Elementary Linear Algebra

SIXTH EDITION

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Contents

*Available online at **college.hmco.com/pic/larsonELA6e.***

A Word from the Authors

Welcome! We have designed *Elementary Linear Algebra*, Sixth Edition, for the introductory linear algebra course.

Students embarking on a linear algebra course should have a thorough knowledge of algebra, and familiarity with analytic geometry and trigonometry. We do not assume that calculus is a prerequisite for this course, but we do include examples and exercises requiring calculus in the text. These exercises are clearly labeled and can be omitted if desired.

Many students will encounter mathematical formalism for the first time in this course. As a result, our primary goal is to present the major concepts of linear algebra clearly and concisely. To this end, we have carefully selected the examples and exercises to balance theory with applications and geometrical intuition.

The order and coverage of topics were chosen for maximum efficiency, effectiveness, and balance. For example, in Chapter 4 we present the main ideas of vector spaces and bases, beginning with a brief look leading into the vector space concept as a natural extension of these familiar examples. This material is often the most difficult for students, but our approach to linear independence, span, basis, and dimension is carefully explained and illustrated by examples. The eigenvalue problem is developed in detail in Chapter 7, but we lay an intuitive foundation for students earlier in Section 1.2, Section 3.1, and Chapter 4.

Additional online Chapters 8, 9, and 10 cover complex vector spaces, linear programming, and numerical methods. They can be found on the student website for this text at *college.hmco.com/pic/larsonELA6e*.

Please read on to learn more about the features of the Sixth Edition.

We hope you enjoy this new edition of *Elementary Linear Algebra*.

Acknowledgments

We would like to thank the many people who have helped us during various stages of the project. In particular, we appreciate the efforts of the following colleagues who made many helpful suggestions along the way:

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We would also like to thank Helen Medley for her careful accuracy checking of the textbook.

On a personal level, we are grateful to our wives, Deanna Gilbert Larson and Susan Falvo, for their love, patience, and support. Also, special thanks go to R. Scott O'Neil.

Ron Larson
David C. Falvo

Theorems and Proofs

THEOREM 2.9 The Inverse of a Product

If A and B are invertible matrices of size n, then AB is invertible and

$$(AB)^{-1} = B^{-1}A^{-1}.$$

Theorems are presented in clear and mathematically precise language.

Key theorems are also available via **PowerPoint® Presentation** on the instructor website. They can be displayed in class using a computer monitor or projector, or printed out for use as class handouts.

Students will gain experience solving **proofs** presented in several different ways:

- Some proofs are presented in **outline form,** omitting the need for burdensome calculations.
- Specialized exercises labeled **Guided Proofs** lead students through the initial steps of constructing proofs and then utilizing the results.
- The proofs of several theorems are left as **exercises,** to give students additional practice.

PROOF To begin, observe that if E is an elementary matrix, then, by Theorem 3.3, the next few statements are true. If E is obtained from I by interchanging two rows, then $|E| = -1$. If E is obtained by multiplying a row of I by a nonzero constant c, then $|E| = c$. If E is obtained by adding a multiple of one row of I to another row of I, then $|E| = 1$. Additionally, by Theorem 2.12, if E results from performing an elementary row operation on I and the same elementary row operation is performed on B, then the matrix EB results. It follows that

$$|EB| = |E|\,|B|.$$

This can be generalized to conclude that $|E_k \cdot \cdot \cdot E_2E_1B| = |E_k| \cdot \cdot \cdot |E_2|\,|E_1|\,|B|$, where E_i is an elementary matrix. Now consider the matrix AB. If A is *nonsingular*, then, by Theorem 2.14, it can be written as the product of elementary matrices $A = E_k \cdot \cdot \cdot E_2E_1$ and you can write

$|A|\,|B|.$

56. **Guided Proof** Prove Theorem 3.9: If A is a square matrix, then $\det(A) = \det(A^T)$.

Getting Started: To prove that the determinants of A and A^T are equal, you need to show that their cofactor expansions are equal. Because the cofactors are $\pm$ determinants of smaller matrices, you need to use mathematical induction.

(i) Initial step for induction: If A is of order 1, then $A = [a_{11}] = A^T$, so $\det(A) = \det(A^T) = a_{11}$.

(ii) Assume the inductive hypothesis holds for all matrices of order $n - 1$. Let A be a square matrix of order n. Write an expression for the determinant of A by expanding by the first row.

(iii) Write an expression for the determinant of A^T by expanding by the first column.

(iv) Compare the expansions in (i) and (ii). The entries of the first row of A are the same as the entries of the first column of A^T. Compare cofactors (these are the $\pm$ determinants of smaller matrices that are transposes of one another) and use the inductive hypothesis to conclude that they are equal as well.

Real World Applications

REVISED! Each chapter ends with a section on **real-life applications** of linear algebra concepts, covering interesting topics such as:

- Computer graphics
- Cryptography
- Population growth and more!

A full listing of the applications can be found in the **Index of Applications** inside the front cover.

EXAMPLE 4 **Forming Uncoded Row Matrices**

Write the uncoded row matrices of size 1×3 for the message MEET ME MONDAY.

SOLUTION Partitioning the message (including blank spaces, but ignoring punctuation) into groups of three produces the following uncoded row matrices.

$$\begin{array}{ccccccccccccccc} [13 & 5 & 5] & [20 & 0 & 13] & [5 & 0 & 13] & [15 & 14 & 4] & [1 & 25 & 0] \\ M & E & E & T & \_ & M & E & \_ & M & O & N & D & A & Y & \_ \end{array}$$

Note that a blank space is used to fill out the last uncoded row matrix.

INDEX OF APPLICATIONS

Conceptual Understanding

CHAPTER OBJECTIVES

- Find the determinants of a 2×2 matrix and a triangular matrix.
- Find the minors and cofactors of a matrix and use expansion by cofactors to find the determinant of a matrix.
- Use elementary row or column operations to evaluate the determinant of a matrix.
- Recognize conditions that yield zero determinants.
- Find the determinant of an elementary matrix.
- Use the determinant and properties of the determinant to decide whether a matrix is singular or nonsingular, and recognize equivalent conditions for a nonsingular matrix.
- Verify and find an eigenvalue and an eigenvector of a matrix.

NEW! **Chapter Objectives** are now listed on each chapter opener page. These objectives highlight the key concepts covered in the chapter, to serve as a guide to student learning.

True or False? In Exercises 62–65, determine whether each statement is true or false. If a statement is true, give a reason or cite an appropriate statement from the text. If a statement is false, provide an example that shows the statement is not true in all cases or cite an appropriate statement from the text.

62. (a) The nullspace of A is also called the solution space of A.

(b) The nullspace of A is the solution space of the homogeneous system $A\mathbf{x} = \mathbf{0}$.

63. (a) If an $m \times n$ matrix A is row-equivalent to an $m \times n$ matrix B, then the row space of A is equivalent to the row space of B.

(b) If A is an $m \times n$ matrix of rank r, then the dimension of the solution space of $A\mathbf{x} = \mathbf{0}$ is $m - r$.

The **Discovery** features are designed to help students develop an intuitive understanding of mathematical concepts and relationships.

Discovery

Let

$$A = \begin{bmatrix} 6 & 4 & 1 \\ 0 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}.$$

Use a graphing utility or computer software program to find A^{-1}. Compare $\det(A^{-1})$ with $\det(A)$. Make a conjecture about the determinant of the inverse of a matrix.

True or False? exercises test students' knowledge of core concepts. Students are asked to give examples or justifications to support their conclusions.

Graphics and Geometric Emphasis

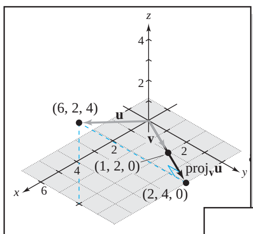
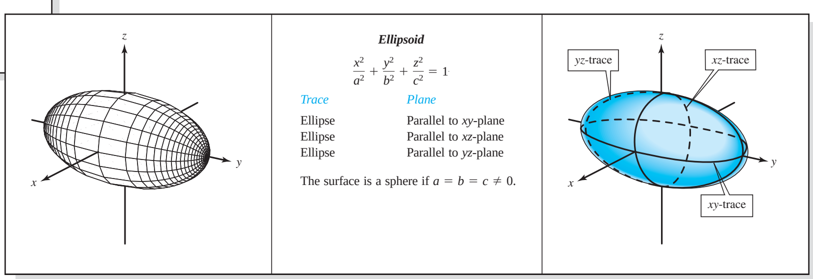


Figure 5.13

Visualization skills are necessary for the understanding of mathematical concepts and theory. The Sixth Edition includes the following resources to help develop these skills:

- **Graphs** accompany examples, particularly when representing vector spaces and inner product spaces.
- **Computer-generated illustrations** offer geometric interpretations of problems.



Problem Solving and Review

53. $\mathbf{u} = (0, 1, \sqrt{2}),\quad \mathbf{v} = (-1, \sqrt{2}, -1)$

54. $\mathbf{u} = (-1, \sqrt{3}, 2),\quad \mathbf{v} = (\sqrt{2}, -1, -\sqrt{2})$

55. $\mathbf{u} = (0, 2, 2, -1, 1, -2),\quad \mathbf{v} = (2, 0, 1, 1, 2, -2)$

56. $\mathbf{u} = (1, 2, 3, -2, -1, -3),\quad \mathbf{v} = (-1, 0, 2, 1, 2, -3)$

57. $\mathbf{u} = (-1, 1, 2, -1, 1, 1, -2, 1),$
$\mathbf{v} = (-1, 0, 1, 2, -2, 1, 1, -2)$

58. $\mathbf{u} = (3, -1, 2, 1, 0, 1, 2, -1),$
$\mathbf{v} = (1, 2, 0, -1, 2, -2, 1, 0)$

In Exercises 59–62, verify the Cauchy-Schwarz Inequality for the given vectors.

59. $\mathbf{u} = (3, 4),\quad \mathbf{v} = (2, -3)$

60. $\mathbf{u} = (-1, 0),\quad \mathbf{v} = (1, 1)$

61. $\mathbf{u} = (1, 1, -2),\quad \mathbf{v} = (1, -3, -2)$

62. $\mathbf{u} = (1, -1, 0),\quad \mathbf{v} = (0, 1, -1)$

In Exercises 63–72, find the angle θ between the vectors.

63. $\mathbf{u} = (3, 1),\quad \mathbf{v} = (-2, 4)$

64. $\mathbf{u} = (2, -1),\quad \mathbf{v} = (2, 0)$

65. $\mathbf{u} = \left(\cos\frac{\pi}{6}, \sin\frac{\pi}{6}\right),\quad \mathbf{v} = \left(\cos\frac{3\pi}{4}, \sin\frac{3\pi}{4}\right)$

83. $\mathbf{u} = \left(-\frac{1}{3}, \frac{2}{3}\right),\quad \mathbf{v} = (2, -4)$

84. $\mathbf{u} = (1, -1),\quad \mathbf{v} = (0, -1)$

85. $\mathbf{u} = (0, 1, 0),\quad \mathbf{v} = (1, -2, 0)$

86. $\mathbf{u} = (0, 1, 6),\quad \mathbf{v} = (1, -2, -1)$

87. $\mathbf{u} = (-2, 5, 1, 0),\quad \mathbf{v} = \left(\frac{1}{4}, -\frac{5}{4}, 0, 1\right)$

88. $\mathbf{u} = \left(4, \frac{3}{2}, -1, \frac{1}{2}\right),\quad \mathbf{v} = \left(-2, -\frac{3}{4}, \frac{1}{2}, -\frac{1}{4}\right)$

In Exercises 89–92, use a graphing utility or computer software program with vector capabilities to determine whether **u** and **v** are orthogonal, parallel, or neither.

89. $\mathbf{u} = \left(-2, \frac{1}{2}, -1, 3\right),\quad \mathbf{v} = \left(\frac{3}{2}, 1, -\frac{5}{2}, 0\right)$

90. $\mathbf{u} = \left(-\frac{21}{2}, \frac{43}{2}, -12, \frac{3}{2}\right),\quad \mathbf{v} = \left(0, 6, \frac{21}{2}, -\frac{9}{2}\right)$

91. $\mathbf{u} = \left(-\frac{3}{4}, \frac{3}{2}, -\frac{9}{2}, -6\right),\quad \mathbf{v} = \left(\frac{3}{8}, -\frac{3}{4}, \frac{9}{8}, 3\right)$

92. $\mathbf{u} = \left(-\frac{4}{3}, \frac{8}{3}, -4, -\frac{32}{3}\right),\quad \mathbf{v} = \left(-\frac{16}{3}, -2, \frac{4}{3}, -\frac{2}{3}\right)$

Writing In Exercises 93 and 94, determine if the vectors are orthogonal, parallel, or neither. Then explain your reasoning.

93. $\mathbf{u} = (\cos\theta, \sin\theta, -1),\quad \mathbf{v} = (\sin\theta, -\cos\theta, 0)$

94. $\mathbf{u} = (-\sin\theta, \cos\theta, 1),\quad \mathbf{v} = (\sin\theta, -\cos\theta, 0)$

REVISED! Comprehensive section and chapter exercise sets give students practice in problem-solving techniques and test their understanding of mathematical concepts. A wide variety of exercise types are represented, including:

- **Writing exercises**
- **Guided Proof exercises**
- **Technology exercises,** indicated throughout the text with .
- **Applications** exercises
- Exercises utilizing **electronic data sets,** indicated by and found on the student website at ***college.hmco.com/pic/larsonELA6e***

Each chapter includes two **Chapter Projects,** which offer the opportunity for group activities or more extensive homework assignments.

Chapter Projects are focused on theoretical concepts or applications, and many encourage the use of technology.

CHAPTER 3 Projects

1 Eigenvalues and Stochastic Matrices

In Section 2.5, you studied a consumer preference model for competing cable television companies. The matrix representing the transition probabilities was

$$P = \begin{bmatrix} 0.70 & 0.15 & 0.15 \\ 0.20 & 0.80 & 0.15 \\ 0.10 & 0.05 & 0.70 \end{bmatrix}.$$

When provided with the initial state matrix X, you observed that the number of subscribers after 1 year is the product PX.

$$X = \begin{bmatrix} 15{,}000 \\ 20{,}000 \\ 65{,}000 \end{bmatrix} \longrightarrow PX = \begin{bmatrix} 0.70 & 0.15 & 0.15 \\ 0.20 & 0.80 & 0.15 \\ 0.10 & 0.05 & 0.70 \end{bmatrix}\begin{bmatrix} 15{,}000 \\ 20{,}000 \\ 65{,}000 \end{bmatrix} = \begin{bmatrix} 23{,}250 \\ 28{,}750 \\ 48{,}000 \end{bmatrix}$$

Cumulative Tests follow chapters 3, 5, and 7, and help students synthesize the knowledge they have accumulated throughout the text, as well as prepare for exams and future mathematics courses.

CHAPTERS 4 & 5 Cumulative Test

Take this test as you would take a test in class. After you are done, check your work against the answers in the back of the book.

1. Given the vectors $\mathbf{v} = (1, -2)$ and $\mathbf{w} = (2, -5)$, find and sketch each vector.
   (a) $\mathbf{v} + \mathbf{w}$ (b) $3\mathbf{v}$ (c) $2\mathbf{v} - 4\mathbf{w}$
2. If possible, write $\mathbf{w} = (2, 4, 1)$ as a linear combination of the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$.
   $\mathbf{v}_1 = (1, 2, 0),\quad \mathbf{v}_2 = (-1, 0, 1),\quad \mathbf{v}_3 = (0, 3, 0)$
3. Prove that the set of all singular 2×2 matrices is not a vector space.

Historical Emphasis

HISTORICAL NOTE

Augustin-Louis Cauchy (1789–1857)
was encouraged by Pierre Simon de Laplace, one of France's leading mathematicians, to study mathematics. Cauchy is often credited with bringing rigor to modern mathematics. To read about his work, visit *college.hmco.com/pic/larsonELA6e.*

NEW! **Historical Notes** are included throughout the text and feature brief biographies of prominent mathematicians who contributed to linear algebra.

Students are directed to the Web to read the full biographies, which are available via **PowerPoint® Presentation.**

Computer Algebra Systems and Graphing Calculators

Technology Note

You can use a graphing utility or computer software program to find the unit vector for a given vector. For example, you can use a graphing utility to find the unit vector for $\mathbf{v} = (-3, 4)$, which may appear as:

```
VECTOR:V          2
 e1=-3
 e2=4
```

```
unitV V
               [-.6 .8]
```

The **Technology Note** feature in the text indicates how students can utilize graphing calculators and computer algebra systems appropriately in the problem-solving process.

NEW! **Online Technology Guide** provides the coverage students need to use computer algebra systems and graphing calculators with this text.

Provided on the accompanying student website, this guide includes **CAS and graphing calculator keystrokes for select examples in the text.** These examples feature an accompanying Technology Note, directing students to the Guide for instruction on using their CAS/graphing calculator to solve the example.

In addition, the Guide provides an **Introduction to MATLAB, Maple, Mathematica, and Graphing Calculators,** as well as a section on **Technology Pitfalls.**

Technology Note

You can use a computer software program or graphing utility with a built-in power regression program to verify the result of Example 10. For example, using the data in Table 5.2 and a graphing utility, a power fit program would result in an answer of (or very similar to) $y \approx 1.00042x^{1.49954}$. Keystrokes and programming syntax for these utilities/programs applicable to Example 10 are provided in the **Online Technology Guide,** available at *college.hmco.com/pic/larsonELA6e.*

EXAMPLE 7 **Using Elimination to Rewrite a System in Row-Echelon Form**

Solve the system.

$$\begin{aligned} x - 2y + 3z &= 9 \\ -x + 3y \quad\quad &= -4 \\ 2x - 5y + 5z &= 17 \end{aligned}$$

Keystrokes for TI-83
Enter the system into matrix A.
To rewrite the system in row-echelon form, use the following keystrokes.
MATRX → ALPHA [A] MATRX ENTER ENTER

Keystrokes for TI-83 Plus
Enter the system into matrix A.
To rewrite the system in row-echelon form, use the following keystrokes.
2nd [MATRX] → ALPHA [A] 2nd [MATRX] ENTER ENTER

Keystrokes for TI-84 Plus
Enter the system into matrix A.
To rewrite the system in row-echelon form, use the following keystrokes.
2nd [MATRIX] → ALPHA [A] 2nd [MATRIX] ENTER ENTER

Keystrokes for TI-86
Enter the system into matrix A.
To rewrite the system in row-echelon form, use the following keystrokes.
2nd [MATRIX] F4 F4 ALPHA [A] ENTER

Part I: Texas Instruments TI-83, TI-83 Plus Graphics Calculator

I.1 Systems of Linear Equations

I.1.1 Basics: Press the ON key to begin using your TI-83 calculator. If you need to adjust the display contrast, first press 2nd, then press and hold ▲ (the *up* arrow key) to increase the contrast or ▼ (the *down* arrow key) to decrease the contrast. As you press and hold ▲ or ▼, an integer between 0 (lightest) and 9 (darkest) appears in the upper right corner of the display. When you have finished with the calculator, turn it off to conserve battery power by pressing 2nd and then OFF.

Check the TI-83's settings by pressing MODE. If necessary, use the arrow key to move the blinking cursor to a setting you want to change. Press ENTER to select a new setting. To start, select the options along the left side of the MODE menu as illustrated in Figure I.1: normal display, floating display decimals, radian measure, function graphs, connected lines, sequential plotting, real number system, and full screen display. Details on alternative options will be given later in this guide. For now, leave the MODE menu by pressing CLEAR.

The **Graphing Calculator Keystroke Guide** offers commands and instructions for various calculators and includes examples with step-by-step solutions, technology tips, and programs.

The Graphing Calculator Keystroke Guide covers TI-83/TI-83 PLUS, TI-84 PLUS, TI-86, TI-89, TI-92, and Voyage 200.

Also available on the student website:

- **Electronic Data Sets** are designed to be used with select exercises in the text and help students reinforce and broaden their technology skills using graphing calculators and computer algebra systems.
- **MATLAB Exercises** enhance students' understanding of concepts using MATLAB software. These optional exercises correlate to chapters in the text.

Additional Resources ■ Get More from Your Textbook

Instructor Resources	*Student Resources*
Instructor Website This website offers instructors a variety of resources, including: ■ **Instructor's Solutions Manual,** featuring complete solutions to all even-numbered exercises in the text. ■ **Digital Art and Figures,** featuring key theorems from the text.	**Student Website** This website offers comprehensive study resources, including: ■ *NEW!* **Online Multimedia eBook** ■ *NEW!* **Online Technology Guide** ■ **Electronic Simulations** ■ **MATLAB Exercises** ■ **Graphing Calculator Keystroke Guide** ■ **Chapters 8, 9, and 10** ■ **Electronic Data Sets** ■ **Historical Note Biographies**
NEW! **HM Testing™ (Powered by Diploma®) "Testing the way you want it"** HM Testing provides instructors with a wide array of new algorithmic exercises along with improved functionality and ease of use. Instructors can create, author/edit algorithmic questions, customize, and deliver multiple types of tests.	**Student Solutions Manual** Contains complete solutions to all odd-numbered exercises in the text.

HM Math SPACE with Eduspace®: Houghton Mifflin's Online Learning Tool (powered by Blackboard®)
This web-based learning system provides instructors and students with powerful course management tools and text-specific content to support all of their online teaching and learning needs. Eduspace now includes:

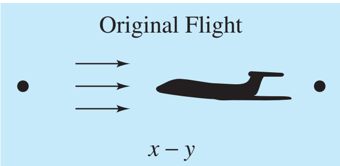
- *NEW!* **WebAssign®** Developed by teachers, for teachers, WebAssign allows instructors to create assignments from an abundant ready-to-use database of algorithmic questions, or write and customize their own exercises. With WebAssign, instructors can: create, post, and review assignments 24 hours a day, 7 days a week; deliver, collect, grade, and record assignments instantly; offer more practice exercises, quizzes and homework; assess student performance to keep abreast of individual progress; and capture the attention of online or distance-learning students.
- SMARTHINKING.com It's how you know — **SMARTHINKING® Live, Online Tutoring** SMARTHINKING provides an easy-to-use and effective online, text-specific tutoring service. A dynamic **Whiteboard** and a **Graphing Calculator** function enable students and e-structors to collaborate easily.

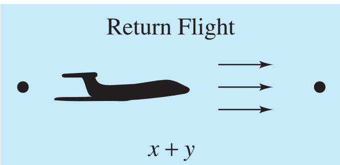
Online Course Content for Blackboard®, WebCT®, and eCollege® Deliver program- or text-specific Houghton Mifflin content online using your institution's local course management system. Houghton Mifflin offers homework and other resources formatted for Blackboard, WebCT, eCollege, and other course management systems. Add to an existing online course or create a new one by selecting from a wide range of powerful learning and instructional materials.

For more information, visit ***college.hmco.com/pic/larson/ELA6e*** or contact your local Houghton Mifflin sales representative.

What Is Linear Algebra?

To answer the question "What is linear algebra?," take a closer look at what you will study in this course. The most fundamental theme of linear algebra, and the first topic covered in this textbook, is the theory of systems of linear equations. You have probably encountered small systems of linear equations in your previous mathematics courses. For example, suppose you travel on an airplane between two cities that are 5000 kilometers apart. If the trip one way against a headwind takes $6\frac{1}{4}$ hours and the return trip the same day in the direction of the wind takes only 5 hours, can you find the ground speed of the plane and the speed of the wind, assuming that both remain constant?





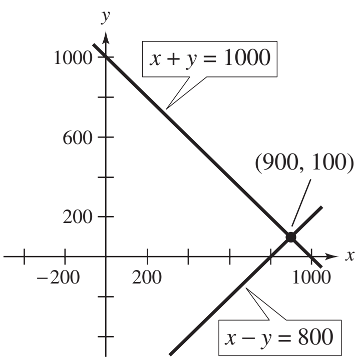
If you let x represent the speed of the plane and y the speed of the wind, then the following system models the problem.

$$\begin{aligned} 6.25(x - y) &= 5000 \\ 5(x + y) &= 5000 \end{aligned}$$

This system of two equations and two unknowns simplifies to

$$\begin{aligned} x - y &= 800 \\ x + y &= 1000, \end{aligned}$$

and the solution is $x = 900$ kilometers per hour and $y = 100$ kilometers per hour. Geometrically, this system represents two lines in the xy-plane. You can see in the figure that these lines intersect at the point $(900, 100)$, which verifies the answer that was obtained.

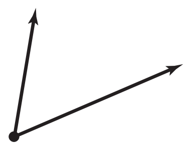


The lines intersect at (900, 100).

Solving systems of linear equations is one of the most important applications of linear algebra. It has been argued that the majority of all mathematical problems encountered in scientific and industrial applications involve solving a linear system at some point. Linear applications arise in such diverse areas as engineering, chemistry, economics, business, ecology, biology, and psychology.

Of course, the small system presented in the airplane example above is very easy to solve. In real-world situations, it is not unusual to have to solve systems of hundreds or even thousands of equations. One of the early goals of this course is to develop an algorithm that helps solve larger systems in an orderly manner and is amenable to computer implementation.

The first three chapters of this textbook cover linear systems and two other computational areas you may have studied before: matrices and determinants. These discussions prepare the way for the central theoretical topic of linear algebra: the concept of a vector space. Vector spaces generalize the familiar properties of vectors in the plane. It is at this point in the text that you will begin to write proofs and learn to verify theoretical properties of vector spaces.



Vectors in the Plane

The concept of a vector space permits you to develop an entire theory of its properties. The theorems you prove will apply to all vector spaces. For example, in Chapter 6 you will study linear transformations, which are special functions between vector spaces. The applications of linear transformations appear almost everywhere—computer graphics, differential equations, and satellite data transmission, to name just a few examples.

Another major focus of linear algebra is the so-called eigenvalue (Ī–gən–value) problem. Eigenvalues are certain numbers associated with square matrices and are fundamental in applications as diverse as population dynamics, electrical networks, chemical reactions, differential equations, and economics.

LINEAR ALGEBRA The branch of algebra in which one studies vector (linear) spaces, linear operators (linear mappings), and linear, bilinear, and quadratic functions (functionals and forms) on vector spaces. *(Encyclopedia of Mathematics, Kluwer Academic Press, 1990)*

Linear algebra strikes a wonderful balance between computation and theory. As you proceed, you will become adept at matrix computations and will simultaneously develop abstract reasoning skills. Furthermore, you will see immediately that the applications of linear algebra to other disciplines are plentiful. In fact, you will notice that each chapter of this textbook closes with a section of applications. You might want to peruse some of these sections to see the many diverse areas to which linear algebra can be applied. (An index of these applications is given on the inside front cover.)

Linear algebra has become a central course for mathematics majors as well as students of science, business, and engineering. Its balance of computation, theory, and applications to real life, geometry, and other areas makes linear algebra unique among mathematics courses. For the many people who make use of pure and applied mathematics in their professional careers, an understanding and appreciation of linear algebra is indispensable.