Indian Institute of Technology Mandi

भारतीय प्रौद्योगिकी संस्थान मण्डी

MA-201: B.Tech. II year

Odd Semester: 2011-12 **Exercise-6 Linear Algebra**

1. Which of the following subsets S of $\mathbb{R}^3$ are LI/LD over the field $\mathbb{R}$?
   (a) $S = \{(1,2,1), (-1,2,0), (5,-1,2)\}$
   (b) $S = \{(1,1,2), (-3,1,0), (1,-1,1), (1,2,-3)\}$
   (c) $S = \{(\frac{1}{2}, \frac{1}{3}, 1), (0,0,0), (2, \frac{3}{4}, -\frac{1}{3})\}$
   (d) $S = \{(1,0,0,0), (1,1,0,0), (1,1,1,1), (0,0,1,1)\}$
   (e) $S = \{(1,1,1,0), (3,2,2,1), (1,1,3,-2), (1,2,6,-5), (1,1,2,6,1)\}$
   (f) $S = \{(1,2,3,0), (-1,7,3,3), (1,-1,1,-1)\}$
   (g) $S = \{x^2-1, x+1, x-1\}$
   (h) $S = \{x, x^3-x, x^4+x^2, x+x^2+x^4+\frac{1}{2}\}$
   (i) $S = \{x, \sin x, \cos x\}$
   (j) $S = \{\sin x, \cos x, \sin x + 1\}$

2. Find a linearly independent subset A of S such that $[A] = [S]$, where S are given bellow:
   (a) $S = \{(1,0,0,0), (1,1,0,0), (1,1,1,1), (0,0,1,1)\}$
   (b) $S = \{(1,-1,2,0), (1,1,2,0), (3,0,0,1), (2,1,-1,0)\}$
   (c) $S = \{(1,1,1,0), (3,2,2,1), (1,1,3,-2), (1,2,6,-5), (1,1,2,6,1)\}$
   (d) $S = \{1, x+x^2, x-x^2, 3x\}$
   (e) $S = \{x, x^3-x, x^4+x^2, x+x^2+x^4+\frac{1}{2}\}$

3. Whenever a set S is LD, locate one of the vector that is in the span of the other. Where set S are
   (a) $S = \{(1,1,2), (-3,1,0), (1,-1,1), (1,2,-3)\}$
   (b) $S = \{(1,0,0,0), (1,1,0,0), (1,1,1,1), (0,0,1,1)\}$
   (c) $S = \{1, x+x^2, x-x^2, 3x\}$
   (d) $S = \{x, \sin x, \cos x\}$
   (e) $S = \{\ln x, \ln x^2, \ln x^3\}$

4. Let $S = \{(1,2,1), (1,1,-1), (4,5,-2)\}$. Determine which of the of the following vectors are in $[S]$:
   (a) (0,0,0) (b) (2,-1,-8) (c) (1,0,1)

5. Let $S = \{x^2, x^2+2x, x^2+2, 1-x\}$. Determine which of the of the following vectors are in $[S]$.

(a) $2x^3 + 3x^2 + 3x + 7$
(b) $x^4 + 7x + 2$
(c) $3x^2 + x + 5$
(d) $x^3 - \frac{3}{2}x^2 + \frac{x}{2}$
(e) $3x + 2$
(f) $x^3 + x^2 + 2x + 3$

6. If S is a nonempty subset of a vector space **V**, prove that $[S] = S$ iff S is a subspace of **V**.

7. What is the span of

   (a) x-axis and y-axis in $\mathbb{R}^3$?
   (b) x-axis and xy-plane in $\mathbb{R}^3$?
   (c) xy-plane and yz-plane in $\mathbb{R}^3$?
   (d) x-axis and the plane $x + y = 0$ in $\mathbb{R}^3$?

8. Find the intersection of the given sets **U** and **W** and determine whether it is a subspace.

   (a) $\mathbf{U} = \{(x_1, x_2) \in \mathbb{R}^2 \mid x_1 \geq 0\}$, $\mathbf{W} = \{(x_1, x_2) \in \mathbb{R}^2 \mid x_1 \leq 0\}$
   (b) $\mathbf{U} = \{f \in \mathscr{C}(-2, 2) \mid f(-1) = 0\}$, $\mathbf{W} = \{f \in \mathscr{C}(-2, 2) \mid f(1) = 0\}$
   (c) $\mathbf{U} = \{f \in \mathscr{C}(-2, 2) \mid \lim_{x\to 1} f(x) = 0\}$ $\mathbf{W} = \{f \in \mathscr{C}(-2, 2) \mid \lim_{x\to 2} f(x) = 1\}$
   (d) $\mathbf{U} = \mathscr{P}$, $\mathbf{W} = \{f \in \mathscr{C}(-\infty, \infty) \mid f(-x) = f(x)\}$

9. Describe $\mathbf{A} + \mathbf{B}$ for the given subsets **A** and **B** of $\mathbb{R}^2$ and determine in each case whether it is a subspace or just a subset of $\mathbb{R}^2$.

   (a) $\mathbf{A} = \{(1, 2), (0, 1)\}, \mathbf{B} = \{(1, 0), (3, -1)\}$
   (b) $\mathbf{A} = \{(\frac{1}{2}, \frac{2}{3}\}$, **B** = segment joining $(-1, 1)$ and $(2, 3)$
   (c) $\mathbf{A} = \{(3, 7), \mathbf{B} = \{t(-1, 2) \mid 0 \leq t \leq 1\}$
   (d) $\mathbf{A} = \{(2, 4), \mathbf{B} = \{(x, y) \mid 2x + 3y = 1\}$
   (e) $\mathbf{A} = \{t(3, 4) \mid 0 \leq t \leq 1\}, \mathbf{B} = \{t(2, 5) \mid 1 \leq t \leq 2\}$
   (f) $\mathbf{A} = \{t(1, 0) \mid$ t is a scalar $\leq 1\}, \mathbf{B} = [(1, 2)]$

10. Describe $\mathbf{A} + \mathbf{B}$ for the given subsets **A** and **B** of $\mathbb{R}^3$. Determine in each case whether $\mathbf{A} + \mathbf{B}$ is a subspace or just a subset of $\mathbb{R}^3$.

    (a) $\mathbf{A} = \{(1, 2, 1)\}, \mathbf{B} = \{t(1, 2, 0) \mid$ t is a scalar $\}$
    (b) $\mathbf{A} = \{(1, -3, 4)\}, \mathbf{B} = [(1, 2, 3)(0, 0, 1)]$
    (c) $\mathbf{A} = \{(\frac{1}{2}, \frac{2}{3}, 1)\}, \mathbf{B} = \{(x, y, z) \mid 2x + 3y + z = 0\}$
    (d) $\mathbf{A} = [(1, 0, -1)], \mathbf{B} = [(2, 5, 8)(2, 3, 4)]$

11. if **U** and **W** are two subspace of a vector space V, prove that $\mathbf{U} + \mathbf{W} = \mathbf{U}$ iff $\mathbf{W} \subset \mathbf{U}$.

12. Let **A** and **B** be two non-empty finite subsets of a vector space **V**. Then prove that

    (a) $[\mathbf{A} \bigcap \mathbf{B}] \subset [\mathbf{A}] \bigcap [\mathbf{B}]$
    (b) $[\mathbf{A} \bigcup \mathbf{B}] = [\mathbf{A}] + [\mathbf{B}]$.