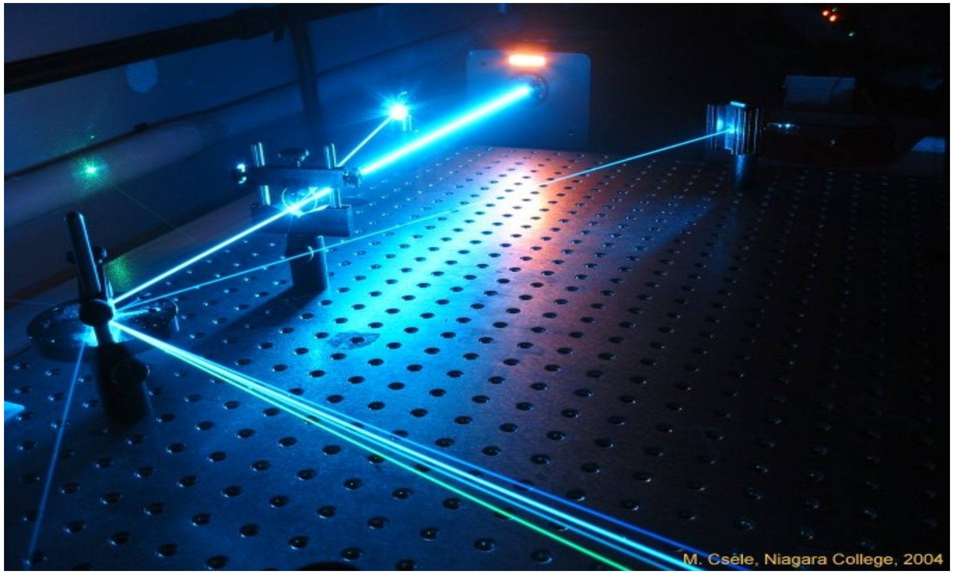
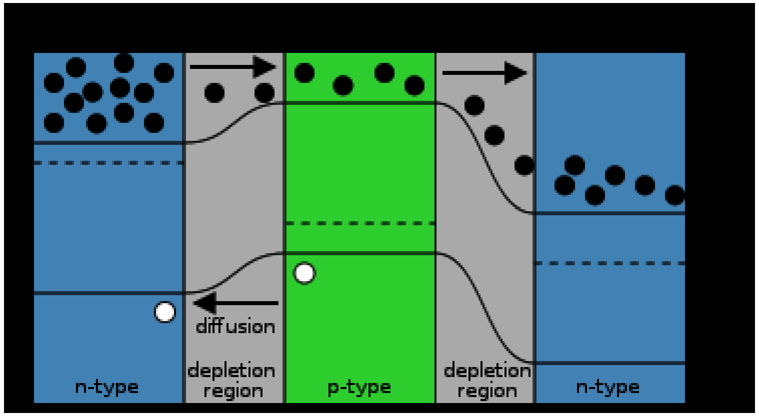
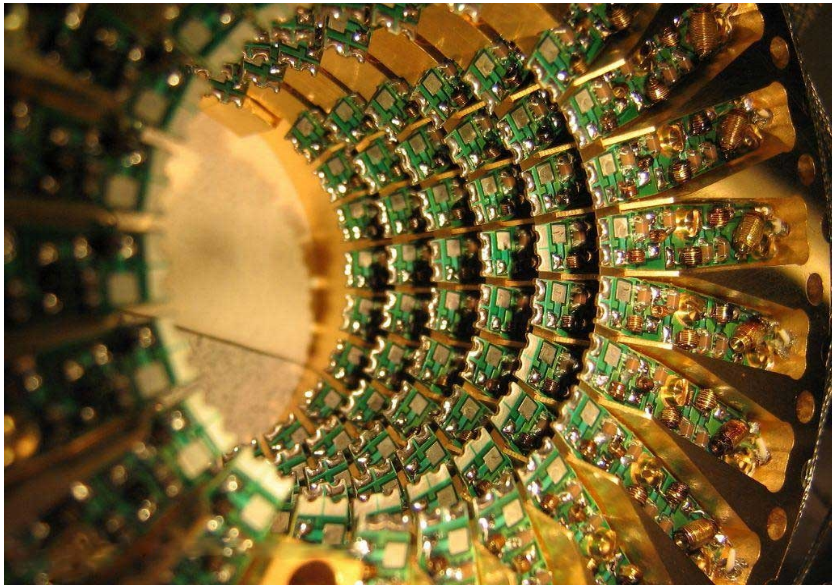
Part II – Introduction to quantum mechanics







Size of objects

Decreasing size of particles

Classical mechanics

Mechanics of planets, and larger objects,
Mechanics of balls, apples simple pendulum etc..

Limiting case mechanics of Cells and macro molecules

$$10^{-9}\, meters\,(nano)$$

Quantum mechanics

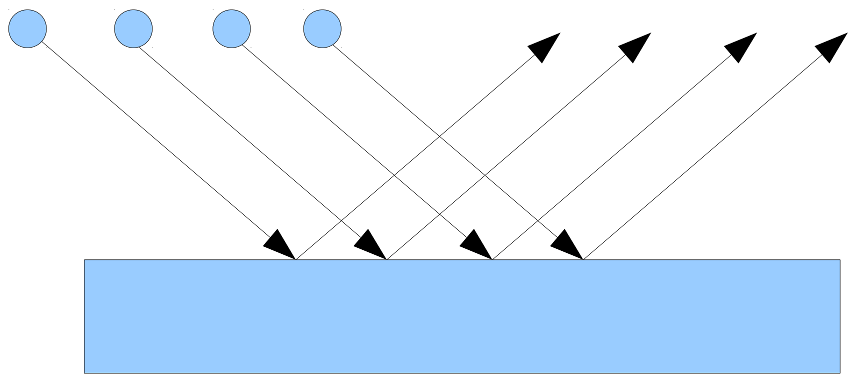
Mechanics of atoms and molecule as limiting case

Mechanics of electron and nucleus, protons and neutrons etc..

Any object that is smaller than atoms

Wave and particle - dual nature of light

Isaac Newton proposed that light constitute collection of particles



Reflection is an example particle nature where it is assumed to be like a bouncing ball.

In contrast Maxwell's work on electro magnetic theory verified the wave nature of light.

In classical description of the world there are two distinct entities, namely, particles and waves therefore there is requirement for treating them separately.

Black-body radiation

Radiation emitted by as a result of its temperature is called thermal radiation

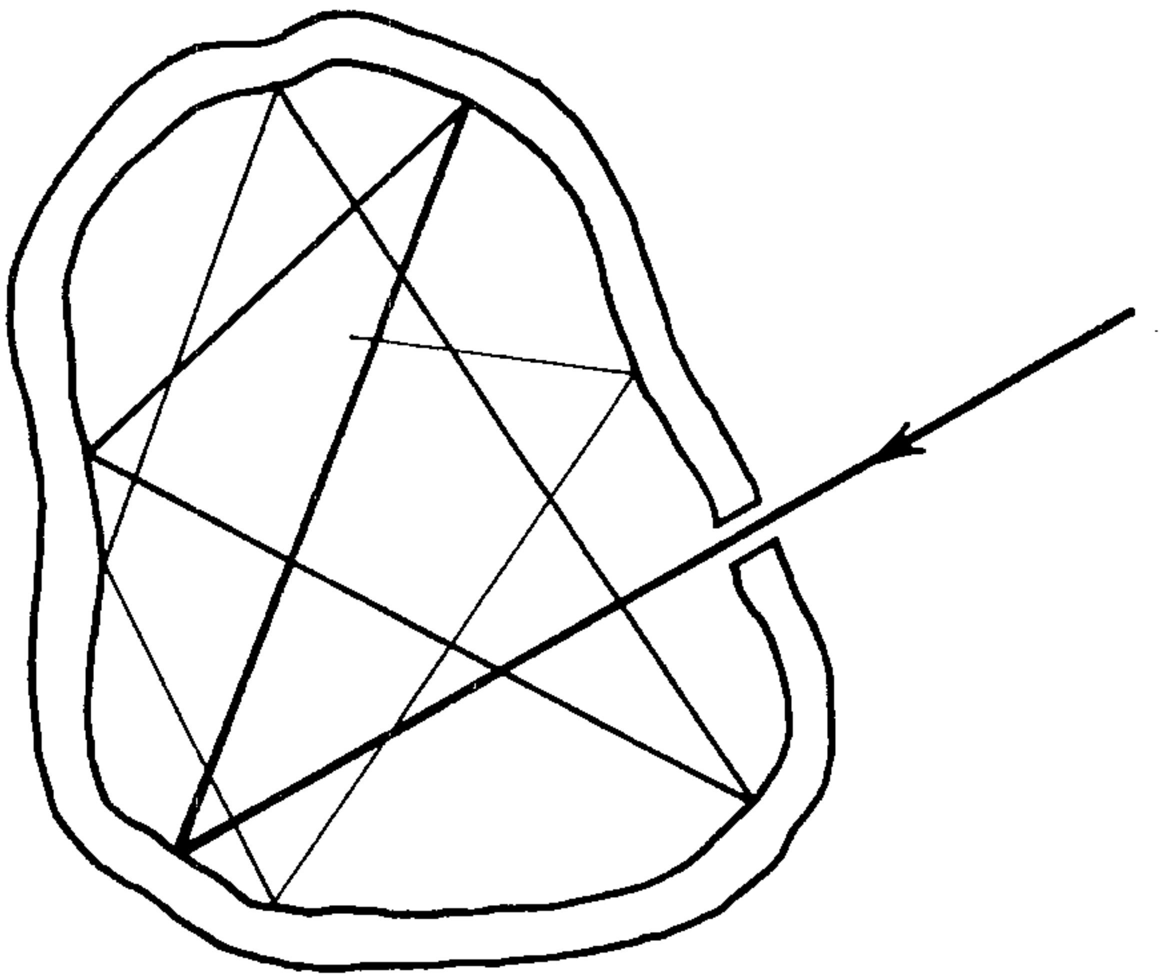
Matter in the condensed state (liquid or solid) emit radiation independent of the properties of materials that depend only on the temperature.

In general the spectrum emitted by a hot body depends on the properties of the body.

However, there is general class of material whose thermal radiation is independent of nature of materials, such bodies also absorb all thermal radiation that are incident on them.

They appear black due to this property of absorption. Example of such a material is carbon black.

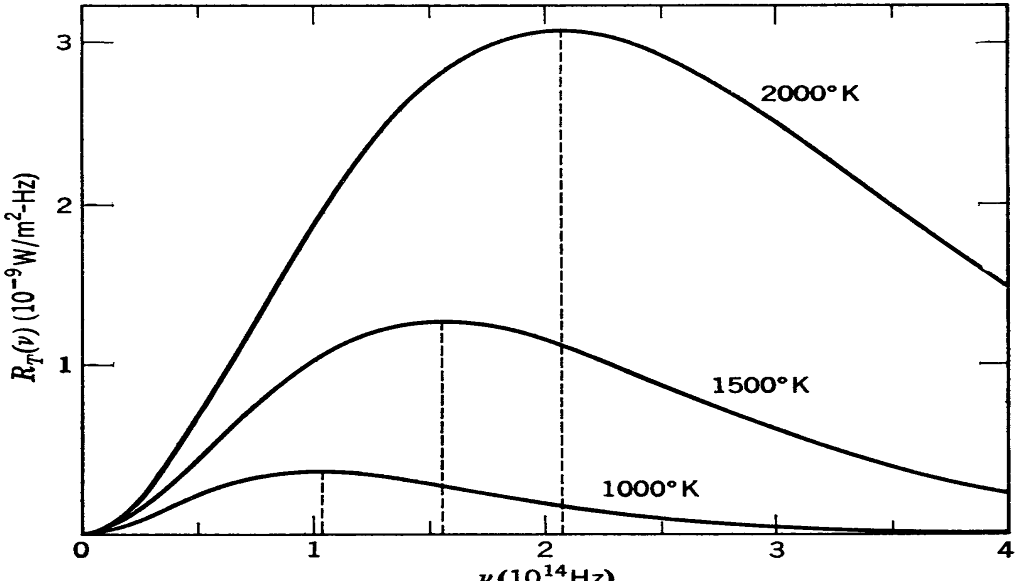
Another class of such material is a cavity with a small hole on it. In this case incident radiation and back and fourth and there is rare chance to escape



It is useful to consider a cavity as black body as it easy to perform theoretical calculation on these system

Spectral distribution of the black body radiation is given by the quantity called spectral radiance $R_T(\nu)$ defined as

energy emitted per unit time in radiation of frequency in interval ν and $\nu + d\nu$ from a unit area of surface at absolute temperature T



In case of cavities it is useful to express spectrum in terms of cavity radiation $\rho_T(\nu)$ defined as energy contained as a unit volume of the cavity at temperature T in the frequency interval ν and $\nu + d\nu$

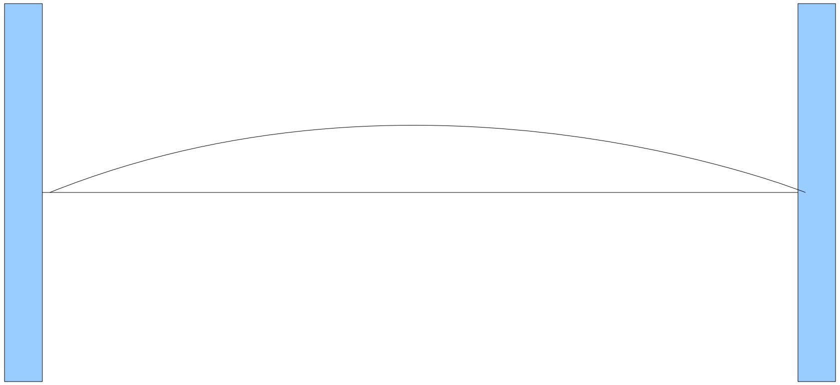
$$\rho_T(\nu) \propto R_T(\nu)$$

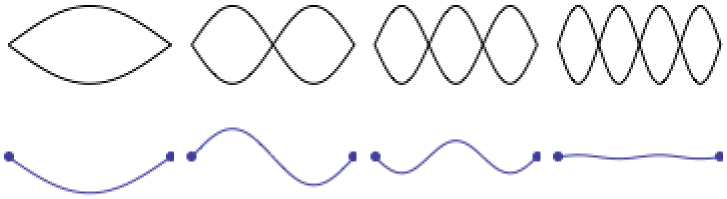
Black-body radiation

As classical theory consider the radiation as particles the results of classical statistical physics may be able to explain the radiation from a black body

Wall of the cavity is maintained at a particular temperature T

Radiation exist inside this cavity in the form of standing waves





Average energy of such a system is proportional to temperature T

Following classical procedure may be used to get radiation coming out a cavity.

(1) Calculate the number of allowed standing frequencies in the range ν between and $\nu + d\nu$ and that depends also on ν

(2) Use classical Kinetic theory to get the average total energy of these waves $\propto T$ (treated as particles)

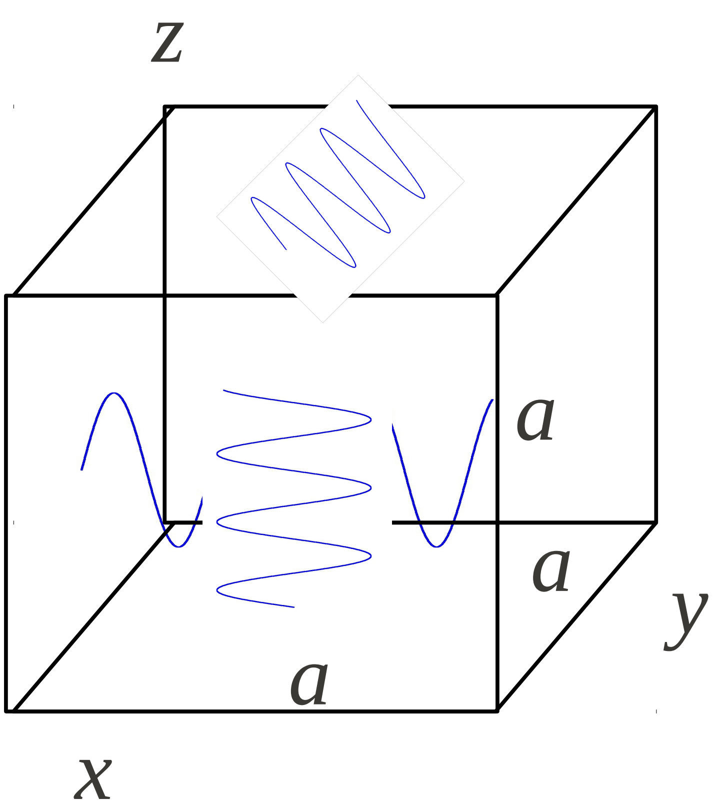
(3) Energy density per unit volume is obtained by dividing by the total volume of the cavity

Let the energy density be $\rho_T(\nu)$ that is to be calculated

For simplicity assume a cubical cavity with box length a for simplicity

The electric field in one of the directions is given by

$$E(x,t)=E_0\sin\left(\frac{2\pi x}{\lambda}\right)\sin(2\pi\nu t)$$



Frequency and the wavelength are related by

$$\nu=\frac{c}{\lambda}$$

The standing wave should satisfy the relation

$$\left(\frac{2x}{\lambda}\right)=n$$ must be an integer $n=1,2,3\ldots$

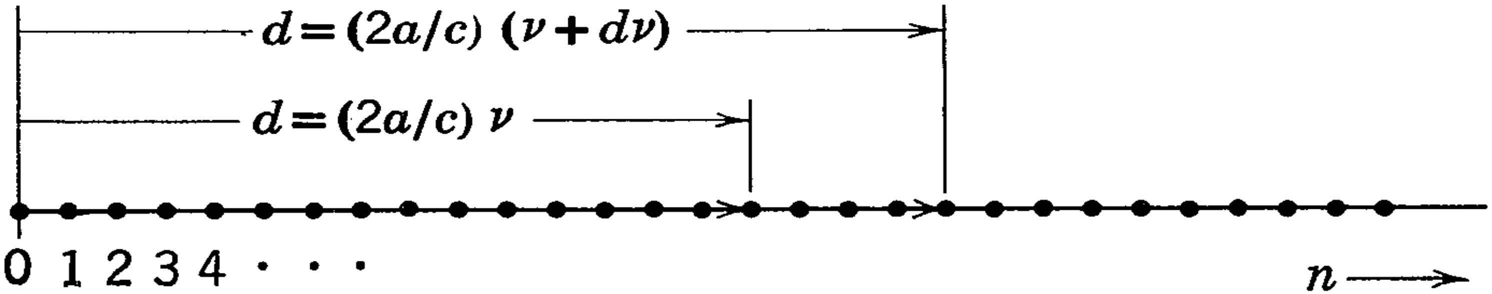
Wavelengths must be integral multiples of a that is $\left(\frac{2a}{\lambda}\right)=n$

Since we are interested in allowed frequencies only using the

relation $\nu=\frac{c}{\lambda}$ and $\left(\frac{2a}{n}\right)=\lambda$

$$\nu=\frac{cn}{2a} \qquad n=1,2,3\ldots$$

Therefore the **allowed frequencies** stands between ν and $\nu+d\nu$



This is called $N(\nu)d\nu$

$$n_1=\frac{2a}{c}\nu \qquad n_2=\frac{2a}{c}(\nu+d\nu)$$

$$N(\nu)d\nu=\frac{4a}{c}d\nu$$

The additional factor of 2 to account for 2 polarization of wave with frequency ν

Now taking the case of three dimensional space any arbitrary wave have three independent components in three orthogonal directions

Let the standing wave is in an arbitrary direction that makes angles α , β , γ with x , y , z directions

$$\left(\frac{2x}{\lambda_x}\right)=n_x \qquad \left(\frac{2y}{\lambda_y}\right)=n_y \qquad \left(\frac{2z}{\lambda_z}\right)=n_z$$

$$n_x=1,2,3\ldots \qquad n_y=1,2,3\ldots \qquad n_z=1,2,3\ldots$$

$$\left(\frac{2a}{\lambda}\right)\cos\alpha=n_x \qquad \left(\frac{2a}{\lambda}\right)\cos\beta=n_y \qquad \left(\frac{2a}{\lambda}\right)\cos\gamma=n_z$$

Squaring and adding both sides of the equation

Squaring and adding both sides of the equation

$$\left(\frac{2a}{\lambda}\right)^2\left(\cos^2\alpha+\cos^2\beta+\cos^2\gamma\right)=\left(n_x^2+n_y^2+n_z^2\right)$$

Direction cosine have the property

$$\left(\cos^2\alpha+\cos^2\beta+\cos^2\gamma\right)=1$$

Therefore $\quad \left(\frac{2a}{\lambda}\right)=\sqrt{n_x^2+n_y^2+n_z^2}$

Now we get the allowed frequencies in the three dimensional case

$$\nu=\frac{c}{\lambda}=\frac{c}{2a}\sqrt{n_x^2+n_y^2+n_z^2}, \qquad \left(\frac{2a}{\sqrt{n_x^2+n_y^2+n_z^2}}\right)=\lambda$$

In this case also the number of allowed frequencies are given by

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$$N(\nu)\,d\nu$$

In this case also the number of allowed frequencies are given by

$$\nu=\frac{c}{2a}\sqrt{n_x^2+n_y^2+n_z^2}=\frac{c}{2a}r$$ where $$r=\sqrt{n_x^2+n_y^2+n_z^2}$$

$$r=\frac{2a}{c}\nu$$

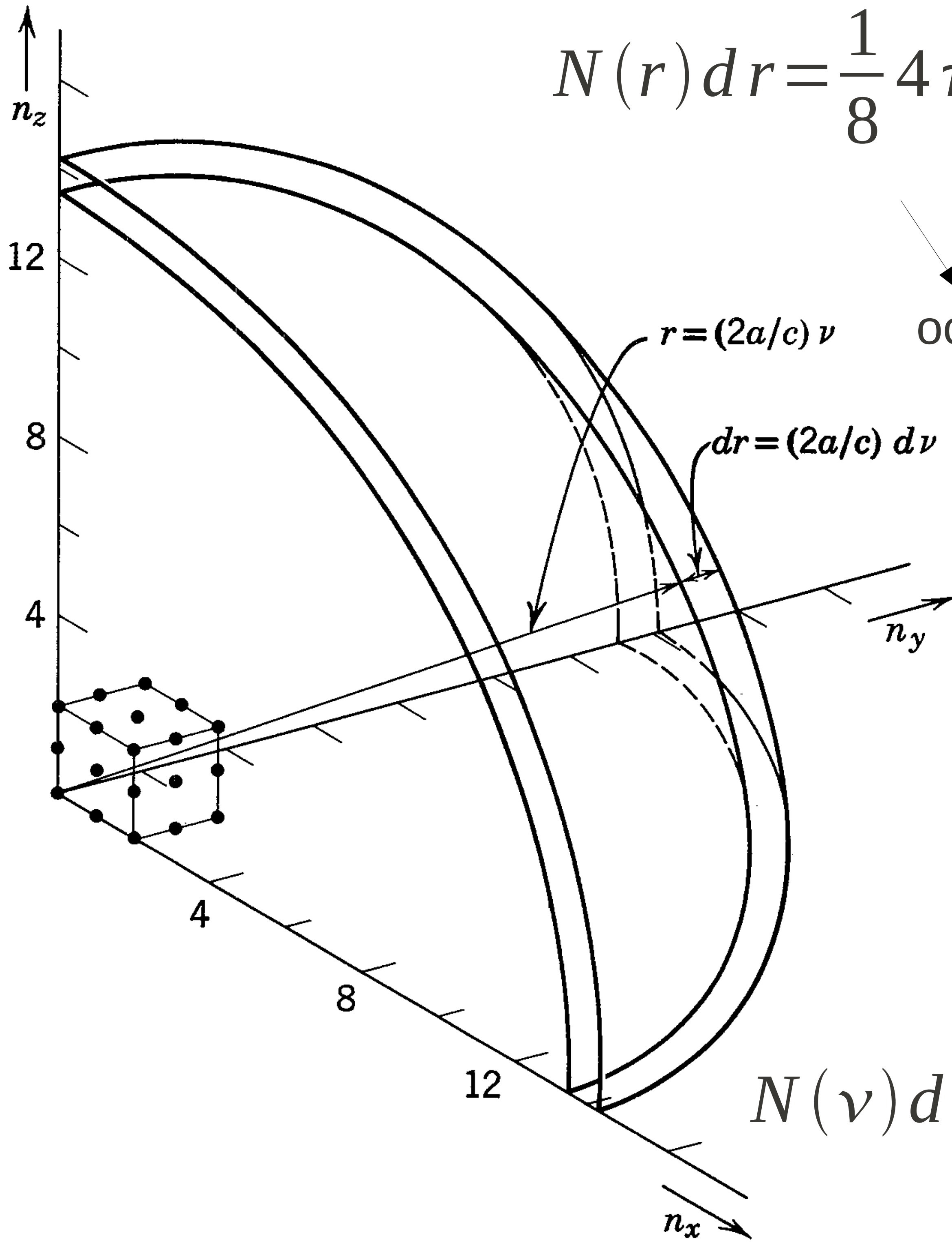
The number of frequencies allowed between $N(\nu)\,d\nu$ is equal to the number of points in the shell $N(r)\,dr$ between r and $r+dr$

$N(r)\,dr$ equal to the volume enclosed in the positive octant of the 3d space

$$N(r)\,dr=\frac{1}{8}4\pi r^2 dr=\frac{\pi r^2 dr}{2}$$

octant

area of sphere



where

$$r=\frac{2a}{c}\nu$$

$$dr=\frac{2a}{c}d\nu$$

using

$$N(r)\,dr=N(\nu)\,d\nu$$

$$N(\nu)\,d\nu=\frac{\pi}{2}\left(\frac{2a}{c}\right)^2\nu^2\left(\frac{2a}{c}\right)d\nu$$

$$N(\nu)d\nu = \frac{\pi}{2}\left(\frac{2a}{c}\right)^2 \nu^2 \left(\frac{2a}{c}\right) d\nu$$

$$= \frac{\pi}{2}\left(\frac{2a}{c}\right)^3 \nu^2 d\nu$$

In this case also two modes of the electromagnetic wave must be considered

$$N(\nu)d\nu = \pi\left(\frac{2a}{c}\right)^3 \nu^2 d\nu$$

We have computed number of standing waves at a frequency interval, if considered as a particle they are at thermal equilibrium with the walls of the container.

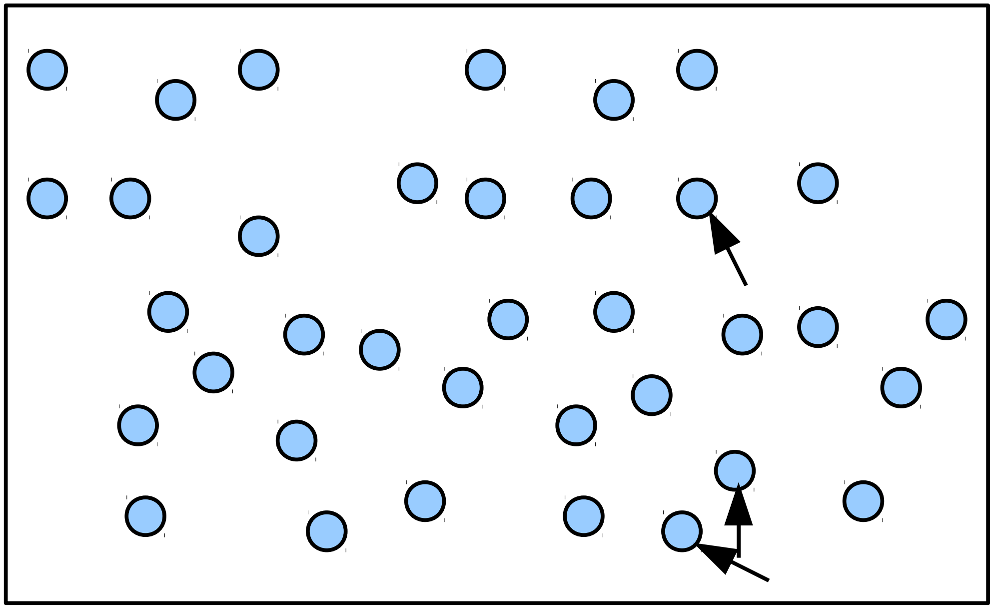
In order to obtain the average energy of the radiation we need a result from **statistical mechanics for a collection of classical particles such as gas**

For collection of classical particles the probability of finding a particle with a particular energy E is given by Maxwell-Boltzmann distribution $P(E)$

$$P(E)=e^{-\frac{E}{k_B T}}$$

(not normalized)

k_B Boltzmann constant



for continuous distribution of energies

$$\bar{E}=\frac{\int_0^\infty E\,P(E)\,dE}{\int_0^\infty P(E)\,dE}$$

Condition for normalization

total probability is equal to 1 for normalized function

for descrete distribution of energies

$$\bar{E}=\frac{\sum_0^\infty E_i P(E_i)}{\sum_0^\infty P(E_i)}$$

sum of probability is equal to 1for normalized function assuming continuous distribution

$$\bar{E}=\frac{\int_0^\infty E\,P(E)\,dE}{\int_0^\infty P(E)\,dE}$$

$$P(E)=e^{-\frac{E}{k_B T}}$$

$$\int_0^\infty E\,P(E)\,dE=\int_0^\infty E\,e^{-\frac{E}{k_B T}}\,dE$$

$$\beta=\frac{1}{k_B T}$$

$$=\int_0^\infty E\,e^{-\beta E}\,dE$$

$$=-\frac{\partial}{\partial\beta}\int_0^\infty e^{-\beta E}\,dE$$

$$=-\frac{\partial}{\partial\beta}\frac{e^{-\beta E}\Big|_0^\infty}{-\beta}\quad=-\frac{\partial}{\partial\beta}\left(\frac{-1}{-\beta}\right)\quad=\frac{1}{\beta^2}$$

$$\bar{E} = \frac{\int_0^\infty E\,P(E)\,dE}{\int_0^\infty P(E)\,dE}$$

$$\int_0^\infty E\,P(E)\,dE = \frac{1}{\beta^2}$$

$$\beta = \frac{1}{k_B T}$$

$$\int_0^\infty P(E)\,dE = \int_0^\infty e^{-\beta E}\,dE$$

$$= \frac{e^{-\beta E}\Big|_0^\infty}{-\beta} = \left(\frac{-1}{-\beta}\right) = \left(\frac{1}{\beta}\right)$$

$$\bar{E} = \frac{\frac{1}{\beta^2}}{\frac{1}{\beta}} = \frac{1}{\beta} = k_B T$$

average energy per particle

Probability of finding a particular particles with a certain energy is $k_B T$

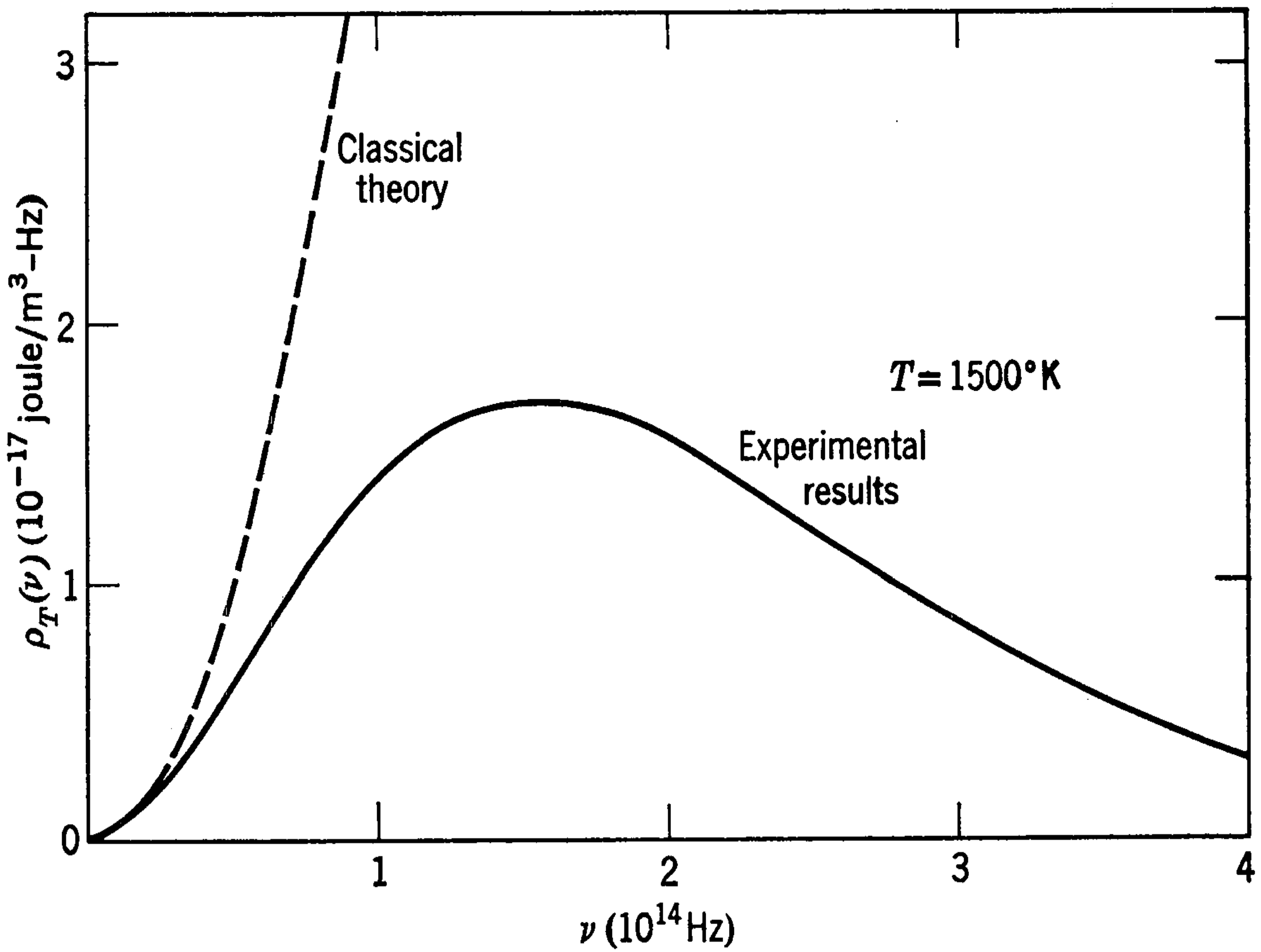
We assume that each frequency correspond to a particular particle for distribution of $N(\nu)d\nu$ particles the probability per unit volume a^3 is give by

$$N(\nu)d\nu=\pi\left(\frac{2a}{c}\right)^3\nu^2 d\nu$$

$$\rho_T d\nu=N(\nu)d\nu=\left(\frac{8\pi\nu^2 a^3 k_B T}{c^3}\right)d\nu/a^3$$

$$=\left(\frac{8\pi\nu^2 k_B T}{c^3}\right)d\nu$$

Result is independent of the shape of the cavity – this is the Rayleigh-Jeans formula for blackbody radiation



The unrealistic behavior predicted by classical theory is called **"ultraviolet catastrophe"**

Planck's theory of cavity radiation

Rayleigh- Jean's law give satisfactory explanation at low frequencies, that means

$$\bar{E} \underset{\nu \to 0}{\longrightarrow} k_B T$$

average total energy approaches $k_B T$ at low frequencies

The discrepancy at high frequency vanish if we get a relation

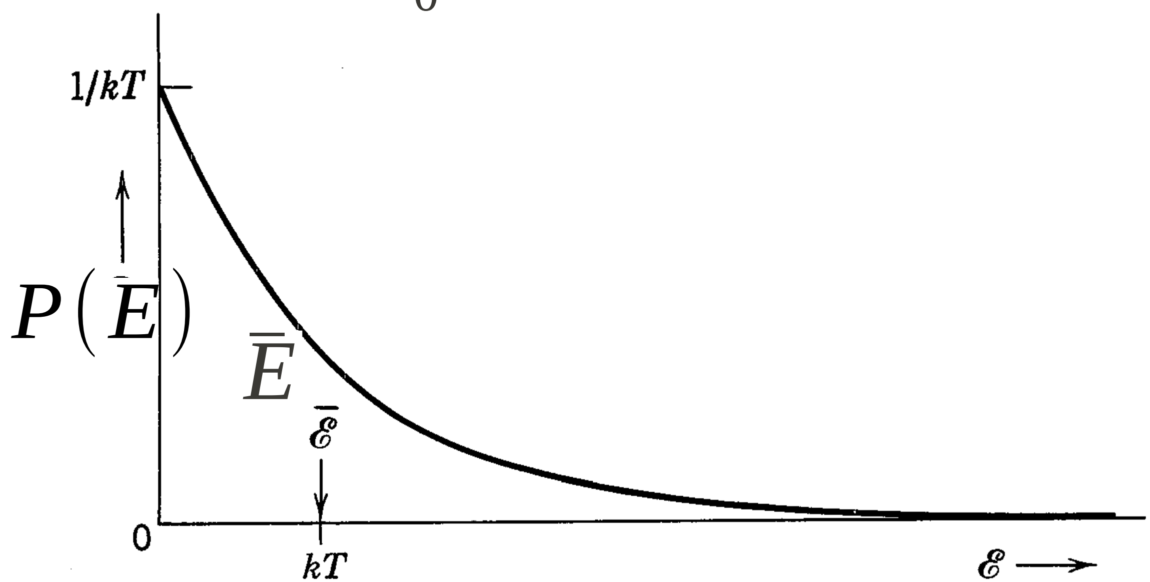
$$\bar{E} \underset{\nu \to \infty}{\longrightarrow} 0$$

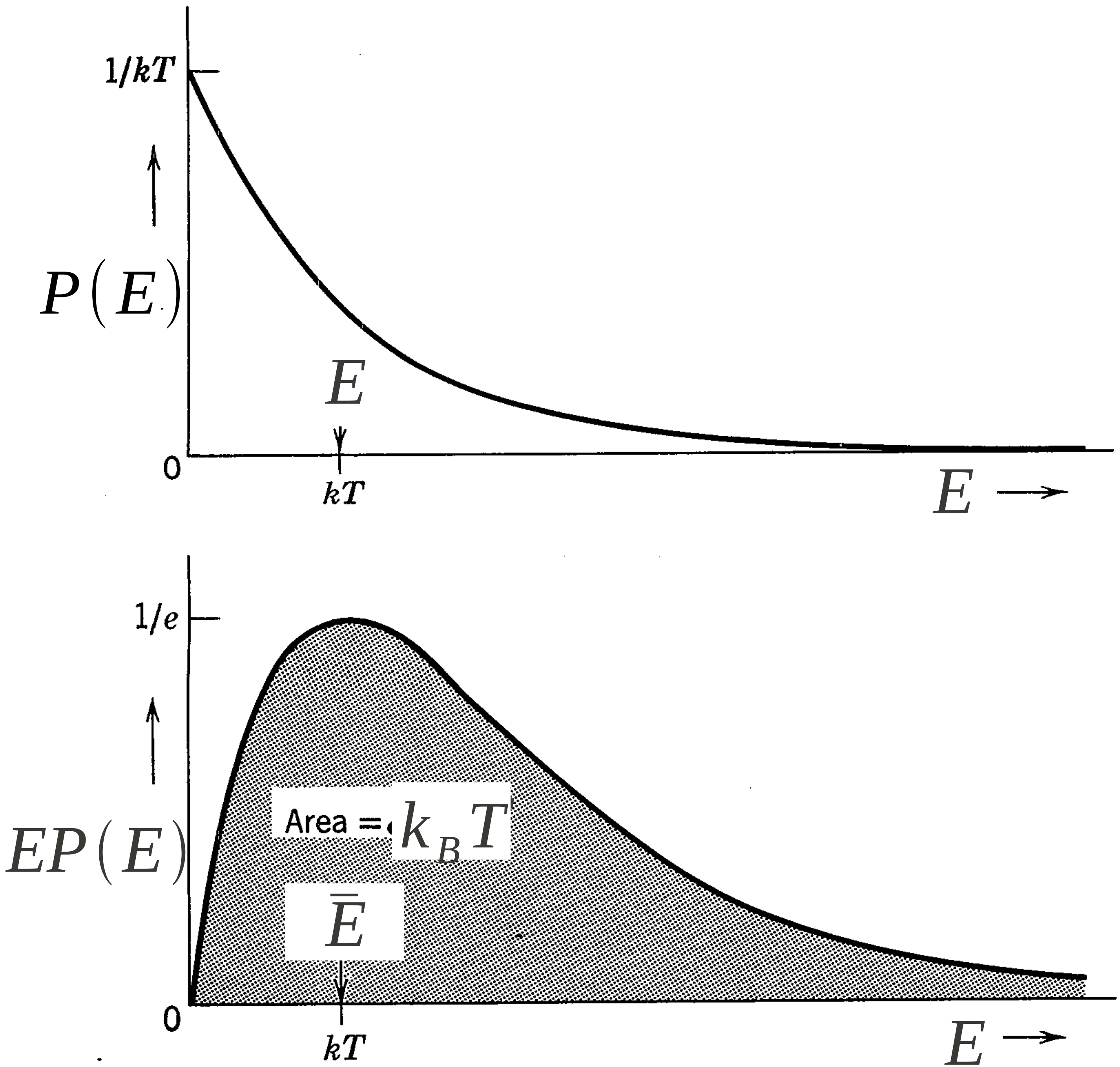
This assumption is against the law of equipartition of energy where each particle have energy independent of frequency

The normalized Boltzmann distribution is given by

$$P(E) = \frac{e^{-\frac{E}{k_B T}}}{k_B T}$$

Since $$\int_0^\infty P(E)\,dE = \int_0^\infty e^{-\beta E}\,dE = k_B T$$




1/kT
P(E)
E
0
kT
E
1/e
EP(E)
Area = kBT
Ē
0
kT
E

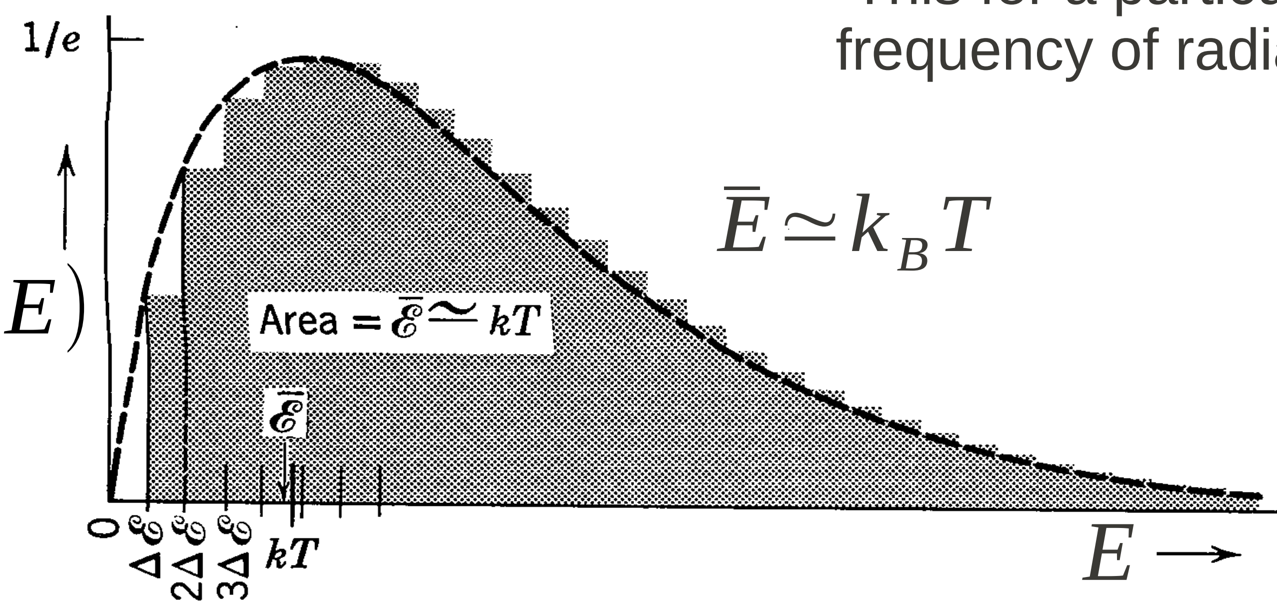
Planck's contribution in identify the fact the energy can take only discrete values instead of a continuum of values that is

$$E = 0, \Delta E, 2\Delta E, 3\Delta E.....$$

as allowed values of energy with ΔE as constant energy interval

This for a particular frequency of radiation ν

$E\,P(E)$

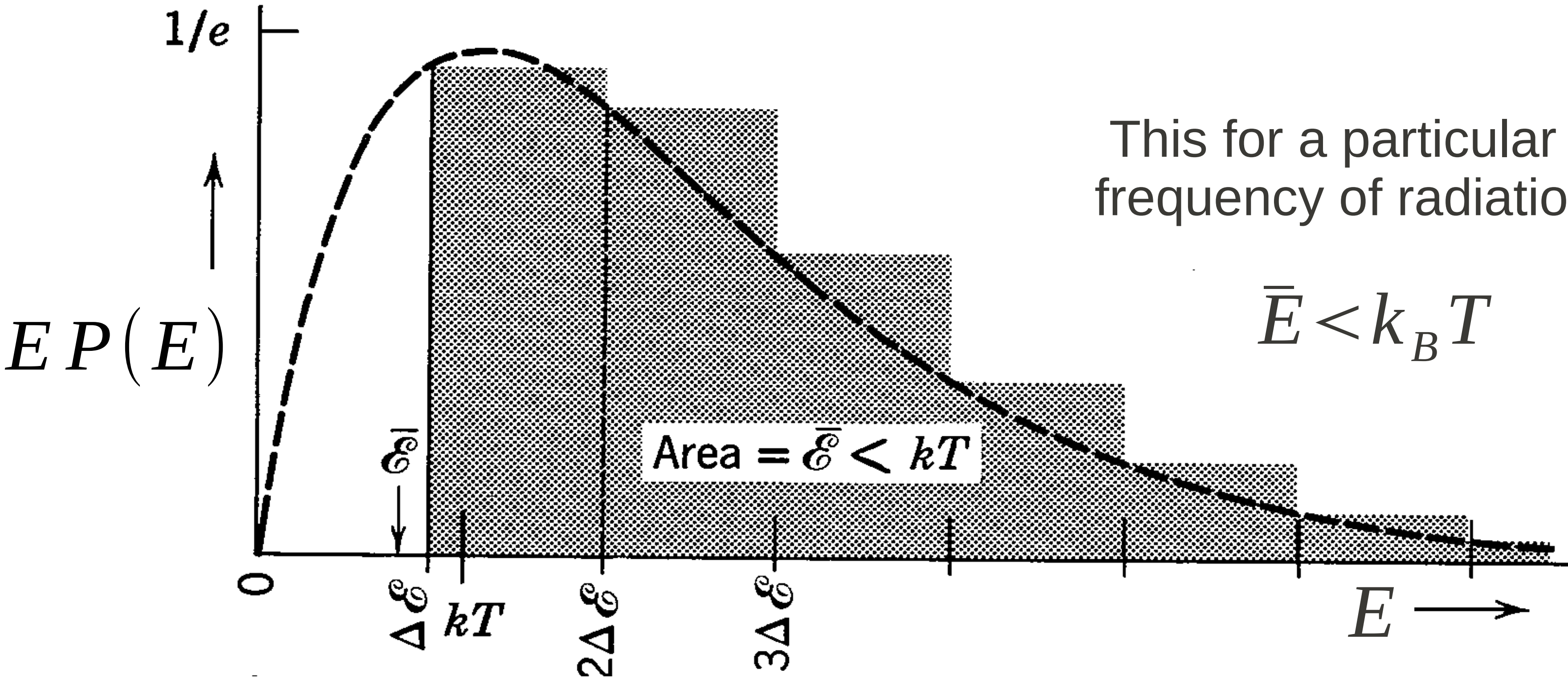


$\bar{E} \simeq k_B T$

$E \longrightarrow$

Take the case when energy interval is small compared to $k_B T$ in the case most of the are is covered as in the case of continuous distribution

$$\Delta E \ll k_B T \qquad \bar{E} \simeq k_B T$$



This for a particular frequency of radiation ν

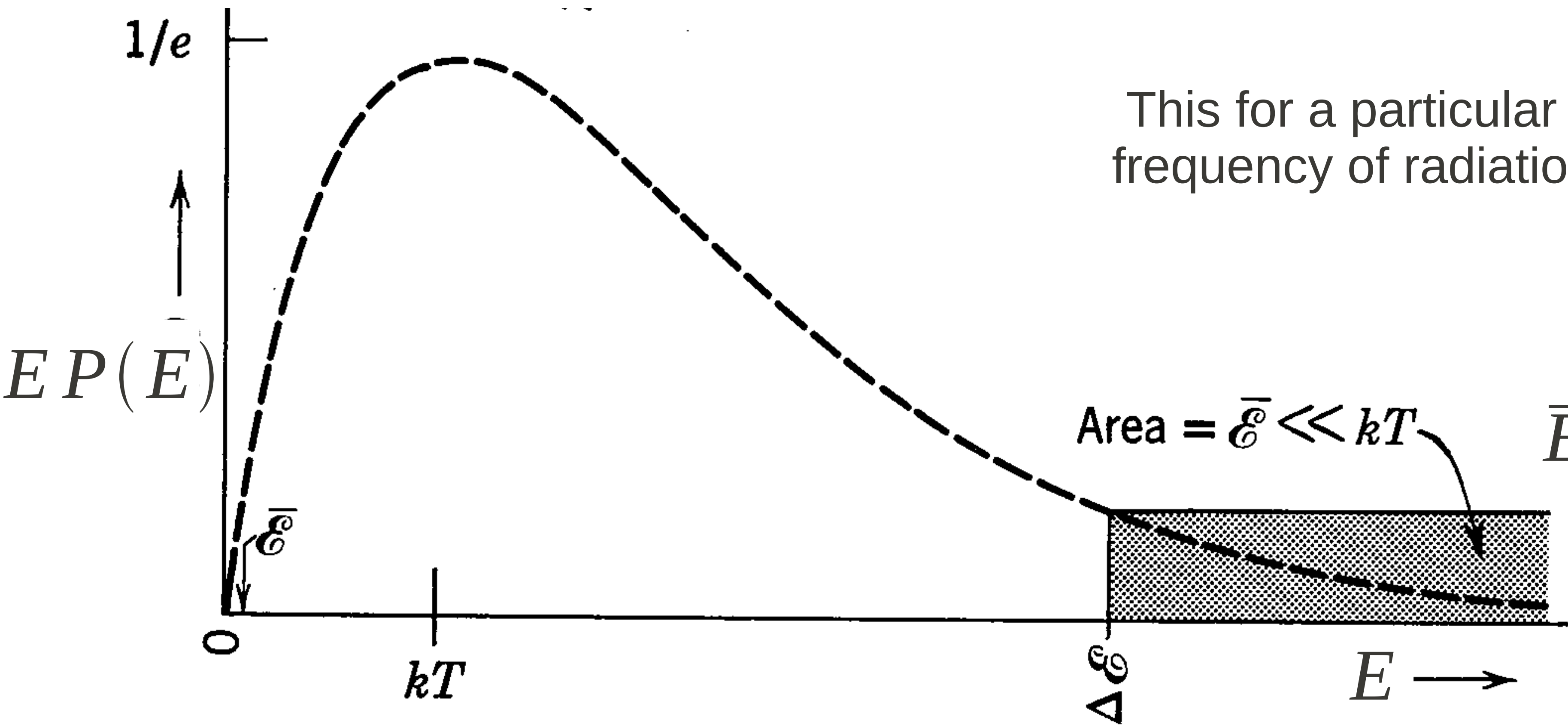
$$\bar{E} < k_B T$$

Take the case when energy interval is comparable to $k_B T$ in this case most of the are is covered as in the case of continuous distribution

$$\Delta E \simeq k_B T$$

$$\bar{E} < k_B T$$

This for a particular frequency of radiation ν



$\bar{E} \ll k_B T$

Take the case when energy interval is greater than $k_B T$ in this case most of the are is covered as in the case of continuous distribution

$$\Delta E \gg k_B T$$

$$\bar{E} \simeq 0$$

In order to satisfy the limiting relations $\bar{E} \underset{\nu \to 0}{\longrightarrow} k_B T$ and $\bar{E} \underset{\nu \to \infty}{\longrightarrow} 0$

We have to keep energy as a increasing function of frequency

$$\Delta E = f(\nu)$$

The simplest possible such relation obtained by Planck is

$$\Delta E = h\nu$$

is the proportionality constant

the numerical work of experimental spectrum allowed Planck to determine the constant of proportionality

$$h = 6.63 \times 10^{-34}\ joule - sec$$

Constant is known as **Planck's constant**

As the energy distribution become discrete we need to evaluate the sum for finding the average energy of the system

$$\bar{E}=\frac{\sum_{i=0}^{\infty} E_i P(E_i)}{\sum_{i=0}^{\infty} P(E_i)}$$

$$P(E)=\frac{e^{-\frac{E}{k_B T}}}{k_B T}$$

With introduction of Planck's constant the discrete distribution

$$E=0, \Delta E, 2\Delta E, 3\Delta E.....$$

becomes

$$E=0, h\nu, 2h\nu, 3h\nu.....$$

Now evaluating Boltzmann distribution for the discrete case

$$\bar{E}=\frac{\sum_{n=0}^{\infty} \frac{nh\nu}{k_B T} e^{-\frac{nh\nu}{k_B T}}}{\sum_{n=0}^{\infty} \frac{1}{k_B T} e^{-\frac{nh\nu}{k_B T}}}$$

$$\bar{E}=\frac{\sum_{n=0}^{\infty}\frac{nh\nu}{k_B T}e^{-\frac{nh\nu}{k_B T}}}{\sum_{n=0}^{\infty}\frac{1}{k_B T}e^{-\frac{nh\nu}{k_B T}}}$$

By substitution we may write this equation in reduced form

$$\bar{E}=k_B T\frac{\sum_{n=0}^{\infty}n\alpha e^{-n\alpha}}{\sum_{n=0}^{\infty}e^{-n\alpha}} \quad \text{where} \quad \alpha=\frac{h\nu}{k_B T}$$

We can simplify this equation using following transformations

$$=k_B T\frac{-\alpha\sum_{n=0}^{\infty}\frac{d}{d\alpha}e^{-n\alpha}}{\sum_{n=0}^{\infty}e^{-n\alpha}}$$

$$\bar{E} = k_B T \frac{-\alpha \sum_{n=0}^{\infty} \frac{d}{d\alpha} e^{-n\alpha}}{\sum_{n=0}^{\infty} e^{-n\alpha}}$$

$$\bar{E} = k_B T \frac{-\alpha \frac{d}{d\alpha} \sum_{n=0}^{\infty} e^{-n\alpha}}{\sum_{n=0}^{\infty} e^{-n\alpha}}$$

$$\alpha = \frac{h\nu}{k_B T}$$

Converting into differential of logarithmic form

$$\bar{E} = -k_B T \alpha \frac{d}{d\alpha} \log_e \left(\sum_{n=0}^{\infty} e^{-n\alpha} \right)$$

$$\bar{E} = -h\nu \frac{d}{d\alpha} \log_e \left(\sum_{n=0}^{\infty} e^{-n\alpha} \right)$$

In this equation the summation may be evaluated as

$$\bar{E} = -h\nu \frac{d}{d\alpha} \log_e \left(\sum_{n=0}^{\infty} e^{-n\alpha} \right)$$

$$\sum_{n=0}^{\infty} e^{-n\alpha} = 1 + e^{-\alpha} + e^{-2\alpha} + \ldots\ldots\ldots\ldots$$

$$\left(1 - e^{-\alpha}\right)^{-1} = 1 + e^{-\alpha} + e^{-2\alpha} + \ldots\ldots\ldots\ldots$$

Therefore we may further simplify as

$$\bar{E} = -h\nu \frac{d}{d\alpha} \log_e \left(1 - e^{-\alpha}\right)^{-1}$$

$$\bar{E} = h\nu \frac{d}{d\alpha} \log_e \left(1 - e^{-\alpha}\right)$$

$$\bar{E} = h\nu \frac{e^{-\alpha}}{\left(1 - e^{-\alpha}\right)}$$

$$\bar{E} = \frac{h\nu}{\left(e^{\alpha} - 1\right)}$$

$$\alpha = \frac{h\nu}{k_B T}$$

$$\bar{E} = \frac{h\nu}{\left(e^{\frac{h\nu}{k_B T}} - 1\right)}$$

Average energy of the electro magnetic standing wave of frequency ν

This equation has correct behavior in the limits

$$\bar{E} \underset{\nu \to 0}{\longrightarrow} k_B T$$

In the low frequency limit

$$\bar{E} = \frac{h\nu}{\left(1 + \frac{h\nu}{k_B T} + ... - 1\right)}$$

$$\bar{E} \underset{\nu \to 0}{\longrightarrow} k_B T$$

$$\simeq \frac{h\nu}{\left(\frac{h\nu}{k_B T}\right)} = k_B T$$

In the high frequency limit $\bar{E} \underset{\nu \to \infty}{\longrightarrow} 0$

$$\bar{E} = \frac{h\nu}{\left(e^{\frac{h\nu}{k_B T}} - 1\right)}$$

$$= \frac{h\nu}{\infty} = 0$$

These are desired limiting behavior obtained from the Planck's distribution

When this distribution law is multiplied by distribution of number of allowed frequencies we get the **Planck's black body spectrum**

$$\rho_T(\nu)\,d\nu = \left(\frac{8\pi\nu^2}{c^3}\right)\frac{h\nu}{\left(e^{\frac{h\nu}{k_B T}}-1\right)}\,d\nu$$

From the standing wave modes

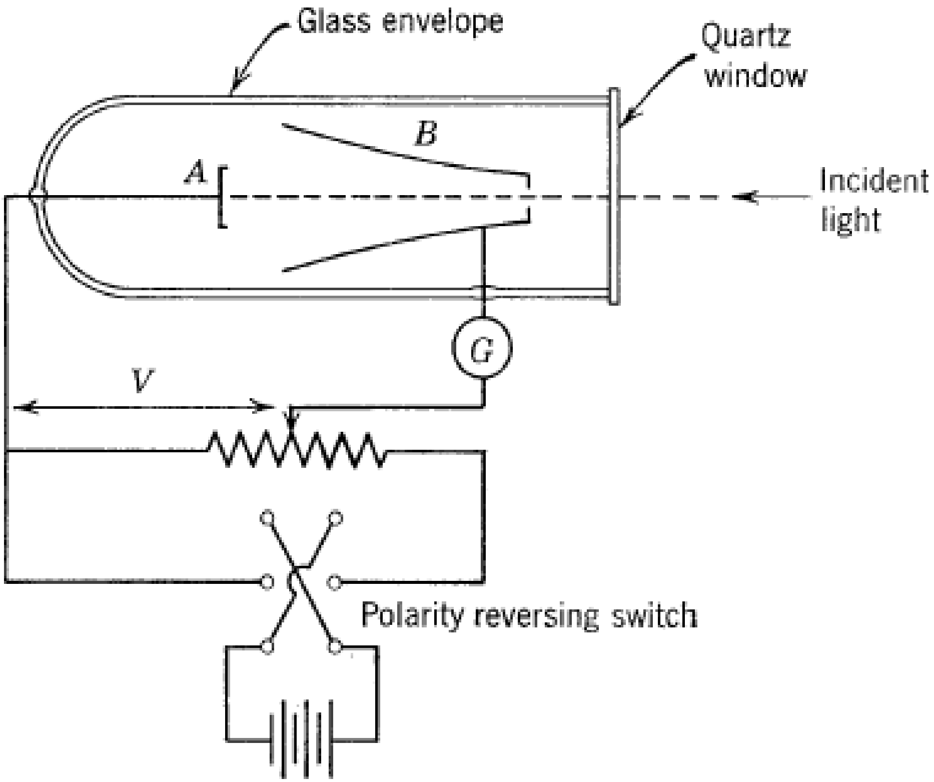
discreteness of the energy values

Quantization of energy of fundamental particles. The energy of the electromagnetic wave theory is defined per unit volume. Here the electromagnetic wave consist of packets of energy with is equal to Planck's constant constant times the frequency – this is the concept of quantization of energy in a wave like particle.

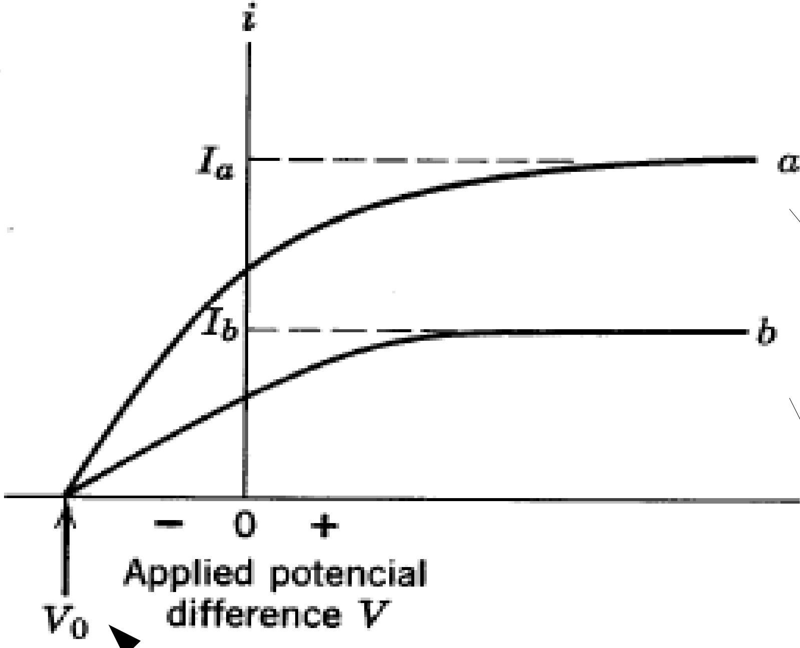
Photoelectric effect

Hertz noted that electric discharge between two electrodes are enhanced when ultraviolet light falls on one of the electrodes.

Later experiments showed this discharge is due to ejection of electrons from the surface of the electrodes.



The effect of ejection electrons from a surface by the radiation is called **photoelectric effect and ejected electrons are called photoelectrons.**

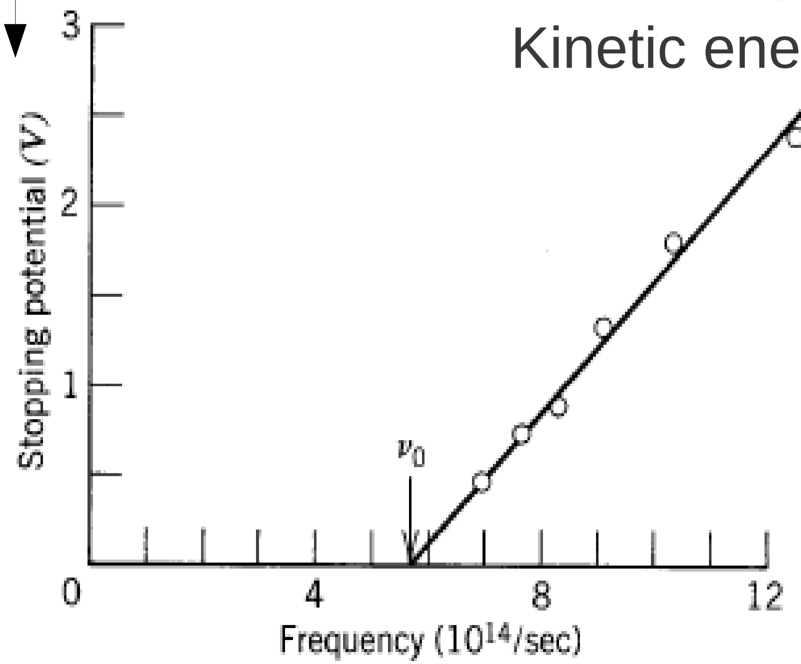


Current voltage characteristics during photoelectric effect

Initial voltage is independent of intensity of light

The I-V characteristics at two intensities

Kinetic energy maximum $K_{max} = eV_0$



Stopping potential is directly proportional to the intensity

According to the classical electro magnetic theory the oscillating electric vector of the electric field $\vec{E}(t)$ increases with intensity of the radiation.

The acceleration electron must be proportional to $e\vec{E}(t)$ which should increase with intensity.

In photoelectric effect the voltage or kinetic energy is independent of intensity.

In reality the photoelectric voltage only depends on frequency ν below the stopping voltage no photoelectric effect occurs no matter what intensity the light has.

When electron is accelerated by the electric field of the radiation, since the cross-section of the atoms are small it would take considerable amount of time between instant of arrival of radiation and ejection of the photon, however, in photoelectric effect is instantaneous.

Einstein's explanation of the photo-electric effect

Planck believed the quantization scheme he proposed works only in the confinement of a cavity, that is, the electromagnetic radiation spreads like a wave outside cavity

Einstein hypothesized that electro magnetic energy moves in bundles of $h\nu$ from the source any distance. That is the energy does not spread by move energy of a beam exist as integral multiples of fundamental packet $nh\nu$.

Every photon is associated with its energy with the relation $h\nu$

With this postulates the kinetic energy of the electron emitted is given by

$$K = h\nu - w$$

energy of incident photon

Work required to eject an electron from the attractive influence of atoms on the surface

Now consider the case where the electron get ejected with a maximum kinetic energy where the interaction of electron with the surface is minimum

$$K_{max} = h\nu - w_0$$

characteristic energy of the metal called **work function** of the material

Work function is minimum energy required to extract an electron from the interaction of the surface constituted of atoms.

In this framework the intensity of the beam does not play a role in the ejection of electrons it only affect amount of current. That is the ν carry energy less than work function then no photoelectric effect occurs what ever be the intensity of the radiation

$$h\nu_0 = w_0$$

critical frequency

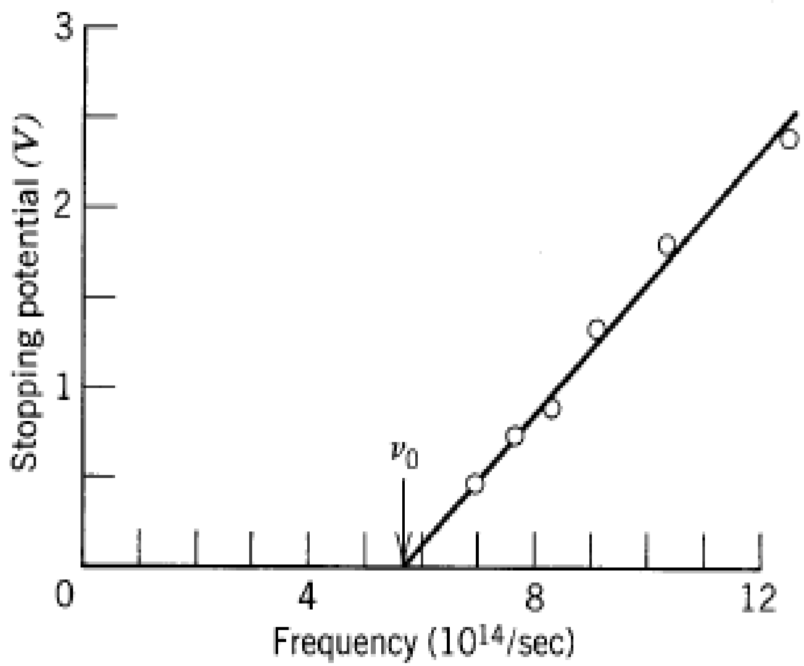
In this relation $K_{max} = h\nu - w_0$ we substitute $K_{max} = eV_0$

to get

$$eV_0 = h\nu - w_0$$

$$V_0 = \frac{h\nu}{e} - \frac{w_0}{e}$$

linear relation between frequency and stopping voltage



Slope of the curve gives the value of $\frac{h}{e}$ which is agreement with Planck's calculation h with in experimental errors

$$h = 6.4 \times 10^{-34}\ joules - sec$$

Also collision of packet and subsequent ejection of electron is instantaneous

de Broglie hypothesis – matter waves

Maurice de Broglie hypothesized that the dual, that is wave- particle, behavior of radiation applies equally well to the larger particles.

According to de Broglie for matter and for radiation as well as the total energy E of the entity is related to the frequency ν of the wave associated with the particle

$$E = h\nu$$

For matter waves concepts energy E and momentum p are connected through the Planck's constant to the frequency ν and wavelength λ in the following form called **de Broglie relation**

$$\lambda = \frac{h}{p}$$

It gives de Broglie wavelength λ associated with any matter with momentum p

Verification of de Broglie hypothesis – Experiments of Davisson and Germer

Electron is one of the smallest particle for which the de Brogle hypothesis can be verified

$$K = \frac{1}{2} m v^2$$

Wave length of an electron whose energy is 100 eV

$$K = e V_0$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2 m K}} = \frac{h}{\sqrt{2 m e V}}$$

$$= \frac{6.6 \times 10^{-34}}{\left(2 \times 9.1 \times 10^{-31} \times 100 \times 1.6 \times 10^{-19}\right)^{\frac{1}{2}}}$$

$$= 1.2 A^o$$

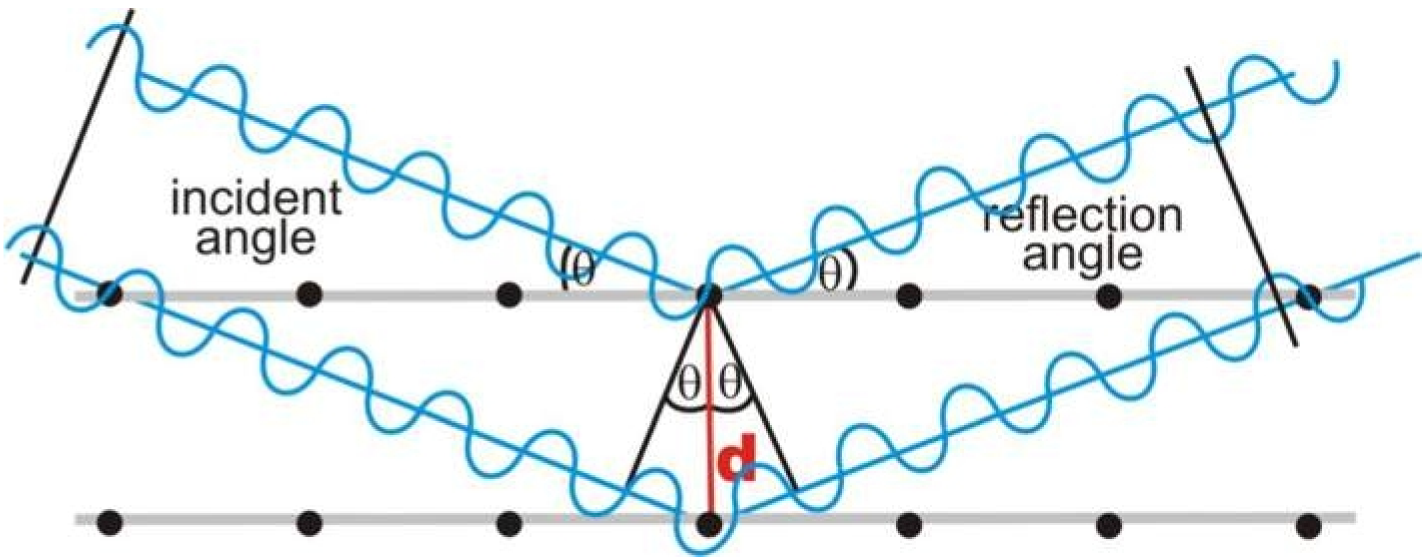
For the study of interference experiments we need optical apparatus that has comparable dimension a of the wave length λ of radiation used

When $\frac{\lambda}{a} \rightarrow 0$ the radiation behave like particles

For the smallest particle who have observable wavelength is electron that requires apparatus which has dimension of few angstroms,

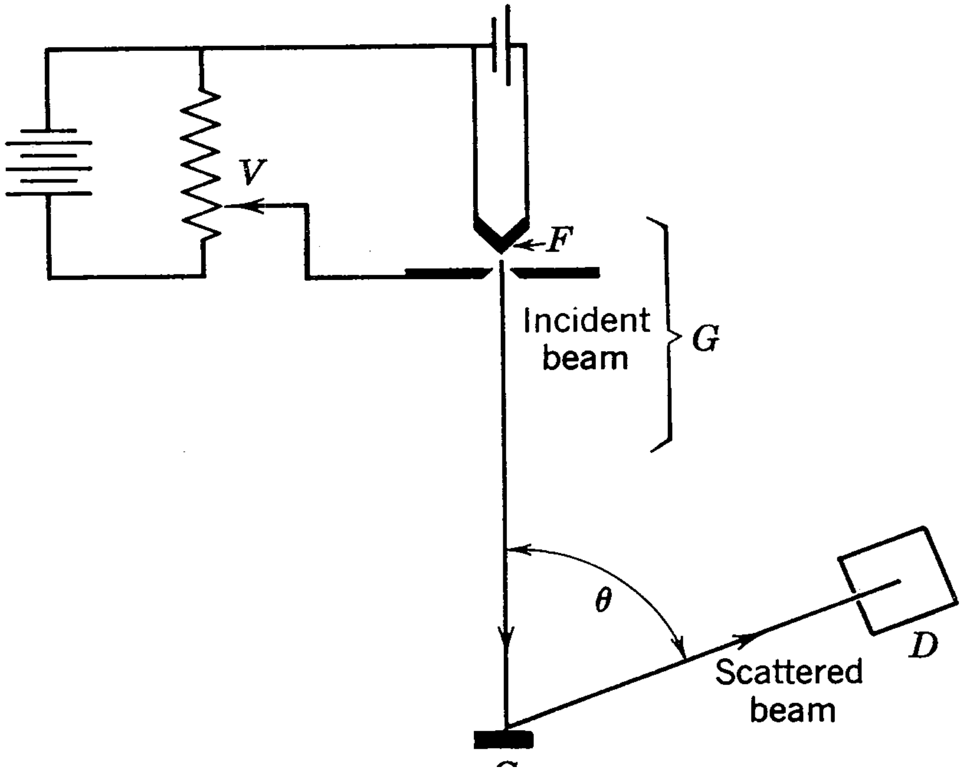
Planes of adjacent layer of atoms have dimension few angstroms are ideal for testing de Broglie hypothesis

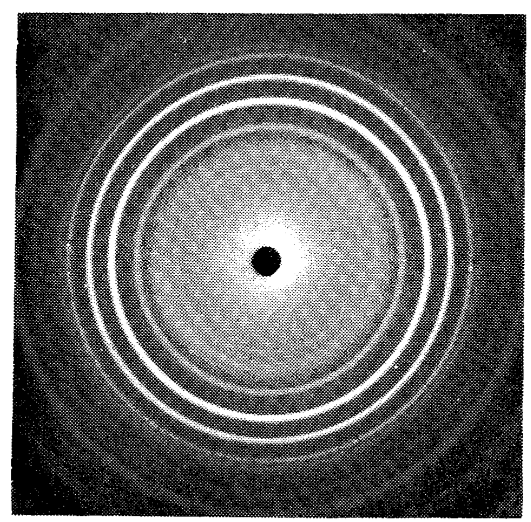
X-rays with similar wave-length is known to diffraction pattern that obeys Bragg's Law

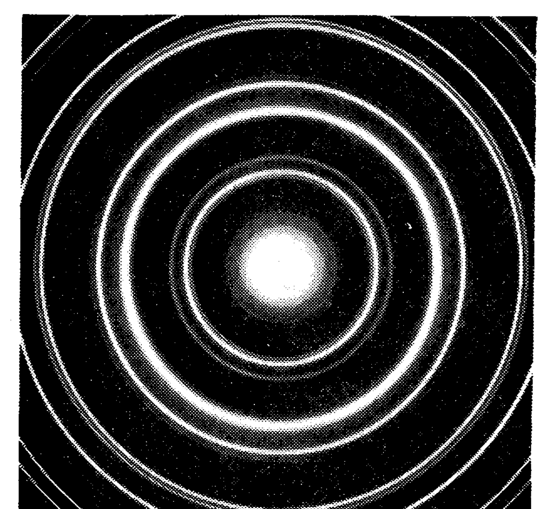


$$2d\sin\theta = n\lambda$$

Electrons from a heated element is accelerated under a potential to have a specific wavelength is used to stud diffraction patters. The peaks of the pattern matches with the predicted wavelength from hypothesis of de Broglie

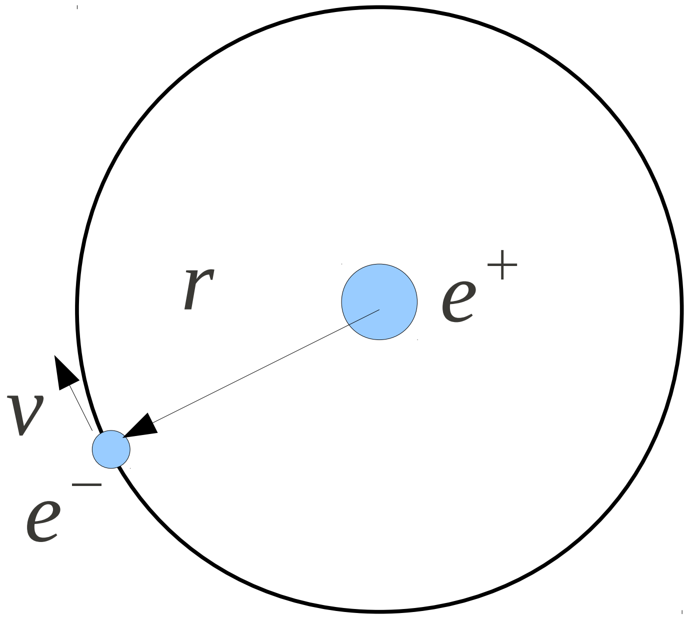






Classical unstable atoms and Bohr model of atom

According to classical electromagnetic theory atoms when charges are accelerated they radiate and energy is dissipated from the system



This is therefore different from the model of solar system where planets move in orbits around sun or star

According the classical physics the electron while accelerating produce radiation and radius of the orbits shrink and finally collapses into nucleus

The experimental results contradict the classical postulates

Atoms are stable, there is no experimental evidence suggesting their annihilation

The spectra produced by the atoms are not continuous as required by the continuous collapse of an electron – instead radiation from the atoms appears as lines.

Bohr postulates

1) Electrons moves in circular orbits

2)Contrary to the expectation from the classical mechanics the orbits should have angular momentum that is integral multiples of

$$n\hbar = n\frac{h}{2\pi}$$

$$L = n\hbar$$

3) The energy of orbits remains constant

3) Electro magnetic energy is emitted when an electron which is moving initially in the level E_i jumps discontinuously to another energy level E_f , the emitted radiation has frequency

$$\nu = \frac{E_i - E_f}{h}$$

Postulates of Bohr give quantitative way to quantify the spectra produced by atoms

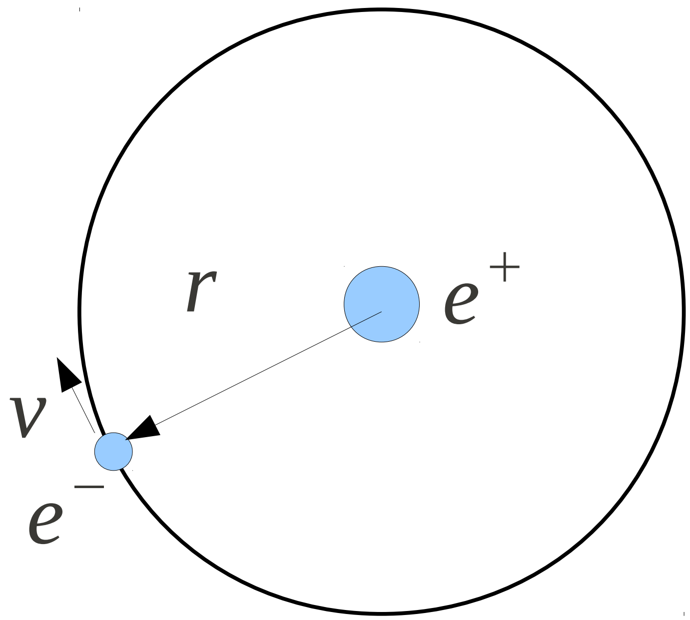
The conditions for mechanical stability in an atom is given by

Atomic number

$$\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r^2} = m\frac{v^2}{r}$$

velocity

radius



Applying the quantization condition

$$mvr = n\hbar$$

$$\Rightarrow v = \frac{n\hbar}{mr}$$

We have the relation

$$\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r^2} = m\frac{v^2}{r}$$

$$\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r} = mv^2$$

We have the relation

$$\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r}=mv^2$$

$$v=\frac{n\hbar}{mr}$$

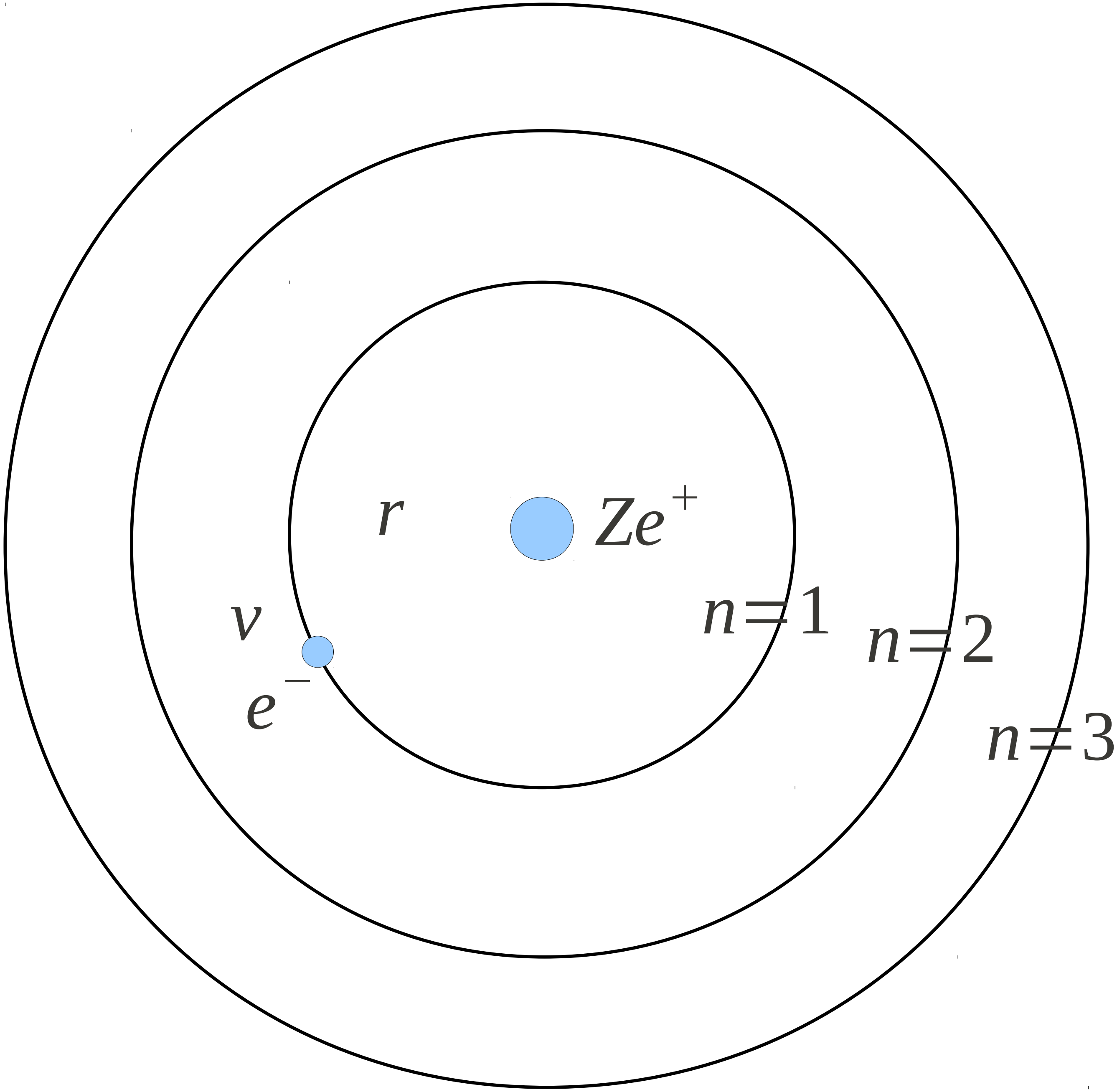
$$\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r}=m\left(\frac{n\hbar}{mr}\right)^2$$

$$\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r}=m\left(\frac{n\hbar}{mr}\right)^2$$

Rearranging for radius

$$r=4\pi\epsilon_0\left(\frac{n^2\hbar^2}{mZe^2}\right)$$

Allowed circular orbits should satisfy the condition, there fore all radius are not permissible the radius reduced



$$v=\frac{n\hbar}{mr}$$

$$r=4\pi\epsilon_0\left(\frac{n^2\hbar^2}{mZe^2}\right)$$

by substitution

$$v=\frac{n\hbar}{mr}=\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{n\hbar}$$

Now we have position and velocity with condition for quantization – we may now compute total energy of the system

The total energy of the electron nucleus system may be computed from their classical definitions

Potential energy of the system is given by

$$V=-\int_{r}^{\infty}\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r^2}dr$$

$$=-\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r}$$

We have $$\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r}=mv^2$$

Kinetic energy of the system is given by

$$\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{2r}=\frac{1}{2}mv^2=K$$

Total energy of the system is given by

$$E = K + V = \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{2r} - \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r}$$

$$= -\left(\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{2r}\right)$$

$$= -K$$

Using condition of quantization on radius $r = 4\pi\epsilon_0\left(\frac{n^2\hbar^2}{mZe^2}\right)$

$$E = -\left(\frac{1}{(4\pi\epsilon_0)^2}\frac{mZ^2e^4}{2\hbar^2}\right)\frac{1}{n^2}$$

The quantization of the orbital angular momentum of the electron leads to a quantization of the total energy.

It now possible test the Bohr postulate that radiation when emitted the frequency emitted is given by

$$\nu = \frac{E_i - E_f}{h}$$

Energy of the n'th energy level is given by

$$E_n = -\left(\frac{1}{(4\pi\epsilon_0)^2} \frac{mZ^2 e^4}{2\hbar^2} \right) \frac{1}{n^2}$$

The frequency of the radiation is now given by

$$\nu = \frac{E_i - E_f}{h}$$

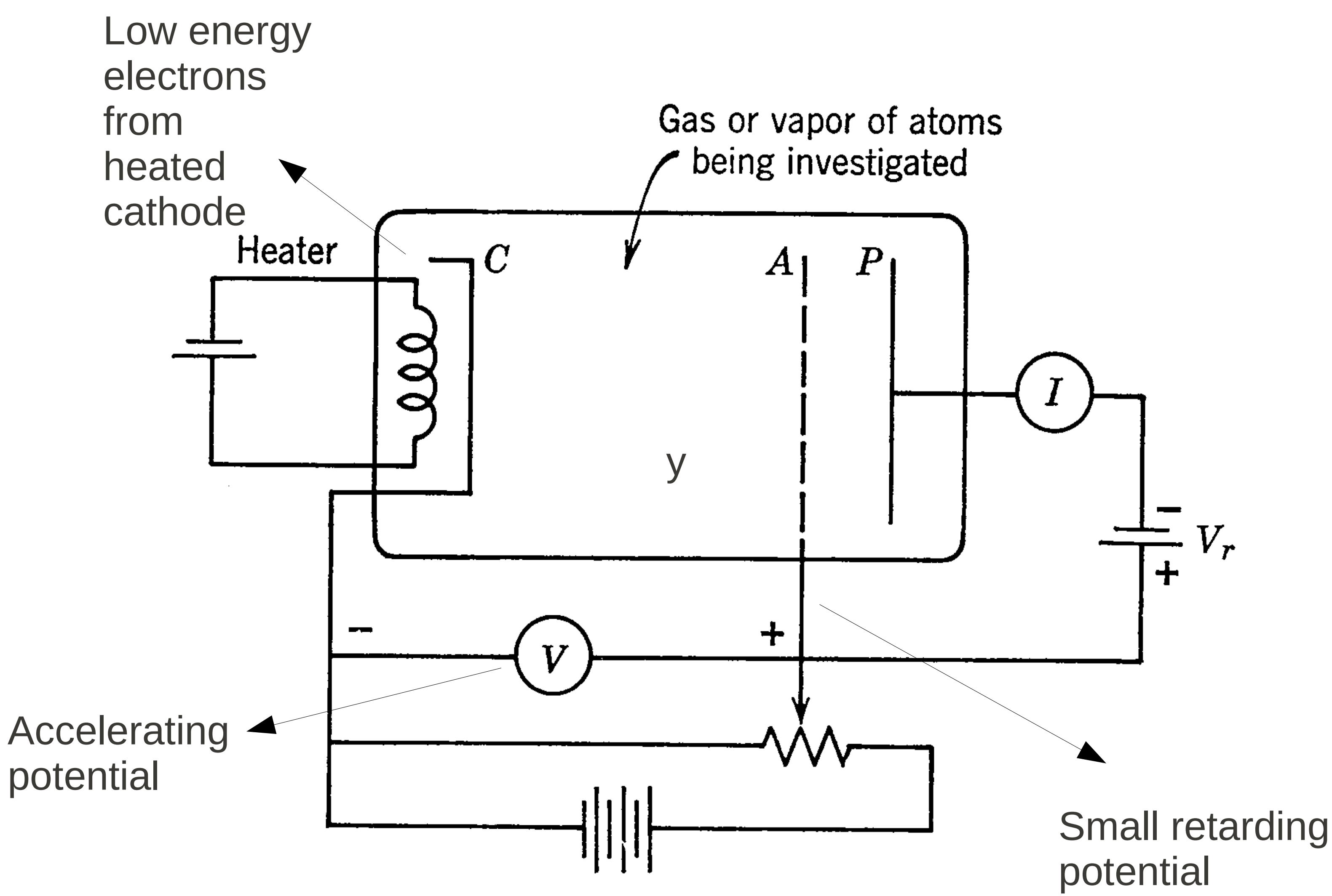
$$= \left(\frac{1}{(4\pi\epsilon_0)^2} \frac{mZ^2 e^4}{4\pi\hbar^3} \right) \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

$$\nu = \left(\frac{1}{\left(4\pi\epsilon_0\right)^2} \frac{m Z^2 e^4}{4\pi\hbar^3} \right) \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

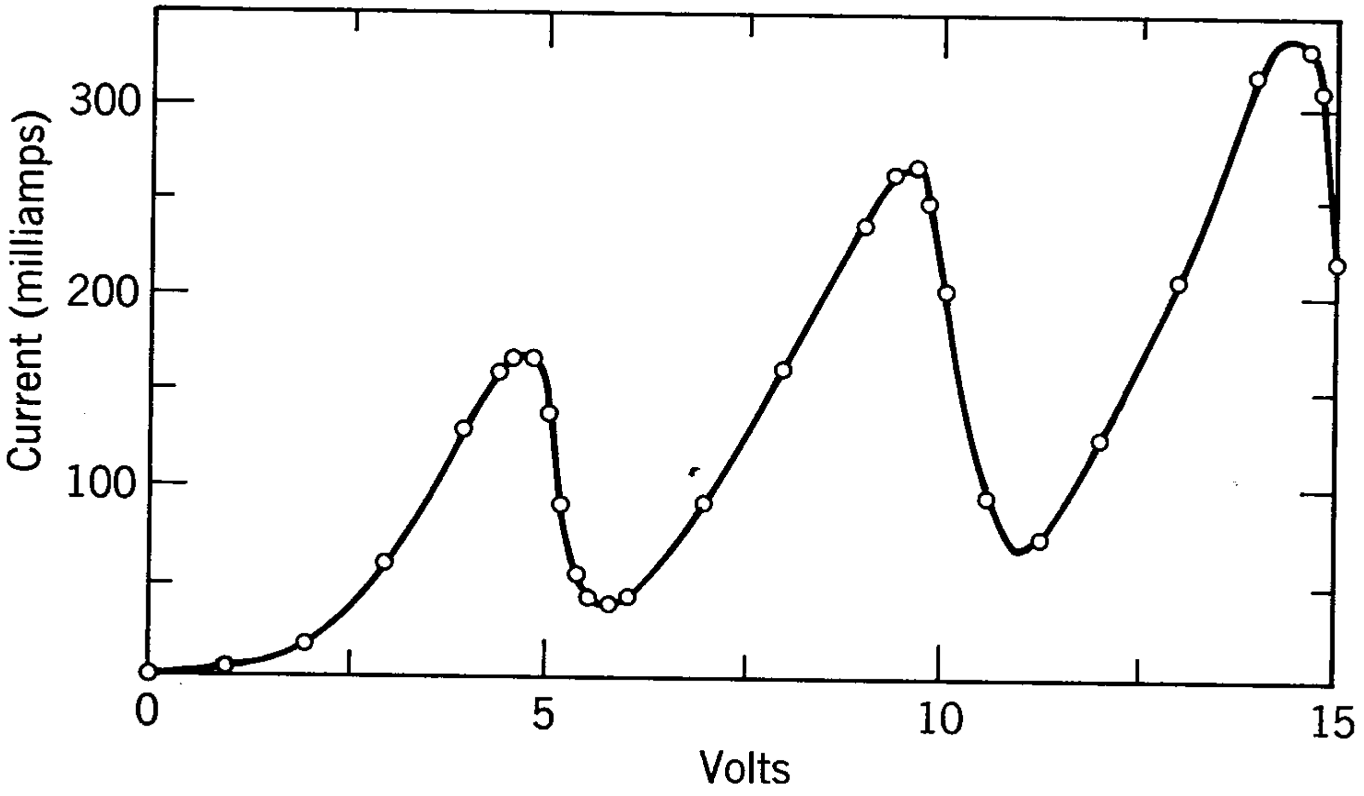
The Bohr model have quantitative agreement with experimental lines in the spectrum

Franck and Hertz experiment

Evidence for quantization of internal energy can be obtained from the experiment devised by Franck and Hertz



First experimental vapor is that of mercury. When voltage is increased the current also increases. The electrons have wavelength and frequency corresponding to the accelerating voltage



Current drops when the frequency of the accelerating electron matches with that of the energy level of the particular molecule. For mercury it is 4.9 eV

Direct method to measure the energy levels