IC111

Linear Algebra

School of Basic Sciences

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Linear Algebra Notes (Version 1.1)

Bachelor of Engineering

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Chapter 1

Vector Spaces

1.1 Group

Binary Operator: A Binary operator on a non-empty set $\mathbf{S}$ is a map from its cartesian product $\mathbf{S} \times \mathbf{S}$ to $\mathbf{S}$. Let $*$ be the binary operation on $\mathbf{S}$ then we have

$$* : \mathbf{S} \times \mathbf{S} \longrightarrow \mathbf{S}$$

Group: A non-empty set G, together with a binary operation $*$ is said to form a group, if it satisfies the following properties.

1. *Associativity:* $a * (b * c) = (a * b) * c \qquad \forall a, b, c \in G.$
2. *Existence of Identity:* $\exists$ an element $e \in G$ such that

   $$a * e = e * a = a \qquad \forall a \in G$$

   where, e is the identity element.
3. *Existence of inverse:* For every $a \in G$, $\exists\, a' \in G$ such that

   $$a * a' = a' * a = e$$

   Here, a' is called an inverse element of a.

Remarks

1. If $*$ is a binary operation on G then G is said to satisfy closure property.
2. Identity element for a group is unique.
3. Inverse of an element is also unique.
4. Existence of right identity and left inverse does not form a group.
5. Existence of left identity and right inverse also does not form a group.
6. In above definition, existence of right identity and right inverse is sufficient to form a group because right identity is also left identity and right inverse is also left inverse.
7. If a' be the inverse element of a then, $(a')' = a$.