

METODO DI GAUSS JORDAN

Sunday, August 22, 2021 2:59 PM

Gauss Jordan

dimostrazione del metodo di Gauss-Jordan

Esercizio: Risolvere il seguente sistema di equazioni lineari con il metodo di riduzione di Gauss-Jordan.

$$\begin{cases} -x_1 + 3x_2 + x_3 = 1 \\ 2x_1 - x_2 - x_3 = 0 \\ 2x_1 + x_2 - 2x_3 - 2x_4 = -1 \end{cases}$$

Passo 1: $\left(\begin{array}{cccc|c} 1 & 3 & 1 & 0 & 1 \\ 2 & -1 & -1 & 0 & 0 \\ 2 & 1 & -2 & -1 & -1 \end{array} \right) \xrightarrow{\text{trasformazione elementare (1): considero la riga che non contiene il termine per } x_1 \text{ e applico } E_1 \leftarrow E_1 - 2E_2}$

Passo 2: $\left(\begin{array}{cccc|c} 1 & 3 & 1 & 0 & 1 \\ 0 & -1 & -1 & 0 & 0 \\ 2 & 1 & -2 & -1 & -1 \end{array} \right) \xrightarrow{\lambda = \frac{0}{-1} = 0 \text{ dunque}} \left(\begin{array}{cccc|c} 1 & 3 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 2 & 1 & -2 & -1 & -1 \end{array} \right) \xrightarrow{\text{trasformazione elementare (2): considero la riga che contiene il termine per } x_2 \text{ e applico } E_2 \leftarrow E_2 - E_1}$

Passo 3: $\left(\begin{array}{cccc|c} 1 & 3 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 2 & 1 & -2 & -1 & -1 \end{array} \right) \xrightarrow{\lambda = \frac{1}{1} = 1 \text{ dunque}} \left(\begin{array}{cccc|c} 1 & 3 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 2 & 1 & -2 & -1 & -1 \end{array} \right) \xrightarrow{\text{trasformazione elementare (3): è necessario permettere la progressione delle righe}} \left(\begin{array}{cccc|c} 1 & 3 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & -1 & -1 \end{array} \right) \xrightarrow{\text{di nuovo:}}$

Passo 4: $\left(\begin{array}{cccc|c} 1 & 3 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & -1 & -1 \end{array} \right) \xrightarrow{\text{trasformazione elementare (4): considero la riga che non contiene il termine per } x_3 \text{ e applico } E_3 \leftarrow E_3 - E_2}$

Passo 5: $\left(\begin{array}{cccc|c} 1 & 3 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & -1 & -1 \end{array} \right) \xrightarrow{\lambda = -\frac{1}{2} = -\frac{1}{2} \text{ (3)}} \left(\begin{array}{cccc|c} 1 & 3 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & -\frac{1}{2} \end{array} \right)$

Esercizio 1 Dato il sistema lineare:

$$\begin{cases} x_1 + x_2 - 2x_3 = 1 \\ 2x_1 + x_2 + x_3 = 0 \\ -2x_1 + x_2 - x_3 = -3 \end{cases}$$

Lo si risolve con il metodo di Gauss:

Passo 1: $\left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 2 & 1 & 1 & 0 \\ -2 & 1 & -1 & -3 \end{array} \right) \xrightarrow{\lambda = \frac{1}{2} = -4}$

Passo 2: $\left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & 1 & 5 & -8 \\ 0 & 1 & -1 & -3 \end{array} \right) \xrightarrow{E_2 \leftarrow E_2 - E_1, E_3 \leftarrow E_3 + 2E_1} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & 0 & 3 & -7 \\ 0 & 1 & -1 & -3 \end{array} \right)$

Passo 3: $\left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & 0 & 3 & -7 \\ 0 & 1 & -1 & -3 \end{array} \right) \xrightarrow{-5+9=4} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & 0 & 3 & -7 \\ 0 & 0 & 4 & -4 \end{array} \right) \xrightarrow{E_3 \leftarrow E_3 + 3E_2} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & 0 & 3 & -7 \\ 0 & 0 & 4 & -4 \end{array} \right)$

Passo 4: $\left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & 0 & 3 & -7 \\ 0 & 0 & 4 & -4 \end{array} \right) \xrightarrow{\lambda = \frac{-2}{4} = -\frac{1}{2}} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & 0 & 3 & -7 \\ 0 & 0 & 4 & -4 \end{array} \right) \xrightarrow{E_2 \leftarrow -E_2} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & 0 & 3 & 7 \\ 0 & 0 & 4 & -4 \end{array} \right)$

Passo 5: $\left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & 0 & 3 & 7 \\ 0 & 0 & 4 & -4 \end{array} \right) \xrightarrow{\lambda = \frac{7}{4} = \frac{7}{4}} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & 0 & 3 & 7 \\ 0 & 0 & 1 & -1 \end{array} \right) \xrightarrow{E_3 \leftarrow \frac{1}{4}E_3} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & 0 & 3 & 7 \\ 0 & 0 & 1 & -1 \end{array} \right)$

$$\begin{cases} x_1 = -1 - x_2 \\ x_2 = -2 \\ x_3 = -1 \end{cases}$$

$$\lambda_1 = -\frac{0}{2} = 0$$

$$\lambda_2 = -\frac{3}{2}$$

①

$$\begin{array}{l} \times 1 \left[\begin{array}{cccc|c} 0 & -1 & 0 & 1 & 1 \\ 2 & 1 & -1 & 2 & 3 \\ 3 & 0 & 1 & 1 & 2 \end{array} \right] E_1 \leftarrow E_1 + 2E_2 \\ \left[\begin{array}{cccc|c} 0 & -1 & 0 & 1 & 1 \\ 0 & 3 & -3 & 4 & 3 \\ 3 & 0 & 1 & 1 & 2 \end{array} \right] E_3 \leftarrow E_3 - \frac{3}{2}E_2 \\ \left[\begin{array}{cccc|c} 0 & -1 & 0 & 1 & 1 \\ 0 & 3 & -3 & 4 & 3 \\ 0 & \frac{3}{2} & \frac{3}{2} & -2 & -\frac{3}{2} \end{array} \right] \end{array} \xrightarrow{\quad} \left[\begin{array}{cccc|c} 0 & -1 & 0 & 1 & 1 \\ 0 & 3 & -3 & 4 & 3 \\ 0 & \frac{3}{2} & \frac{3}{2} & -2 & -\frac{3}{2} \end{array} \right]$$

②

$$\left[\begin{array}{cccc|c} 0 & -1 & 0 & 1 & 1 \\ 2 & 1 & -1 & 2 & 3 \\ 0 & -\frac{3}{2} & \frac{3}{2} & -2 & -\frac{3}{2} \end{array} \right] E_2 \leftarrow \frac{1}{2}E_2 \xrightarrow{\quad} \left[\begin{array}{cccc|c} 0 & -1 & 0 & 1 & 1 \\ 1 & \frac{1}{2} & -\frac{1}{2} & 1 & \frac{3}{2} \\ 0 & -\frac{3}{2} & \frac{3}{2} & -2 & -\frac{3}{2} \end{array} \right]$$

③

$$\left[\begin{array}{cccc|c} 0 & -1 & 0 & 1 & 1 \\ 1 & \frac{1}{2} & -\frac{1}{2} & 1 & \frac{3}{2} \\ 0 & -\frac{3}{2} & \frac{3}{2} & -2 & -\frac{3}{2} \end{array} \right] E_1 \leftrightarrow E_2 \xrightarrow{\quad} \left[\begin{array}{cccc|c} 1 & \frac{1}{2} & -\frac{1}{2} & 1 & \frac{3}{2} \\ 0 & -1 & 0 & 1 & 1 \\ 0 & -\frac{3}{2} & \frac{3}{2} & -2 & -\frac{3}{2} \end{array} \right]$$

(4)

$$\lambda_1 = \frac{1}{2} \quad \lambda_2 = -\frac{3}{2}$$

$$\left[\begin{array}{cccc|c} 1 & \frac{1}{2} & -\frac{1}{2} & 1 & \frac{3}{2} \\ 0 & -1 & 0 & 1 & 1 \\ 0 & -\frac{3}{2} & \frac{5}{2} & -2 & -\frac{5}{2} \end{array} \right] \xrightarrow{\begin{array}{l} E_1 \leftarrow E_1 + \frac{1}{2}E_2 \\ E_3 \leftarrow E_3 - \frac{3}{2}E_2 \end{array}} \left[\begin{array}{cccc|c} 1 & 0 & -\frac{1}{2} & \frac{3}{2} & 2 \\ 0 & -1 & 0 & 1 & 1 \\ 0 & 0 & \frac{5}{2} & -\frac{3}{2} & -4 \end{array} \right]$$

(5)

$$\left[\begin{array}{cccc|c} 1 & 0 & -\frac{1}{2} & \frac{3}{2} & 2 \\ 0 & -1 & 0 & 1 & 1 \\ 0 & 0 & \frac{5}{2} & -\frac{3}{2} & -4 \end{array} \right] \xrightarrow{E_2 \leftarrow -E_2} \left[\begin{array}{cccc|c} 1 & 0 & -\frac{1}{2} & \frac{3}{2} & 2 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & \frac{5}{2} & -\frac{3}{2} & -4 \end{array} \right]$$

(6)

$$\lambda_1 = -\frac{-\frac{1}{2}}{\frac{5}{2}} = -\frac{1}{2} \cdot \frac{2}{5} = +\frac{2}{10} = \frac{1}{5}$$

$$\lambda_2 = \frac{0}{\frac{5}{2}} = 0$$

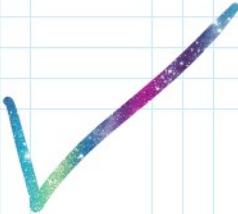
$$\left[\begin{array}{cccc|c} 1 & 0 & -\frac{1}{2} & \frac{3}{2} & 2 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & \frac{5}{2} & -\frac{3}{2} & -4 \end{array} \right] \xrightarrow{\begin{array}{l} E_1 \leftarrow E_1 + \frac{1}{5}E_3 \\ E_2 \leftarrow E_2 + 0E_3 \end{array}} \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{4}{5} & \frac{6}{5} \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & \frac{5}{2} & -\frac{3}{2} & -4 \end{array} \right]$$

$$2 - \frac{1}{5} 4$$

(7)

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{4}{5} & \frac{6}{5} \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & \frac{5}{2} & -\frac{3}{2} & -4 \end{array} \right] \xrightarrow{E_3 \leftarrow \frac{2}{5}E_3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{4}{5} & \frac{6}{5} \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 1 & -\frac{3}{5} & -\frac{8}{5} \end{array} \right]$$

- 4 $\frac{2}{5}$



(1)

$$\left[\begin{array}{ccc|c} 2 & -1 & 1 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 & 1 \\ 1 & 1 & 1 & 3 \\ 2 & 2 & 1 & 5 \end{array} \right] \xrightarrow{\begin{array}{l} E_2 \leftarrow E_2 - \frac{1}{4}E_1 \\ E_3 \leftarrow E_3 - \frac{1}{2}E_1 \\ E_4 \leftarrow E_4 - E_1 \end{array}} \left[\begin{array}{ccc|c} 2 & -1 & 1 & 1 \\ 0 & \frac{3}{4} & -\frac{1}{4} & \frac{3}{4} \\ 0 & \frac{3}{2} & \frac{1}{2} & \frac{5}{2} \\ 0 & 3 & 0 & 4 \end{array} \right]$$

$$\lambda_2 = -\frac{1/2}{2} = -\frac{1}{4}$$

$$\lambda_3 = -\frac{1}{2}$$

$$\lambda_4 = -1$$

(2)

$$\left[\begin{array}{ccc|c} 2 & -1 & 1 & 1 \\ 0 & \frac{3}{4} & -\frac{1}{4} & \frac{3}{4} \\ 0 & \frac{3}{2} & \frac{1}{2} & \frac{5}{2} \\ 0 & 3 & 0 & 4 \end{array} \right] \xrightarrow{E_1 \leftarrow \frac{1}{2}E_1} \left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{3}{4} & -\frac{1}{4} & \frac{3}{4} \\ 0 & \frac{3}{2} & \frac{1}{2} & \frac{5}{2} \\ 0 & 3 & 0 & 4 \end{array} \right]$$

(3)

(3)

$$\left[\begin{array}{cccc|c} 1 & -\frac{1}{2} & 1_2 & 1/2 \\ 0 & \frac{3}{4} & -\frac{1}{4} & \frac{3}{4} \\ 0 & \frac{3}{2} & \frac{1}{2} & 5/2 \\ 0 & 3 & 0 & 4 \end{array} \right] \quad \begin{array}{l} E_1 \leftarrow E_1 + \frac{2}{3} E_2 \\ E_3 \leftarrow E_3 - 2E_2 \\ E_4 \leftarrow E_4 - 4E_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{3} & 1 \\ 0 & \frac{3}{4} & -\frac{1}{4} & \frac{3}{4} \\ 0 & -\frac{3}{2} & 1 & -\frac{4}{3} \\ 0 & 0 & 1 & 1 \end{array} \right]$$

$$\lambda_1 = \frac{2}{3} \quad \lambda_3 = -2 \quad \lambda_4 = -4$$

(4)

$$\left[\begin{array}{cccc|c} 1 & 0 & \frac{1}{3} & 1 \\ 0 & \frac{3}{4} & -\frac{1}{4} & \frac{3}{4} \\ 0 & -\frac{3}{2} & 1 & -\frac{4}{3} \\ 0 & 0 & 1 & 1 \end{array} \right] \quad \begin{array}{l} E_2 \leftarrow \frac{4}{3} E_2 \\ E_3 \leftarrow E_4 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{3} & 1 \\ 0 & 1 & -\frac{1}{3} & 1 \\ 0 & 0 & 1 & 1 \\ 0 & -\frac{3}{2} & 1 & -\frac{4}{3} \end{array} \right]$$

(5)

$$\left[\begin{array}{cccc|c} 1 & 0 & \frac{1}{3} & 1 \\ 0 & 1 & -\frac{1}{3} & 1 \\ 0 & 0 & 1 & 1 \\ 0 & -\frac{3}{2} & 1 & -\frac{4}{3} \end{array} \right] \quad \begin{array}{l} E_1 \leftarrow E_1 - \frac{1}{3} E_3 \\ E_2 \leftarrow E_2 + \frac{1}{3} E_3 \\ E_4 \leftarrow E_4 - E_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{2}{3} \\ 0 & 1 & 0 & \frac{4}{3} \\ 0 & 0 & 1 & 1 \\ 0 & -\frac{3}{2} & 0 & -\frac{7}{3} \end{array} \right]$$

$$\lambda_1 = -\frac{1}{3} \quad \lambda_2 = \frac{1}{3} \quad \lambda_3 = -1$$

(6)

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{2}{3} \\ 0 & 1 & 0 & \frac{4}{3} \\ 0 & 0 & 1 & 1 \\ 0 & -\frac{3}{2} & 0 & -\frac{7}{3} \end{array} \right] \quad E_4 \leftarrow -\frac{2}{3} E_4$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{2}{3} \\ 0 & 1 & 0 & \frac{4}{3} \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -\frac{7}{3} \end{array} \right]$$

$$-\frac{2}{3} \left(-\frac{7}{3} \right) = +\frac{14}{9}$$



$$\left[\begin{array}{cccc|c} 2 & \frac{1}{2} & \frac{1}{3} & 1 & 1 \\ 1 & -1 & 2 & 1 & 1 \\ 3 & 2 & -1 & 2 & 1 \end{array} \right] \quad \begin{array}{l} E_1 \leftarrow \frac{1}{2} E_1 \times \\ E_2 \leftarrow E_2 - \frac{1}{2} E_1 \\ E_3 \leftarrow E_3 - \frac{3}{2} E_1 \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & \frac{1}{4} & \frac{1}{6} & \frac{1}{2} & \frac{1}{2} \\ 0 & -\frac{5}{4} & \frac{11}{6} & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{5}{4} & -\frac{3}{2} & \frac{1}{2} & -\frac{1}{2} \end{array} \right]$$

$$\lambda_1 = \frac{1}{2} \quad \lambda_2 = -\frac{1}{2} \quad \lambda_3 = -\frac{3}{2}$$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{4} & \frac{1}{6} & \frac{1}{2} \\ 0 & -\frac{5}{4} & \frac{11}{6} & \frac{1}{2} \\ 0 & \frac{5}{4} & -\frac{3}{2} & \frac{1}{2} \end{array} \right] \quad \begin{aligned} E_1 &\leftarrow E_1 + \frac{1}{5}E_2 \\ E_2 &\leftarrow -\frac{1}{5}E_2 \\ E_3 &\leftarrow E_3 + E_2 \end{aligned}$$

$$\lambda_1 = +\frac{1}{5}, \quad \lambda_2 = -\frac{4}{5}, \quad \lambda_3 = 1$$

$$\left[\begin{array}{cccc|c} 1 & 0 & \frac{8}{15} & \frac{3}{5} & \frac{3}{5} \\ 0 & 1 & -\frac{22}{15} & -\frac{2}{5} & -\frac{2}{5} \\ 0 & 0 & \frac{10}{3} & 1 & 0 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & \frac{8}{15} & \frac{3}{5} & \frac{3}{5} \\ 0 & 1 & -\frac{22}{15} & -\frac{2}{5} & -\frac{2}{5} \\ 0 & 0 & \frac{10}{3} & 1 & 0 \end{array} \right] \quad \begin{aligned} E_1 &\leftarrow E_1 - \frac{4}{25}E_3 \\ E_2 &\leftarrow E_2 + \frac{11}{25}E_3 \\ E_3 &\leftarrow \frac{3}{10}E_3 \end{aligned}$$

$$\lambda_1 = -\frac{4}{25}, \quad \lambda_2 = \frac{11}{25}, \quad \lambda_3 = \frac{3}{10}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{11}{25} & \frac{3}{5} \\ 0 & 1 & 0 & \frac{1}{25} & -\frac{2}{5} \\ 0 & 0 & 1 & \frac{3}{10} & 0 \end{array} \right]$$

$$\lambda_1 = \frac{1}{2}, \quad \lambda_2 = \frac{1}{2}, \quad \lambda_3 = -\frac{1}{4}$$

$$\left[\begin{array}{ccc|c} 1 & \frac{3}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{5}{2} & \frac{3}{2} & -\frac{3}{2} \\ 0 & \frac{5}{4} & \frac{3}{4} & -\frac{3}{4} \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 2 & 3 & -1 & -1 & 5 \\ -1 & 1 & 2 & -1 & 6 \\ \frac{1}{2} & 2 & \frac{1}{2} & -1 & 3 \end{array} \right]$$

$$\begin{aligned} E_1 &\leftarrow \frac{1}{2}E_1 & \text{①} \\ E_2 &\leftarrow E_2 + \frac{1}{2}E_1 & \checkmark \\ E_3 &\leftarrow E_3 - \frac{1}{4}E_1 \end{aligned}$$

$\lambda_1 = 3, \quad \lambda_2 = 2$

$$\left[\begin{array}{cccc|c} 0 & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & | & \frac{1}{4} \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & \frac{2}{5} & -\frac{7}{5} & | & \frac{-13}{5} \\ 0 & 1 & \frac{3}{5} & -\frac{3}{5} & | & \frac{17}{5} \\ 0 & 0 & 0 & 0 & | & \frac{-5}{2} \end{array} \right]$$

← ←



x

$$0 \leftarrow -3 - \frac{1}{4} \cdot L_1$$

$$\lambda_1 = -\frac{3}{5}, \quad \lambda_2 = \frac{2}{5}, \quad \lambda_3 = -\frac{1}{2}.$$

$$E_1 \leftarrow E_1 - \frac{3}{5} E_2$$

$$E_2 \leftarrow \frac{2}{5} E_2$$



$$E_3 \leftarrow E_3 - \frac{1}{2} E_2.$$

SONO Felice 😊

③



x

$$\left[\begin{array}{cccc|c} 2 & -1 & 0 & -2 & | & 1 \\ 0 & 2 & 1 & 2 & | & 3 \\ 3 & 0 & 1 & -1 & | & 2 \end{array} \right]$$

$E_1 \leftarrow \frac{1}{2} E_1$
 $E_2 \leftarrow \bar{E}_2$
 $\bar{E}_3 \leftarrow E_3 - \frac{3}{2} E_1$

$$\left[\begin{array}{cccc|c} 1 & -\frac{1}{2} & 0 & -1 & | & \frac{1}{2} \\ 0 & 2 & 1 & 2 & | & 3 \\ 0 & +\frac{3}{2} & 1 & -2 & | & \frac{1}{2} \end{array} \right]$$

$$\lambda_1 = \frac{1}{2}, \quad \lambda_2 = 0, \quad \lambda_3 = -\frac{3}{2}$$

$$\left[\begin{array}{cccc|c} 1 & -\frac{1}{2} & 0 & -1 & | & \frac{1}{2} \\ 0 & 2 & 1 & 2 & | & 3 \\ 0 & +\frac{3}{2} & 1 & -2 & | & \frac{1}{2} \end{array} \right]$$

$E_1 \leftarrow \bar{E}_1 + \frac{1}{6} \bar{E}_2$
 $\bar{E}_2 \leftarrow \frac{1}{2} \bar{E}_2$
 $\bar{E}_3 \leftarrow \bar{E}_3 - \frac{3}{4} \bar{E}_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & \frac{1}{6} & -\frac{1}{2} & | & \frac{5}{6} \\ 0 & 1 & \frac{1}{2} & 1 & | & \frac{3}{2} \\ 0 & 0 & \frac{1}{4} & \frac{7}{2} & | & \frac{3}{8} \end{array} \right]$$

$$\lambda_1 = \frac{1}{4}, \quad \lambda_2 = \frac{1}{2}, \quad \lambda_3 = -\frac{3}{4}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & \frac{1}{4} & -\frac{1}{2} & \frac{5}{8} \\ 0 & 1 & \frac{1}{2} & 1 & \frac{3}{2} \\ 0 & 0 & \frac{1}{4} & \frac{7}{2} & \frac{3}{8} \end{array} \right] \quad \begin{aligned} \bar{E}_1 &\leftarrow \bar{E}_1 - \bar{E}_3 \\ \bar{E}_2 &\leftarrow \bar{E}_2 - 2\bar{E}_3 \\ \bar{E}_3 &\leftarrow 4\bar{E}_3 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -4 & \frac{7}{16} \\ 0 & 1 & 0 & -6 & \frac{3}{4} \\ 0 & 0 & 1 & 14 & \frac{3}{2} \end{array} \right]$$

$$\lambda_1 = -1 \quad \lambda_2 = -2 \quad \lambda_3 = 4$$

$$\frac{3}{2} - \frac{3}{8} \cdot 2 = \frac{3}{2} - \frac{6}{8}$$



$$\left[\begin{array}{ccccc|c} 2 & 2 & 0 & -1 & 3 \\ 0 & 1 & -2 & 1 & 4 \\ 1 & 1 & -\frac{2}{3} & 0 & 2 \end{array} \right] \quad \begin{aligned} \bar{E}_1 &= \bar{E}_1 - \frac{1}{2} \\ \bar{E}_2 &= \bar{E}_2 \\ \bar{E}_3 &= \bar{E}_3 - \frac{1}{2}\bar{E}_1 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & -\frac{1}{2} & \frac{3}{2} \\ 0 & 1 & -2 & 1 & 4 \\ 0 & 0 & -\frac{2}{3} & \frac{1}{2} & \frac{1}{2} \end{array} \right]$$

$$\lambda_1 = \frac{1}{2} \quad \lambda_2 = 0 \quad \lambda_3 = -\frac{1}{2}$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & -\frac{1}{2} & \frac{3}{2} \\ 0 & 1 & -2 & 1 & 4 \\ 0 & 0 & -\frac{2}{3} & \frac{1}{2} & \frac{1}{2} \end{array} \right] \quad \begin{aligned} \bar{E}_1 &= \bar{E}_1 - \bar{E}_2 \\ \bar{E}_2 &= \bar{E}_2 \\ \bar{E}_3 &= \bar{E}_3 \end{aligned}$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 2 & -\frac{3}{2} & -\frac{5}{2} \\ 0 & 1 & -2 & 1 & 4 \\ 0 & 0 & -\frac{2}{3} & \frac{1}{2} & \frac{1}{2} \end{array} \right]$$

$$\lambda_1 = -1 \quad \lambda_2 = 1 \quad \lambda_3 = 0$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 2 & -\frac{3}{2} & -\frac{5}{2} \\ 0 & 1 & -2 & 1 & 4 \\ 0 & 0 & -\frac{2}{3} & \frac{1}{2} & \frac{1}{2} \end{array} \right] \quad \begin{aligned} \bar{E}_1 &= \bar{E}_1 + 3\bar{E}_3 \\ \bar{E}_2 &= \bar{E}_2 - 3\bar{E}_3 \\ \bar{E}_3 &= -\frac{3}{2}\bar{E}_3 \end{aligned}$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & -\frac{1}{2} & \frac{5}{2} \\ 0 & 0 & 1 & -\frac{3}{4} & -\frac{3}{4} \end{array} \right]$$

$$\lambda_1 = 3 \quad \lambda_2 = -3 \quad \lambda_3 = -\frac{3}{2}$$



$$\lambda_1 = 3 \quad \lambda_2 = -3 \quad \lambda_3 = -\frac{5}{2}$$



x

$$\left[\begin{array}{cccc|c} 2 & 2 & 0 & -1 & 3 \\ 0 & 1 & -2 & 1 & 4 \\ 1 & 1 & -\frac{2}{3} & 0 & 2 \end{array} \right]$$



$$\left[\begin{array}{cccc|c} 2 & 2 & 0 & -1 & 3 \\ 0 & 1 & -2 & 1 & 4 \\ 1 & 1 & -\frac{2}{3} & 0 & 2 \end{array} \right] \quad \begin{aligned} E_1 &= \frac{1}{2} \bar{E}_1 \\ E_2 &= E_2 \\ \bar{E}_3 &= \bar{E}_3 - \frac{1}{2} \bar{E}_1 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & -\frac{1}{2} & \frac{3}{2} \\ 0 & 1 & -2 & 1 & 4 \\ 0 & 0 & -\frac{2}{3} & \frac{1}{2} & \frac{1}{2} \end{array} \right]$$

$$\lambda_1 = \frac{1}{2} \quad \lambda_2 = 0 \quad \lambda_3 = -\frac{1}{2}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & -\frac{1}{2} & \frac{3}{2} \\ 0 & 1 & -2 & 1 & 4 \\ 0 & 0 & -\frac{2}{3} & \frac{1}{2} & \frac{1}{2} \end{array} \right] \quad \begin{aligned} E_1 &= E_1 - E_2 \\ \bar{E}_2 &= E_2 \\ \bar{E}_3 &= E_3 \end{aligned}$$

$$\lambda_1 = -1 \quad \lambda_2 = 1 \quad \lambda_3 = 0$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 2 & -\frac{3}{2} & -\frac{5}{2} \\ 0 & 1 & -2 & 1 & 4 \\ 0 & 0 & -\frac{2}{3} & \frac{1}{2} & \frac{1}{2} \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 2 & -\frac{3}{2} & -\frac{5}{2} \\ 0 & 1 & -2 & 1 & 4 \\ 0 & 0 & -\frac{2}{3} & \frac{1}{2} & \frac{1}{2} \end{array} \right] \quad \begin{aligned} E_1 &= \bar{E}_1 + 3\bar{E}_3 \\ E_2 &= \bar{E}_1 - 3\bar{E}_3 \\ E_3 &= -\frac{3}{2}\bar{E}_3 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & -\frac{1}{2} & \frac{5}{2} \\ 0 & 0 & 1 & -\frac{3}{4} & -\frac{3}{4} \end{array} \right]$$

$$\lambda_1 = 3 \quad \lambda_2 = -3 \quad \lambda_3 = -\frac{3}{2}$$



$$\left[\begin{array}{ccccc|c} 2 & 0 & -1 & 3 & -1 & 3 \\ 0 & 1 & 2 & -1 & 0 & 4 \\ 1 & \frac{1}{2} & \frac{1}{2} & 1 & -\frac{1}{2} & 3 \end{array} \right] \quad \begin{aligned} E_1 &= \frac{1}{2}E_1 \\ E_2 &= E_2 \\ E_3 &= E_3 - \frac{1}{2}E_1 \end{aligned}$$

$$\lambda_1 = \frac{1}{2} \quad \lambda_2 = 0 \quad \lambda_3 = -\frac{1}{2}$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & -\frac{1}{2} & \frac{3}{2} & -\frac{1}{2} & \frac{3}{2} \\ 0 & 1 & 2 & -1 & 0 & 4 \\ 0 & \frac{1}{2} & 1 & -\frac{1}{2} & 0 & \frac{3}{2} \end{array} \right]$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & -\frac{1}{2} & \frac{3}{2} & -\frac{1}{2} & \frac{3}{2} \\ 0 & 1 & 2 & -1 & 0 & 4 \\ 0 & \frac{1}{2} & 1 & -\frac{1}{2} & 0 & \frac{3}{2} \end{array} \right] \quad \begin{aligned} E_1 &= E_1 \\ E_2 &= E_2 \\ E_3 &= E_3 - \frac{1}{2}E_2 \end{aligned}$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & -\frac{1}{2} & \frac{3}{2} & -\frac{1}{2} & \frac{3}{2} \\ 0 & 1 & 2 & -1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} \end{array} \right]$$

$$\lambda_1 = 0 \quad \lambda_2 = 1 \quad \lambda_3 = -\frac{1}{2}$$

Il sistema non ammette soluzioni

