PROGRAMMAZIONE LINEARE

Saturday, August 21, 2021 5:13 PM

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① Risolvere il programma con il metodo grafico → Basta disegnare le rette

$$\begin{cases} X_1 + 3X_2 = 6 \\ -X_1 + X_2 = 1 \\ X_1 = 3 \end{cases}$$

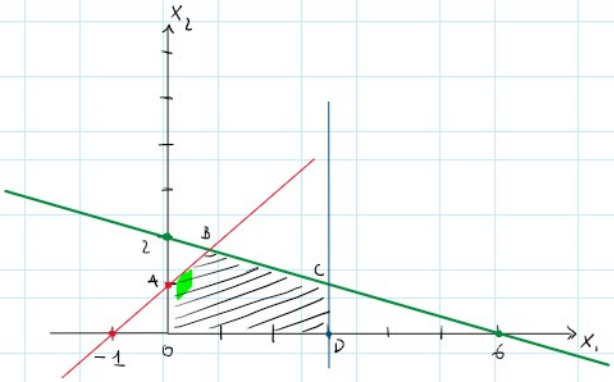
$X_1, X_2 \geq 0$

$X_1 + 3X_2 = 6$

X_1	X_2
0	2
6	0

$-X_1 + X_2 = 1$

X_1	X_2
-1	0
0	1



② Trasformare il programma in forma standard

$$\max z = X_1 + 4X_2$$
$$X_1 + 3X_2 + X_3 = 6$$
$$-X_1 + X_2 + X_4 = 1$$
$$X_1 + X_5 = 3$$

$X_1, X_2, X_3, X_4, X_5 \geq 0$

③ Elencare le basi ammissibili del programma in forma standard

Calcolo le coordinate dei vertici

O (0,0)
A (0,1)
B (3/4, 7,4)
C (3,1)
D (3,1)

→ Mettere a sistema le rette per trovare il punto (vedi immagine)

O (0,0)
A (0,1)
B (3/4, 7/4)
C (3,1)
D (3,1)

↳ Mettere a sistema le rette per trovare il punto (vedi immagine)

$$\begin{cases} x_1 + 3x_2 + x_3 = 6 \\ -x_1 + x_2 + x_4 = 1 \\ x_1 + x_5 = 3 \end{cases}$$

Vertice			
0	$x_3=6$	$x_4=1$	$x_5=3$
A	$x_2=3$	$x_4=1$	$x_5=3$
B	$x_1=3/4$	$x_3=3/4$	$x_4=3/4$
C	$x_1=3$	$x_3=1$	$x_4=3$
D	$x_1=3$	$x_3=3$	$x_4=4$

Vertice			
0			
A			
B			
C			
D			
E			

- O(0,0) $\begin{cases} x_3=6 \\ x_4=1 \\ x_5=3 \end{cases}$
- A(0,1) $\begin{cases} 3+x_3=6 \\ 1+x_4=1 \\ x_5=3 \end{cases} \Rightarrow \begin{cases} x_3=3 \\ x_4=0 \\ x_5=3 \end{cases}$
- B$(\frac{3}{4}, \frac{7}{4})$ $\begin{cases} \frac{3}{4}+3\frac{7}{4}+x_3=6 \\ -\frac{3}{4}+\frac{7}{4}+x_4=1 \\ \frac{3}{4}+x_5=3 \end{cases} \Rightarrow \begin{cases} x_3=6-\frac{24}{4}=0 \\ x_4=0 \\ x_5=\frac{9}{4} \end{cases}$
- C(3,1) $\begin{cases} 3+3+x_3=6 \\ -3+1+x_4=1 \\ 3+x_5=3 \end{cases} \Rightarrow \begin{cases} x_3=0 \\ x_4=3 \\ x_5=0 \end{cases}$
- D(3,0) $\begin{cases} 3+x_3=6 \\ -3+x_4=1 \\ 3+x_5=3 \end{cases} \Rightarrow \begin{cases} x_3=3 \\ x_4=4 \\ x_5=0 \end{cases}$

$$\begin{cases} x_1 + 3x_2 + x_3 = 6 \\ -x_1 + x_2 + x_4 = 1 \\ x_1 + x_5 = 3 \end{cases}$$

④ Calcolare soluzione ottima con algoritmo del simplesso

$$\begin{cases} x_1 + 3x_2 + x_3 = 6 \\ -x_1 + x_2 + x_4 = 1 \\ x_1 + x_5 = 3 \end{cases}$$ → forma standard

$$\left[\begin{array}{cc|ccc|c} 1 & 3 & 1 & 0 & 0 & 6 \\ -1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

Rapporti: 2, 1, $\nexists$

$\max z = x_1 + 4x_2$ → Costo ridotto maggiore

$x_3 = -x_1 - 3x_2 + 6$

$x_4 = x_1 - x_2 + 1$

$x_5 = -x_1 + 3$

$x_3\ x_4\ x_5 \rightarrow x_3\ x_2\ x_5$

Effettuo un'operazione di pivot (perno = 1) per cambiare base da $\{x_3, x_4, x_5\}$ a $\{x_2, x_3, x_5\}$

$$\left[\begin{array}{cc|ccc|c} 1 & 3 & 1 & 0 & 0 & 6 \\ -1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

$E_1 \leftarrow E_1 - 3E_2$

$E_3 \leftarrow E_3 + 0E_2$

$\lambda_1 = -3 \quad \lambda_3 = 0$

$$\left[\begin{array}{cc|ccc|c} 4 & 0 & 1 & -3 & 0 & 3 \\ -1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

La nuova base è $\{x_2, x_3, x_5\}$

$x_2 = x_1 - x_4 + 1$
$x_3 = -4x_1 + 3x_4 + 3$
$x_5 = -x_1 + 3$

Trovo la nuova funzione obiettivo

max $z = x_1 + 4x_2$

$\max z = x_1 + 4(x_1 - x_4 + 1)$

$\max z = x_1 + 4x_1 - 4x_4 + 4$

$\max z = 5x_1 - 4x_4 + 4$ → Nuova funzione obiettivo
Lo NON è ancora ottima.

$$\left[\begin{array}{ccccc|c} 4 & 0 & 1 & -3 & 0 & 3 \\ -1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 3 \end{array}\right] \begin{matrix} 3/4 \\ // \\ 3 \end{matrix}$$

$E_2 \leftarrow E_2 + \frac{1}{4}E_1$
$E_3 \leftarrow E_3 - \frac{1}{4}E_1$
$E_1 \leftarrow \frac{1}{4}E_1$

$\lambda_2 = \frac{1}{4}$ $\lambda_3 = -\frac{1}{4}$

$$\left[\begin{array}{ccccc|c} 1 & 0 & \frac{1}{4} & -\frac{3}{4} & 0 & 3/4 \\ 0 & 1 & \frac{1}{4} & \frac{1}{4} & 0 & 7/4 \\ 0 & 0 & -\frac{1}{4} & \frac{3}{4} & 1 & \frac{9}{4} \end{array}\right]$$

La base è cambiata da $\{x_1, x_3, x_5\}$ a $\{x_1, x_2, x_5\}$

Calcolo le x

$x_1 = -\frac{1}{4}x_3 + \frac{3}{4}x_4 + \frac{3}{4}$
$x_2 = -\frac{1}{4}x_3 - \frac{1}{4}x_4 + \frac{7}{4}$
$x_5 = \frac{1}{4}x_3 - \frac{3}{4}x_4 + \frac{9}{4}$

f. obiettivo

$\max z = 5x_1 - 4x_4 + 4$
$\max z = 5\left(-\frac{1}{4}x_3 + \frac{3}{4}x_4 + \frac{3}{4}\right) - 4x_4 + 4$
$\max z = -\frac{5}{4}x_3 + \frac{15}{4}x_4 + \frac{15}{4} - 4x_4 + 4$
$\max z = -\frac{5}{4}x_3 - \frac{1}{4}x_4 + \frac{31}{4}$

① Metodo grafico

$\max z = x_1 + 2x_2$

$$2x_1+3x_2 \geq 4$$
$$2x_1+x_2 \leq 6$$
$$x_2 \leq 3$$
$$x_1, x_2 \geq 0$$

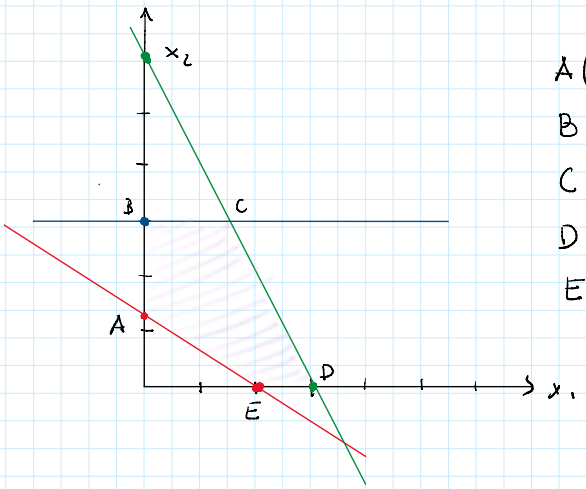
$2x_1+3x_2=4$

x_1	x_2
2	0
0	4/3

$2x_1+x_2=6$

x_1	x_2
3	0
0	6

$x_2=3$



$A\left(0, \frac{4}{3}\right)$
$B(0,3)$
$C\left(\frac{3}{2}, 3\right)$
$D(3,0)$
$E(2,0)$

② Forma standard

$$\max z = x_1+2x_2$$

$$2x_1+3x_2 \geq 4$$
$$2x_1+x_2 \leq 6$$
$$x_2 \leq 3$$

$\Rightarrow$

$$\max z = x_1+2x_2$$
$$2x_1+3x_2-x_3=4$$
$$2x_1+x_2+x_4=6$$
$$x_2+x_5=3$$

③ Basi ammissibili

$A\left(0, \frac{4}{3}\right)$
$B(0,3)$
$C\left(\frac{3}{2}, 3\right)$
$D(3,0)$
$E(2,0)$

$$\begin{cases} 2x_1+3x_2-x_3=4 \\ 2x_1+x_2+x_4=6 \\ x_2+x_5=3 \end{cases}$$

- $A(0, 4/3)$

$$\begin{cases} 0x_1+4-x_3=4 \\ 4/3+x_4=6 \\ 4/3+x_5=3 \end{cases} \quad \begin{cases} x_3=0 \\ x_4=6-\frac{4}{3}=\frac{14}{3} \\ x_5=\frac{5}{3} \end{cases}$$

- B(0,3) $\begin{cases} 9 - x_3 = 4 \\ 3 + x_4 = 6 \\ 3 + x_5 = 3 \end{cases} \begin{cases} x_3 = 5 \\ x_4 = 3 \\ x_5 = 0 \end{cases}$

- C(3/2, 3) $\begin{cases} 3 + 9 - x_3 = 4 \\ 3 + 3 + x_4 = 6 \\ 3 + x_5 = 3 \end{cases} \begin{cases} x_3 = 8 \\ x_4 = 0 \\ x_5 = 0 \end{cases}$

- D(3,0) $\begin{cases} 6 - x_3 = 4 \\ 6 + x_4 = 6 \\ x_5 = 3 \end{cases} \begin{cases} x_3 = 2 \\ x_4 = 0 \\ x_5 = 3 \end{cases}$

- E(2,0) $\begin{cases} 4 - x_3 = 4 \\ 4 + x_4 = 6 \\ x_5 = 3 \end{cases} \begin{cases} x_3 = 0 \\ x_4 = 2 \\ x_5 = 3 \end{cases}$

- $x_2 \quad x_4 \quad x_5$
  $x_2 \quad x_3 \quad x_4$
  $x_1 \quad x_2 \quad x_3$
  $x_1 \quad x_3 \quad x_5$
  $x_1 \quad x_4 \quad x_5$

(4) Simplesso

Dobbiamo portare in base $\{x_2, x_4, x_5\}$

$\max z = x_1 + 2x_2$

$2x_1 + 3x_2 - x_3 = 4$
$2x_1 + x_2 + x_4 = 6$
$x_2 + x_5 = 3$

$$2x_1 + x_2 + x_4 = 6$$
$$x_2 + x_5 = 3$$

$$\left[\begin{array}{ccccc|c} 2 & 3 & -1 & 0 & 0 & 4 \\ 2 & 1 & 0 & 1 & 0 & 6 \\ 0 & 1 & 0 & 0 & 1 & 3 \end{array}\right] \qquad \lambda_2 = -\frac{1}{3} \quad \lambda_3 = -\frac{1}{3} \quad \lambda_1 = \frac{1}{3}$$

$E_2 \leftarrow E_2 - \frac{1}{3}E_1$
$E_3 \leftarrow E_3 - \frac{1}{3}E_1$
$E_1 \leftarrow \frac{1}{3}E_1$

$$\left[\begin{array}{ccccc|c} 2/3 & 1 & -\frac{1}{3} & 0 & 0 & 4/3 \\ 4/3 & 0 & 1/3 & 1 & 0 & 14/3 \\ -\frac{2}{3} & 0 & 1/3 & 0 & 1 & 5/3 \end{array}\right]$$

$$\begin{cases} x_2 = -\frac{2}{3}x_1 + \frac{1}{3}x_3 + \frac{4}{3} \\ x_4 = -\frac{4}{3}x_1 - \frac{1}{3}x_3 + \frac{14}{3} \\ x_5 = \frac{2}{3}x_1 - \frac{1}{3}x_3 + \frac{5}{3} \end{cases}$$

$$\max z = x_1 + 2\left(-\frac{2}{3}x_1 + \frac{1}{3}x_3 + \frac{4}{3}\right)$$
$$\max z = x_1 - \frac{4}{3}x_1 + \frac{2}{3}x_3 + \frac{8}{3}$$

$$\max z = -\frac{1}{3}x_1 + \frac{2}{3}x_3 + \frac{8}{3}$$

Soggetto a

$$x_2 = -\frac{2}{3}x_1 + \frac{1}{3}x_3 + \frac{4}{3}$$
$$x_4 = -\frac{4}{3}x_1 - \frac{1}{3}x_3 + \frac{14}{3}$$
$$x_5 = \frac{2}{3}x_1 - \frac{1}{3}x_3 + \frac{5}{3}$$

$$\left[\begin{array}{ccccc|c} 2/3 & 1 & -\frac{1}{3} & 0 & 0 & 4/3 \\ 4/3 & 0 & 1/3 & 1 & 0 & 14/3 \\ -\frac{2}{3} & 0 & 1/3 & 0 & 1 & 5/3 \end{array}\right] \begin{array}{l} \\ \\ 14 \\ 5 \end{array}$$

$$\left[\begin{array}{ccccc|c} 2/3 & 1 & -\frac{1}{3} & 0 & 0 & 4/3 \\ 4/3 & 0 & 1/3 & 1 & 0 & 14/3 \\ -\frac{2}{3} & 0 & 1/3 & 0 & 1 & 5/3 \end{array}\right]$$

$\lambda_1 = 1 \quad \lambda_2 = -1 \quad \lambda_3 = 3$

$E_1 \leftarrow E_1 + E_3$
$E_2 \leftarrow E_2 - E_3$
$E_3 \leftarrow 3E_3$

$$x_2 = -x_5 + 3$$
$$x_3 = 2x_1 - 3x_5 + 5$$
$$x_4 = -2x_1 + x_5 + 3$$

$$\left[\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & 1 & 3 \\ 2 & 0 & 0 & 1 & -1 & 3 \\ -2 & 0 & 1 & 0 & 3 & 5 \end{array}\right]$$

$$\max z = -\frac{1}{3}x_1 + \frac{2}{3}x_3 + \frac{8}{3}$$

$$\max z = -\frac{1}{3}x_1 + \frac{2}{3}(2x_1 - 3x_5 + 5) + \frac{8}{3}$$

$$\max z = -\frac{1}{3}x_1 + \frac{4}{3}x_1 - 2x_5 + \frac{10}{3} + \frac{8}{3}$$

$$\max z = x_1 - 2x_5 + 6$$

Sogg. a

$$x_2 = -x_5 + 3$$
$$x_3 = 2x_1 - 3x_5 + 5$$
$$x_4 = -2x_1 + x_5 + 3$$

$$\left[\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & 1 & 3 \\ 2 & 0 & 0 & 1 & -1 & 3 \\ -2 & 0 & 1 & 0 & 3 & 5 \end{array}\right] \quad 3/2$$

$$\lambda_1 = 0 \quad \lambda_2 = \frac{1}{2} \quad \lambda_3 = 1$$

$$\left[\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & 1 & 3 \\ 2 & 0 & 0 & 1 & -1 & 3 \\ -2 & 0 & 1 & 0 & 3 & 5 \end{array}\right]$$

$E_1 \leftarrow E_1 + 0E_2$

$E_2 \leftarrow \frac{1}{2}E_2$

$E_3 \leftarrow E_3 + E_2$

$$x_1 = -\frac{1}{2}x_4 + \frac{1}{2}x_5 + \frac{3}{2}$$
$$x_2 = -x_5 + 3$$
$$x_3 = -x_4 - 2x_5 + 8$$

$$\left[\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & 1 & 3 \\ 1 & 0 & 0 & 1/2 & -1/2 & 3/2 \\ 0 & 0 & 1 & 1 & 2 & 8 \end{array}\right]$$

$$\max z = x_1 - 2x_5 + 6$$
$$\max z = -\frac{1}{2}x_4 + \frac{1}{2}x_5 + \frac{3}{2} - 2x_5 + 6$$

$$\max z = -\frac{1}{2}x_4 - \frac{3}{2}x_5 + \frac{15}{2}$$

Soggetto a

$$x_1 = -\frac{1}{2}x_4 + \frac{1}{2}x_5 + \frac{3}{2}$$
$$x_2 = -x_5 + 3$$
$$x_3 = -x_4 - 2x_5 + 8$$

Soluzione

⑤ Titolo

$$\max z = -\frac{1}{2}x_4 - \frac{3}{2}x_5 + \frac{15}{2}$$

Soggetto a

$$x_1 = -\frac{1}{2}x_4 + \frac{1}{2}x_5 + \frac{3}{2}$$
$$x_2 = -x_5 + 3$$
$$x_3 = -x_4 - 2x_5 + 8$$

A $(0, \frac{4}{3})$
B $(0,3)$
C $(3/2, 3)$
D $(3,0)$
E $(2,0)$

$$z = 4x_1 + x_2 = 4\left(-\frac{1}{2}x_4 + \frac{1}{2}x_5 + \frac{3}{2}\right) - x_5 + 3$$
$$= -2x_4 + 2x_5 - x_5 + 6 + 3 = -2x_4 + x_5 + 9$$

↳ perde di ottimalità

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① Metodo grafico

$\max z = x_1 + 2x_2$

Sogg. a:

$2x_1 + x_2 \geq 3$

$3x_1 + x_2 \leq 6$

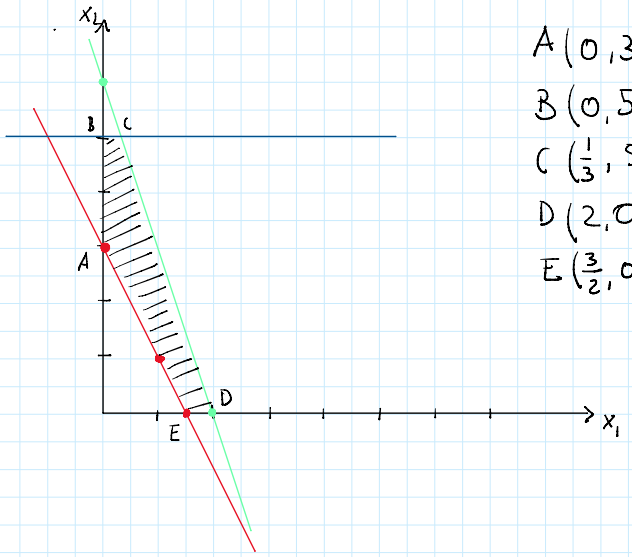
$x_2 \leq 5$

$2x_1 + x_2 = 3$

x_1	x_2
0	3
1	1
3	0

$3x_1 + x_2 = 6$

x_1	x_2
2	0
0	6



$A(0,3)$
$B(0,5)$
$C(\frac{1}{3},5)$
$D(2,0)$
$E(\frac{3}{2},0)$

② Forma standard

$$\max z = x_1 + 2x_2$$

Soggetto

$$2x_1 + x_2 - x_3 \leq 3$$
$$3x_1 + x_2 + x_4 = 6$$
$$x_2 + x_5 = 5$$

③ Basi ammissibili

$A(0,3)$
$B(0,5)$
$C(\frac{1}{3},5)$
$D(2,0)$
$E(\frac{3}{2},0)$

$$\begin{cases} 2x_1 + x_2 - x_3 \leq 3 \\ 3x_1 + x_2 + x_4 = 6 \\ x_2 + x_5 = 5 \end{cases}$$

$A(0,3)$

$$\begin{cases} 0 + 3 - x_3 = 3 \\ 0 + 3 + x_4 = 6 \\ 3 + x_5 = 5 \end{cases} \quad \begin{cases} x_3 = 0 \\ x_4 = 3 \\ x_5 = 2 \end{cases} \quad \{x_2, x_4, x_5\}$$

$B(0,5)$

$$\begin{cases} 0 + 5 - x_3 = 3 \\ 0 + 5 + x_4 = 6 \\ 5 + x_5 = 5 \end{cases} \quad \begin{cases} x_3 = 2 \\ x_4 = 1 \\ x_5 = 0 \end{cases} \quad \{x_2, x_3, x_4\}$$

C $\left(\frac{1}{3}, 5\right)$ $\begin{cases} \frac{2}{3} + 5 - x_3 = 3 \\ 1 + 5 + x_4 = 6 \\ 5 + x_5 = 5 \end{cases}$ $\begin{cases} x_3 = \frac{8}{3} \\ x_4 = 0 \\ x_5 = 0 \end{cases}$ $\{x_1, x_2, x_3\}$

D $(2,0)$ $\begin{cases} 4 + 0 - x_3 = 3 \\ 6 + 0 + x_4 = 6 \\ 0 + x_5 = 5 \end{cases}$ $\begin{cases} x_3 = 1 \\ x_4 = 0 \\ x_5 = 5 \end{cases}$ $\{x_1, x_3, x_5\}$

E $\left(\frac{3}{2}, 0\right)$ $\begin{cases} 3 - x_3 = 3 \\ \frac{9}{2} + x_4 = 6 \\ x_5 = 5 \end{cases}$ $\begin{cases} x_3 = 0 \\ x_4 = \frac{3}{2} \\ x_5 = 5 \end{cases}$ $\{x_1, x_4, x_5\}$

→

x_2	x_4	x_5
x_2	x_3	x_4
x_1	x_2	x_3
x_1	x_3	x_5
x_1	x_4	x_5

$\begin{cases} 2x_1 + x_2 - x_3 \leq 3 \\ 3x_1 + x_2 + x_4 = 6 \\ x_2 + x_5 = 5 \end{cases}$

④ Simplesso

$\max z = x_1 + 2x_2$

Soggetto

$2x_1 + x_2 - x_3 \leq 3$

$3x_1 + x_2 + x_4 = 6$

$x_2 + x_5 = 5$

$$\left[\begin{array}{ccccc|c} 2 & 1 & -1 & 0 & 0 & 3 \\ 3 & 1 & 0 & 1 & 0 & 6 \\ 0 & 1 & 0 & 0 & 1 & 5 \end{array}\right]$$

$\lambda_2 = -1 \quad \lambda_3 = -1$

$E_1 \leftarrow E_1$

$E_2 \leftarrow E_2 - E_1$

$E_3 \leftarrow E_3 - E_1$

↓

$$\left[\begin{array}{ccccc|c} 2 & 1 & -1 & 0 & 0 & 3 \\ 1 & 0 & 1 & 1 & 0 & 3 \\ -2 & 0 & 1 & 0 & 1 & 2 \end{array}\right]$$

$$\left[\begin{array}{ccccc|c} 2 & 1 & -1 & 0 & 0 & 3 \\ 1 & 0 & 1 & 1 & 0 & 3 \\ -2 & 0 & 1 & 0 & 1 & 2 \end{array}\right]$$

$\max z = x_1 + 2x_2$

$\max z = x_1 + 2(-2x_1 + x_3 + 3)$

$\max z = x_1 - 4x_1 + 2x_3 + 6$

$\max z = -3x_1 + 2x_3 + 6$

Soggetto a:

$x_2 = -2x_1 + x_3 + 3$

$x_4 = -x_1 - x_3 + 3$

$x_5 = 2x_1 - x_3 + 2$

$$\left[\begin{array}{ccccc|c} 2 & 1 & -1 & 0 & 0 & 3 \\ 1 & 0 & 1 & 1 & 0 & 3 \\ -2 & 0 & \boxed{1} & 0 & 1 & 2 \end{array}\right] \begin{array}{l} // \\ 3 \\ 2 \end{array}$$

$\lambda_1 = 1 \quad \lambda_2 = -1 \quad \lambda_3 = 1$

$$\left[\begin{array}{ccccc|c} 2 & 1 & -1 & 0 & 0 & 3 \\ 1 & 0 & 1 & 1 & 0 & 3 \\ -2 & 0 & \boxed{1} & 0 & 1 & 2 \end{array}\right]$$

$E_1 \leftarrow E_1 + E_3$

$E_2 \leftarrow E_2 - E_3$

$E_3 \leftarrow E_3$

$x_2 = -x_5 + 5$

$x_3 = 2x_1 - x_5 + 2$

$x_4 = -3x_1 + x_5 + 1$

$$\left[\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & 1 & 5 \\ 3 & 0 & 0 & 1 & -1 & 1 \\ -2 & 0 & 1 & 0 & 1 & 2 \end{array}\right]$$

$\max z = -3x_1 + 2x_3 + 6$

$\max z = -3x_1 + 2(2x_1 - x_5 + 2) + 6$

$\max z = -3x_1 + 4x_1 - 2x_5 + 4 + 6$

$\max z = \boxed{x_1} - 2x_5 + 10$

$$\left[\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & 1 & 5 \end{array}\right] //$$

$$\max z = x_1 - 2x_5 + 10$$

$$x_2 = -x_5 + 5$$
$$x_3 = 2x_1 - x_5 + 2$$
$$x_4 = -3x_1 + x_5 + 1$$

$$\left[\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & 1 & 5 \\ 3 & 0 & 0 & 1 & -1 & 1 \\ -2 & 0 & 1 & 0 & 1 & 2 \end{array}\right] \quad // \quad \frac{1}{3} \quad //$$

$$\left[\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & 1 & 5 \\ 3 & 0 & 0 & 1 & -1 & 1 \\ -2 & 0 & 1 & 0 & 1 & 2 \end{array}\right]$$

$\lambda_1 = 0 \quad \lambda_2 = \frac{1}{3} \quad \lambda_3 = \frac{2}{3}$

$E_1 \leftarrow E_1 + 0E_2$
$E_2 \leftarrow \frac{1}{3} E_2$
$E_3 \leftarrow E_3 + \frac{2}{3} E_2$

$$x_1 = -\frac{1}{3}x_4 + \frac{1}{3}x_5 + \frac{1}{3}$$
$$x_2 = -x_5 + 5$$
$$x_3 = -\frac{2}{3}x_4 - \frac{1}{3}x_5 + \frac{8}{3}$$

$$\left[\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & 1 & 5 \\ 1 & 0 & 0 & \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 1 & \frac{2}{3} & \frac{1}{3} & \frac{8}{3} \end{array}\right] \quad \{x_1, x_2, x_3\}$$

$$\max z = x_1 - 2x_5 + 10$$
$$\max z = -\frac{1}{3}x_4 + \frac{1}{3}x_5 + \frac{1}{3} - 2x_5 + 10$$
$$\max z = -\frac{1}{3}x_4 - \frac{5}{3}x_5 + \frac{31}{3}$$

5) Cambio di base

$$\min z = -2x_1 + x_2$$
$$\max z = 2x_1 - x_2$$

$$x_1 = -\frac{1}{3}x_4 + \frac{1}{3}x_5 + \frac{1}{3}$$
$$x_2 = -x_5 + 5$$

$$\max z = 2\left(-\frac{1}{3}x_4 + \frac{1}{3}x_5 + \frac{1}{3}\right) - (-x_5 + 5) =$$
$$\max z = -\frac{2}{3}x_4 + \frac{2}{3}x_5 + \frac{1}{3} + x_5 - 5$$
$$\max z = -\frac{2}{3}x_4 + \frac{5}{3}x_5 - \frac{14}{3}$$

→ Non è ottima

① Metodo grafico

$\max z = x_1 + 2x_2$

s.a.

$2x_1 + 3x_2 \geq 4$

$2x_1 - x_2 \leq 6$

$x_2 \leq 3$

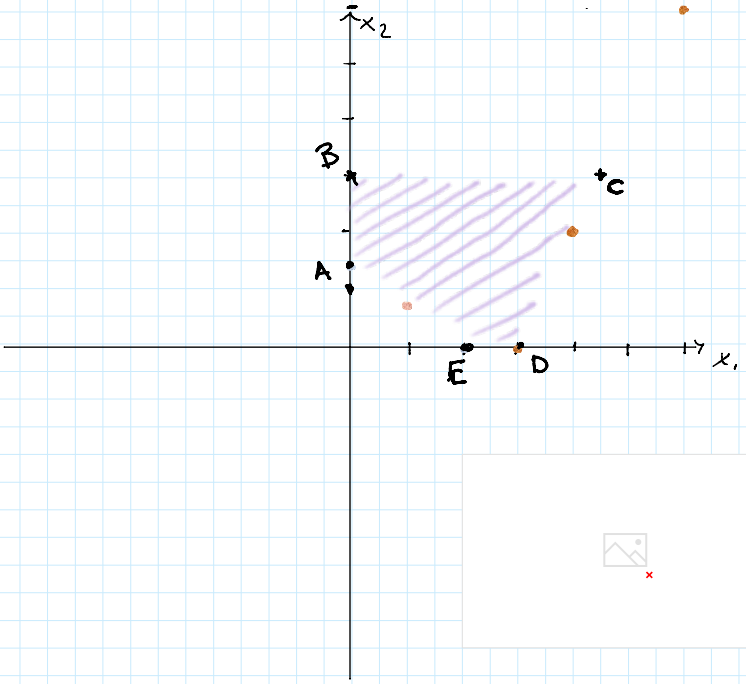
$2x_1 + 3x_2 = 4$

x_1	x_2
2	0
0	$\frac{4}{3}$
1	$\frac{2}{3}$

$2x_1 - x_2 = 6$

x_1	x_2
3	0
6	6
4	2

$x = 3$



$A\left(0, \frac{4}{3}\right)$

$B(0,3)$

$C\left(\frac{9}{2}, 3\right)$

$D(3,0)$

$E(2,0)$

② LA FORMA STANDARD:

$\max z = x_1 + 2x_2$

$2x_1 + 3x_2 \geq 4$

$2x_1 - x_2 \leq 6$

$x_2 \leq 3$

$\max z = x_1 + 2x_2$

$2x_1 + 3x_2 - x_3 = 4$

$2x_1 - x_2 + x_4 = 6$

$x_2 + x_5 = 3$

$x_1, x_2, x_3, x_4, x_5 \geq 0$.

(3) LE Basi ammissibili:

A $\left(0, \frac{4}{3}\right)$
B $(0,3)$
C $\left(\frac{9}{2}, 3\right)$
D $(3,0)$
E $(2,0)$

$$\max z = x_1 + 2x_2$$
$$2x_1 + 3x_2 - x_3 = 4$$
$$2x_1 - x_2 + x_4 = 6$$
$$x_2 + x_5 = 3$$

• A $\left(0, \frac{4}{3}\right)$ •

$$\begin{cases} 2\cdot 0 + 4 - x_3 = 4 \\ 0 - \frac{4}{3} + x_4 = 6 \\ \frac{4}{3} + x_5 = 3 \end{cases} \Rightarrow \begin{cases} x_3 = 0 \\ x_4 = 6 + \frac{4}{3} = \frac{22}{3} \\ x_5 = 3 - \frac{4}{3} = \frac{9-4}{3} = \frac{5}{3} \end{cases}$$

• B $(0,3)$ •

$$\begin{cases} 2\cdot 0 + 3 - x_3 = 4 \\ 2\cdot 0 - 3 + x_4 = 6 \\ 3 + x_5 = 3 \end{cases} \rightarrow \begin{array}{l} -x_3 = 4 - 3 \rightarrow x_3 = 5 \\ x_4 = 6 - 3 = 3 \\ x_5 = 0 \end{array}$$

• C $\left(\frac{9}{2}, 3\right)$ •

$$\begin{cases} 2\cdot\frac{9}{2} + 3 - x_3 = 4 \\ 2\cdot\frac{9}{2} - 3 + x_4 = 6 \\ 3 + x_5 = 3 \end{cases} \rightarrow \begin{cases} 6 + 3 - x_3 = 4 \rightarrow x_3 = 11 \\ 12 + x_3 = 6 \rightarrow x_3 = -6 \\ x_5 = 0 \end{cases}$$

• D $(3,0)$

$$\begin{cases} 6 + 0 - x_3 = 4 \\ 6 - 0 + x_4 = 6 \\ 0 + x_5 = 3 \end{cases} \rightarrow \begin{cases} -x_3 = 4 - 6 \rightarrow x_3 = 2 \\ x_4 = 0 \\ x_5 = 3 \end{cases}$$

• E $(2,0)$

$$\begin{cases} 4 + 0 - x_3 = 4 \\ 4 - 0 + x_4 = 6 \end{cases} \rightarrow \begin{cases} -x_3 = 0 \rightarrow x_3 = 0 \\ x_4 = 2 \end{cases}$$

$$\begin{cases} 4 + 0 - x_3 = 4 \\ 4 - 0 + x_4 = 6 \\ 0 + x_5 = 3 \end{cases} \rightarrow \begin{cases} x_6 - 0 \rightarrow x_5 = 0 \\ x_4 = 2 \\ x_5 = 3 \end{cases}$$

(3). IL METODO DEL SIMPLESSO:

$$\begin{aligned} 2x_1 + 3x_2 - x_3 &= 4 \\ 2x_1 - x_2 + x_4 &= 6 \\ x_2 + x_5 &= 3 \end{aligned}$$

$$\left(\begin{array}{ccccc|c} 2 & \textcircled{3} & -1 & 0 & 0 & 4 \\ 2 & -1 & 0 & 1 & 0 & 6 \\ 0 & 1 & 0 & 0 & 1 & 3 \end{array}\right)$$

$\max z = x_1 + 2x_2$

$\lambda_1 = \frac{1}{3} \quad \lambda_2 = \frac{1}{3} \quad \lambda_3 = -\frac{1}{3}$

$$\left(\begin{array}{ccccc|c} 2/3 & 1 & -\frac{1}{3} & 0 & 0 & 4/3 \\ 8/3 & 0 & -\frac{1}{3} & 1 & 0 & 22/3 \\ -\frac{2}{3} & 0 & \frac{1}{3} & 0 & 1 & 5/3 \end{array}\right)$$

$E_1 \leftarrow \frac{1}{3} E_1$

$E_2 \leftarrow E_2 + \frac{1}{3} E_1$

$E_3 \leftarrow E_3 - \frac{1}{3} E_1$

- ADESSO CREABBIAMO 3 COLONNE RIDOTTE POSSIAMO APPLICARE IL SIMPLESSO

$$\begin{cases} x_2 = \frac{4}{3} - \frac{2}{3} x_1 + \frac{1}{3} x_3 \\ x_4 = 22/3 - 8/3 x_1 + \frac{1}{3} x_3 \\ x_5 = \frac{5}{3} + \frac{2}{3} x_1 - \frac{1}{3} x_3 \end{cases}$$

$\max z = x_1 + 2x_2$

$\max z = x_1 + 2\left(\frac{4}{3} - \frac{2}{3} x_1 + \frac{1}{3} x_3\right)$

$\max z = x_1 + \frac{8}{3} - \frac{4}{3} x_1 + \frac{2}{3} x_3$

$\max z = \frac{8}{3} - \frac{1}{3} x_1 + \frac{2}{3} x_3$

$$\left(\begin{array}{ccccc|c} 2/3 & 1 & -\frac{1}{3} & 0 & 0 & 4/3 \\ 8/3 & 0 & -\frac{1}{3} & 1 & 0 & 22/3 \\ -\frac{2}{3} & 0 & \textcircled{\frac{1}{3}} & 0 & 1 & 5/3 \end{array}\right) \quad \frac{1}{3}$$

$\lambda_1 = 1 \quad \lambda_2 = 1 \quad \lambda_3 = 3$

$E_1 \leftarrow E_1 + E_3$

$E_2 \leftarrow E_2 + E_3$

$E_3 \leftarrow 3E_3$

ESCE x_5 ed ENTRA x_3

$$\left(\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & 1 & 9/3 \\ 2 & 0 & 0 & \textcircled{1} & 1 & 9 \\ & & & & & \end{array}\right)$$

$$\left(\begin{array}{ccccc|c} 2 & 0 & 0 & 1 & 1 & 9 \\ -2 & 0 & 1 & 0 & 3 & 5 \end{array}\right)$$

x_3

$$\begin{cases} x_2 = \frac{9}{3} - x_5 \\ x_4 = 9 - 2x_1 - x_5 \\ x_3 = 5 + 2x_1 - 3x_5 \end{cases}$$

$$\max z = \frac{8}{3} - \frac{1}{3}x_1 + \frac{2}{3}x_3 .$$

$$\max z = \frac{8}{3} - \frac{1}{3}x_1 + \frac{2}{3}(5 + 2x_1 - 3x_5)$$

$$\max z = \frac{8}{3} - \frac{1}{3}x_1 + \frac{10}{3} + \frac{4}{3}x_1 - 2x_5$$

$$\boxed{\max z = 6 + x_1 - 2x_5}$$

$$\left(\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & 1 & 9/3 \\ 2 & 0 & 0 & 1 & 1 & 9 \\ -2 & 0 & 1 & 0 & 3 & 5 \end{array}\right)$$

$\lambda_1 = 0 \quad \lambda_2 = \frac{1}{2} \quad \lambda_3 = 1$

$E_1 \leftarrow E_1$

$E_2 \leftarrow \frac{1}{2}E_2$

$E_3 \leftarrow E_3 + E_2 .$

$$\left(\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & 1 & 9/3 \\ 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 9/2 \\ 0 & 0 & 1 & 1 & 4 & 14 \\ \times & \times & \times & & & \end{array}\right)$$

USCITO x_4 ED ENTRATO x_1 .

$$\begin{cases} x_2 = \frac{9}{3} - x_5 \\ x_1 = \frac{9}{2} - \frac{1}{2}x_4 - \frac{1}{2}x_5 \\ x_3 = 14 - 4x_5 - x_4 \end{cases}$$

$$\boxed{\max z = 6 + x_1 - 2x_5}$$

$$\max z = 6 + \left(\frac{9}{2} - \frac{1}{2}x_4 - \frac{1}{2}x_5\right) - 2x_5 .$$

$$\boxed{\max z = \frac{21}{2} - \frac{1}{2}x_4 - \frac{5}{2}x_5}$$

(i)

$\max z = x_1 + x_2$

$2x_1 + 3x_2 \leq 6$

$-x_1 + 2x_2 \leq 2$

$2x_1 \leq 5$

$2x_1 + 3x_2 \leq 6$

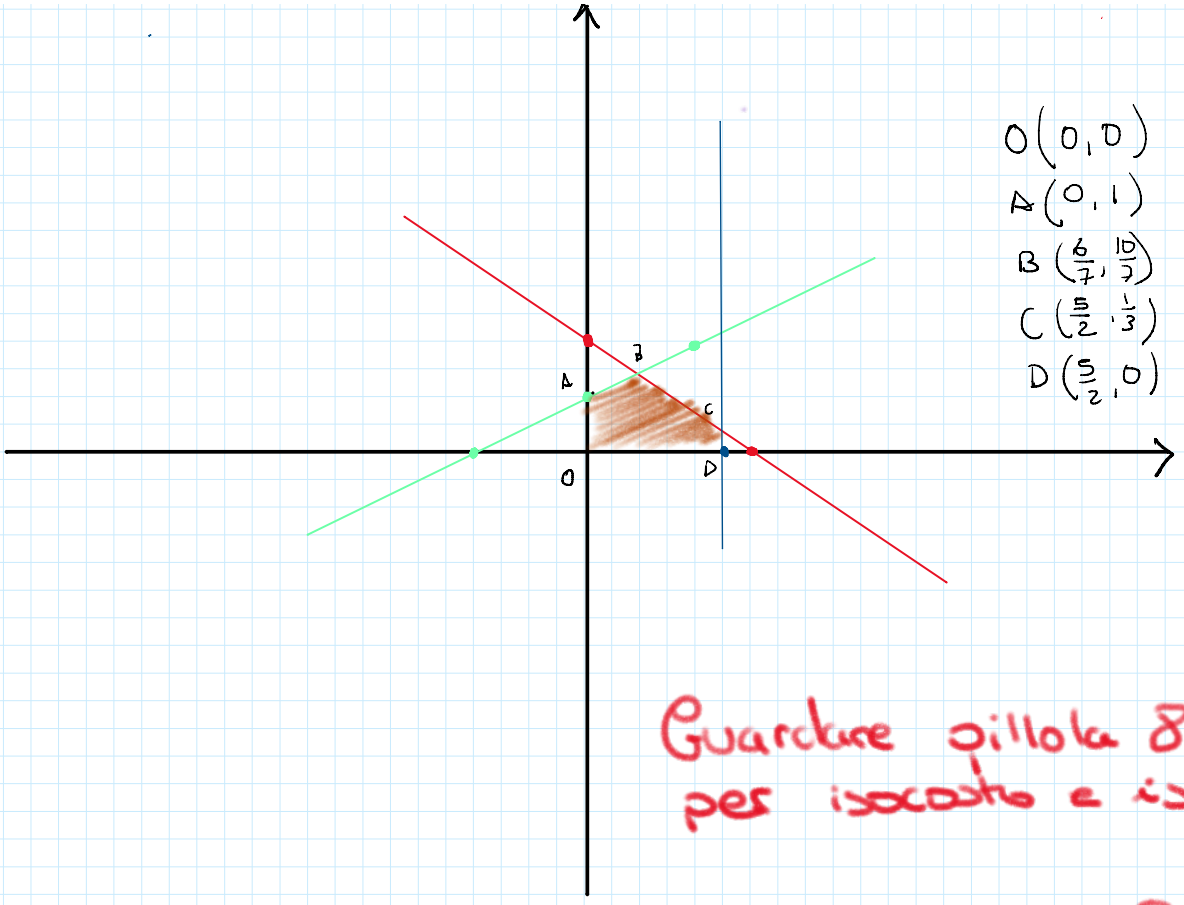
x_1	x_2
0	2
3	0

$-x_1 + 2x_2 \leq 2$

x_1	x_2
0	1
-2	0
4	3

→ Guardare Pillola 7

$x_1 \leq \frac{5}{2}$



$O\left(0,0\right)$
$A\left(0,1\right)$
$B\left(\frac{6}{7},\frac{10}{7}\right)$
$C\left(\frac{5}{2},-\frac{1}{3}\right)$
$D\left(\frac{5}{2},0\right)$

Guardare pillola 8
per isocosto e isoprofitto

Vedere Teorema fondam.
della PL

↓

Pillole 7 e 8

(ii)

$$\max z = x_1 + x_2$$

$$\begin{aligned} 2x_1 + 3x_2 &\leq 6 \\ -x_1 + 2x_2 &\leq 2 \\ 2x_1 &\leq 5 \end{aligned}$$

$\Longrightarrow$

$$\max z = x_1 + x_2$$

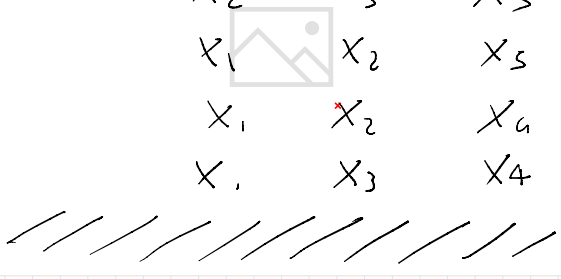
$$\begin{aligned} 2x_1 + 3x_2 + x_3 &= 6 \\ -x_1 + 2x_2 + x_4 &= 2 \\ 2x_1 + x_5 &= 5 \end{aligned}$$

$$x_{1\ldots5} \geq 0$$

(iii)

$x_3 \quad x_4 \quad x_5$

x_3	x_4	x_5
x_2	x_3	x_5
x_1	x_2	x_5
x_1	x_2	x_4
x_1	x_3	x_4



$$\max \; z = x_1 + x_2$$
$$2x_1 + 3x_2 + x_3 = 6$$
$$-x_1 + 2x_2 + x_4 = 2$$
$$2x_1 + x_5 = 5$$

$O(0,0)$ $\quad A(0,1)$ $\quad B\left(\frac{6}{7}, \frac{10}{7}\right)$ $\quad C\left(\frac{5}{2}, \frac{1}{3}\right)$ $\quad D\left(\frac{5}{2}, 0\right)$

$O(0,0)$

$$0 + 0 + x_3 = 6$$
$$0 + 0 + x_4 = 2$$
$$0 + 0 + x_5 = 5$$

$A(0,1)$

$$0 + 3 + x_3 = 6 \quad x_3 = 3$$
$$0 + 2 + x_4 = 2 \quad x_4 = 0$$
$$0 + x_5 = 5 \quad x_5 = 5$$

$B\left(\frac{6}{7}, \frac{10}{7}\right)$

$$\frac{12}{7} + \frac{30}{7} + x_3 = 6 \quad x_3 = 0$$
$$-\frac{6}{7} + \frac{20}{7} + x_4 = 2 \quad x_4 = 0$$
$$\frac{12}{7} + x_5 = 5 \quad x_5 = \frac{23}{7}$$

$C\left(\frac{5}{2}, \frac{1}{3}\right)$

$$5 + 1 + x_3 = 6 \quad x_3 = 0$$
$$-\frac{5}{2} + \frac{2}{3} + x_4 = 2 \quad x_4 = \frac{23}{6}$$
$$5 + x_5 = 5 \quad x_5 = 0$$

$D\left(\frac{5}{2}, 0\right)$

$$5 + 0 + x_3 = 6 \quad x_3 = 3$$
$$-\frac{5}{2} + x_4 = 2 \quad x_4 = \frac{9}{2}$$
$$5 + x_5 = 5 \quad x_5 = 0$$

(iv)

$$\left[\begin{array}{ccccc|c} 2 & 3 & 1 & 0 & 0 & 6 \\ -1 & 2 & 0 & 1 & 0 & 2 \\ 2 & 0 & 0 & 0 & 1 & 5 \end{array}\right] \begin{array}{l} 3 \\ 1 \\ \frac{5}{2} \end{array}$$

$$\max \; z = x_1 + x_2$$
$$2x_1 + 3x_2 + x_3 = 6$$
$$-x_1 + 2x_2 + x_4 = 2$$
$$2x_1 + x_5 = 5$$

Cambio di base da $\{x_3, x_4, x_5\}$ a $\{x_1, x_3, x_4\}$

$$\left[\begin{array}{ccccc|c} 2 & 3 & 1 & 0 & 0 & 6 \\ -1 & 2 & 0 & 1 & 0 & 2 \\ 2 & 0 & 0 & 0 & 1 & 5 \end{array}\right] \begin{array}{l} E_1 \leftarrow E_1 - E_3 \\ E_2 \leftarrow E_2 + \frac{1}{2}E_3 \\ E_3 \leftarrow \frac{1}{2}E_3 \end{array}$$

$$\left[\begin{array}{ccccc|c} 0 & 3 & 1 & 0 & -1 & 1 \\ 0 & 2 & 0 & 1 & \frac{1}{2} & \frac{9}{2} \\ 1 & 0 & 0 & 0 & \frac{1}{2} & \frac{5}{2} \end{array}\right]$$

$\lambda_1 = -1 \quad \lambda_2 = \frac{1}{2} \quad \lambda_3 = \frac{1}{2}$

$$x_1 = -\frac{1}{2}x_5 + \frac{5}{2}$$
$$x_3 = -3x_2 + x_5 + 1$$
$$x_4 = -2x_2 - \frac{1}{2}x_5 + \frac{9}{2}$$

$$\max \; z = x_1 + x_2$$
$$\max \; z = -\frac{1}{2}x_5 + \frac{5}{2} + x_2$$
$$\max z = \frac{5}{2} + x_2 - \frac{1}{2}x_5$$

$$\left[\begin{array}{ccccc|c} 0 & 3 & 1 & 0 & -1 & 1 \\ 0 & 2 & 0 & 1 & \frac{1}{2} & \frac{9}{2} \\ 1 & 0 & 0 & 0 & \frac{1}{2} & \frac{5}{2} \end{array}\right] \begin{array}{c} \frac{1}{3} \\ \frac{9}{4} \\ // \end{array}$$

Cambio di base da $\{x_1, x_3, x_4\}$ a $\{x_1, x_2, x_4\}$

$$\left[\begin{array}{ccccc|c} 0 & 3 & 1 & 0 & -1 & 1 \\ 0 & 2 & 0 & 1 & \frac{1}{2} & \frac{9}{2} \\ 1 & 0 & 0 & 0 & \frac{1}{2} & \frac{5}{2} \end{array}\right] \quad \begin{array}{l} E_1 \leftarrow \frac{1}{3} E_1 \\ E_2 \leftarrow E_2 - \frac{2}{3} E_1 \\ E_3 \leftarrow E_3 \end{array} \quad \left[\begin{array}{ccccc|c} 0 & 1 & \frac{1}{3} & 0 & -\frac{1}{3} & \frac{1}{3} \\ 0 & 0 & -\frac{2}{3} & 1 & 7/6 & \frac{23}{6} \\ 1 & 0 & 0 & 0 & \frac{1}{2} & \frac{5}{2} \end{array}\right]$$

$\lambda_1 = \frac{1}{3} \quad \lambda_2 = -\frac{2}{3} \quad \lambda_3 = 0$

$x_1 = -\frac{1}{2} x_5 + \frac{5}{2}$

$x_2 = -\frac{1}{3} x_3 + \frac{1}{3} x_5 + \frac{1}{3}$

$x_4 = \frac{2}{3} x_3 - \frac{7}{6} x_5 + \frac{23}{6}$

$\max z = \frac{5}{2} + x_2 - \frac{1}{2} x_5$

$\max z = \frac{5}{2} - \frac{1}{3} x_3 + \frac{1}{3} x_5 + \frac{1}{3} - \frac{1}{2} x_5$

$$\boxed{\max z = \frac{17}{6} - \frac{1}{3} x_3 - \frac{1}{6} x_5}$$

La base ottima è $\{x_1, x_2, x_4\}$ → Vertice $C\left(\frac{5}{2}, \frac{1}{3}\right)$

L'ottimo è $\frac{17}{6} = \frac{5}{2} + \frac{1}{3}$

---

① $\min z = -x_1 + 2x_2$

$2x_1 + 3x_2 = 5 \qquad x_1 - 2x_2 = 4$

(1)

$\min z = -x_1 + 2x_2$

$2x_1 + 3x_2 \geq 5$

$x_1 - 2x_2 \leq 4$

$x_1 \leq 5$

$2x_1 + 3x_2 = 5$

x_1	x_2
1	1
2	$\frac{1}{3}$

$x_1 - 2x_2 = 4$

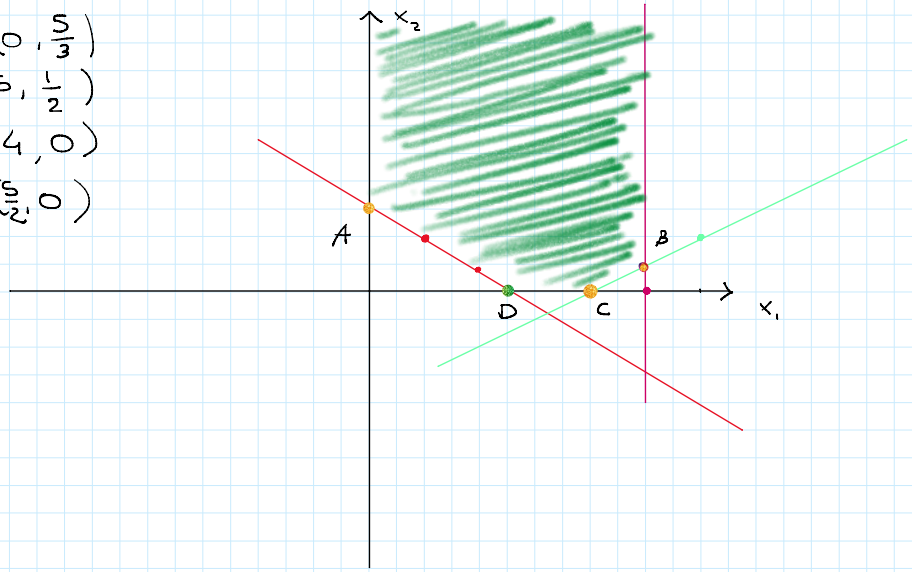
x_1	x_2
6	1
4	0

$A\left(0, \frac{5}{3}\right)$

$B\left(5, \frac{1}{2}\right)$

$C(4, 0)$

$D\left(\frac{5}{2}, 0\right)$



(2)

$\min z = -x_1 + 2x_2$

$2x_1 + 3x_2 \geq 5$

$x_1 - 2x_2 \leq 4$

$x_1 \leq 5$

$\Rightarrow$

$\max z = x_1 - 2x_2$

$2x_1 + 3x_2 - x_3 = 5$

$x_1 - 2x_2 + x_4 = 4$

$x_1 + x_5 = 5$

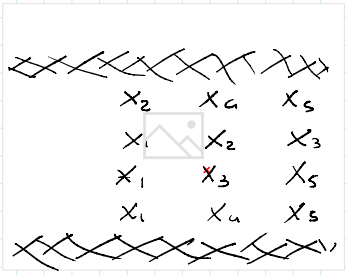
(3)

$A\left(0, \frac{5}{3}\right)$

$B\left(5, \frac{1}{2}\right)$

$C(4, 0)$

$D\left(\frac{5}{2}, 0\right)$



(A)

$5 - x_3 = 5 \quad x_3 = 0$

$-\frac{10}{3} + x_4 = 4 \quad x_4 = \frac{22}{3}$

$x_5 = 5 \quad x_5 = 5$

(B)

$10 + \frac{3}{2} - x_3 = 5 \quad x_3 = -\frac{13}{2}$

$5 - 1 + x_4 = 4 \quad x_4 = 0$

$5 + x_5 = 5 \quad x_5 = 0$

(C)

$8 + 0 - x_3 = 5 \quad x_3 = 3$

(D)

$5 + 0 - x_3 = 5 \quad x_3 = 0$

$4 - 0 + x_4 = 4 \qquad x_4 = 0$

$4 + x_5 = 5 \qquad x_5 = 1$

$\frac{5}{2} - 0 + x_4 = 4 \qquad x_4 = \frac{3}{2}$

$\frac{5}{2} + x_5 = 5 \qquad x_5 = \frac{5}{2}$

(4) $\max z = x_1 - 2x_2$

$2x_1 + 3x_2 - x_3 = 5$

$x_1 - 2x_2 + x_4 = 4$

$x_1 + x_5 = 5$

$$\left[\begin{array}{ccccc|c} 2 & 3 & -1 & 0 & 0 & 5 \\ 1 & -2 & 0 & 1 & 0 & 4 \\ 1 & 0 & 0 & 0 & 1 & 5 \end{array}\right]$$

$$\left[\begin{array}{ccccc|c} 2 & 3 & -1 & 0 & 0 & 5 \\ 1 & -2 & 0 & 1 & 0 & 4 \\ 1 & 0 & 0 & 0 & 1 & 5 \end{array}\right] \begin{array}{l} E_1 = \frac{1}{3}E_1 \\ E_2 = E_2 + \frac{2}{3}E_1 \\ E_3 = E_3 \end{array} \left[\begin{array}{ccccc|c} \frac{2}{3} & 1 & -\frac{1}{3} & 0 & 0 & \frac{5}{3} \\ \frac{7}{3} & 0 & -\frac{2}{3} & 1 & 0 & \frac{22}{3} \\ 1 & 0 & 0 & 0 & 1 & 5 \end{array}\right]$$

$\lambda_1 = \frac{1}{3} \qquad \lambda_2 = \frac{2}{3} \qquad \lambda_3 = 0$

$$x_2 = -\frac{2}{3}x_1 + \frac{1}{3}x_3 + \frac{5}{3}$$

$$x_4 = -\frac{7}{3}x_1 + \frac{2}{3}x_3 + \frac{22}{3}$$

$$x_5 = -x_1 + 5$$

$\max z = x_1 - 2\left(-\frac{2}{3}x_1 + \frac{1}{3}x_3 + \frac{5}{3}\right)$

$\max z = \frac{7}{3}x_1 - \frac{2}{3}x_3 - \frac{10}{3}$

$\frac{5}{3} \quad \frac{3}{2} \qquad \frac{22}{8} \quad \frac{3}{7}$

Cambio di base da $\{x_2, x_4, x_5\}$ a $\{x_1, x_4, x_5\}$

$$\left[\begin{array}{ccccc|c} \frac{2}{3} & 1 & -\frac{1}{3} & 0 & 0 & \frac{5}{3} \\ \frac{7}{3} & 0 & -\frac{2}{3} & 1 & 0 & \frac{22}{3} \\ 1 & 0 & 0 & 0 & 1 & 5 \end{array}\right] \begin{array}{l} E_1 = \frac{3}{2}E_1 \\ E_2 = E_2 - \frac{7}{2}E_1 \\ E_3 = E_3 - \frac{3}{2}E_1 \end{array} \left[\begin{array}{ccccc|c} 1 & \frac{3}{2} & -\frac{3}{6} & 0 & 0 & \frac{5}{2} \\ 0 & -\frac{7}{2} & \frac{1}{2} & 1 & 0 & \frac{3}{2} \\ 0 & -\frac{3}{2} & \frac{1}{2} & 0 & 1 & \frac{5}{2} \end{array}\right]$$

$\lambda_1 = \frac{3}{2} \qquad \lambda_2 = -\frac{7}{2} \qquad \lambda_3 = -\frac{3}{2}$

$$x_1 = -\frac{3}{2}x_2 + \frac{3}{6}x_3 + \frac{5}{2}$$

$$x_4 = \frac{7}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}$$

$$x_5 = \frac{3}{2}x_2 - \frac{1}{2}x_3 + \frac{5}{2}$$

$\max z = \frac{7}{3}x_1 - \frac{2}{3}x_3 - \frac{10}{3}$

$\max z = \frac{7}{3}\left(-\frac{3}{2}x_2 + \frac{3}{6}x_3 + \frac{5}{2}\right) - \frac{2}{3}x_3 - \frac{10}{3}$

$\max z = -\frac{21}{26}x_2 + \frac{21}{18}x_3 + \frac{35}{6} - \frac{2}{3}x_3 - \frac{10}{3}$

$\max z = -\frac{7}{2}x_2 + \frac{1}{2}x_3 + \frac{5}{2}$

$$\left[\begin{array}{ccccc|c} 1 & \frac{3}{2} & -\frac{3}{6} & 0 & 0 & \frac{5}{2} \\ 0 & -\frac{7}{2} & \frac{1}{2} & 1 & 0 & \frac{3}{2} \end{array}\right]$$

$$\left[\begin{array}{ccccc|c} 1 & \frac{3}{2} & -\frac{1}{6} & 0 & 0 & \frac{5}{2} \\ 0 & -\frac{7}{2} & \frac{1}{2} & 1 & 0 & \frac{3}{2} \\ 0 & -\frac{3}{2} & \frac{1}{2} & 0 & 1 & \frac{5}{2} \end{array}\right] \begin{array}{l} 1 \\ 3 \\ 5 \end{array}$$

$\frac{3}{2}z$ $\quad$ $\frac{5}{2}z$

Cambio base da $\{x_1, x_4, x_5\}$ a $\{x_1, x_3, x_5\}$

$$\left[\begin{array}{ccccc|c} 1 & \frac{3}{2} & -\frac{1}{2} & 0 & 0 & \frac{5}{2} \\ 0 & -\frac{7}{2} & \frac{1}{2} & 1 & 0 & \frac{3}{2} \\ 0 & -\frac{3}{2} & \frac{1}{2} & 0 & 1 & \frac{5}{2} \end{array}\right] \begin{array}{l} E_1 = E_1 + E_2 \\ E_2 = 2E_2 \\ E_3 = E_3 - E_2 \end{array} \qquad \left[\begin{array}{ccccc|c} 1 & -2 & 0 & 1 & 0 & 4 \\ 0 & -7 & 1 & 2 & 0 & 3 \\ 0 & 5 & 0 & -1 & 1 & 1 \end{array}\right]$$

$\lambda_1 = +1 \quad \lambda_2 = 2 \quad \lambda_3 = -1$

$$x_1 = 2x_2 - x_4 + 4$$
$$x_3 = 7x_2 - 2x_4 + 3$$
$$x_5 = -5x_2 + x_4 + 1$$

$$\max z = -\frac{7}{2}x_2 + \frac{1}{2}x_3 + \frac{5}{2}$$

$$\max z = -\frac{7}{2}x_2 + \frac{1}{2}(7x_2 - 2x_4 + 3) + \frac{5}{2}$$

$$\max z = -\frac{7}{2}x_2 + \frac{7}{2}x_2 - x_4 + \frac{3}{2} + \frac{5}{2}$$

$$\max z = 0x_2 - x_4 + 4$$

Soluzione ottima sulla base $\{x_1, x_3, x_5\}$ (vertice C)

L'ottimo ha valore $4 + 0 = 4$

Soggetto a:

$$x_1 = 2x_2 - x_4 + 4$$
$$x_3 = 7x_2 - 2x_4 + 3$$
$$x_5 = -5x_2 + x_4 + 1$$

(5)

$$\max z = 2x_1 + x_2$$
$$\max z = 2(2x_2 - x_4 + 4) + x_2$$
$$\max z = 4x_2 - 2x_4 + 8 + x_2$$
$$\max z = 5x_2 - 2x_4 + 8$$

Si perde ottimalità

$\min z = 2x_1 - x_2$

$x_1 + 2x_2 \geq 4$

$-3x_1 + x_2 \leq 4$

$x_2 \leq 5$

$x_1 + 2x_2 = 4$

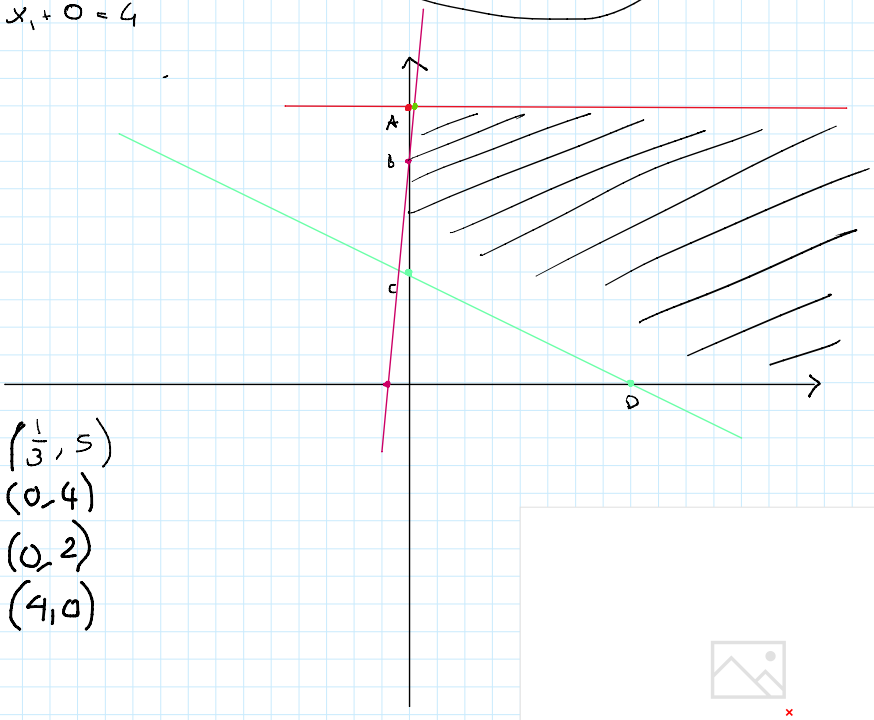
$-3x_1 + x_2 = 4$

$x_2 = 5$

x_1	x_2
0	2
4	0

x_1	x_2
0	4
$-\frac{4}{3}$	0

$-3x_1 + 0 = 4$



A $\left(\frac{1}{3}, 5\right)$

B $(0, 4)$

C $(0, 2)$

D $(4, 0)$

② Forma Standard

$\min z = 2x_1 - x_2$

$x_1 + 2x_2 \geq 4$

$-3x_1 + x_2 \leq 4$

$x_2 \leq 5$

$\Rightarrow$

$\max z = -2x_1 + x_2$

$x_1 + 2x_2 - x_3 = 4$

$-3x_1 + x_2 + x_4 = 4$

$x_2 + x_5 = 5$

③ Basi ammissibili

$\max z = -2x_1 + x_2$

$x_1 + 2x_2 - x_3 = 4$

$-3x_1 + x_2 + x_4 = 4$

$x_2 + x_5 = 5$

A $(\frac{1}{3}, 5)$

B $(0, 4)$

C $(0, 2)$

D $(4, 0)$

(C)

$4 - x_3 = 4 \quad x_3 = 0$

$2 + x_4 = 4 \quad x_4 = 2$

$2 + x_5 = 5 \quad x_5 = 3$

(D)

$4 - x_3 = 4 \quad x_3 = 0$

$-12 + x_4 = 4 \quad x_4 = 16$

$4 + x_5 = 5 \quad x_5 = 1$

(A)

$\frac{1}{3} + 10 - x_3 = 4 \quad x_3 = -\frac{19}{3}$

$-1 + 5 + x_4 = 4 \quad x_4 = 0$

$5 + x_3 = 5 \quad x_3 = 0$

(B)

$8 - x_3 = 4 \quad x_3 = 4$

$4 + x_4 = 4 \quad x_4 = 0$

$4 + x_5 = 5 \quad x_5 = 1$

→

x_1	x_2	x_3
x_2	x_3	x_5
x_2	x_4	x_5
x_1	x_4	x_5

④ Simplesso

$\max z = -2x_1 + x_2$

$x_1 + 2x_2 - x_3 = 4$

$-3x_1 + x_2 + x_4 = 4$

$x_2 + x_5 = 5$

$$\left[\begin{array}{ccccc|c} 1 & 2 & -1 & 0 & 0 & 4 \\ -3 & 1 & 0 & 1 & 0 & 4 \\ 0 & 1 & 0 & 0 & 1 & 5 \end{array}\right]$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & -1 & 0 & 0 & 4 \\ -3 & 1 & 0 & 1 & 0 & 4 \\ 0 & 1 & 0 & 0 & 1 & 5 \end{array}\right] \quad \begin{array}{l} E_1 = \frac{1}{2}E_1 \\ E_2 = E_2 - \frac{1}{2}E_1 \\ E_3 = E_3 - \frac{1}{2}E_1 \end{array}$$

$$\left[\begin{array}{ccccc|c} \frac{1}{2} & 1 & -\frac{1}{2} & 0 & 0 & 2 \\ -\frac{7}{2} & 0 & \frac{1}{2} & 1 & 0 & 2 \\ -\frac{1}{2} & 0 & \frac{1}{2} & 0 & 1 & 3 \end{array}\right]$$

$\lambda_1 = \frac{1}{2} \quad \lambda_2 = -\frac{1}{2} \quad \lambda_3 = -\frac{1}{2}$

$$x_2 = -\frac{1}{2}x_1 + \frac{1}{2}x_3 + 2$$

$$x_4 = \frac{7}{2}x_1 - \frac{1}{2}x_3 + 2$$

$$x_5 = \frac{1}{2}x_1 - \frac{1}{2}x_3 + 3$$

$\max z = -2x_1 + x_2$

$\max z = -2x_1 + \left(-\frac{1}{2}x_1 + \frac{1}{2}x_3 + 2\right)$

$\max z = -2x_1 - \frac{1}{2}x_1 + \frac{1}{2}x_3 + 2$

$\max z = -\frac{5}{2}x_1 + \frac{1}{2}x_3 + 2$

$$\left[\begin{array}{ccccc|c} \frac{1}{2} & 1 & -\frac{1}{2} & 0 & 0 & 2 \\ -\frac{7}{2} & 0 & \frac{1}{2} & 1 & 0 & 2 \\ -\frac{1}{2} & 0 & \frac{1}{2} & 0 & 1 & 3 \end{array}\right] \quad \begin{array}{l} E_1 = E_1 + E_2 \\ E_2 = 2E_2 \\ E_3 = E_3 - E_2 \end{array}$$

$$\left[\begin{array}{ccccc|c} -3 & 1 & 0 & 1 & 0 & 4 \\ -7 & 0 & 1 & 2 & 0 & 4 \\ 3 & 0 & 0 & -1 & 1 & 1 \end{array}\right]$$

$\lambda_1 = 1 \quad \lambda_2 = 2 \quad \lambda_3 = -1$

$$x_2 = 3x_1 - x_4 + 4$$

$$x_3 = 7x_1 - 2x_4 + 4$$

$$x_5 = -3x_1 + x_4 + 1$$

$\max z = -\frac{5}{2}x_1 + \frac{1}{2}x_3 + 2$

$\max z = -\frac{5}{2}x_1 + \frac{1}{2}\left(7x_1 - 2x_4 + 4\right) + 2$

$\max z = -\frac{5}{2}x_1 + \frac{7}{2}x_1 - x_4 + 2 + 2$

$\max z = +x_1 - x_4 + 4$

$$\left[\begin{array}{ccccc|c} -3 & 1 & 0 & 1 & 0 & 4 \\ -7 & 0 & 1 & 2 & 0 & 4 \\ 3 & 0 & 0 & -1 & 1 & 1 \end{array}\right] \quad \begin{array}{l} E_1 = E_1 + E_3 \\ E_2 = E_2 + \frac{7}{3}E_3 \\ E_3 = \frac{1}{3}E_3 \end{array}$$

$$\left[\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & 1 & 5 \\ 0 & 0 & 1 & -\frac{1}{3} & \frac{7}{3} & \frac{19}{3} \\ 1 & 0 & 0 & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{array}\right]$$

$\lambda_1 = 1 \quad \lambda_2 = \frac{7}{3} \quad \lambda_3 = \frac{1}{3}$

$\max z = +x_1 - x_4 + 4$

$\max z = \frac{1}{3}x_4 - \frac{1}{3}x_5 + \frac{1}{3} - x_4 + 4$

$\max z = \frac{1}{3}x_4 - \frac{1}{3}x_5 + \frac{1}{3} - x_4 + 4$

$\max z = -\frac{2}{3}x_4 - \frac{1}{3}x_5 + \frac{13}{3}$

$x_1 = \frac{1}{3}x_4 - \frac{1}{3}x_5 + \frac{1}{3}$

$x_2 = -x_5 + 5$

$x_3 = \frac{1}{3}x_4 - \frac{7}{3}x_5 + \frac{19}{3}$