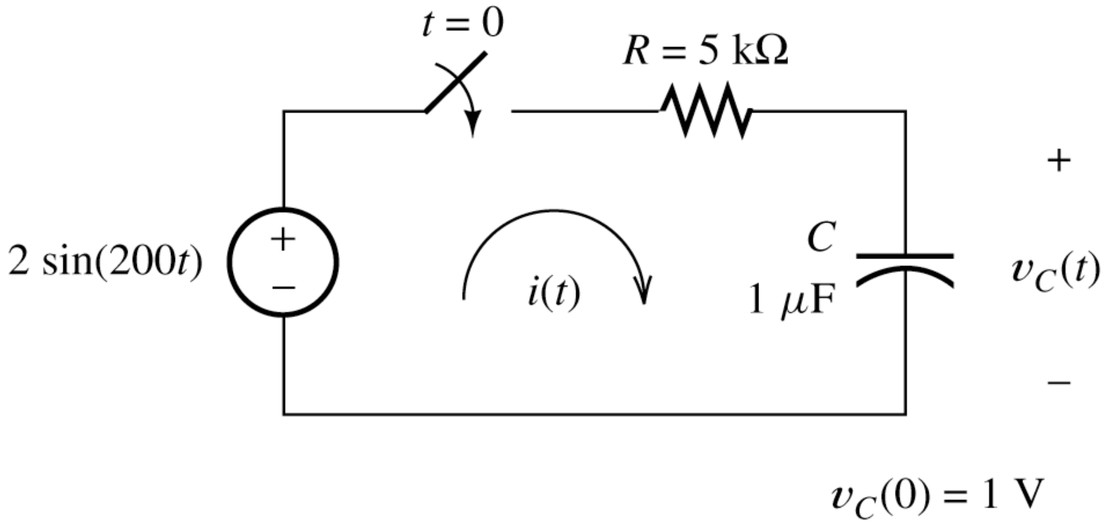
RC and RL Circuits With General Sources

- The general solution consists of two parts:
  - Particular Solution: (also called the forced response) is any expression that satisfies the equation.
  - Complementary Solution: (also called the natural response) is one that satisfies the initial conditions
    - Obtained by solving the homogeneous equation (obtained by setting the forcing function to zero)

Step-by-Step Solution

1. Write the circuit equation and reduce it to a first-order differential equation.
2. Find a particular solution. The details of this step depend on the form of the forcing function.
3. Obtain the complete solution by adding the particular solution to the complementary solution, which contains the arbitrary constant K.
4. Use initial conditions to find the value of K.

Example Exercise



Solution

By KVL

$$Ri(t)+\frac{1}{C}\int_0^t i(t)\,dt+v_C(0)-2\sin(200t)=0$$

Differentiating

$$R\frac{di(t)}{dt}+\frac{1}{C}i(t)=400\cos(200t)$$

$$i_p(t)=A\cos(200t)+B\sin(200t)$$

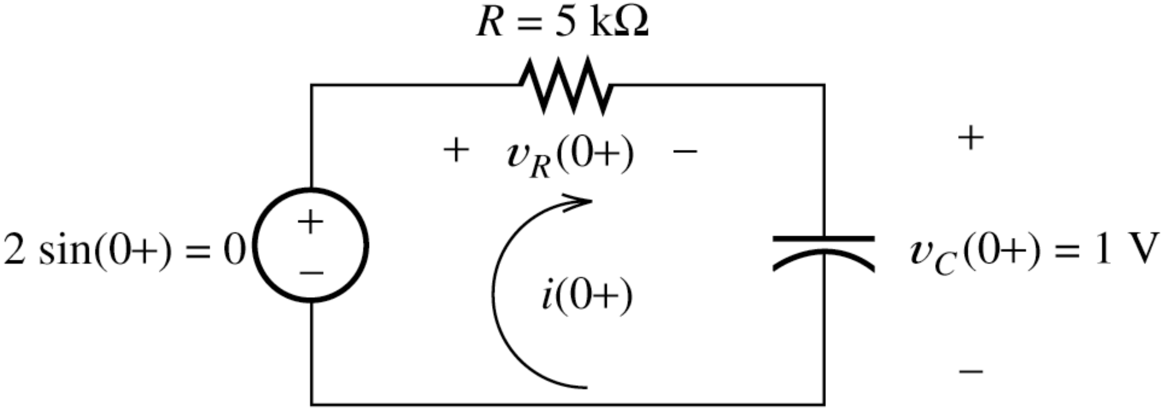
$$=200\cos(200t)+200\sin(200t)\mu A$$

$$i_c(t)=Ke^{-t/RC}$$

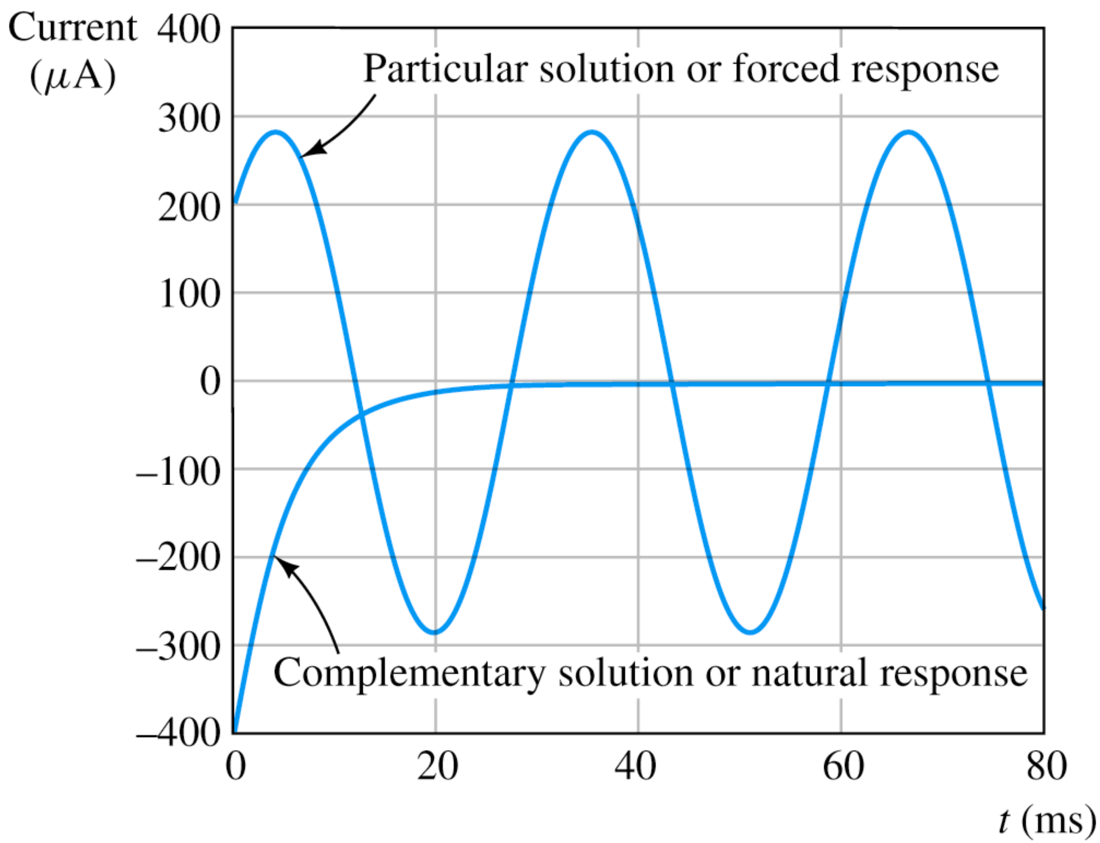
$$=-400e^{-t/RC}\mu A$$

Initial Condition

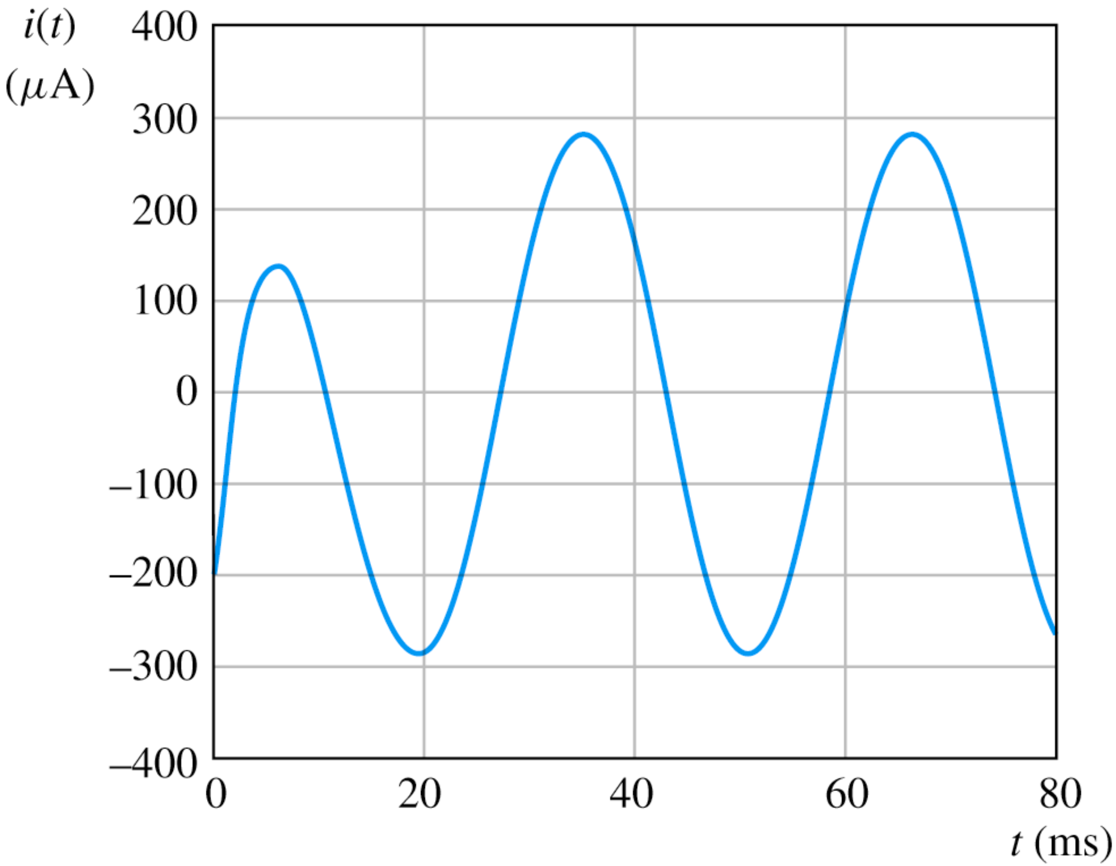
- Immediately after switch close



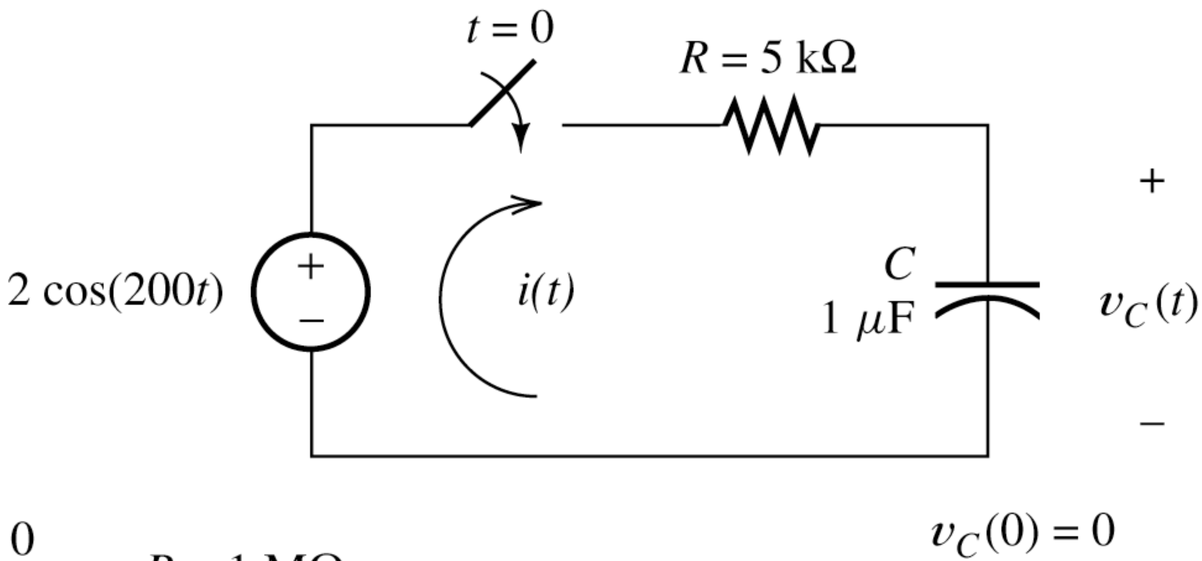
Particular and Complementary Solutions

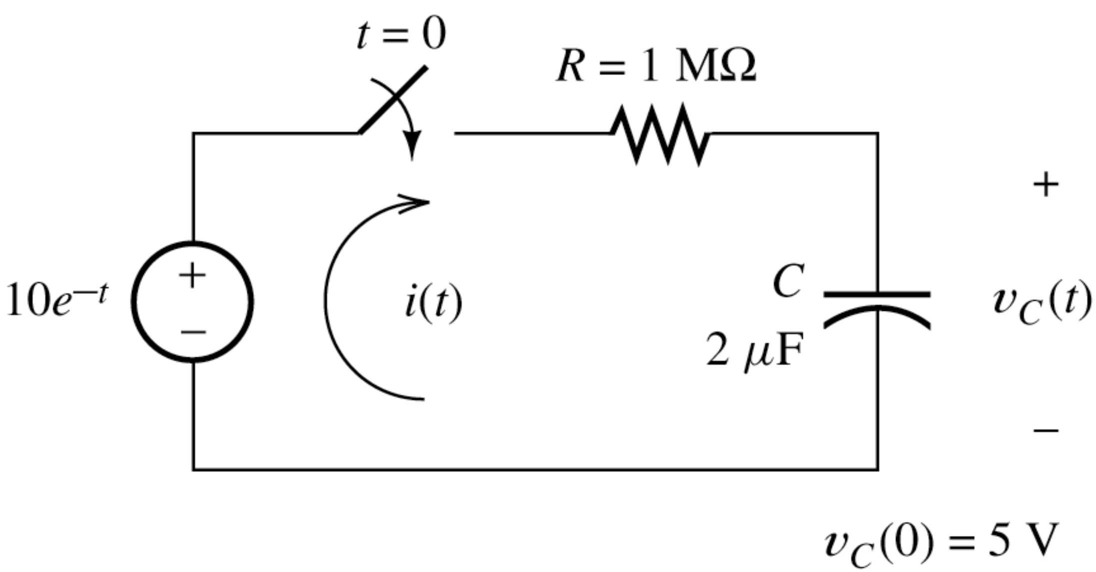


Complete Solution



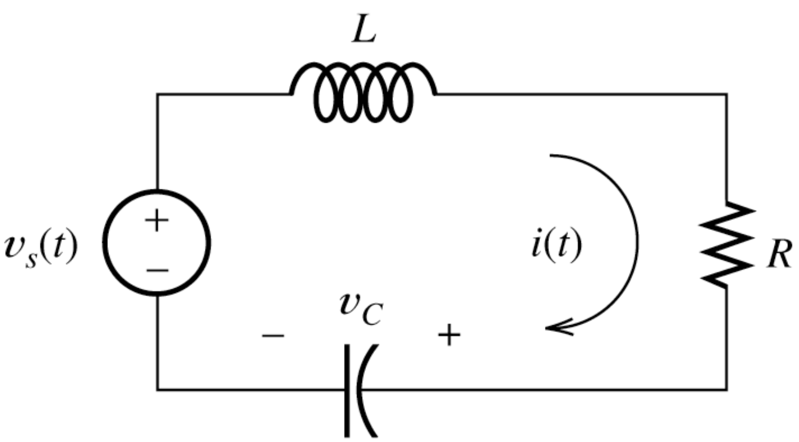
Example Exercises



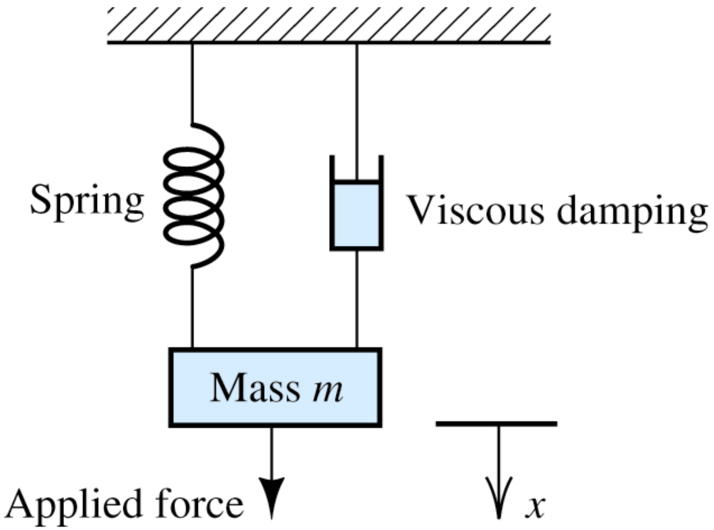


Second Order Circuits

- Series RLC Circuit and its mechanical analog



(a) Electrical circuit



(b) Mechanical analog

Second-order Circuits

$$L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int_0^t i(t)dt + v_C(0) = v_s(t)$$

Define damping coefficient $\alpha = \frac{R}{2L}$

and undamped resonant frequency $\omega_0 = \frac{1}{\sqrt{LC}}$

and forcing function $f(t) = \frac{1}{L}\frac{dv_s(t)}{dt}$

Second-order Circuits (Contd.)

$$\frac{d^2 i(t)}{dt^2} + 2\alpha \frac{di(t)}{dt} + \omega_0^2 i(t) = f(t)$$

- Particular Solution/Forced Response
  - Replace inductance by short circuit and capacitance by open circuit and then solve.
- Complementary Solution
  - Use trial solution $i_c(t) = Ke^{st}$ to obtain characteristic equation

$$s^2 + 2\alpha s + w_0^2 = 0$$

Second-order Circuits (Contd.)

- Define damping ratio $\zeta = \dfrac{\alpha}{\omega_0}$
- Roots of the characteristic equation

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \qquad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

- 3 cases

Over Damped

- If $\zeta > 1$ (or equivalently, if $\alpha > \omega_0$), the roots of the characteristic equation are real and distinct. Then, the complementary solution is

$$i_c(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}$$

Critically Damped

- If $\zeta = 1$ (or equivalently, if $\alpha = \omega_0$), the roots are real and equal. Then the complementary solution is

$$x_c(t) = K_1 e^{s_1 t} + K_2 t e^{s_1 t}$$

Under Damped

ζ < 1 (or equivalently, if α < ω0), the roots are complex. (By the term complex, we mean that roots involve j, the square root of −1.) The complementary solution is of the form

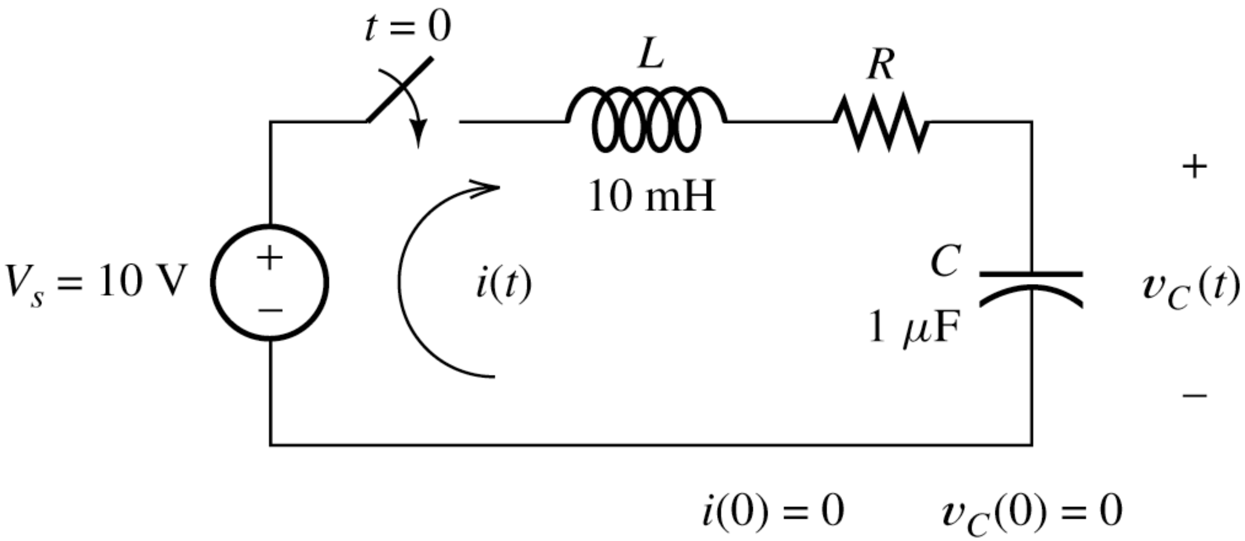
$$i_c(t) = K_1 e^{-\alpha t}\cos(\omega_n t) + K_2 e^{-\alpha t}\sin(\omega_n t)$$

Where the natural frequency is given by

$$\omega_n = \sqrt{\omega_0^2 - \alpha^2}$$

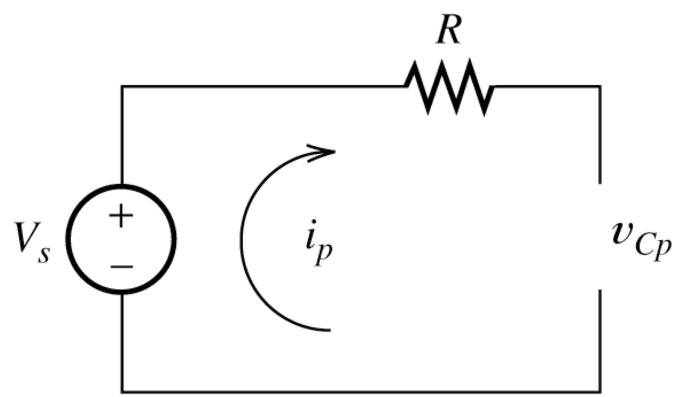
In electrical engineering, we use j rather than i to stand for square root of −1, because we use i for current.

Example Exercise

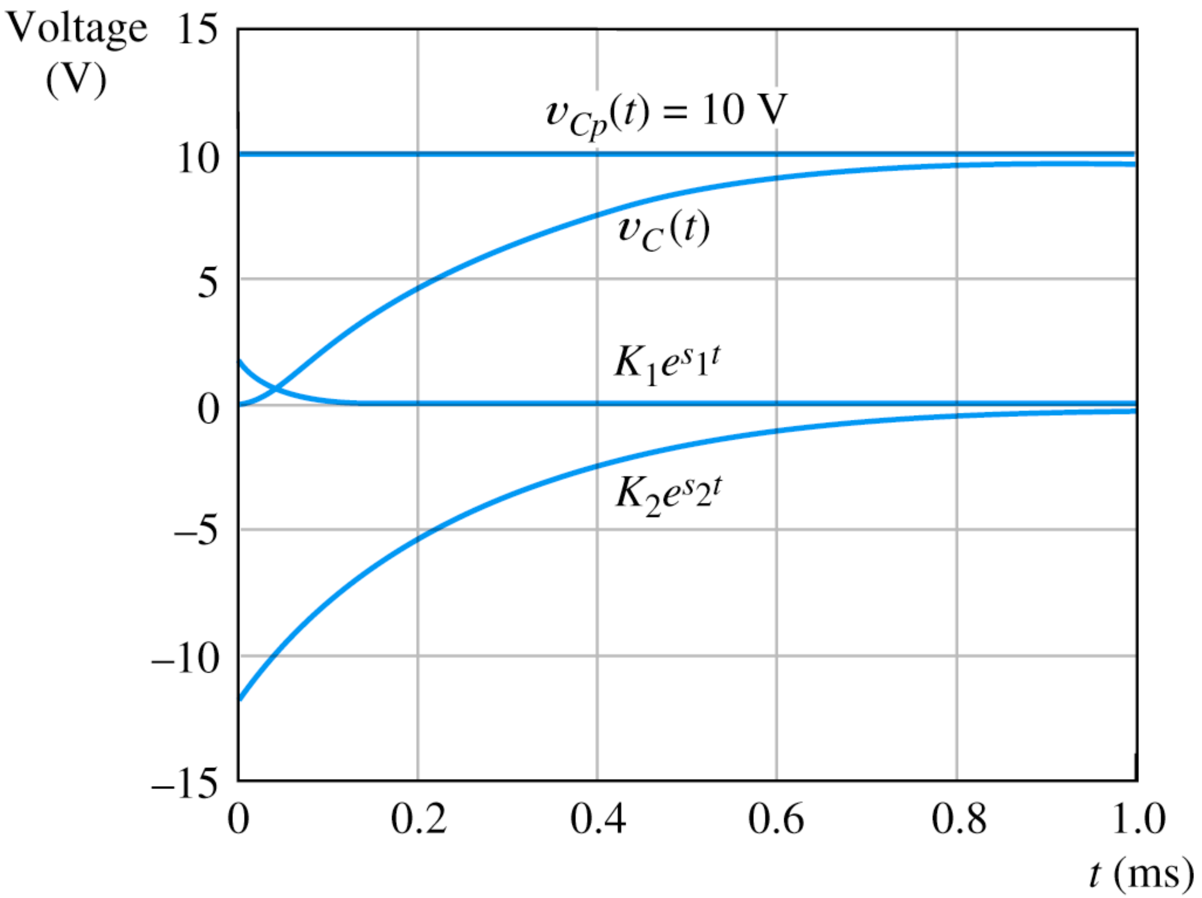


R=300/200/100 Ω

Steady State

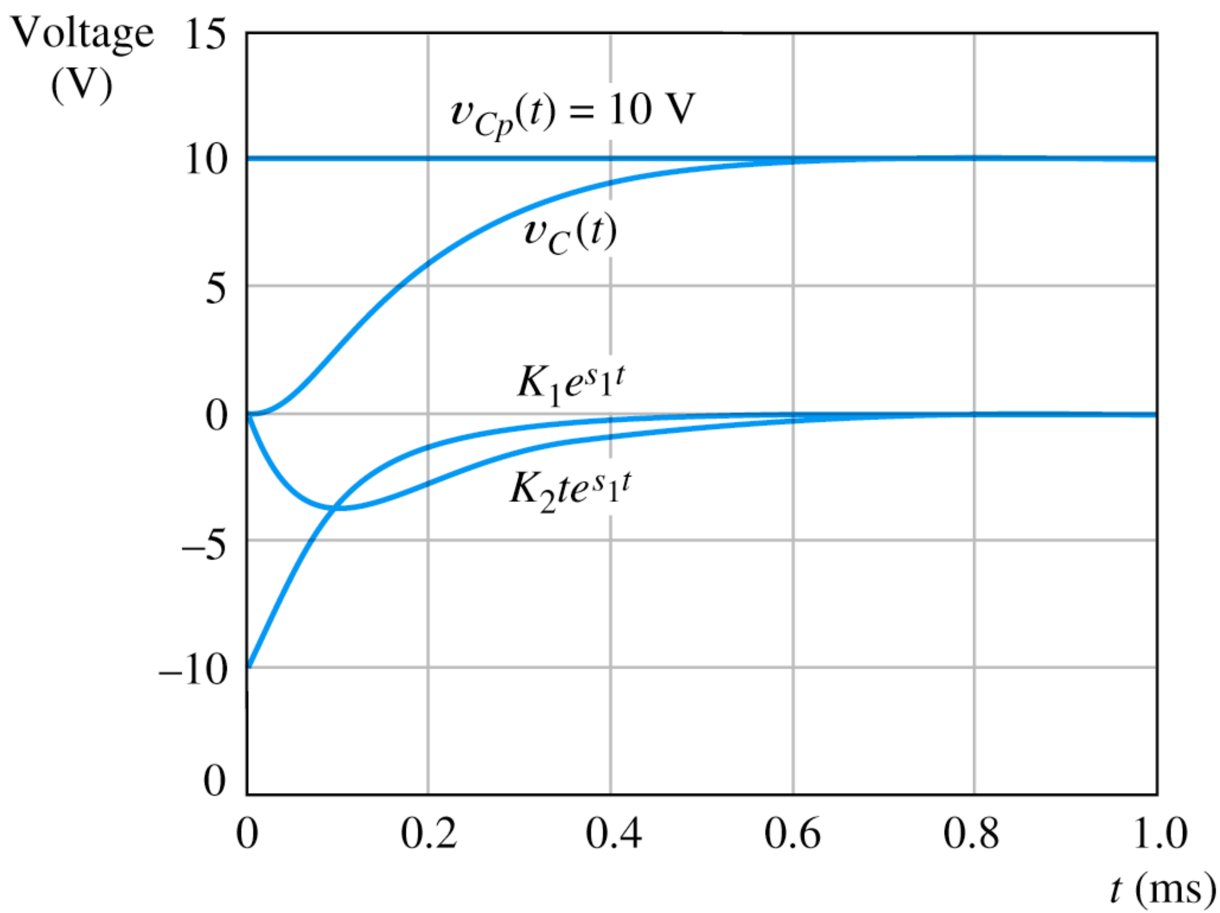


R=300 Ohms



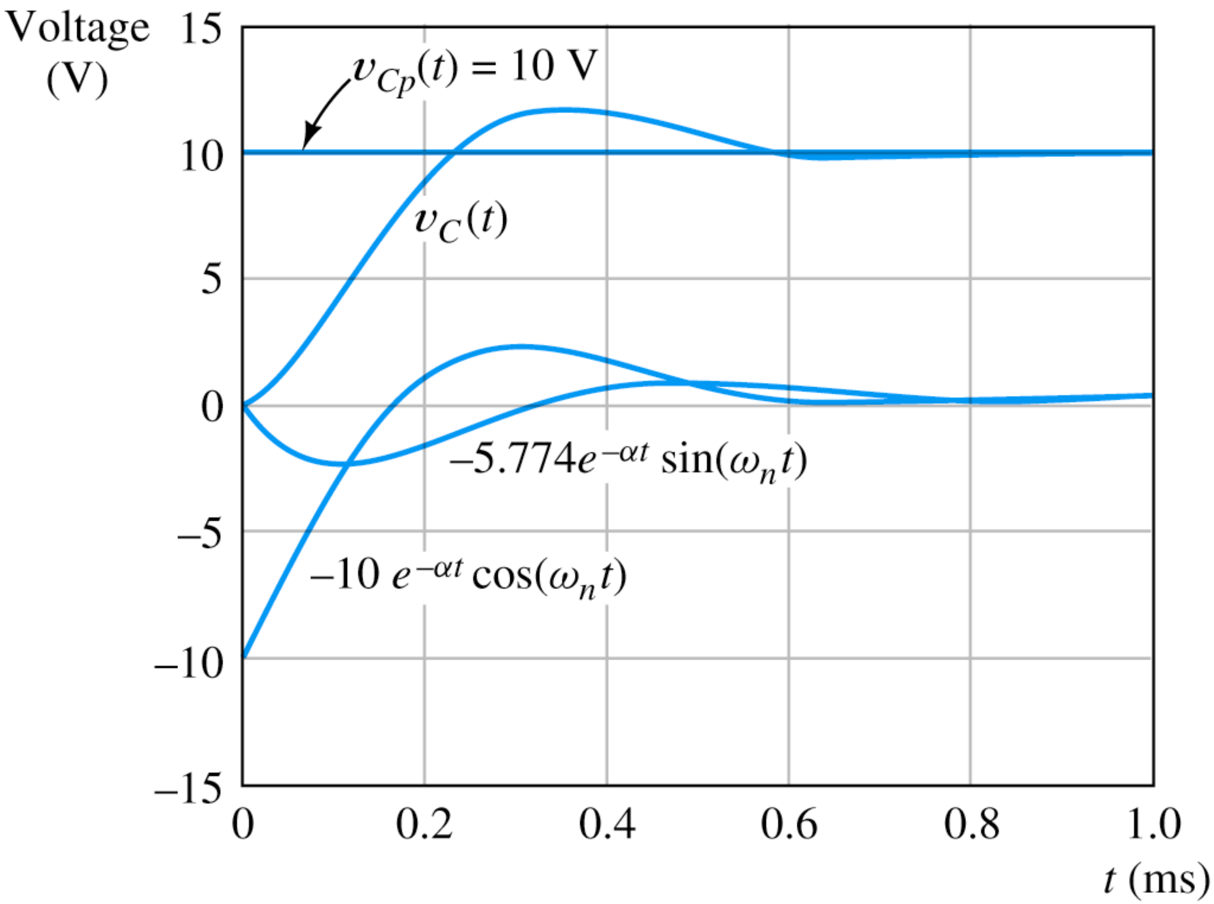
Solution for $R = 300\ \Omega$.

R=200 Ohms



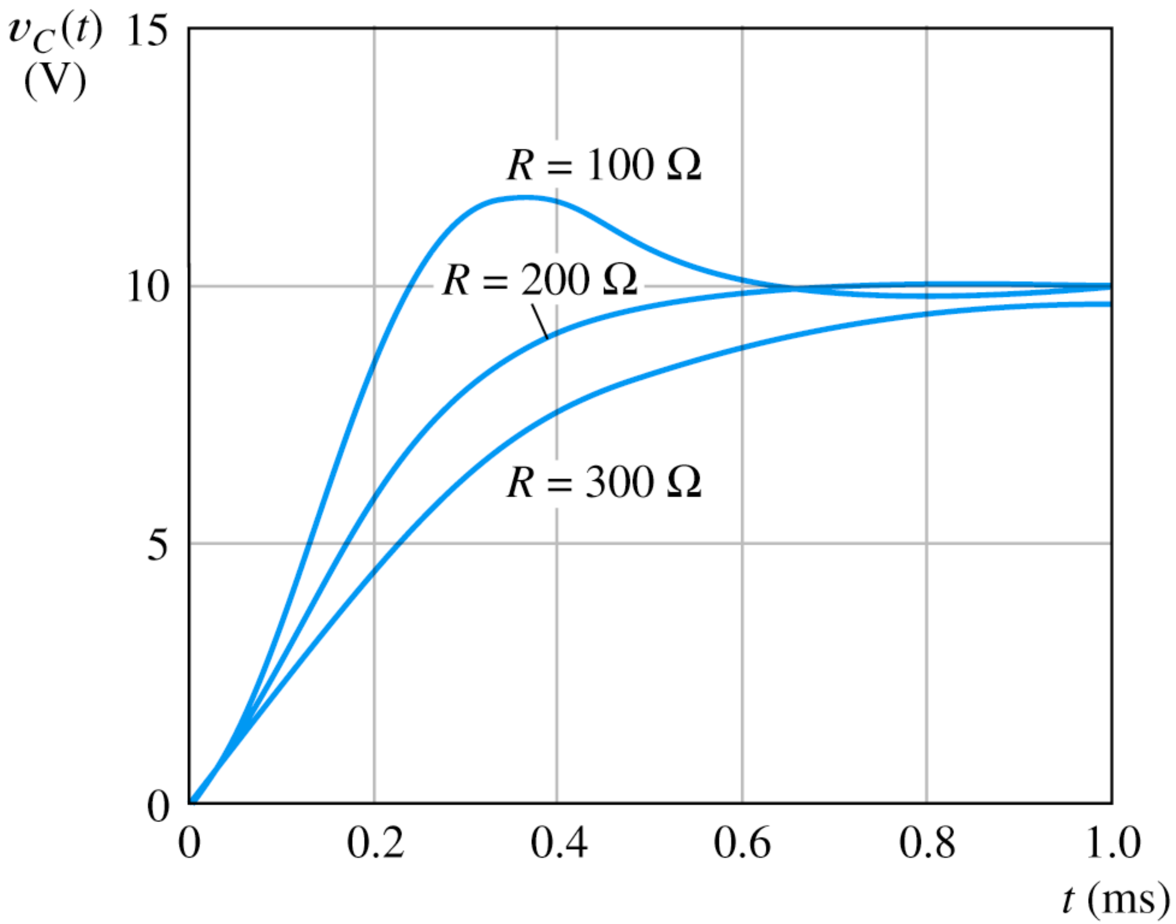
Solution for $R = 200\ \Omega$.

R=100 Ohms



Solution for $R = 100\ \Omega$.

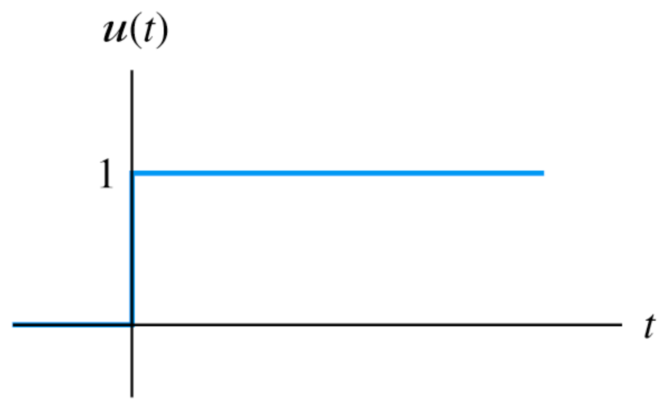
3 Solutions

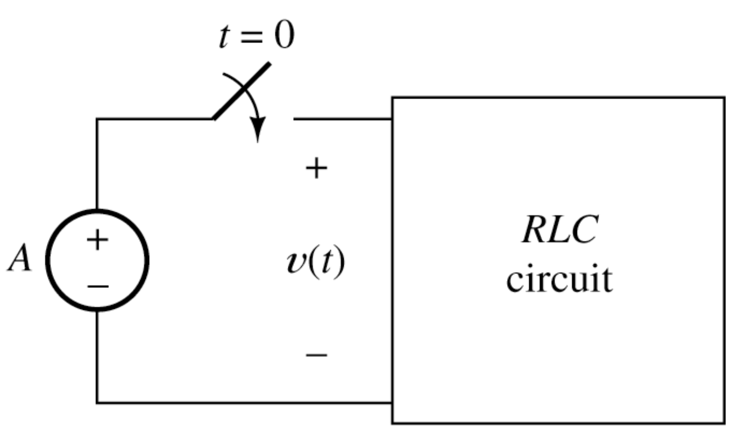


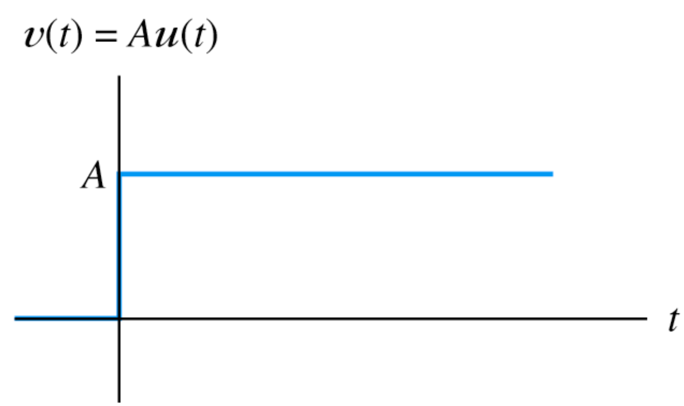
Solutions for all three resistances.

Step Function

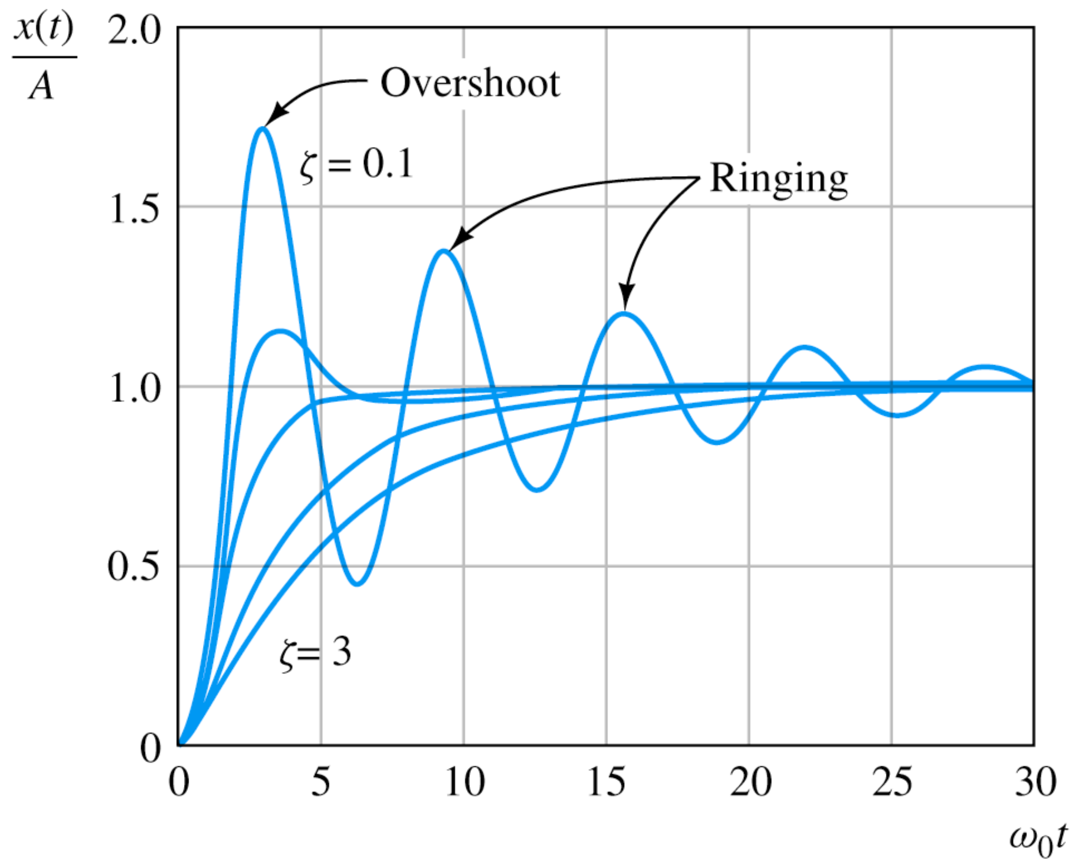
Model for DC source by closing a switch





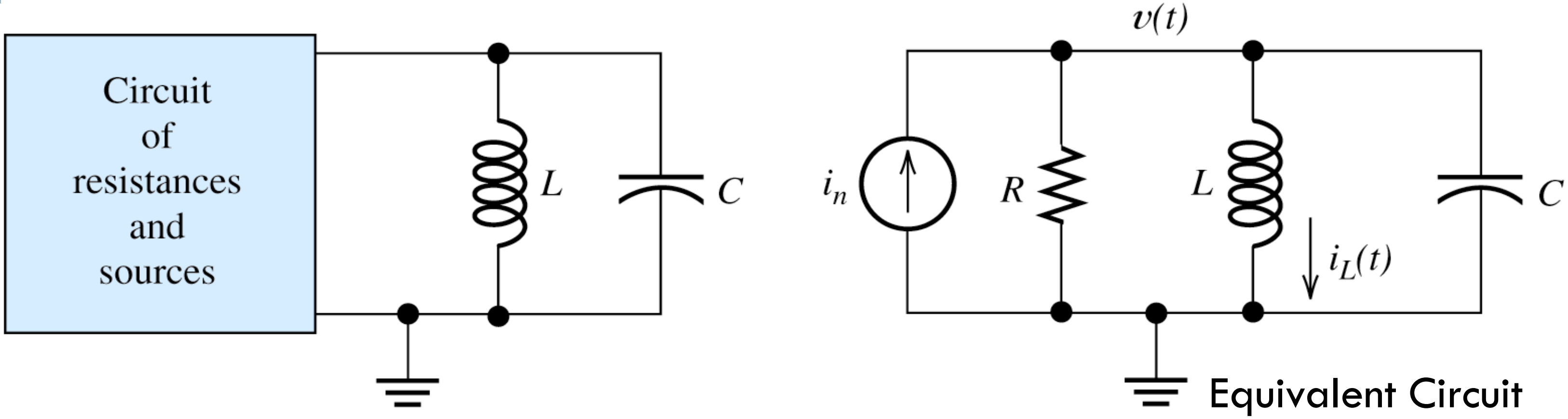


Normalized Step Response



Normalized step responses for second-order systems described by Equation 4.99 with damping ratios of $\zeta = 0.1$, 0.5, 1, 2, and 3. The initial conditions are assumed to be $x(0) = 0$ and $x'(0) = 0$.

Parallel LC Circuit



- KCL

$$C\frac{dv(t)}{dt}+\frac{1}{R}v(t)+\frac{1}{L}\int_0^t v(t)\,dt+i_L(0)=i_n(t)$$

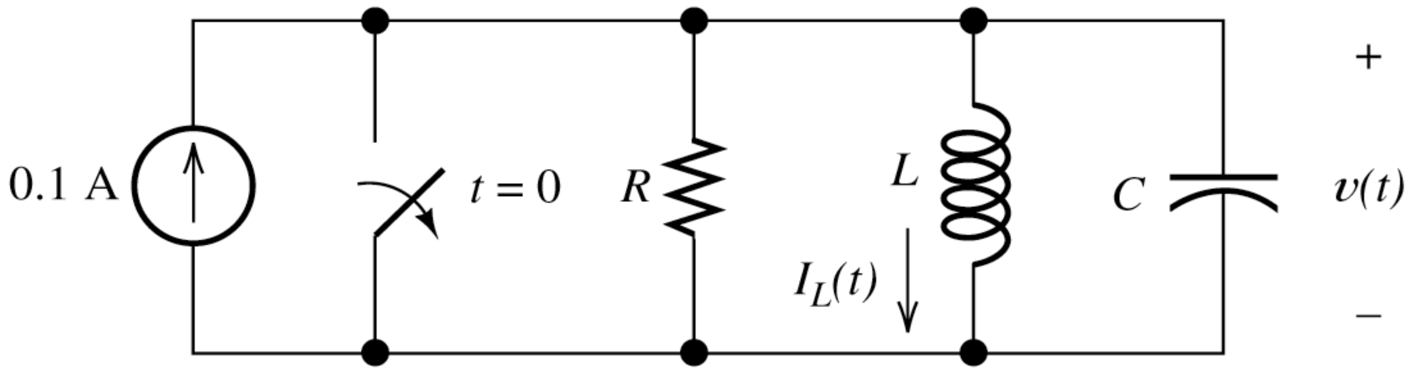
- Define

$$\alpha=\frac{1}{2RC},\omega_0=\frac{1}{\sqrt{LC}},f(t)=\frac{1}{C}\frac{di_n(t)}{dt}$$

- Similar to series LC circuit

Example Exercise

- Initial v(0-) and i_L(0-)=0
- R=25/50/250 Ohms



$L = 1$ mH $C = 0.1\ \mu$F