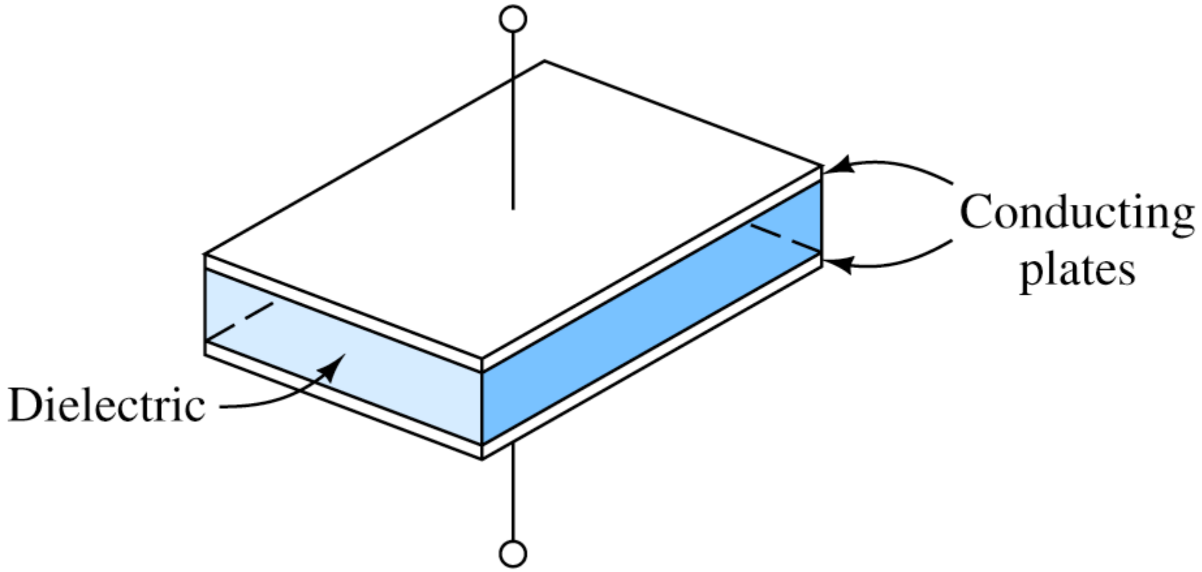
INDUCTANCE AND CAPACITANCE

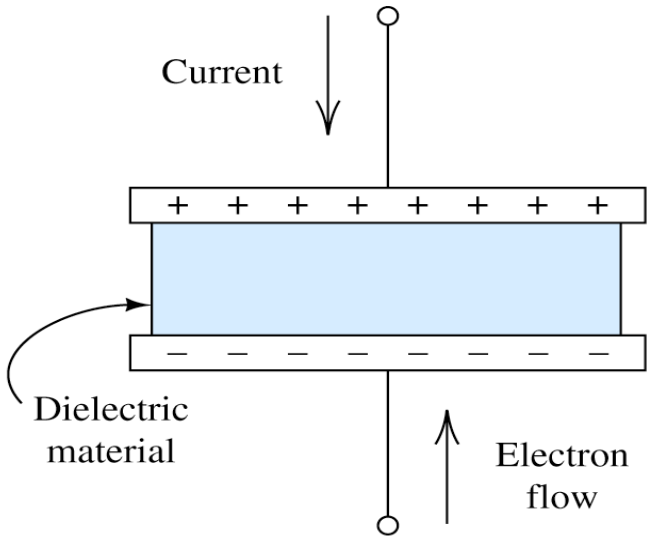
Introduction

- Energy storage elements
- Passive
- Capacitance
  - Energy in electric fields
- Inductance
  - Energy in magnetic fields

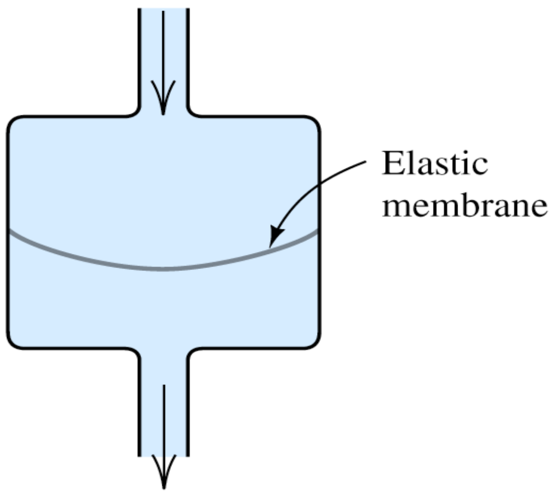
Parallel Plate Capacitor Construction



Capacitance



(a) As current flows through a capacitor, charges of opposite sign collect on the respective plates



(b) Fluid-flow analogy for capacitance

- As charge builds up, voltage appears across capacitor
- Charge accumulated on one plate is stored in the capacitor

Capacitance

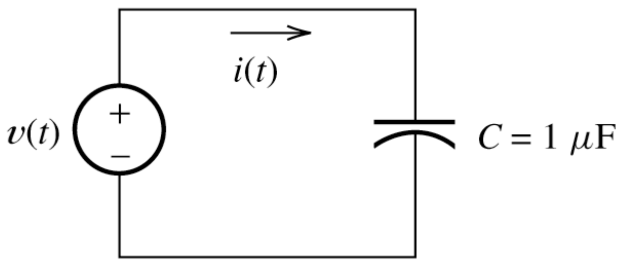
$$q = Cv$$

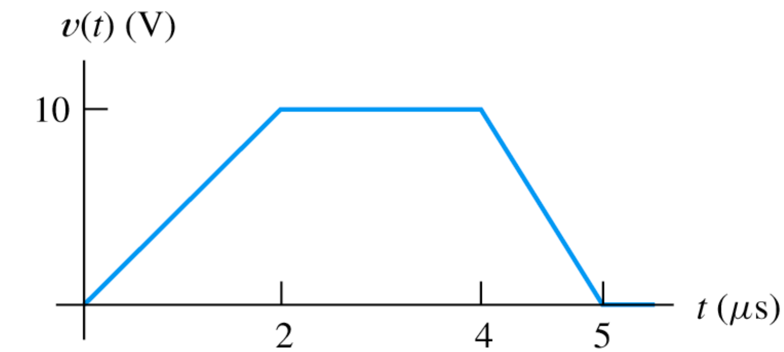
$$q(t) = \int_{t_0}^{t} i(t)dt + q(t_0)$$

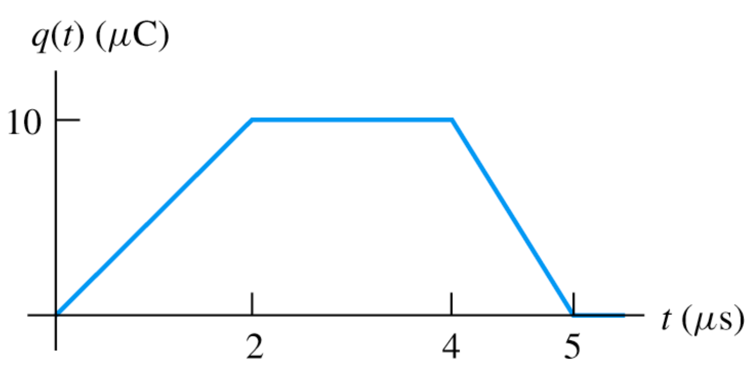
$$i = \frac{dq}{dt} = C\frac{dv}{dt}$$

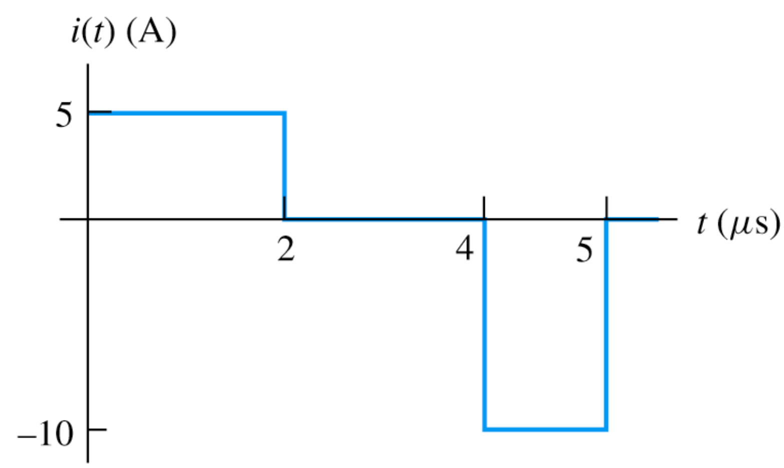
$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(t)dt + v(t_0)$$

Example Exercise

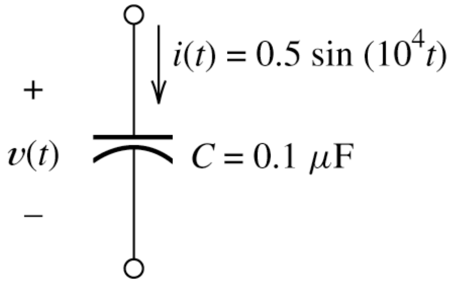


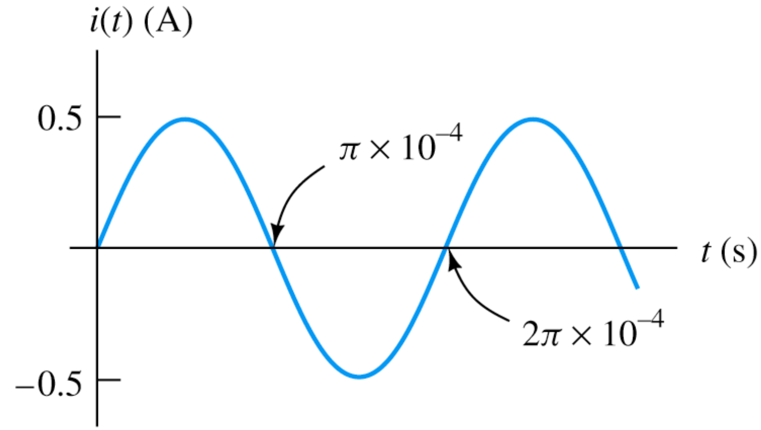


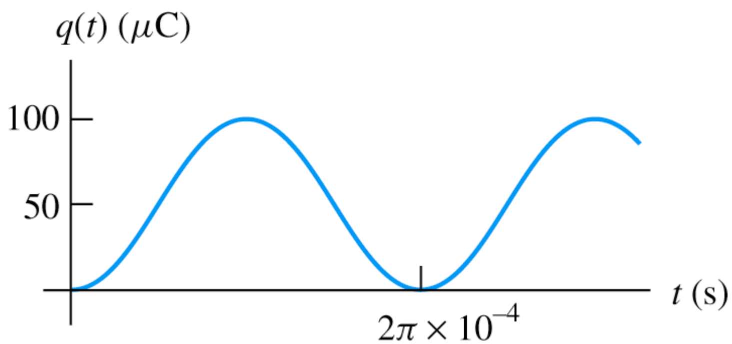


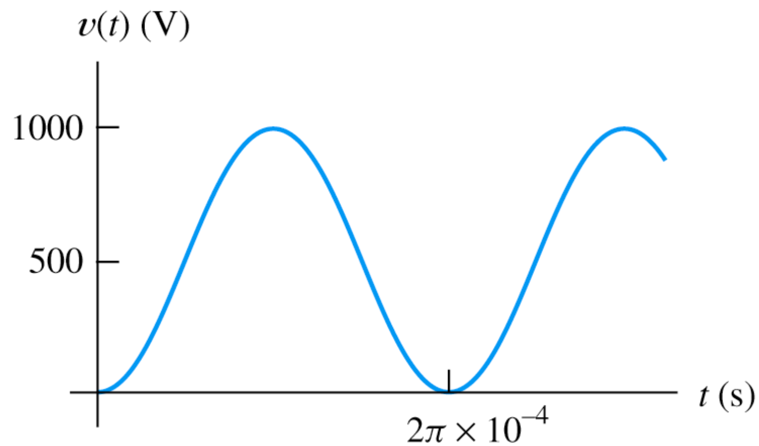


Example Exercise









Stored Energy and Power

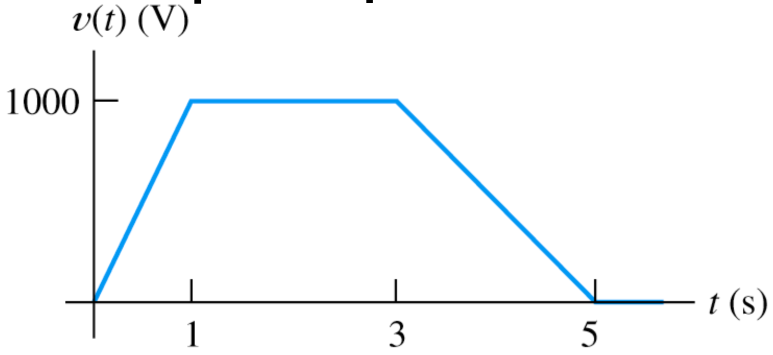
$$p(t) = v(t)i(t) = Cv\frac{dv}{dt}$$

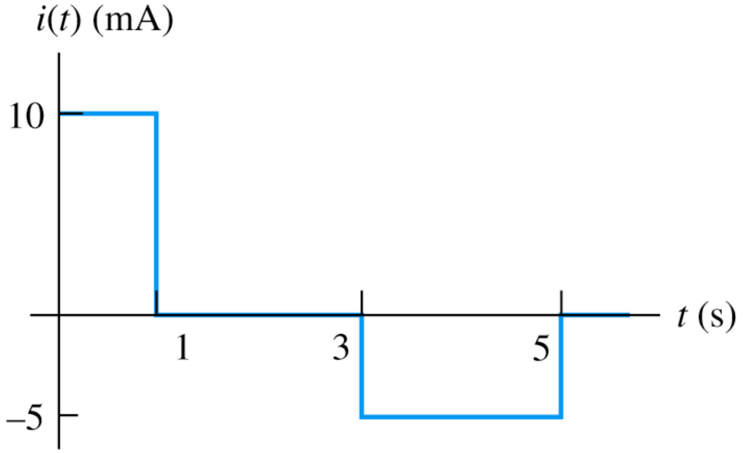
$$w(t) = \int_{t_0}^{t} p(t)dt = \int_{t_0}^{t} Cv\frac{dv}{dt}dt = \int_{0}^{v(t)} Cv\,dv = \frac{1}{2}Cv^2(t)$$

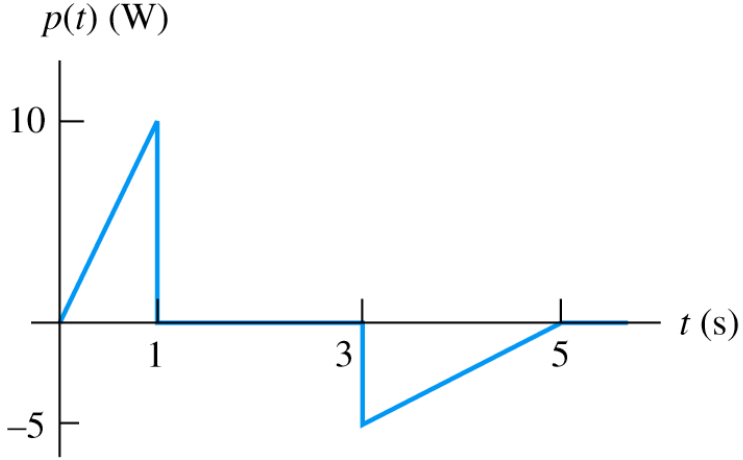
$$w(t) = \frac{1}{2}Cv^2(t) = \frac{1}{2}v(t)q(t) = \frac{q^2(t)}{2C}$$

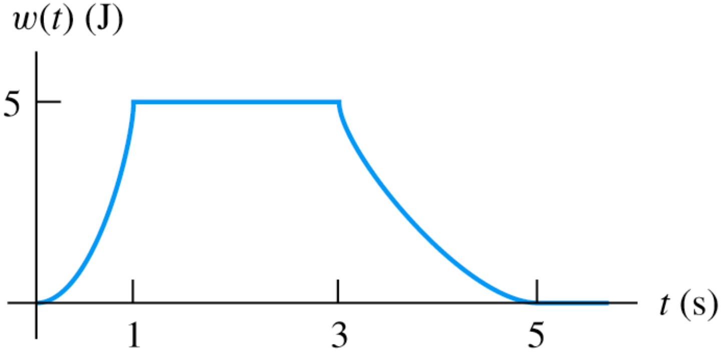
Example Exercise

- 10 μF capacitor



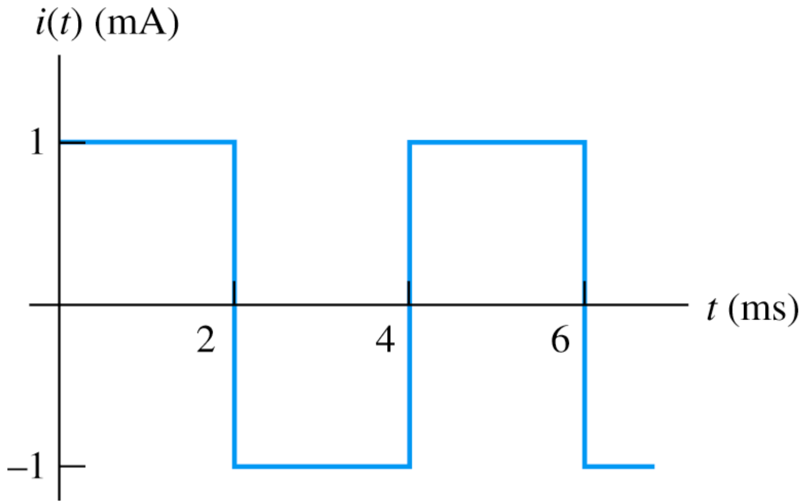




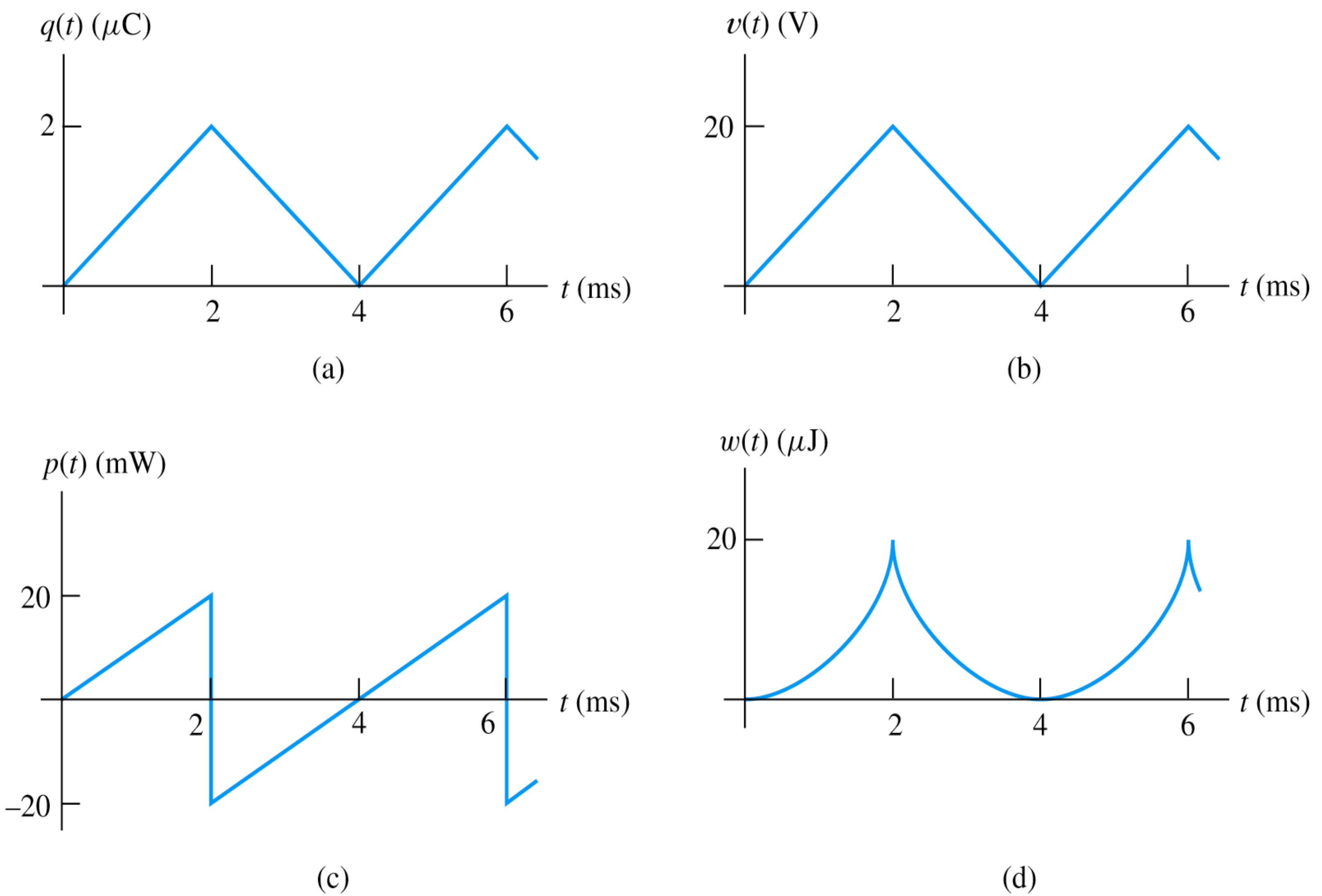


Example Exercise

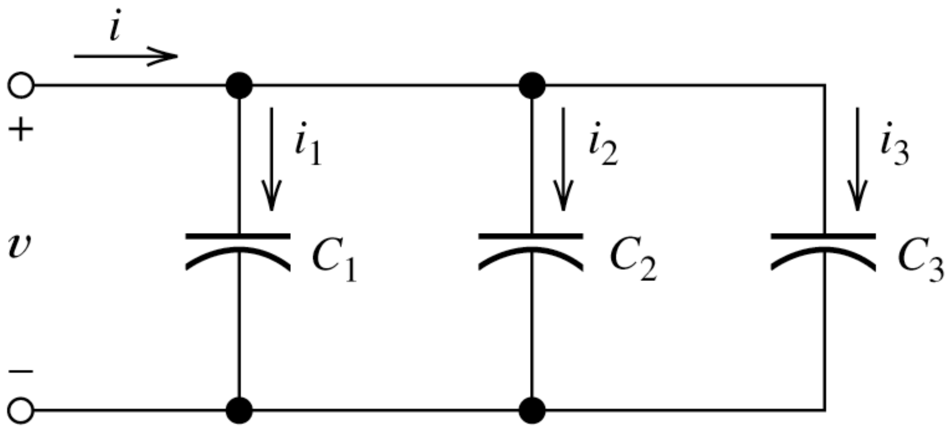
- 0.1μF capacitor



Example Exercise (Solution)



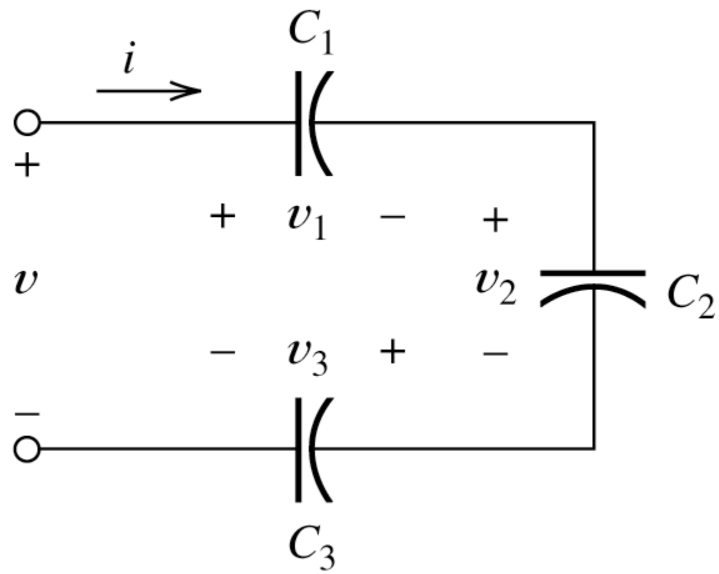
Capacitance in Parallel



$$C_{eq} = C_1 + C_2 + C_3$$

- Why?
  - Hint: $i = \dfrac{dq}{dt} = C\dfrac{dv}{dt}$

Capacitance in Series



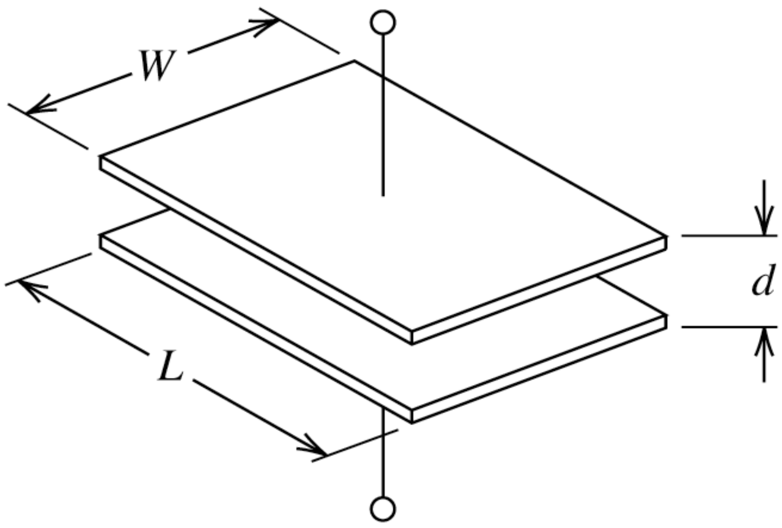
$$C_{eq} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3}$$

- Why?

Physical Characteristics of Capacitors

- Ideal parallel plate
- Practical
  - Rolled
  - Electrolytic
  - Parasitic effects

Parallel-Plate Capacitor



$$C = \frac{\varepsilon A}{d} \qquad A = WL$$

If d << W and L

$$\varepsilon_0 \cong 8.85 \times 10^{-12}\ \mathrm{F/m}$$

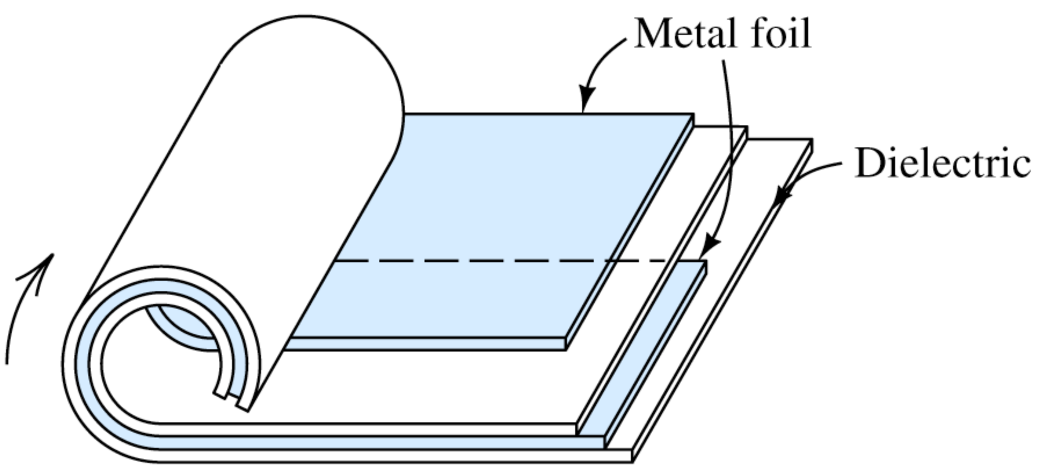
Dielectric constant for vacuum

$$\varepsilon = \varepsilon_r \varepsilon_0$$

With relative dielectric constant

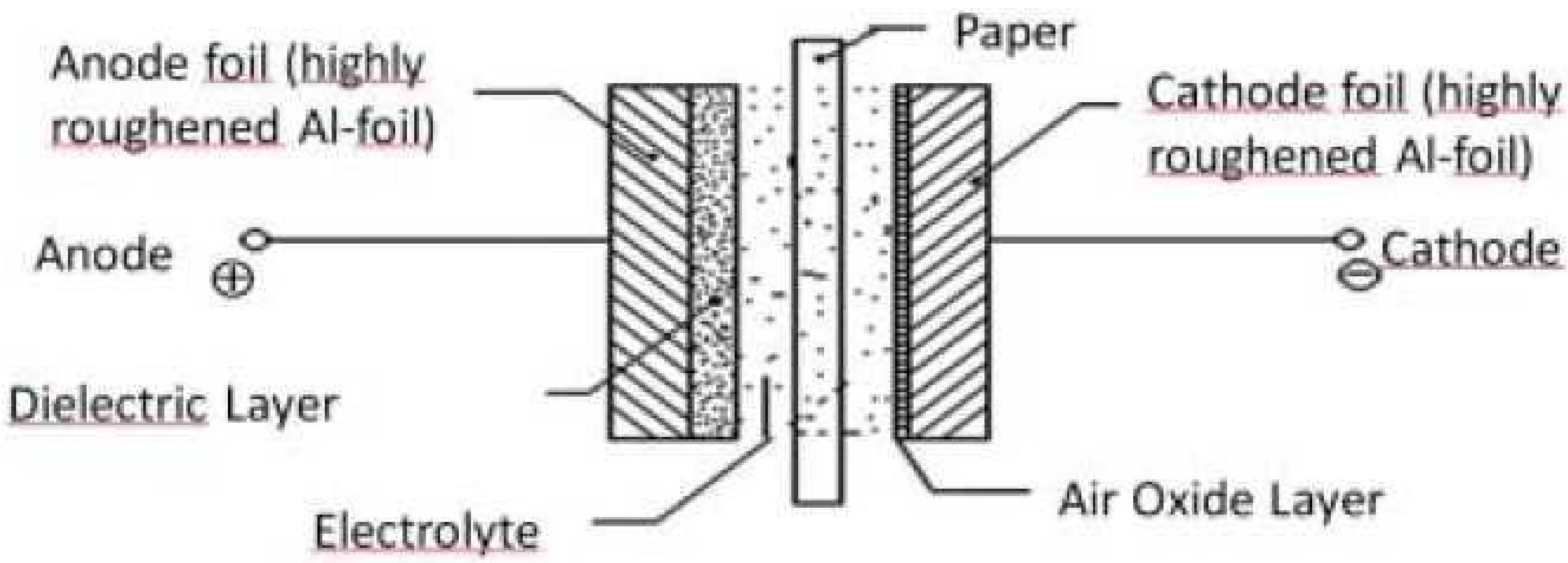
Practical Capacitors

- Dimension too large for ideal
- Rolled up metal plates
- Dielectric materials become conductors at high electric field intensity

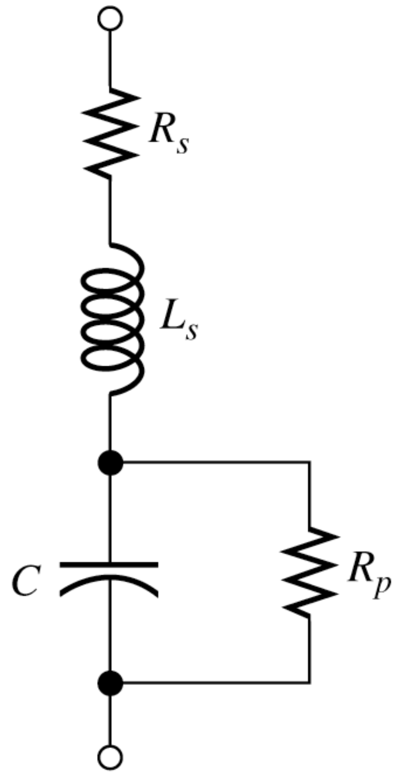


Electrolytic Capacitors

- One plate is metallic, dielectric is oxide layer and other "plate" is electrolytic solution

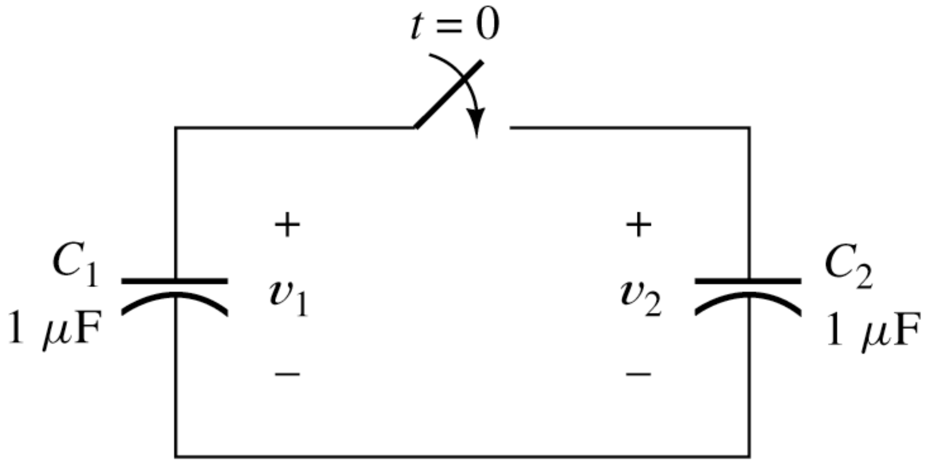


Parasitic Effects



Example Exercise

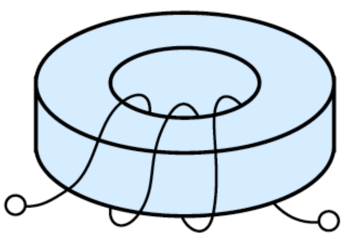
- Before t=0, v_1=100V, v_2=0V. Compute total energy stored before and after t=0.



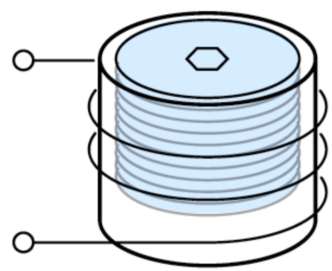
- Why are they different?

Inductance

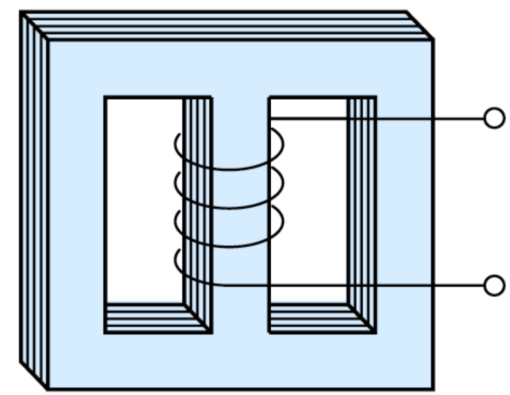
- Coil of wire along some form
- Current flow through coil creates flux
- Time varying flux induces voltage across coil



(a) Toriodal inductor



(b) Coil with an iron-oxide slug that can be screwed in or out to adjust the inductance

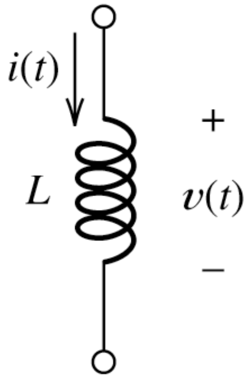


(c) Inductor with a laminated iron core

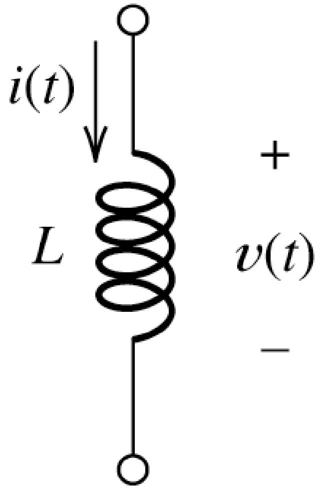
Inductance

- Unit of Henries (H) equivalent to volt seconds per ampere.
- As current increases, energy is stored in the magnetic field.

$$v(t) = L\frac{di}{dt}$$



Inductance



$$v(t) = L\,\frac{di}{dt}$$

$$v(t) = L\frac{di}{dt}$$

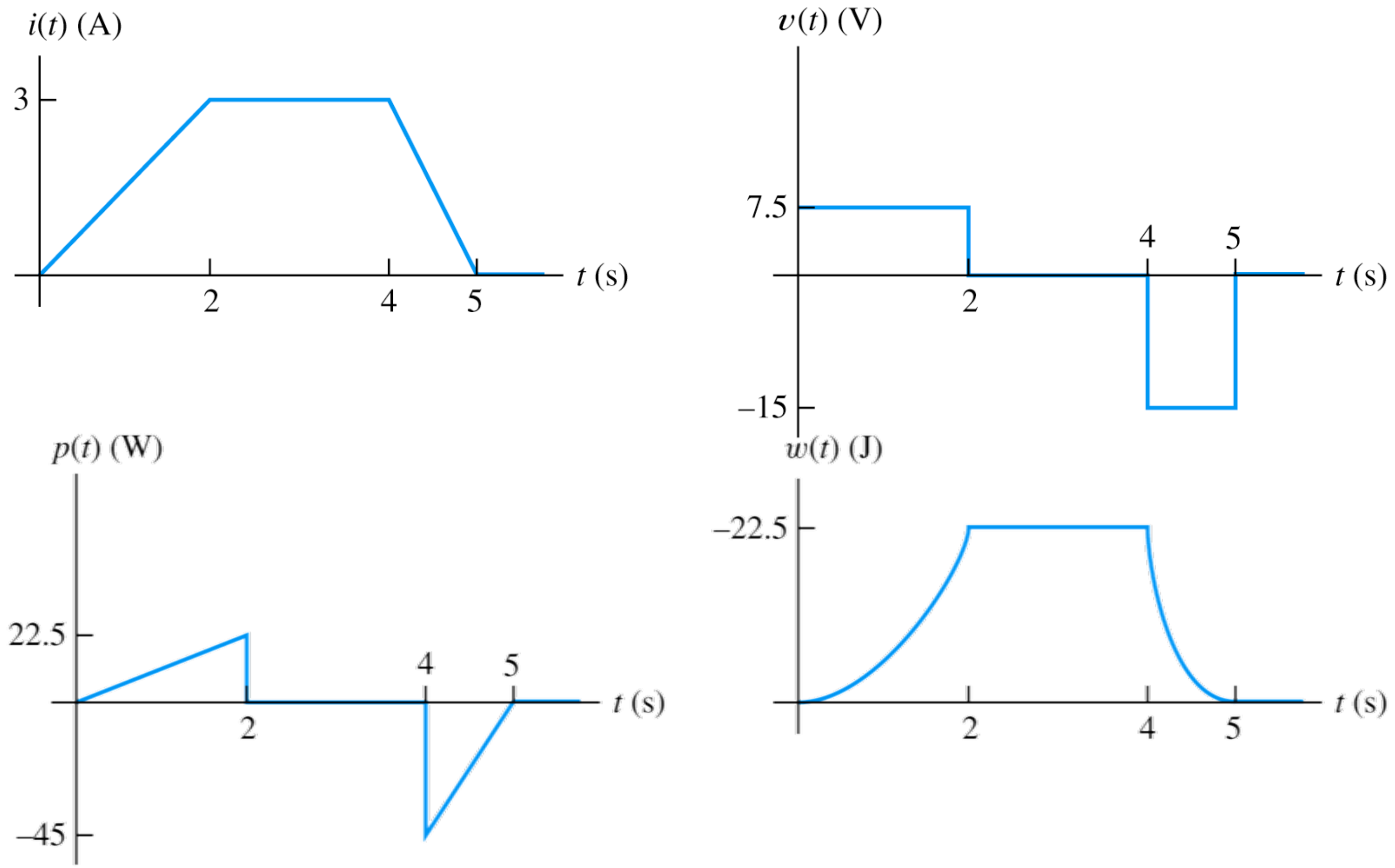
$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(t)dt + i(t_0)$$

$$p(t) = Li(t)\frac{di}{dt}$$

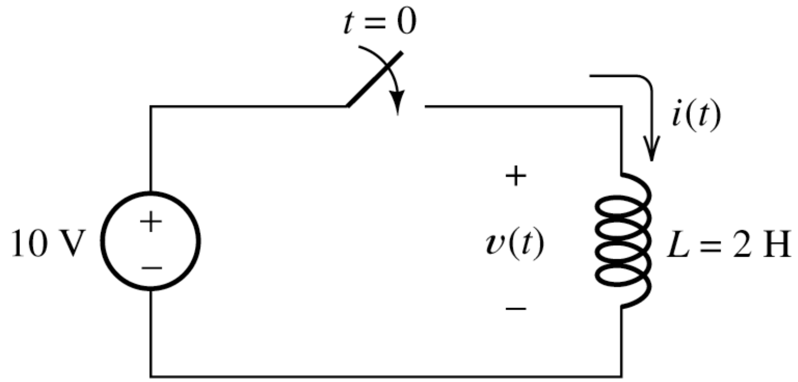
$$w(t) = \frac{1}{2}Li^2(t)$$

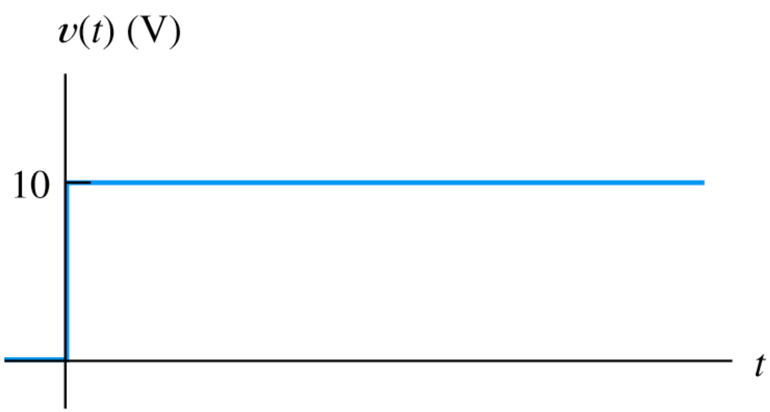
Example Exercise

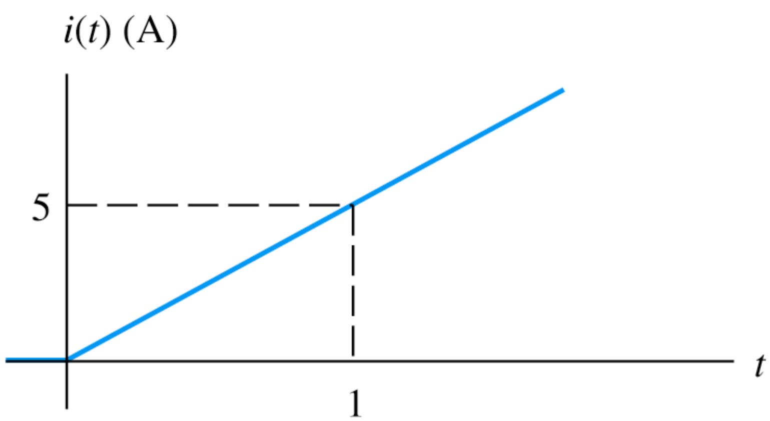
- Current through a 5 H inductor. Plot V, p, w.



Example Exercise

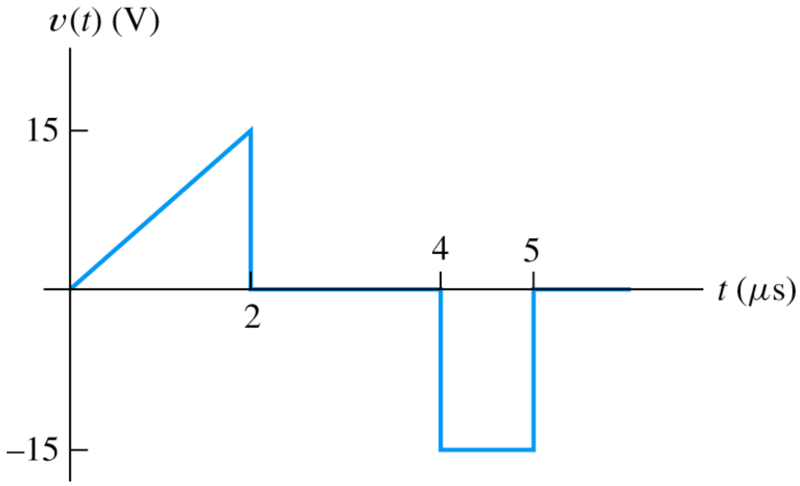


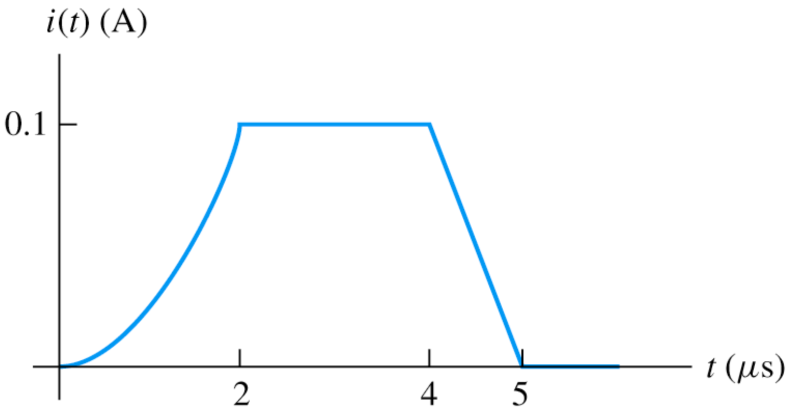




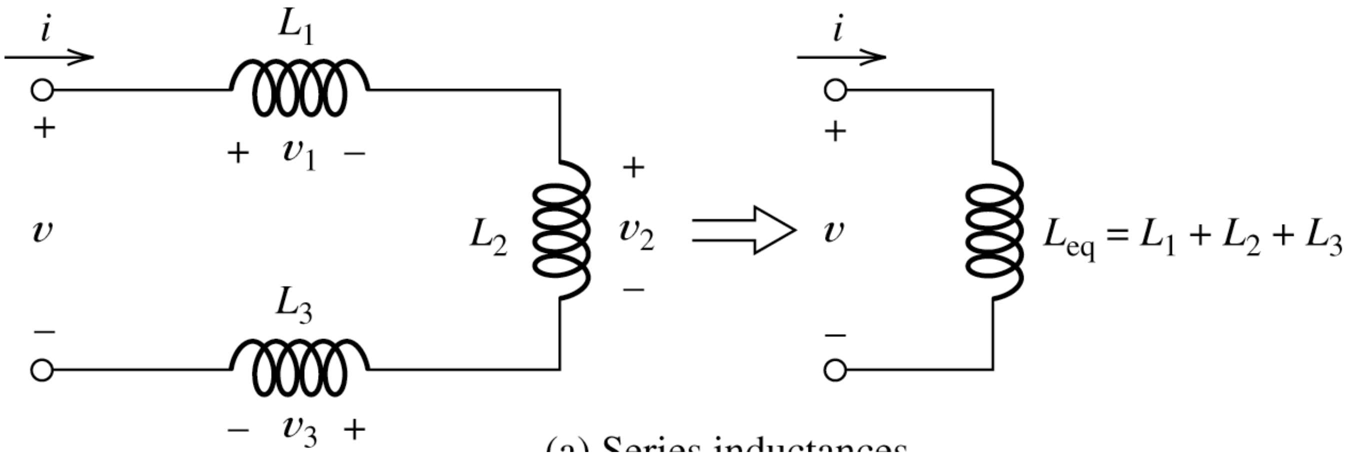
Example Exercise

- Voltage across 150 μH. Plot current



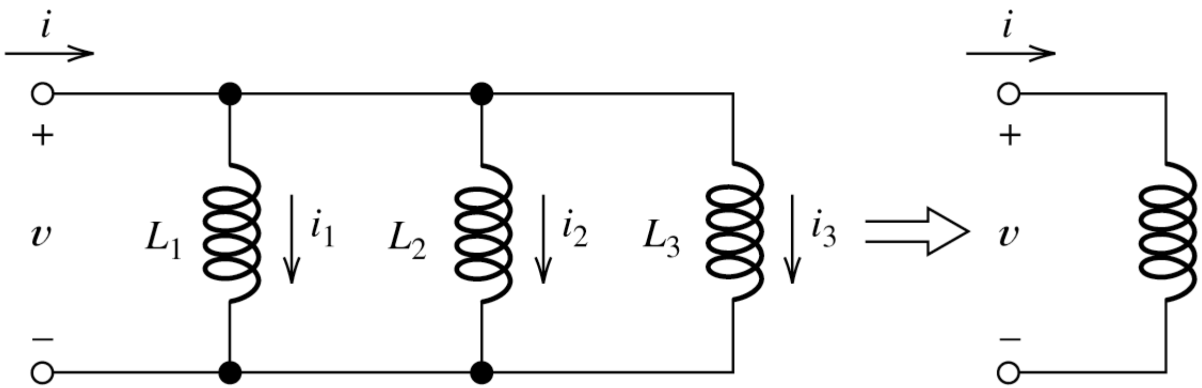


Inductance in Series



- Why?

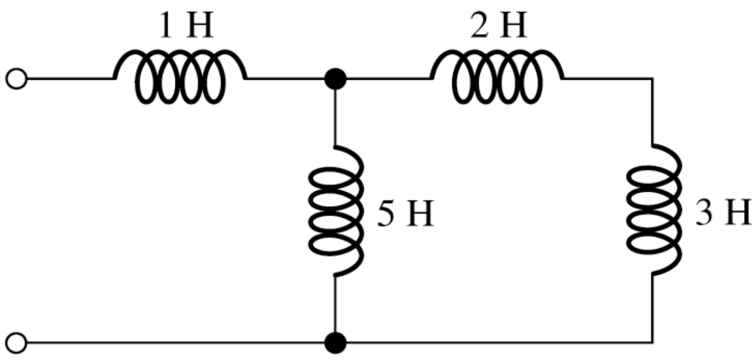
Inductance in Parallel

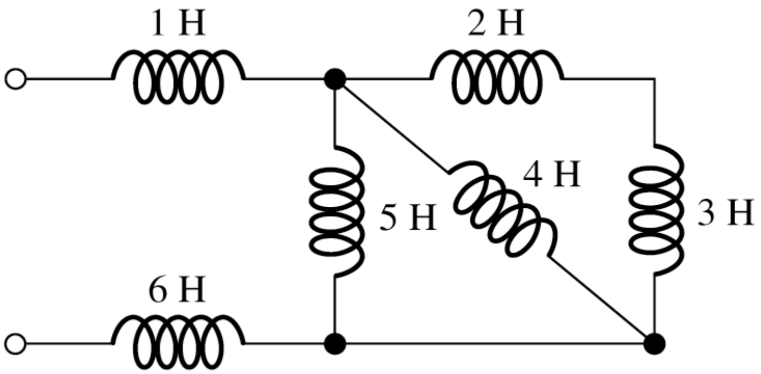


$$L_{eq} = \frac{1}{1/L_1 + 1/L_2 + 1/L_3}$$

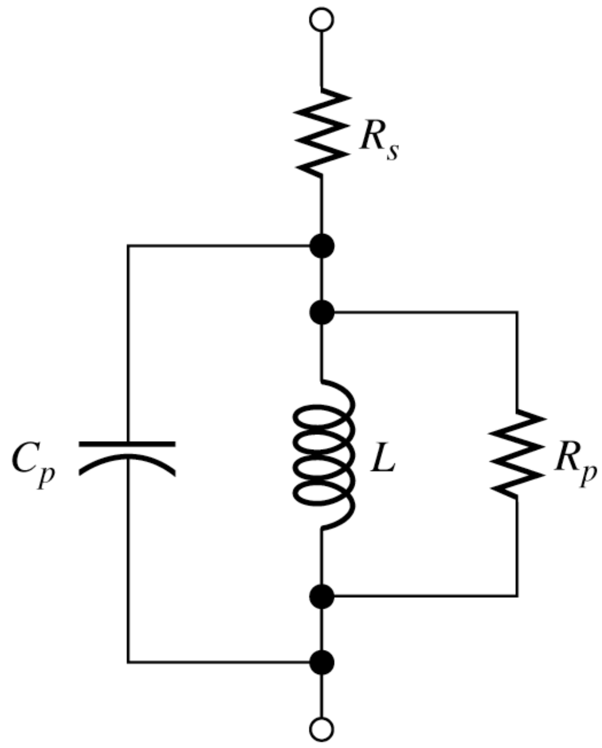
- Why?

Example Exercise



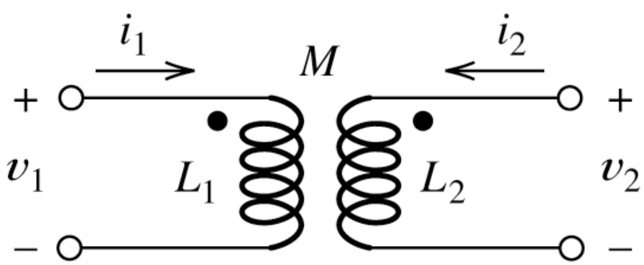


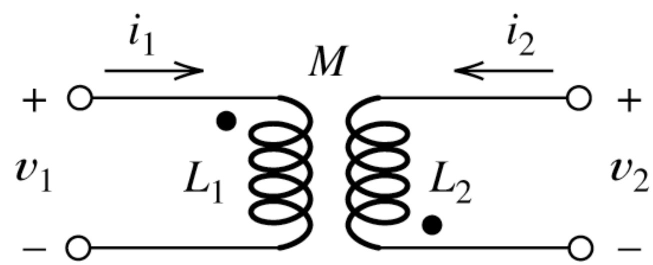
Parasitic Effects



Mutual Inductance

- Magnetic flux produced by one coil links the others (aid or oppose)





$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

$$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$v_2 = -M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

Linear Variable Differential Transformer

- With the core centered, $v_o(t)=0$
- Ideally $v_o(t)=Kx\cos(\omega t)$ where x is the displacement of the core