IC 110: Engineering Mathematics; Practice sheet

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1. If $u = x + y + z,\ uv = y + z,\ uvw = z$, show that $J(x, y, z) = \frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2 v$.

2. If $u_1 = \frac{x_2 x_3}{x_1},\ u_2 = \frac{x_1 x_3}{x_2},\ u_3 = \frac{x_1 x_2}{x_3}$, show that $\frac{\partial(u_1,u_2,u_3)}{\partial(x,y,z)} = 4$.

3. If $u = 2xy,\ v = x^2 - y^2,\ x = r\cos\theta,\ y = r\sin\theta$, find $\frac{\partial(u,v)}{\partial(r,\theta)}$.

4. If $U = x + y - z,\ V = x - y + z,\ W = x^2 + y^2 + z^2 - 2yz$, show that U, V, W are connected by a functional relation, and find the functional relation.

5. The root of the equation in λ
$$(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$$
are u, v, w. Show that
$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = -2\frac{(y-z)(z-x)(x-y)}{(v-w)(w-u)(u-v)}.$$

6. Find the points (x, y), where the function $xy(1 - x - y)$ is either maximum or minimum.

7. The temperature T at any point (x, y, z) in space is $T = 400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$.

8. Find the maximum value of $x^m y^n z^p$ subject to the condition $x + y + z = a$.

9. Divide 24 into three parts such that the continued product of the first, square of the second, and the cube of the third part may be maximum.

10. Investigate the maximum and minimum radii vector of the sector of surface of elasticity
$$(x^2 + y^2 + z^2)^2 = a^2x^2 + b^2y^2 + c^2z^2,$$
made by the plane $lx + my + nz = 0$.