Karnaugh Maps (K maps)

What is Karnaugh[1] maps?

•A graphical method of simplifying logic equations or truth tables—also called a K map.

•Theoretically can be used for any number of input variables—practically limited to 5 or 6 variables.

- Karnaugh maps provide an alternative way of simplifying logic circuits.
- Instead of using Boolean algebra simplification techniques, you can transfer logic values from a Boolean statement or a truth table into a Karnaugh map.

The truth table values are placed in the K map. Shown here is a two-variable map.

A	B	X	
0	0	1	$\rightarrow \bar{A}\bar{B}$
0	1	0	
1	0	0	
1	1	1	$\rightarrow AB$

$$\left\{ x = \bar{A}\bar{B} + AB \right\}$$

	$\bar{B}$	B
$\bar{A}$	1	0
A	0	1

- ***These are grids that all you to easily find the simplest algebraic expression for a truth table.***
- ***K-Maps consists of one square for each possible minterm in a function.***

Cell = 2^n , where n is a number of variables

For the case of 2 variables, we form a map consisting of $2^2 = 4$ cells as shown in Figure

B \ A	0	1
0	$A+B$	$\overline{A}+B$
1	$A+\overline{B}$	$\overline{A}+\overline{B}$

Maxterm

B \ A	0	1
0	00 **0**	10 **2**
1	01 **1**	11 **3**

B \ A	0	1
0	$\overline{A}\,\overline{B}$	$A\overline{B}$
1	$\overline{A}B$	AB

Minterm

- The arrangement of 0's and 1's within the map helps you to visualise the logic relationships between the variables and leads directly to a simplified Boolean statement.

- Karnaugh maps, or K-maps, are often used to simplify logic problems with 2, 3 or 4 ,5,6 variables.

MIN and MAX Terms

MIN Term : Product of single letter term containing all the variables within a system either with or without bar is called MIN term.

Example : $\bar{A}\bar{B}$, $\bar{A}B$, $A\bar{B}$ and AB.

MIN terms are plotted as 1's in the K map, meaning that the function is true for that MIN term.

MAX Term : Sum of single letter term containing all the variables within a system either with or without bar is called MAX term.

Example : $\bar{A}+\bar{B}$, $\bar{A}+B$, $A+\bar{B}$ and A+B.

MAX terms are plotted as 0's in the K map, meaning that the function is false for that MAX term.

Sum of Product Form (SOP) : If all the MIN terms are added or ORed together they form Sum Of Product equation .

Example : $Y(ABC) = \bar{A}B+A\bar{B}C+AC$

Product of Sum Form (POS) : If all the MAX terms are multiplied or ANDed together they form Product Of Sum equation.

Example, $Y(ABC) = (A+B).(\bar{A}+B+C).(\bar{A}+\bar{B})$

MIN and MAX Terms from Truth Table

Table
(MIN and MAX Terms)

Inputs				MIN Term	MAX Term
A	B	C	D		
0	0	0	0	$\bar{A}\bar{B}\bar{C}\bar{D}$	$A+B+C+D$
0	0	0	1	$\bar{A}\bar{B}\bar{C}D$	$A+B+C+\bar{D}$
0	0	1	0	$\bar{A}\bar{B}C\bar{D}$	$A+B+\bar{C}+D$
0	0	1	1	$\bar{A}\bar{B}CD$	$A+B+\bar{C}+\bar{D}$
0	1	0	0	$\bar{A}B\bar{C}\bar{D}$	$A+\bar{B}+C+D$
0	1	0	1	$\bar{A}B\bar{C}D$	$A+\bar{B}+C+\bar{D}$
0	1	1	0	$\bar{A}BC\bar{D}$	$A+\bar{B}+\bar{C}+D$
0	1	1	1	$\bar{A}BCD$	$A+\bar{B}+\bar{C}+\bar{D}$
1	0	0	0	$A\bar{B}\bar{C}\bar{D}$	$\bar{A}+B+C+D$
1	0	0	1	$A\bar{B}\bar{C}D$	$\bar{A}+B+C+\bar{D}$
1	0	1	0	$A\bar{B}C\bar{D}$	$\bar{A}+B+\bar{C}+D$
1	0	1	1	$A\bar{B}CD$	$\bar{A}+B+\bar{C}+\bar{D}$
1	1	0	0	$AB\bar{C}\bar{D}$	$\bar{A}+\bar{B}+C+D$
1	1	0	1	$AB\bar{C}D$	$\bar{A}+\bar{B}+C+\bar{D}$
1	1	1	0	$ABC\bar{D}$	$\bar{A}+\bar{B}+\bar{C}+D$
1	1	1	1	$ABCD$	$\bar{A}+\bar{B}+\bar{B}+\bar{D}$

Maxterm/Minterm Corresponding to Each Cell of K-Map

If A,B,C,D are variables then the binary numbers formed by them are taken as AB,ABC and ABCD.

$B \backslash A$	0	1
0	$\bar{A}\bar{B}$	$A\bar{B}$
1	$\bar{A}B$	AB

(a)

$B \backslash A$	0	1
0	$A+B$	$\bar{A}+B$
1	$A+\bar{B}$	$\bar{A}+\bar{B}$

(b)

$C \backslash AB$	00	01	11	10
0	$\bar{A}\bar{B}\bar{C}$	$\bar{A}B\bar{C}$	$AB\bar{C}$	$A\bar{B}\bar{C}$
1	$\bar{A}\bar{B}C$	$\bar{A}BC$	ABC	$A\bar{B}C$

(c)

$C \backslash AB$	00	01	11	10
0	$A+B+C$	$A+\bar{B}+C$	$\bar{A}+\bar{B}+C$	$\bar{A}+B+C$
1	$A+B+\bar{C}$	$A+\bar{B}+\bar{C}$	$\bar{A}+\bar{B}+\bar{C}$	$\bar{A}+B+\bar{C}$

(d)

$CD \backslash AB$	00	01	11	10
00	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}B\bar{C}\bar{D}$	$AB\bar{C}\bar{D}$	$A\bar{B}\bar{C}\bar{D}$
01	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}B\bar{C}D$	$AB\bar{C}D$	$A\bar{B}\bar{C}D$
11	$\bar{A}\bar{B}CD$	$\bar{A}BCD$	$ABCD$	$A\bar{B}CD$
10	$\bar{A}\bar{B}C\bar{D}$	$\bar{A}BC\bar{D}$	$ABC\bar{D}$	$A\bar{B}C\bar{D}$

(e)

$CD \backslash AB$	00	01	11	10
00	$A+B+C+D$	$A+\bar{B}+C+D$	$\bar{A}+\bar{B}+C+D$	$\bar{A}+B+C+D$
01	$A+B+C+\bar{D}$	$A+\bar{B}+C+\bar{D}$	$\bar{A}+\bar{B}+C+\bar{D}$	$\bar{A}+B+C+\bar{D}$
11	$A+B+\bar{C}+\bar{D}$	$A+\bar{B}+\bar{C}+\bar{D}$	$\bar{A}+\bar{B}+\bar{C}+\bar{D}$	$\bar{A}+B+\bar{C}+\bar{D}$
10	$A+B+\bar{C}+D$	$A+\bar{B}+\bar{C}+D$	$\bar{A}+\bar{B}+\bar{C}+D$	$\bar{A}+B+\bar{C}+D$

(f)

Representation of Truth Table on K-Map

- The output Y is logic 1 corresponding to the rows 1,2,4 and 7
- The equation in terms of SOP $Y = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$
- The output Y is logic 0 corresponding to the rows 0,3,5 and 6
- The equation in terms of POS

$$Y = (A + B + C)(A + \bar{B} + \bar{C})(\bar{A} + B + \bar{C})(\bar{A} + \bar{B} + C)$$

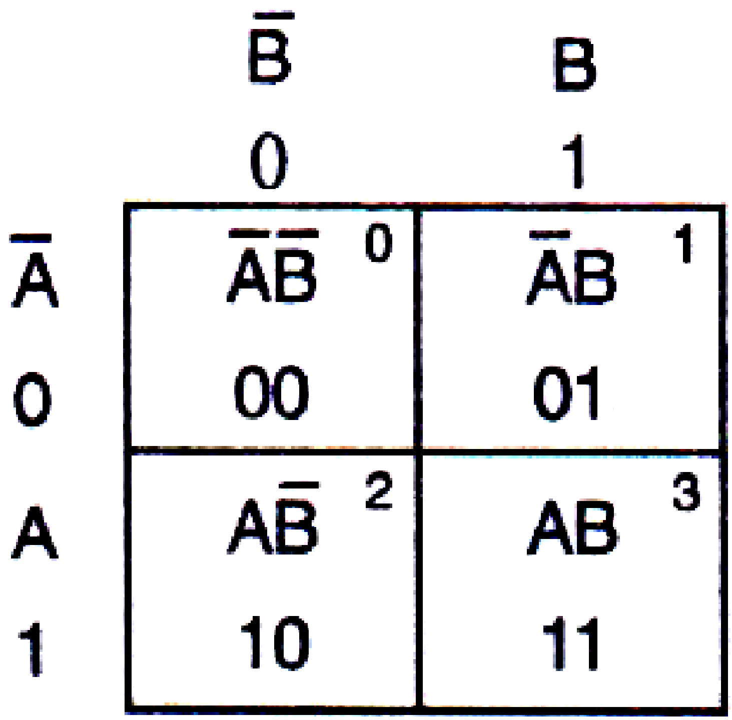
Truth Table for 3 variable system

Row No.	Inputs			Output
	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	1
3	0	1	1	0
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	1

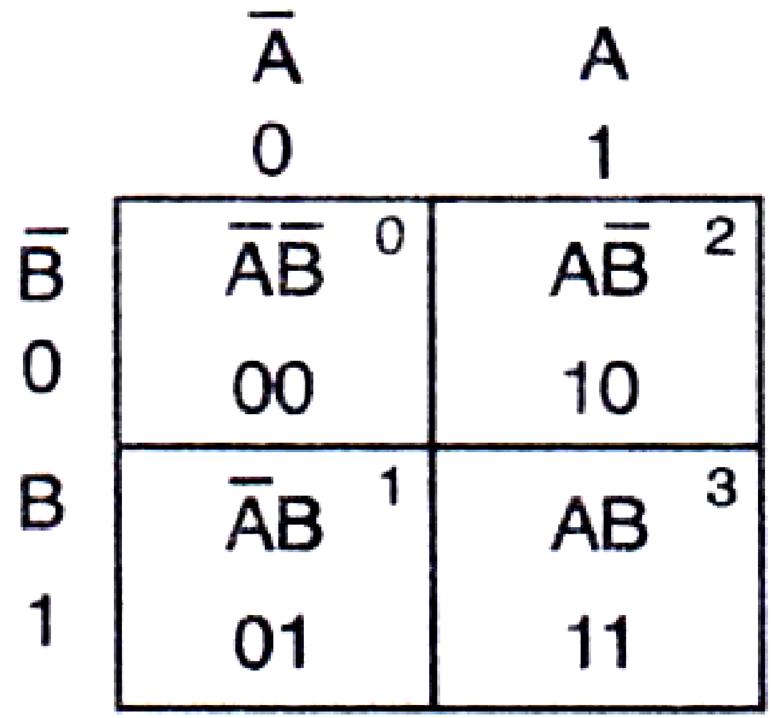
K-Map for 3 Variable System

AB → C ↓	00	01	11	10
0	0 0	2 1	6 0	4 1
1	1 1	3 0	7 1	5 0

Two Variable K Map



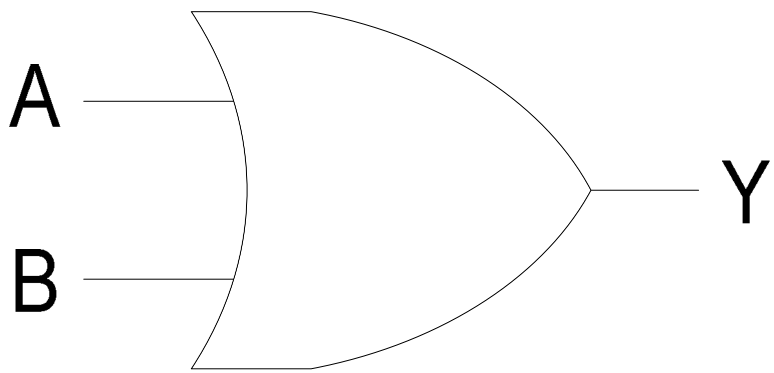
(a)



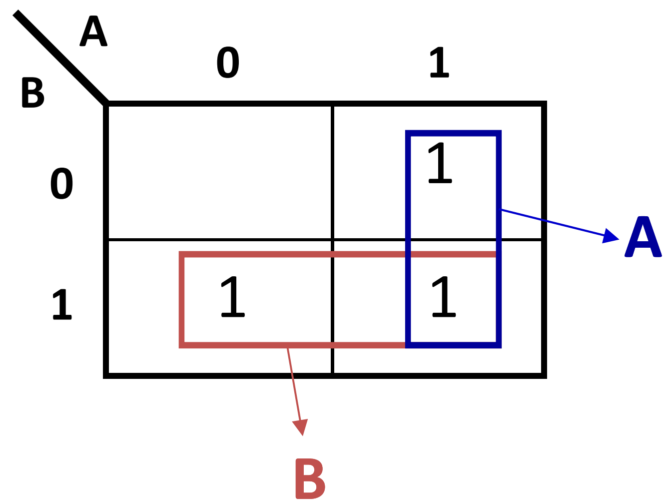
(b)

Example

2-variable Karnaugh maps are trivial but can be used to introduce the methods. The map for a 2-input OR gate looks like this:



A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1



A+B

Four-variable K-Map.

A	B	C	D	X	
0	0	0	0	0	
0	0	0	1	1	$\rightarrow \overline{A}\,\overline{B}\,\overline{C}D$
0	0	1	0	0	
0	0	1	1	0	
0	1	0	0	0	
0	1	0	1	1	$\rightarrow \overline{A}B\overline{C}D$
0	1	1	0	0	
0	1	1	1	0	
1	0	0	0	0	
1	0	0	1	0	
1	0	1	0	0	
1	0	1	1	0	
1	1	0	0	0	
1	1	0	1	1	$\rightarrow AB\overline{C}D$
1	1	1	0	0	
1	1	1	1	1	$\rightarrow ABCD$

$$\left\{ \begin{aligned} X = {} & \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}B\overline{C}D \\ & + AB\overline{C}D + ABCD \end{aligned} \right\}$$

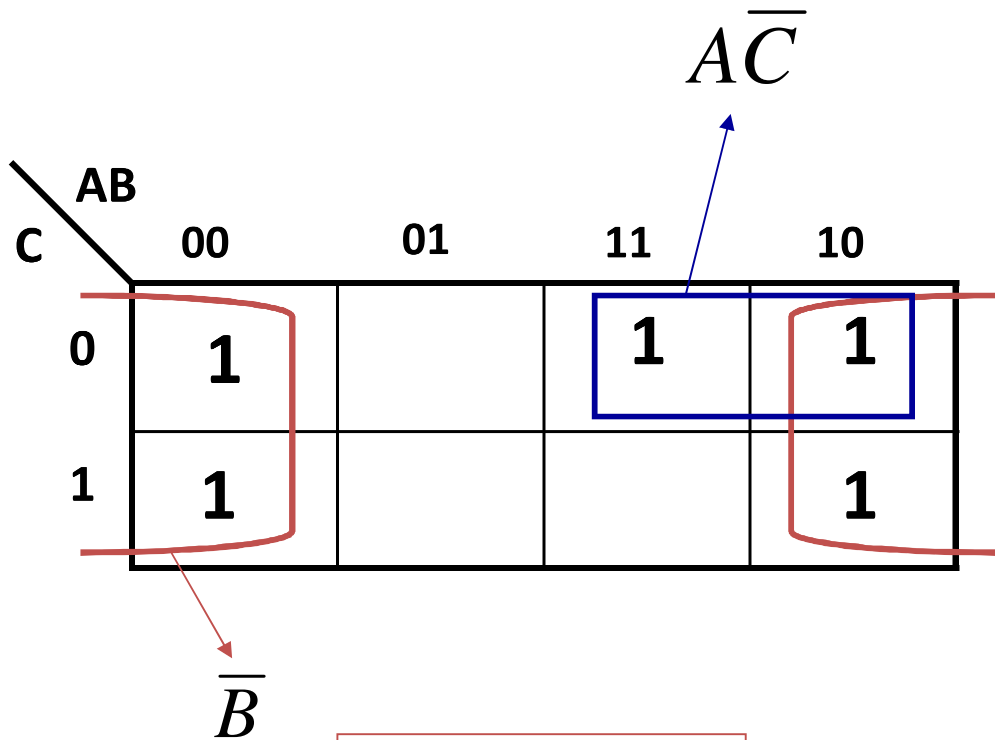
	$\overline{C}\,\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$
$\overline{A}\,\overline{B}$	0	1	0	0
$\overline{A}B$	0	1	0	0
AB	0	1	1	0
$A\overline{B}$	0	0	0	0

Adjacent K map square differ in only one variable both horizontally and vertically.

A SOP expression can be obtained by **OR**ing all squares that contain a 1.

Example

A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0



$$\boxed{\overline{B} + A\overline{C}}$$

When the largest possible groups have been looped, only the common terms are placed in the final expression.

Looping may also be wrapped between top, bottom, and sides.

Complete K map simplification process:

- Construct the K map, place 1s as indicated in the truth table.

-*Loop 1s that are not adjacent to any other 1s.*

-*Loop 1s that are in pairs.*

-*Loop 1s in octets even if they have already been looped.*

-*Loop quads that have one or more 1s not already looped.*

-*Loop any pairs necessary to include 1st not already looped.*

-*Form the* ***OR*** *sum of terms generated by each loop.*

When a variable appears in both complemented and uncomplemented form within a loop, that variable is eliminated from the expression.

Variables that are the same for all squares of the loop must appear in the final expression.

Three Variable K Map

A three variable K map has variables say A , B, C and compliment of these variables. With three variables there can be maximum of 8 combinations (2^n, where $n = 3$). The K map for the three variable is shown in the Fig.

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0	1	3	2
A	4	5	7	6

(a)

or

	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}$	0	2	6	4
C	1	3	7	5

(b)

Fig. : 3 Variable K map

Four Variable K Map

A four variable K map can have variables (say A,B,C and D). AB and CD variables can be placed either on the vertical or horizontal side. With four variables there can be 16 possible combinations (2^n, where n = 4). The possible K maps of four variables is shown in Fig.

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	3	2
$\bar{A}B$	4	5	7	6
AB	12	13	15	14
$A\bar{B}$	8	9	11	10

(a)

or

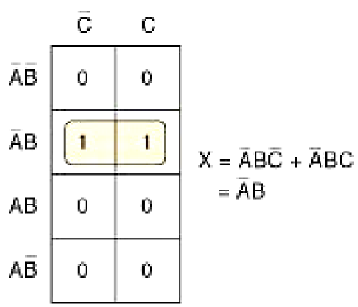
	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	0	4	12	8
$\bar{C}D$	1	5	13	9
CD	3	7	15	11
$C\bar{D}$	2	6	14	10

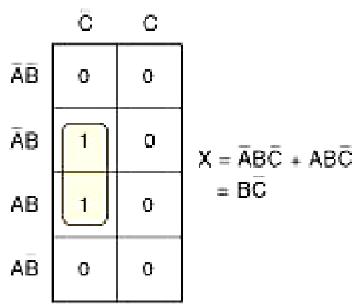
(b)

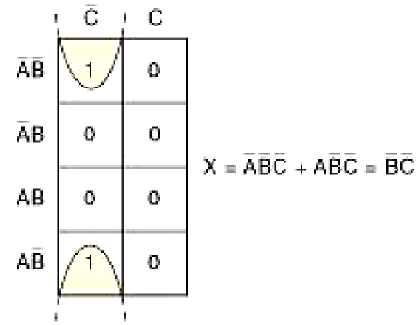
Fig. 4 Variable K-map

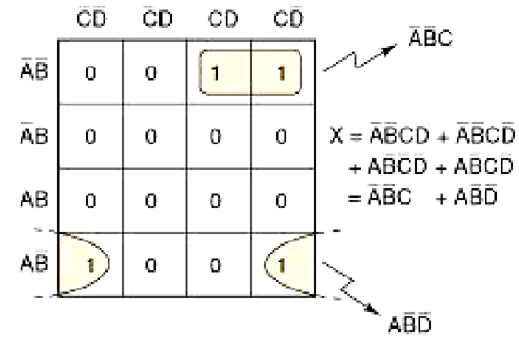
In a four variable K map, group of 2,4 and 8 MIN terms can be made for the purpose of reducing the equation.

Looping 1s in adjacent groups of 2, 4, or 8 will result in further simplification.



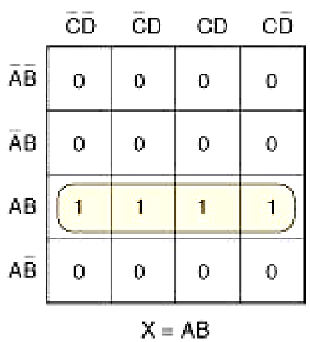




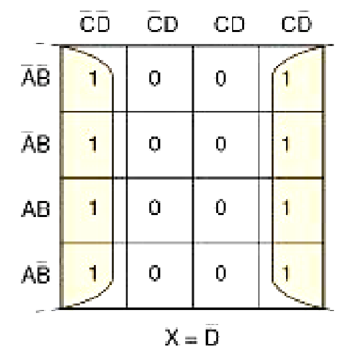


Looping groups of 2 (Pairs)

Groups of 4 (Quads)



Groups of 8 (Octets)



Example : Solve the equation $\bar{A}\bar{B} + \bar{A}B + AB$ using

i) Boolean Algebra

ii) K map

Solution : **i) Boolean Equation**

$$Y = \bar{A}\bar{B} + \bar{A}B + AB$$

$$= \bar{A}(\bar{B} + B) + AB$$

$$= \bar{A} + AB \qquad (\because \bar{B} + B = 1)$$

Multiplying A with $1 + B$ $\qquad (\because 1 + B = 1)$

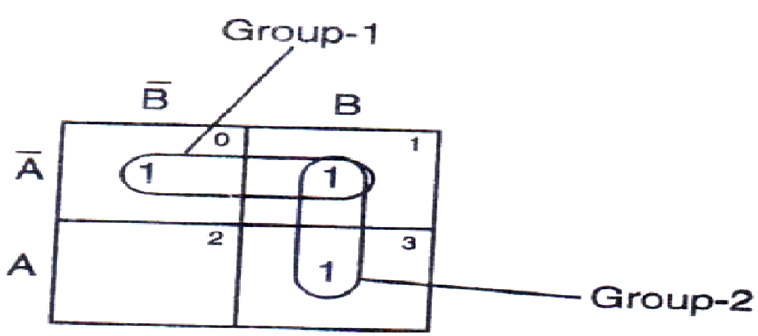
$$= \bar{A}(1 + B) + AB$$

$$= \bar{A} + \bar{A}B + AB$$

$$= \bar{A} + B(\bar{A} + A)$$

$$= \bar{A} + B \qquad (\because \bar{A} + A = 1)$$

ii) K map



The variable $\bar{A}$ is common in Group 1 and the variable common in Group-2 is $\bar{B}$. The final result is $Y = \bar{A} + B$