IC 110: Engineering Mathematics; Practice sheet 3

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1. Evaluate the following:

   (a) $\iint\limits_{\substack{0\le x\le a\\ 0\le y\le b}} ye^{xy}\,dxdy$ (b) $\iint\limits_{\substack{0\le x\le a\\ 0\le y\le b}} \frac{dxdy}{\sqrt{c^2+(x-y)^2}}$

   (c) $\iint\limits_{\substack{y\le x\le 8-y\\ 2\le y\le 4}} ydxdy$ (d) $\iint\limits_{\substack{0\le x\le 1\\ x^2\le y\le x}} ydxdy$

2. Evaluate $\iint\limits_A xydxdy$ where A is the domain bounded by the x-axis, ordinate x = 2a and the arc of the parabola $x^2=4ay$.
3. Evaluate $\iint\limits_A xydxdy$ where A is the region common to the circles $x^2+y^2 = x$, $x^2+y^2 = y$.
4. Evaluate $\iint\limits_A x^{1/2}y^{1/2}(1-x-y)^3dxdy$ over the region A bounded by the triangle with vortices (0, 0), (1, 0) and (0, 1).
5. Integrate f(x, y) = x/y over the region in the first quadrant bounded by the lines y = x, y = 2x, x = 1 and x = 2.
6. Evaluate $\iint ydxdy$ over A, where A is the region bounded by the parabolas:

   (a) $y^2 = 4x$ and $x^2 = 4y$

   (b) $y^2 = x$ and $x^2 = y$
7. Evaluate the following integrals:

   (a) $\iint xdxdy$ over the region bounded by $y = x^2$ and $y = x^3$.

   (b) $\iint ydxdy$ over the region bounded by $y = x^2$ and $y = x^3$.

   (c) $\iint x^2dxdy$ over the region bounded by y = x, y = 2x, x = 2.

   (d) $\iint ydxdy$ over the region above y = 0, bounded by $y^2 = 4x$ and $y^2 = 5 – x$.

   (e) $\iint xydxdy$ over the domain bounded by y – x = 0, x + y = 1 and y = 0.

   (f) $\iint dxdy$ over the region lying between y = 2x and $y = x^2$ lying to the left of x = 1.

   (g) $\iint dxdy$ over the region lying in the first quadrant and bounded by $y^2 = x^3$ and y = x.

8. Write an equivalent double integral with order of integration reversed for $\int_0^{\sqrt{2}}\int_{-\sqrt{4-2y^2}}^{\sqrt{4-2y^2}} ydxdy$.

   Check your answer by evaluating both the double integrals.
9. Write an equivalent double integral with order of integration reversed:

   (a) $\int_0^1\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3ydxdy$ (b) $\int_0^a\int_x^{a^2/x} (x+y)dxdy$

10. Evaluate $\iint \sqrt{4-x^2-y^2}\,dxdy$ over the semicircle $x^2+y^2 = 2x$ in the positive quadrant.

11. Evaluate $\iiint (x+y+z+1)^2 dxdydz$ throughout the region defined by $x \geq 0, y \geq 0, z \geq 0, x+y+z \leq 1.$

12. Evaluate the following triple integrals

(a) $\iiint\limits_{x^2+y^2+z^2 \leq 1} z^2 dxdydz$ (b) $\iiint\limits_{x^2+y^2+z^2 \leq 1} (z^2+z)dxdydz$

(c) $\iiint\limits_{\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1} xdxdydz$ (d) $\iiint\limits_{\substack{x^2+y^2 \leq z^2 \\ x^2+y^2+z^2 \leq 1}} xdxdydz$

13. Test the series for convergence

$(a) \sum_{n=1}^{\infty} \frac{(-1)^n 3^n + n}{2^n - n^3}$ $(b) \sum_{n=1}^{\infty} \sin(\frac{1}{n})$ $(c) \sum_{n=1}^{\infty} \frac{e^{-1/n}}{n}$

14. How many term of the series

$\sum_{n=1}^{\infty} (-1)^n e^{-n}$, does one need to take for the error to be less than 10^{-10} ?

15. Does the series converges or diverges?

$(a) \sum_{n=1}^{\infty} \frac{n!(n+1)!}{(3n)!}$ $(b) \sum_{n=1}^{\infty} (-1)^n \cos(\frac{1}{n})$

$(c) \sum_{n=1}^{\infty} \frac{n+5}{n\sqrt{n+3}}$ $(d) \sum_{n=1}^{\infty} \frac{3+\cos n}{e^n}$

16. Consider the sequence defined by $a_n = \frac{(-1)^n + n}{(-1)^n - n}$. Does this sequence converge and, if it is does, to what limit?

17. Find the value of the series

$$\sum_{n=1}^{\infty} \frac{1+2^n}{3^{n-1}}$$

18. Does the series converge absolutely, converge conditionally, or diverge?

$(a) \sum_{n=0}^{\infty} (-1)^n \frac{1}{\sqrt{n^2+1}}$ $(b) \sum_{n=1}^{\infty} (-1)^n \frac{n!}{\pi^n}$

19. For each of the following, say whether it converges or diverges and explain why.

$(a) \sum_{n=1}^{\infty} \frac{n^3}{n^5+3}$ $(b) \sum_{n=1}^{\infty} \frac{3^n}{4^n+4}$ $(c) \sum_{n=1}^{\infty} \frac{n}{2^n}$

20. For what values of p does the series $\sum_{n=1}^{\infty} \frac{n^p}{n^3+2}$ converges?

21. Determine whether the series is convergent or divergent using the test of your choice. Make sure you state the test used and all of the criteria needed.

$(a) \sum_{n=1}^{\infty} \frac{n}{n^3+1}$ $(b) \sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n+1}$ $(c) \sum_{n=1}^{\infty} \frac{\cos 3n}{1+(1.2)^n}$

22. Check the convergence /divergence of $\sum_{n=1}^{\infty} \frac{2}{n^2+3+4n}$.