IIT Mandi

School of Basic Sciences

IC121:Mechanics of particles and waves,

Tutorial – 1

1) Using the transformation rules of vector rotation prove the familiar trigonometric identities, $\sin(\theta+\phi)=\sin\theta\cos\phi+\cos\theta\sin\phi$ and $\cos(\theta+\phi)=\cos\theta\cos\phi-\sin\theta\sin\phi$.

2) Show that scalar product of two dimensional vectors $\vec{a}\cdot\vec{b}$ is invariant under rotation.

3) Find the gradient of following functions
   (a) $x^2+y^3+z^4$
   (b) $x^2y^3z^4$
   (c) $e^x\sin(y)\ln(z)$

4) Find the directional derivative (gradient) of $f(x,y)=x^2\sin 2y$ at point $(1,\pi/2)$ in the direction $v=3\hat{x}-4\hat{y}$.

5) Find the derivative of $f(x,y,z)=x^3-xy^2-z$ at $P_o(1,1,0)$ in the direction of $\vec{v}=2\hat{x}-3\hat{y}+6\hat{k}$, in what direction does f change most rapidly at P_0 .

6) Let $\vec{\nabla}\phi=(1+2xy)\hat{x}+(x^2+3y^2)\hat{y}.$ Find the associated scalar field.

7) Prove that divergence of a Curl is always zero. Also prove that curl of gradient is also zero always. (Hint: prove it for arbitrary vector field and scalar field by term by term expansion)

8) Let $T=xy^2$. Show that $\int_a^b \vec{\nabla}T\cdot d\vec{r}=T(\vec{b})-T(\vec{a})$ (independent of path) between points $a=(0,0,0)$ and $\vec{b}=(2,1,0)$ via two paths (i) path connecting two points along straight line (ii) first move parallel to the x axis till $x=2$ and then move parallel to y axis till $y=1$.

9) Calculate the divergence of the following fields
   (i) $\vec{v}=e^x(\cos y\,\hat{x}+\sin y\,\hat{y})$
   (ii) $\vec{v}=x^2\hat{x}+y^2\hat{y}+z^2\hat{z}$
   (iii) $\vec{v}=v_1(y,z)\hat{x}+v_2(z,x)\hat{y}+v_3(x,y)\hat{z}$

10) Find the Laplacian $\nabla^2 f$ for the following scalar fields
   (i) $f=4x^2+9y^2+z^2$
   (ii) $f=e^{2x}\sin 2y$
   (iii) $f=xy/z$

11) A vector field $\vec{A}$ and space curve $\vec{r}=\vec{r}(t)$ are given by $\vec{A}=(3x^2-6xy)\hat{x}+(2y+3xy)\hat{y}+(1-4xyz^2)\hat{z}$ and $\vec{r}(t)=t\,\hat{x}+t^2\,\hat{y}+t^3\,\hat{z}$ evaluate the

line integral $\int \vec{A}\cdot\vec{d}\,\vec{r}$ in the limits $t=0$ and $t=2$ (find the values of x, y, z in termds of t by comparing with $\vec{r}$).

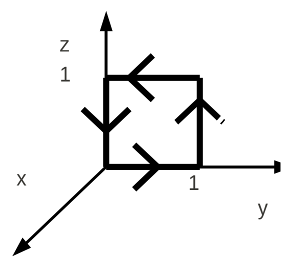
12) Show that the vector field $\vec{A}=(2xy+z^3)\hat{x}+(x^2+2y)\hat{y}+(3xz^2-2)\hat{z}$ is independent of the path from $(1,-1,1)$ to $(2,1,2)$ for the integral $\int \vec{A}\cdot d\vec{r}$. Find the potential function $\phi(x,y,z)$.

13) What is a conservative force field? Is the force field $\vec{F}=(3xy-y)\hat{x}-x\hat{y}+(3/2)x^2\hat{z}$ is conservative ? If yes determine the potential V and the work A to be performed to move a particle from point $(1,1,1)$ to $(2,2,2)$.

14) Check the divergence theorem using the function $\vec{v}=y^2\hat{x}+(2xy+z^2)\hat{y}+(2yz)\hat{z}$, the unit cube placed at the origin ((0,0,0) ,(1,1,1) are diagonal points of the cube in the positive quadrant)

15) Find the curl $\vec{\nabla}\times v$ of the following functions
(i) $\vec{v}=2y\hat{x}+5x\hat{y}$
(ii) $v=xyz(x\hat{x}+y\hat{y}+z\hat{z})$
(iii) $\vec{v}=v_1(x)\hat{x}+v_2(y)\hat{y}+v_3(z)\hat{z}$

16) Suppose $\vec{v}=(2xz+3y^2)\hat{y}+(4yz^2)\hat{z}$. Check Stoke's theorem for the square surface shown in the figure



17) Calculate the volume integral of $T=xyz^2$ over the prism shown in the figure