IC-150
Computation for Engineers
Numerical Methods

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Numerical Methods

- Used for:
  - Solution of algebraic equations
  - Approximation of functions
  - Differentiation and integration of functions
  - Solution of differential equations
  - Statistical analysis of data

Numerical Methods

- Used because:
  - Analytical solution extremely difficult for a complex function
  - Analytical solution may require evaluation of esoteric functions
  - Mathematical functions may not be analytical
  - Function may be in the form of pairs of data
    - Eg. Given experimental data for column design:

Column radius (m)	1.2	1.5	1.8	2.0	2.95
Max Load (tons)	10.3	15.6	20.3	32.7	43.5

  - What is column radius for 35 tons?

Numerical Errors

- Source of Errors
  - Approximate evaluation of functions
    - $\pi = 22/7 = 3.14285714\ldots$
  - Representation of numbers in a finite number of bits
  - Round-off error, eg correct to 3 decimals:
    - $2.000 + 0.77\times10^{-6} = 2.00000077 = 2.000$
- Reduction of Error
  - Iterative solutions -- repeat until error < ε
    - Efficiency of convergence
    - Stability --- may never converge

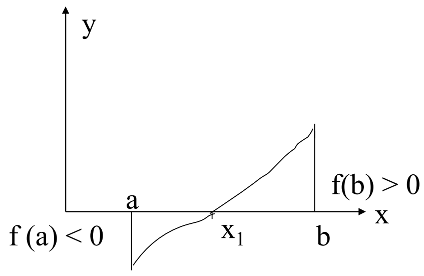
Fundamental Motifs

- Several techniques for a given problem
- Technique of choice depends on nature of the data
- One technique with modifications may be used to solve several different problems
- Error analysis essential to determine reliability of the computed results

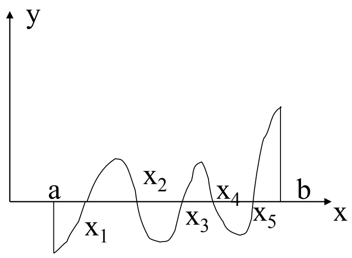
Root Finding: f(x)=0

Method 1: The Bisection method

Thm: If f(x) is continuous in [a,b] and if f(a)f(b)<0, then there is at least one root of f(x)=0 in (a,b).



Single root in (a,b)



Multiple roots in (a,b)

The Bisection Method

- Find an interval $[x_0, x_1]$ such that $f(x_0)f(x_1) < 0$
  - This may not be easy. Use your knowledge of the physical phenomenon that the equation represents.

- In each iteration, cut the interval into half
  - Examine the sign of the function at the mid point

$$m = \frac{x_0 + x_1}{2}$$

  - If f(m) = 0, x is the root
  - If $f(m) \neq 0$ and $f(x_0)f(x) < 0$, root lies in $[x_0, m]$
  - Otherwise root lies in $[m, x_1]$
- Repeat the process until convergence (length of interval < ε)

Number of Iterations and Error Tolerance

- Length of the interval (where the root lies) after n iterations

$$e_n = \frac{x_1 - x_0}{2^n}$$

- We can fix the number of iterations so that the root lies within an interval of chosen length ∈ (error tolerance).

$$e_n \leq \in \implies n \geq \frac{\ln(x_1 - x_0) - \ln \in}{\ln 2}$$

- If n satisfies this, root lies within a distance of ε/2 of the actual root

- Though the root lies in a small interval, $|f(x)|$ may not be small if $f(x)$ has a large slope.

- Conversely if $|f(x)|$ small, x may not be close to the root if $f(x)$ has a small slope.

- So, we use both these facts for the termination criterion. We first choose an error tolerance on $f(x)$: $|f(x)| < \in$ and K the maximum number of iterations.

Pseudo code (Bisection Method)

1. Input $\in > 0$, $K > 0$, $x_1 > x_0$ so that $f(x_0)\, f(x_1) < 0$.
   Compute $f_0 = f(x_0)$.
   $k = 1$ (iteration count)
2. Do
   {
   (a) Compute $m = \left(\dfrac{x_0 + x_1}{2}\right)$, and $f = f(m)$
   (b) If $f \times f_0 < 0$, set $x_1 = m$
   otherwise set $x_0 = m$
   (c) Set $k = k+1$
   } while $|f| > \in$ and $k \leq K$
3. Set root = m

Bisection Method Example

$f(x) = x^3 - 2 = 0$, $\varepsilon = 10^{-4}$

$x_0 = 1$, $x_1 = 2$

k	x_0	x_1	m	f(m)
1	1	2	1.5	1.375
2	1	1.5	1.25	-0.4688
3	1.25	1.5	1.375	0.5996
4	1.25	1.375	1.3125	0.2610
5	1.25	1.3125	1.2813	0.1033

- After 13 iterations, m = 1.2599 (to 4 decimal places)

- Using $n \geq \dfrac{\ln(x_1 - x_0) - \ln \in}{\ln 2}$, $n \geq 12.29$

Example using Scilab

Find roots of $f(x) = x^3 - 2 = 0$

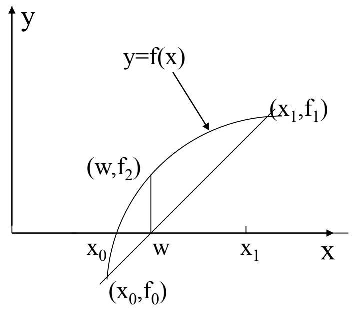
```
→ roots(poly([-2 0 0 1],'x', "coeff"))
 ans  =

  - 0.6299605 + 1.0911236i
  - 0.6299605 - 1.0911236i
    1.259921
→
```

False Position Method (Regula Falsi)

- Root may lie near end of interval with smaller value of $|f|$
- Instead of bisecting the interval $[x_0, x_1]$, choose the point where the straight line through the end points meets the x-axis, say w
- Bracket the root with $[x_0, w]$ or $[w, x_1]$ depending on sign of $f(w)$

False Position Method



Straight line through (x_0,f_0) ,(x_1,f_1) :

$$y = f_1 + \frac{f_1 - f_0}{x_1 - x_0}(x - x_1)$$

New end point w:

$$w = x_1 - \left(\frac{x_1 - x_0}{f_1 - f_0}\right) f_1$$

False Position Method (Pseudo Code)

1. Choose $\in > 0$ (tolerance on $|f(x)|$)
   $K > 0$ (maximum number of iterations)
   $k = 1$ (iteration count)
   x_0 ,x_1 (so that $f_0\, f_1 < 0$)
2. do {
   a. Compute $w = x_1 - \left(\frac{x_1 - x_0}{f_1 - f_0}\right) f_1$ and $f = f(w)$

   b. If $f_0 \times f < 0$ set $x_1 = w$, $f_1 = f$

   else set $x_0 = w$, $f_0 = f$

   c. $k = k+1$

   } while ($|f| \geq \in$) and ($k \leq K$)

4. The root is $x = w$

Regula Falsi Example

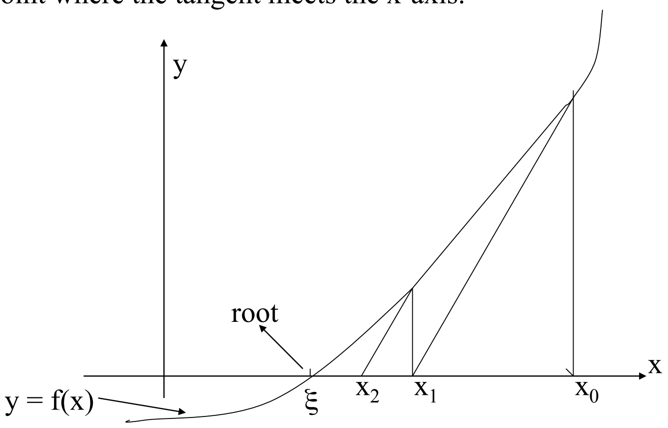
$f(x) = x^3 - 2 = 0$, $\varepsilon = 10^{-4}$

$x_0 = 1$, $x_1 = 2$

k	x_0	x_1	$f(x_0)$	$f(x_1)$	w	f(w)
0	1.0	2.0	-1.0	6.0	1.1429	-0.5071
1	1.1429	2.0	-0.5071	6.0	1.2097	-0.2298
2	1.2097	2.0	-0.2298	6.0	1.2389	-0.0987
3	1.2389	2.0	-0.0987	6.0	1.2512	-0.0412
4	1.2512	2.0	-0.0412	6.0	1.2563	-0.0172
9	1.2598	2.0	-0.0003	6.0	1.2607	0.0039
10	1.2598	1.2607	-0.0003	0.0039	1.2599	-0.0003
11	1.2599	1.2607	-0.0003	0.0039	1.2600	0.0002

Newton-Raphson *or* Newton's Method

At an approximation x_k to the root, the curve is approximated by the tangent to the curve at x_k and the next approximation x_{k+1} is the point where the tangent meets the x-axis.



Tangent at (x_k, f_k) :

$$y = f(x_k) + f'(x_k)(x - x_k)$$

This tangent cuts the x-axis at x_{k+1}

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

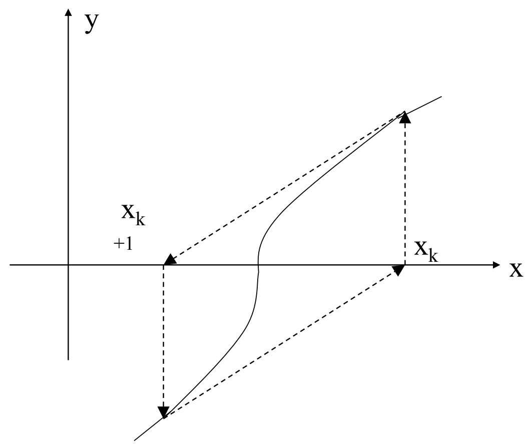
Warning : If $f'(x_k)$ is very small, method fails.

- Two function Evaluations per iteration

Newton's Method - Pseudo code

1. Choose $\in > 0$ (function tolerance $|f(x)| < \in$)
   $m > 0$ (Maximum number of iterations)
   x_0 - initial approximation
   k - iteration count
   Compute $f(x_0)$
2. Do { $q = f'(x_0)$ (evaluate derivative at x_0)
   $x_1 = x_0 - f_0/q$
   $x_0 = x_1$
   $f_0 = f(x_0)$
   $k = k+1$
   }
3. While ($|f_0| \geq \in$) and ($k \leq m$)
4. The root is $x = x_1$

Getting caught in a cycle of Newton's Method



Alternate iterations fall at the same point .

No Convergence.

Newton's Method for finding the square root of a number $x = \sqrt{a}$

$f(x) = x^2 - a^2 = 0$

$$x_{k+1} = x_k - \frac{x_k^2 - a^2}{2x_k}$$

Example : a = 5 , initial approximation $x_0 = 2$.

$x_1 = 2.25$

$x_2 = 2.236111111$

$x_3 = 2.236067978$

$x_4 = 2.236067978$

IC150 Lecture 28: Root Finding – Secant Method

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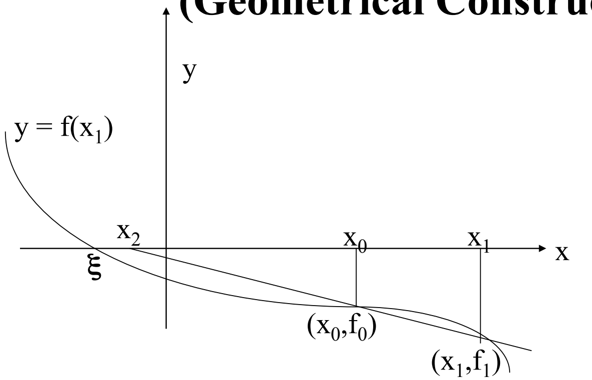
Problems with Newton's Method

- If $|f'(x)|$ is very small, accuracy is difficult to obtain
- Depending on the initial estimate, any one of the roots may be found (answer may not have physical significance)
  - Use bisection to get close to desired root, then Newton's method for fast convergence
- May get caught in an infinite cycle

The secant Method

- Newton's Method requires 2 function evaluations (f, f').
- The Secant Method requires only 1 function evaluation and converges as fast as Newton's Method at a simple root.
- Start with two points x_0,x_1 near the root (no need for bracketing the root as in Bisection Method or Regula Falsi Method) .
- x_{k-1} is dropped once x_{k+1} is obtained.

The Secant Method (Geometrical Construction)



- Two initial points x_0, x_1 are chosen
- The next approximation x_2 is the point where the straight line joining (x_0,f_0) and (x_1,f_1) meet the x-axis
- Take (x_1,x_2) and repeat.

The secant Method (Pseudo Code)

1. Choose $\in > 0$ (function tolerance $|f(x)| \le \in$)
   $m > 0$ (Maximum number of iterations)
   x_0 , x_1 (Two initial points near the root)
   $f_0 = f(x_0)$
   $f_1 = f(x_1)$
   $k = 1$ (iteration count)
2. Do {
   $$x_2 = x_1 - \left(\frac{x_1 - x_0}{f_1 - f_0}\right) f_1$$
   $x_0 = x_1$
   $f_0 = f_1$
   $x_1 = x_2$
   $f_1 = f(x_2)$
   $k = k+1$ }
3. while ($|f_1| \ge \in$) and ($m \le k$)

On Convergence

\# The false position method in general converges faster than the bisection method.

\# But not always, shown by counter examples

\# The bisection method and the false position method are guaranteed to converge

\# The secant method and the Newton-Raphson method are not guaranteed to converge

Order of Convergence

\# **A measure of how fast an algorithm converges**

Let ξ be the actual root: f(ξ) = 0

Let x_k be the approximate root at the kth iteration .
Error at the kth iteration, $e_k = |x_k - \xi|$

The algorithm converges with order p if there exists α such that

$$e_{k+1} = \alpha\, e_k^p$$

Order of Convergence of

- # Bisection method p = 1 (linear convergence)
- # False position - generally super linear (1 < p < 2)
- # Secant method $\frac{1+\sqrt{5}}{2} = 1.618$ (super linear)
- # Newton Raphson method p = 2 quadratic

Machine Precision

The smallest positive float $\in_M$ that can be added to one and produce a sum that is greater than one.

Pseudo code to find 'Machine Epsilon'

1. Set $\in_M = 1$
2. Do

   {

   $\in_M = \in_M / 2$

   $x = 1 + \in_M$

   }
3. While (x > 1)
4. $\in_M = 2 \in_M$

IC150 Lecture 29: Approximation & Interpolation

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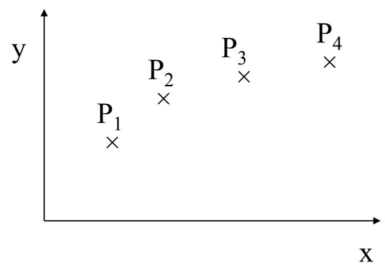
Approximation & Interpolation

Reasons to approximate value of a function:

- Difficult or impossible to evaluate the function analytically, eg. sine, log, etc.
- Have only a table of values and must interpolate
- Faster to compute approx function than original
- Function defined implicitly rather than by an equation

Interpolation



Given the data:

(x_k, y_k) , $k = 1, 2, 3, \ldots, n$,

find a function f which we can use to predict the value of y at points other than the samples

1. f(x) may pass through all the data points:

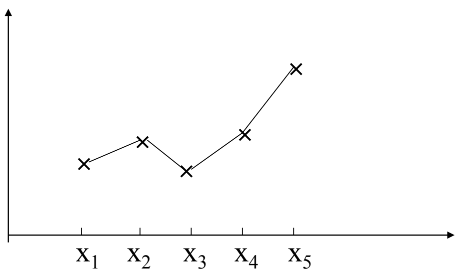
$$f(x_k) = y_k, \qquad 1 \le k \le n$$

2. f(x) need not pass through any of the data points:

Need to control error $|f(x_k) - y_k|$

1. Piecewise Linear Interpolation

A straight line segment is used between each adjacent pair of data points



$$f_k(x) = y_k + \frac{x - x_k}{x_{k+1} - x_k}(y_{k+1} - y_k) \qquad 1 \le k \le n$$

Simple and computationally efficient

1. Polynomial Interpolation

For the data set (x_k, y_k), $k = 1, \ldots, n$,

we find the *one* polynomial of degree (n - 1) subject to the n interpolation constraints $f(x_k) = y_k$

$$\mathbf{f(x)} = \sum_{k=1}^{n} \mathbf{a}_k \mathbf{x}^{k-1}$$

$$\begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \cdots & x_2^{n-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^{n-1} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

Not feasible for large data sets, since the *condition number* increases rapidly with increasing n.

Lagrange Interpolating Polynomial

The Lagrange interpolating polynomial of degree k, $f(x)$ is constructed as follows:

1. Caculate the Lagrangian multipliers $Q_k(x)$ each of which is a polynomial of degree n-1 that is non-zero at only the one base point x_k

   Normalise by $Q_k(x_k)$

$$Q_k(x) = \prod_{i=0, i\neq k}^{n} (x - x_i) / \prod_{i=0, i\neq k}^{n} (x_k - x_i)$$

$$Q_k(x) = \frac{(x - x_1)(x - x_2)\ldots(x - x_{k-1})(x - x_{k+1})\ldots(x - x_n)}{(x_k - x_1)(x_k - x_3)\ldots(x_k - x_{k-1})(x_k - x_{k+1})(x_k - x_n)}$$

- Each $Q_k(x)$ is a polynomial of degree (n - 1)
- $Q_k(x_j) = 1$, $j = k$

  $= 0$, $j \neq k$
- The polynomial curve that passes through the data set (x_k, y_k), $k = 1, 2, \ldots, n$ is

  $f(x) = y_1 Q_1(x) + y_2 Q_2(x) + \ldots + y_n Q_n(x)$
- Polynomial is written directly without having to solve a system of equations

Lagrange interpolations (Pseudo Code)

Choose x, the point where the function value is required

```
    y = 0
for  i = 1 to n
        p = y1
        for j = 1 to n
            if ( i ≠ j )
                p = p * (x - xj) / (xi - xj)
        end for
    y = y + product
End for
```

Lagrange Interpolation Example

- Given the following base points, estimate sin 23° to five decimal places:

i	x_i	y_i
0	20°	0.34202
1	22°	0.37641
2	24°	0.40674
3	26°	0.43837

$$Q_k(x) = \prod_{i=0, i\neq k}^{n} (x - x_i) / \prod_{i=0, i\neq k}^{n} (x_k - x_i)$$

$Q_0(x) = -0.0625$

$Q_1(x) = -0.5625$

$Q_2(x) = -0.5625$

$Q_3(x) = -0.0625$

$f(x) = y_1 Q_1(x) + y_2 Q_2(x) + \ldots + y_n Q_n(x)$

$= (0.34202)(-0.0625) +$
$(0.37461)(0.5625) +$
$(0.40674)(0.5625) +$
$(0.43837)(-0.0625)$

$= 0.39074$

True value: 0.39073, discrepancy due to accumulation of round-off error.

IC150 Lecture 30: Curve Fitting

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2. Least Squares Fit

If the number of samples is large or the dependant variable contains measurement noise, it is often better to find a function that minimizes an error criterion such as

$$E = \sum_{k=1}^{n} [f(x_k) - y_k]^2$$

A function that minimizes E is called the Least Squares Fit

Depending on the nature of the function we have:

- linear regression
- polynomial regression (quadratic, cubic, …)
- exponential regression, etc.

2. Minimax

Least squares approximation gives good fit overall, but may have large deviation from one point

Minimize the maximum deviation from y_k

Also known as *Chebyshev* or *optimal polynomial* approximation

$$E = \max_k | f(x_k) - y_k |$$

Straight Line Fit *or* Linear Regression

- To fit a straight line through the n points $(x_1, y_1), (x_2, y_2), \ldots ,(x_n, y_n)$
- Assume $f(x) = a_1 + a_2x$

Error
$$E = \sum_{k=1}^{n} [f(x_k) - y_k]^2$$
$$= \sum_{k=1}^{n} [a_1 + a_2x_k - y_k]^2$$

- Find a_1, a_2 which minimize E

$$\frac{\partial E}{\partial a_1} = 2\sum_{k=1}^{n} (a_1 + a_2x_k - y_k) = 0$$

$$\frac{\partial E}{\partial a_2} = 2\sum_{k=1}^{n} (a_1 + a_2x_k - y_k)x_k = 0$$

$$\frac{\partial E}{\partial a_1} = 2\sum_{k=1}^{n}(a_1 + a_2 x_k - y_k) = 0$$

$$\frac{\partial E}{\partial a_2} = 2\sum_{k=1}^{n}(a_1 + a_2 x_k - y_k)x_k = 0$$

Solve:

$$\begin{bmatrix} n & \sum x_k \\ \sum x_k & \sum x_k^2 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum y_k \\ \sum x_k y_k \end{bmatrix}$$

Straight Line Fit (example)

Fit a straight line through the five points

(0, 2.10), (1, 2.85), (2, 1.10), (3, 3.20), (4, 3.90)

$a_{11} = n = 5$

$a_{12} = \sum x_k = 0 + 1 + 2 + 3 + 4 = 0$

$a_{21} = a_{12}$

$a_{22} = \sum x_k^2 = 1 + 4 + 9 + 16 = 30$

$b_1 = \sum y_k = 2.10 + 2.85 + 1.10 + 3.20 = 13.15$

$b_2 = \sum x_k y_k = 2.85 + 2(1.10) + 3(3.20) + 4(3.90) = 30.25$

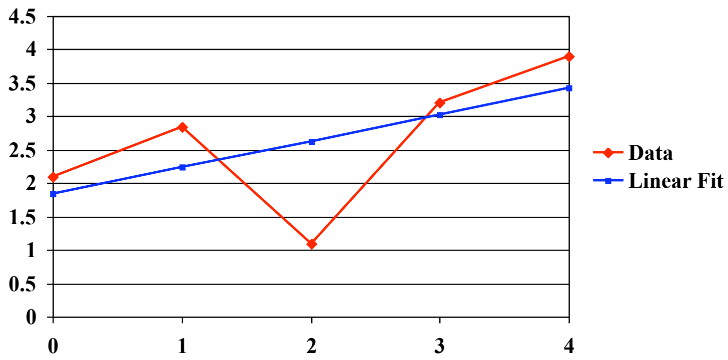
$$\begin{bmatrix} 5 & 10 \\ 10 & 30 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 13.15 \\ 30.25 \end{bmatrix}$$

$a_1 = 1.84$, $a_2 = 0.395$, $f(x) = 1.84 + 0.395x$

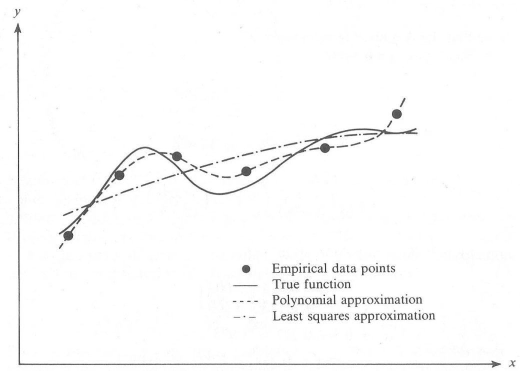
Straight Line Fit (example)

Data points: (0, 2.10), (1, 2.85), (2, 1.10), (3, 3.20), (4, 3.90)

Linear fit: $f(x) = 1.84 + 0.395x$



Two Kinds of Curve Fitting



Data Representation

Integers - Fixed Point Numbers

Decimal System - base 10 uses 0,1,2,…,9

$(396)_{10} = (6 \times 10^0) + (9 \times 10^1) + (3 \times 10^2) = (396)_{10}$

Binary System - base 2 uses 0,1

$(11001)_2 = (1 \times 2^0) + (0 \times 2^1) + (0 \times 2^2) + (1 \times 2^3) + (1 \times 2^4) = (25)_{10}$

Decimal to Binary Conversion

Convert $(39)_{10}$ to binary form

base = 2

2	39	
2	19	+ Remainder 1
2	9	+ Remainder 1
2	4	+ Remainder 1
2	2	+ Remainder 0
2	1	+ Remainder 0
	0	+ Remainder 1

Put the remainder in reverse order

$(100111)_2 = (1 \times 2^0) + (1 \times 2^1) + (1 \times 2^2) + (0 \times 2^3) + (0 \times 2^4)$

$+ (1 \times 2^5) = (39)_{10}$

Largest number that can be stored in m-digits

base - 10 : $(99999\ldots9) = 10^m - 1$

base - 2 : $(11111\ldots1) = 2^m - 1$

m = 3 $(999) = 10^3 - 1$

$(111) = 2^3 - 1$

Limitation: Memory cells consist of 8 bits (1 byte) multiples, each position containing 1 binary digit

- Common cell lengths for integers : k = 16 bits

  k = 32 bits

Sign - Magnitude Notation

First bit is used for a sign

0 - non negative number
1 - negative number

The remaining bits are used to store the binary magnitude of the number.

Limit of 16 bit cell : $(32767)_{10}$

Limit of 32 bit cell : $(2\ 147\ 483\ 647)_{10}$

Two's Complement notation

Definition : The two's complement of a negative integer I in a k - bit cell :

Two's Complement of I $= 2^k + I$

(Eg) : Two's Complement of $(-3)_{10}$ in a 3 - bit cell

$= (2^3 - 3)_{10} = (5)_{10} = (101)_2$

$(-3)_{10}$ will be stored as 101

Storage Scheme for storing an integer I in a k - bit cell in Two's Complement notation

$$\text{Stored Value } C = \begin{cases} I\ ,\quad I \geq 0\ , \text{ first bit} = 0 \\ 2^k + I\ ,\quad I < 0 \end{cases}$$

The Two's Complement notation admits one more negative number than the sign - magnitude notation.

(Eg) Take a 2 bit cell (k = 2)

Range in Sign - magnitude notation : $2^1 - 1 = 1$

$-1 = 11$

$1 = 01$

Range in Two's Compliment notation

Two's Compliment of $-1 = 2^2 - 1 = (3)_{10} = (11)_2$

Two's Compliment of $-2 = 2^2 - 2 = (2)_{10} = (10)_2$

Two's Compliment of $-3 = 2^2 - 2 = 0$ - Not possible

Floating Point Numbers

Integer Part + Fractional Part

Decimal System - base 10

235 . 7846

Binary System - base 2

10011 . 11101

Fractional Part $(0.7846)_{10} = \frac{7}{10} + \frac{8}{10^2} + \frac{4}{10^3} + \frac{6}{10^4}$

Fractional Part $(0.11101)_2 = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{0}{2^4} + \frac{1}{2^5}$

Binary Fraction → Decimal Fraction

(10.11)

Integer Part $(10)_2 = 0.2^0 + 1.2^1 = 2$

Fractional Part $(11)_2 = \frac{1}{2} + \frac{1}{2^2} = 0.5 + 0.25 = 0.75$

Decimal Fraction = $(2.75)_{10}$

Decimal Fraction → Binary Fraction

Convert $(0.9)_{10}$ to binary fraction

$$
\begin{array}{ll}
0.9 & \\
\times 2 & \\
\hline
0.8 & + \text{ integer part } 1 \\
\times 2 & \\
\hline
0.6 & + \text{ integer part } 1 \\
\times 2 & \\
\hline
0.2 & + \text{ integer part } 1 \\
\times 2 & \\
\hline
0.4 & + \text{ integer part } 0 \\
\times 2 & \\
\hline
0.8 & + \text{ integer part } 0
\end{array}
$$

Repetition

$(0.9)_{10} = (0.1\overline{1100})_2$

Scientific Notation (Decimal)

$0.0000747 = 7.47 \times 10^{-5}$

$31.4159265 = 3.14159265 \times 10$

$9{,}700{,}000{,}000 = 9.7 \times 10^{9}$

Binary

$(10.01)_2 = (1.001)_2 \times 2^1$

$(0.110)_2 = (1.10)_2 \times 2^{-1}$

Computer stores a binary approximation to x

$$x \approx \pm\, q \times 2^n$$

q is mantissa , n is exponent

$$(-39.9)_{10} = (-100111.1\ 1100)_2$$
$$= (-1.0001111\ 1100)_2 \times (2^5)_{10}$$

Decimal Value of stored number $(-39.9)_{10}$

$$= -1.\underbrace{001111\ 1100\ 1100\ 1100\ 11001}_{23\text{ bit}}$$

32 bits : First bit for sign

Next 8 bits for exponent

23 bits for mantissa

$$= -39.\ 900001525\ \ 878\ \ 906\ \ 25$$

… Floating Point

- Normalisation
  - Decimal point and base of mantissa, exponent not represented
  - high-order bit is always 1
  - it is implicit (saves 1 bit storage per number)
- Exponent
  - Excess-128 notation
- Suppose use base-16:
  - Eg: $-a.8 \times 16^{-100} = -1010.1000 \times 2^{-400}$
  - High-order digit must be represented (4 bits)
  - Exponent range is 16^8 rather than 2^8
- Most commonly used is IEEE format

Round off Errors can be reduced by Efficient Programming Practice

The number of operations (multiplications and additions) must be kept minimum. (Complexity theory)

An Example of Efficient Programming

Problem : Evaluate the value of the Polynomial.

$P(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$

at a given x.

Requires 13 mulitiplications and 4 additions.

Bad Programme!

An Efficient method (Horner's method)

$P(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$

$= a_0 + x(a_1 + x(a_2 + x(a_3 + xa_4)))$

Requires 4 multiplications and 4 additions.

Pseudo-code for an n^{th} degree polynomial

Input a's, n, x

```
p = 0
for i = n, n-1, … , 0
  {
        p = x × p + a[i]
  }
```

Summary

- Finding the root(s) of an equation
  - Bisection, regula falsi, Newton's, secant
- Fitting curves to data:
  - Exact: Lagrange interpolation
  - Least squares fit: linear regression (also polynomial, exponential, etc)
    - OpenOffice functions: LINEST, LOGEST, TREND
- Several techniques for a given problem
- Technique of choice depends on nature of the data
- Error analysis essential to determine reliability of the computed results

References

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