Indian Institute of Technology Mandi

IC 110: B.Tech. I year

Odd Semester 2013-2014

Tutorial-4 (Sequence, Series and Jacobian)

1. Prove or disprove if $\sum a_n$ is convergent, then $\sum (a_n)^{1/2}$ is also convergent, where a_n is positive for all n.
2. Prove or disprove if $\sum a_n$ is convergent, then $\sum |a_n|$ is also convergent, where a_n is positive for all n.
3. By comparison test, show that $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$ converges for all$n \in N$.
4. By comparison test, show that $\sum_{n=1}^{\infty} \frac{1}{n!}$ converges for all$n \in N$.
5. Show that series is convergent/divergent, (a) $\sum_{n=1}^{\infty} cos\ n$ (b) $\sum_{n=1}^{\infty} \frac{cos\ n}{n^2}$.
6. Determine the boundedness and monotonicity of the sequence.
$$\frac{4n}{\sqrt{4n^2+1}}$$
7. State whether the sequence converges, and, if it does, find the limit.
$$(1+\frac{1}{n})^{n/2}$$
8. State whether the sequence converges as $n \to \infty$; if it does, find the limit.
$$\frac{2^{3n-1}}{7^{n+2}}$$
9. Find two divergent series $\sum a_n$ and $\sum b_n$ such that $\sum a_n b_n$ converge.
10. If $\sum a_k^2$ and $\sum b_k^2$ both converges, prove $\sum a_k b_k$ converges.
11. If $\sum_{n=1}^{\infty} a_n$ converges, prove $\sum \frac{a_n}{1+a_n}$ converges.
12. Calculate the Jacobian determinant of $T(u,v) = < u^2 - v, u^2 + v >$.
13. Prove (i) $\frac{\partial(x,y)}{\partial(u,v)} = -\frac{\partial(y,x)}{\partial(u,v)}$ (ii) $\frac{\partial(x,x)}{\partial(u,v)} = \frac{\partial(y,y)}{\partial(u,v)} = 0$.
14. Prove that if $f(u,v), g(u,v) and\ h(u,v)$ are differentiable then prove that
(i)$\frac{\partial(f+g,h)}{\partial(u,v)} = \frac{\partial(f,h)}{\partial(u,v)} + \frac{\partial(g,h)}{\partial(u,v)}$
(ii) $\frac{\partial(fg,h)}{\partial(u,v)} = \frac{\partial(f,h)}{\partial(u,v)} g + f\frac{\partial(g,h)}{\partial(u,v)}$
15. Show that $\frac{\partial(kf,g)}{\partial(u,v)} = k\frac{\partial(f,g)}{\partial(u,v)}$, Where k is constant.