# IIT Mandi

School of Basic Sciences

*IC-121:Mechanics of particles and waves*

*Tutorial – 3*

1) A certain oscillator satisfy the the equation $\ddot{x}+4x=0$ . Initially the particle is at the point $x=\sqrt{3}$ when it is projected toward the origin with speed $2$ . Show that the subsequent motion is given by the equation $x=\sqrt{3}\cos 2t-\sin 2t$ . Deduce the amplitude of the oscillation. How long it will take the particle to first reach the origin?

2) Arrive at equation of motion for a simple pendulum from principle of conservation of energy with length of the pendulum $l$ and having mass $m$ . The pendulum is released from and angle $\theta$ to the vertical.

3) In the problem mentioned above find the equations of motion from balancing of the balance of torques.

4) A mass $m$ oscillate on a spring with spring constant $k$ . The amplitude is $d$ at $t=0$ . At the moment when the mass is at position $x=d/2$ while moving right it collides and sticks to another mass m. What is the amplitude of the new oscillation.

5) A particle of mass 5 kg moves along the $x$ direction under the influence of two forces 1) A force toward the origin with the value $40\,N/m$ and a frictional forces of $200\,N$ for $v=10\mathrm{m}/s$ . Let $x(t=0)=20\,m$ and velocity $\dot{x}(t=0)=0\mathrm{m}/s$ . Find the differential equation of motion, and the solution of the problem. Find also amplitude, period and frequency of the vibration, ratio's of two successive amplitudes.

6) An over damped harmonic oscillator satisfies the equation $\ddot{x}+10\dot{x}+16\mathrm{x}=0$ At time $t=0$ the particle is projected from the point $x=1$ toward the origin with speed $u$ . Find the solution of the problem. Show that the particle will reach the origin at some later time $t$ if $\frac{u-2}{u-8}=e^{6\mathrm{t}}$ . How large must $u$ such that the particle pass through the origin?

7) The exponential damping factor $\gamma$ of a spring of a suspension system is one tenth of the critical value. If the damping frequency is $\omega_0$ , (a) find the resonance frequency (b) quality factor (c)phase angle $\phi$ , when the system is driven at a frequency $\omega=\omega_0/2$ , (d) steady state amplitude of this frequency.

8) A critically damped oscillator with natural frequency $\omega$ and damping coefficient $\gamma$ starts at position $x_0>0$ . What is the maximum initial speed directed toward the origin and not to cross origin?

9) Find the driven response of the of the damped linear oscillator for the case in which driving force $F(t)$ is periodic with period $2\pi$ and takes the values i $F(t)=F_0$ n the interval $(0<t<\pi)$ and $F(t)=-F_0$ in the interval $(\pi<t<2\pi)$ .

10) A particle $P$ of mass $3\mathrm{m}$ is suspended from a fixed point $O$ by a light linear spring with strength $\alpha$ A second particle $Q$ of mass $2\mathrm{m}$ is in turn suspended from $P$ by a second

spring of same strength . The system moves in the vertical straight line through $O$ . Find the normal frequencies and the form of normal modes of the system.

11) Consider two masses $m$ connected to each other and to two walls by springs. The three springs have the same spring constant $k$ . Find the most general solution for the positions of masses as function to time. What are the normal coordinates? What are normal modes?

12) Derive the equation of motion for system of $n$ springs of spring constant k connected to each other by a mass point having mass $m$ . Let the length of the each spring be $h$ by balancing of the forces on each mass. Express the equation of motion in terms of in terms of the total length of the system $L$ effective spring constant $K$ total mass of the $n$ masses $M$ . In the limit of the distance between masses approaches zero show that this equation of motion lead to wave equation $\frac{\partial^2 u(x,t)}{\partial t^2}=\frac{K L^2}{M}\frac{\partial^2 u(x,t)}{\partial x^2}$ .