Nome & Cognome: ______________________________

Algebra Lineare, Esame Finale
Febbraio 8, 2024

- Tutto il lavoro deve essere unicamente vostro.
- L'utilizzo di calcolatrici è vietato.
- L'esame dura 2 ore.
- Scrivete il vostro nome su tutte le pagine, nel caso qualche foglio si staccasse.
- Controllate di avere tutte le 8 pagine dell'esame.
- Ogni domanda a risposta multipla vale 1 punto.
- Le risposte alle domande aperte valgono 11 punti l'una.
- Le domande aperte verranno corrette solo a chi totalizzi almeno 6 punti su 10 nella parte a crocette.

Buon Lavoro!

PER FAVORE MARCATE LE RISPOSTE CON UNA X, non un cerchio!

1.	(a)	~~(b)~~	(c)	(d)	(e)	SI
2.	(a)	(b)	(c)	~~(d)~~	(e)	SI
3.	(a)	(b)	(c)	~~(d)~~	(e)	SI
4.	(a)	(b)	(c)	~~(d)~~	(e)	NO — Distrazione
5.	(a)	(b)	(c)	~~(d)~~	(e)	SI
6.	(a)	(b)	(c)	~~(d)~~	(e)	SI
7.	~~(a)~~	(b)	(c)	(d)	(e)	SI
8.	(a)	(b)	(c)	~~(d)~~	(e)	NO → Distrazione
9.	~~(a)~~	(b)	(c)	(d)	(e)	NO
10.	(a)	~~(b)~~	(c)	(d)	(e)	SI

7

$$\begin{array}{r} 7 + \\ 11 \\ 7 \\ \hline 25 \end{array}$$

→ (2)

Non scrivere qua sotto!

Risp. Multiple ______________________

Risp. Aperte ______________________

Totale ______________________

Nome & Cognome: ______________________________

Risposta multipla

1.(1 pt.) Trovare il numero $z \in \mathbb{C}$ tale che $(i-1)(i-2)z = (i+1)(i+2)(i+3)$?

(a) $z = 10 + 3i$. (b) $z = i - 3$. (c) $z = 30 + 10i$.
(d) $z = 3i - 1$. (e) $z = 1 - 30i$.

2.(1 pt.) Quale degli seguenti insiemi **non** è un sottospazio di $\mathbb{R}_2[x]$:

(a) $\{p(x) \in \mathbb{R}_2[x] \mid p(0) = 0\}$. (b) $\{(t+s)x^2 - tx - s \mid s, t \in \mathbb{R}\}$.
(c) $\{p(x) = ax^2 + bx + c \mid a = 2c, b = 0\}$. (d) $\{(1+t)x^2 + tx \mid t \in \mathbb{R}\}$.
(e) $\{p(x) \in \mathbb{R}_2[x] \mid p(1) = 0 = p(2)\}$.

3.(1 pt.) I polinomi

$$p_1(x) = x^2 + x + 1, \qquad p_2(x) = x^2 + x - 1, \qquad p_3(x) = x - 2$$

formano una base di $\mathbb{R}_2[x]$. Il vettore delle coordinate di $q(x) = (x+1)^2$ in questa base è:

(a) $(1, 2, 1)$. (b) $(2p_1, -p_2, p_3)$. (c) $(3, -2, -1)$.
(d) $(2, -1, 1)$. (e) $(x^2, 2x, 1)$.

4.(1 pt.) Date le matrici

$$A = \begin{pmatrix} \sqrt{2} & 0 & -\sqrt{2} \\ \sqrt{3} & \sqrt{3} & 0 \\ \sqrt{5} & 0 & \sqrt{5} \end{pmatrix}, \quad B = \begin{pmatrix} \sqrt{3} & -\sqrt{2} & 0 \\ 0 & \sqrt{2} & \sqrt{5} \\ \sqrt{3} & 0 & \sqrt{5} \end{pmatrix}$$

calcolare $\mathrm{tr}(AB)$:

(a) 30. (b) $(\sqrt{2} + \sqrt{3} + \sqrt{5})^2$. (c) 5.
(d) $\sqrt{30}$. (e) $\sqrt{5}$.

1) $(i^2-2i-i+2)\cdot z = (i^2+2i+i+2)(i+3)$

$(-1-3i+2)\cdot z = (-1+3i+2)(i+3)$

$(-3i+1)\cdot z = (3i+1)(i+3)$

$= 3i^2+9i+i+3$

$= \cancel{-3}+9i+i\cancel{+3}$

$= \frac{10\cancel{i}}{-3\cancel{i}} \Rightarrow -\frac{10}{3}$

2) (a) $\begin{bmatrix}0\\0\\0\end{bmatrix}$ ✓

(b) $\begin{bmatrix}tx^2+sx^2\\-tx\\-s\end{bmatrix} = \begin{bmatrix}t+s\\-t\\-s\end{bmatrix}$ $\begin{bmatrix}0+0\\0\\0\end{bmatrix}$ ✓ $\begin{bmatrix}-2\\-1\\-1\end{bmatrix}$ ✓

c) $\begin{bmatrix}2c\\0\\c\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$ $\begin{bmatrix}4\\0\\2\end{bmatrix}\begin{bmatrix}2\\0\\1\end{bmatrix} = \begin{bmatrix}6\\0\\3\end{bmatrix} \cdot \frac{1}{3} = \begin{bmatrix}2\\0\\1\end{bmatrix}$ ✓

c) $p_1 = \begin{bmatrix}1\\1\\1\end{bmatrix}$ $p_2 = \begin{bmatrix}1\\1\\-1\end{bmatrix}$ $p_3 = \begin{bmatrix}0\\1\\-2\end{bmatrix}$ $q(x) = (x+1)(x+1)$

$= x^2+x+x+1$

$= x^2+2x+1$ $- \begin{bmatrix}1\\2\\1\end{bmatrix}$

$$\begin{cases}\lambda_1+\lambda_2 = 1\\ \lambda_1+\lambda_2+\lambda_3 = 2\\ \lambda_1-\lambda_2-2\lambda_3 = 1\end{cases}$$

$$\begin{cases}\lambda_1 = 1-\lambda_2\\ 1-\cancel{\lambda_2}+\cancel{\lambda_2}+\lambda_3 = 2\\ 1-\lambda_2-\lambda_2-2\lambda_3 = 1\end{cases} \Rightarrow \begin{cases}//\\ \lambda_3 = 1\\ 1-2\lambda_2-2 = 1\end{cases} \Rightarrow \begin{cases}//\\ \lambda_3 = 1\\ -2\lambda_2 = \cancel{1}+2\cancel{-1}\end{cases} \Rightarrow \begin{cases}\lambda_1 = 2\\ \lambda_2 = -1\\ \lambda_3 = 1\end{cases}$$

$\frac{\cancel{-2}\lambda_2}{\cancel{-2}} = \frac{\cancel{2}}{\cancel{2}} - 1$

$\begin{bmatrix}2\\2\\2\end{bmatrix} + \begin{bmatrix}-1\\-1\\+1\end{bmatrix} + \begin{bmatrix}0\\1\\-2\end{bmatrix} = \begin{bmatrix}1\\2\\1\end{bmatrix}$ ✓

(d)

(3) $AB = \begin{bmatrix}0 & \\ & 0\\ & \end{bmatrix}$

$11 = (\sqrt{2}, 0, -\sqrt{2}) \cdot \begin{pmatrix}\sqrt{3}\\0\\\sqrt{3}\end{pmatrix}$

$= \cancel{\sqrt{2\cdot 3}} - \cancel{\sqrt{2\cdot 3}} = 0$

$22 = (\sqrt{3}, \sqrt{3}, 0) \begin{pmatrix}-\sqrt{2}\\\sqrt{2}\\0\end{pmatrix}$

$= -\cancel{\sqrt{3\cdot 2}} + \cancel{\sqrt{3\cdot 2}} = 0$

$33 = (\sqrt{5}, 0, \sqrt{5}) \begin{pmatrix}\sqrt{3}\\0\\\sqrt{5}\end{pmatrix}$

$= \sqrt{3\cdot 5} + \sqrt{3\cdot 5}$

$= \sqrt{25+15} = \sqrt{30} = \sqrt{3\cdot 2\cdot 5}$

$=$ (d)

$(\sqrt{5}, 0, \sqrt{5}) \begin{pmatrix}0\\\sqrt{5}\\\sqrt{5}\end{pmatrix}$

$= \sqrt{5}\cdot\sqrt{5}$

$= 5$

2.)

d) $\begin{bmatrix} 1+t \\ t \\ 0 \end{bmatrix} \overset{t=0}{=} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \overset{t=-1}{=} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$ $\quad 0_V \notin \{(1+t)x^2+tx \mid t \in \mathbb{R}$

$(i^2-2i-i+2)\cdot z = (i^2+2i+i+2)(i+3)$

$(-1-3i+2)\cdot z = (-1+3i+2)(i+3)$

$(-3i+1)\cdot z = (3i+1)(i+3)$

$= 3i^2+9i+i+3$

$\cancel{-3}+10i\cancel{+3}$

- $(i-3)$

$(-3i+1)(i-3) = 10i$

$-3i^2+9i+i-3 \qquad 3i^2+9i+i+3$

$10i\ \cancel{-3+3}=$

- $(3i-1)$

$(-3i+i)(3i-i)$

$-9i^2+3i+3i^2-i$

$9-3+4i$

- $(-3i+1)(10+3i)$

$-30i-9i^2+10+3i$

$9+10-27i$

- $(-3i+1)(1-30i)$

$-3i+90i^2+1-30i$

$= 90i^2-33i+1$

$-90-33i+1$

- $(-3i+1)(30+10i)$

$-90i-10i^2+30+10i$

$-9i-i^2+30+i$

$-i^2+3-8i$

$-1+3-8i$

Nome & Cognome: ______________________________

5.(1 pt.) La composizione $S \circ T$ di $S : \mathbb{R}^3 \to \mathbb{R}^2$, $S(x, y, z) = (x + y, 2z)$ e $T : \mathbb{R}^2 \to \mathbb{R}^3$, $T(x, y) = (x + y, x - y, y)$ è:

(a) $(x, y, z) \mapsto (2x, 2y, 2z)$.

(b) $(x, y) \mapsto (x + 2y, 2x - 2y)$.

(c) $(x, y, z) \mapsto (x + y + 2z, x + y - 2z, 2z)$.

~~(d)~~ $(x, y) \mapsto (2x, 2y)$.

(e) Non è ben definito.

6.(1 pt.) Il nucleo dell'applicazione lineare $T : M(n) \to M(n)$, $T(A) = A + A^T$:

(a) Consiste in matrici diagonali.

(b) Consiste nella matrice zero.

(c) Consiste di matrici simmetriche.

~~(d)~~ Consiste in matrici antisimmetriche.

(e) È vuoto.

7.(1 pt.) La forma quadratica $q(x) = 2x_1^2 + 2x_1x_2 + 2x_3^2$ si può scrivere come $q_S(x)$ con matrice S uguale a:

~~(a)~~ $\begin{pmatrix} 2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$. (b) $\begin{pmatrix} 2 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. (c) $\begin{pmatrix} 2 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$.

(d) $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix}$. (e) $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 2 \end{pmatrix}$.

8.(1 pt.) In $\mathbb{R}_2[x]$, sia dato il prodotto

$$\langle p(x), q(x) \rangle = p(0)q(0) + p(1)q(1) + p(-1)q(-1).$$

Rispetto a questo prodotto, l'angolo tra $p(x) = x$ e $q(x) = x^2$ è:

(a) x^3. (b) $\pi/4$. (c) 0. ~~(d)~~ 1. (e) $\pi/2$.

5.) $S(\overbrace{x+y}^{x'}, \overbrace{x-y}^{y'}, \overbrace{y}^{z'})$

$= (x+y+x-y, 2y)$

$= (2x, 2y)$ (d)

6) = $A + {}^tA = 0$

(a) $\begin{pmatrix}1 & \\ & 1\end{pmatrix} + \begin{pmatrix}1 & \\ & 1\end{pmatrix} = \begin{pmatrix}2 & \\ & 2\end{pmatrix}$ ✗

b)

c) $\begin{pmatrix}1 & 2\\ 2 & 1\end{pmatrix} + \begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix} = \begin{bmatrix}2 & 4\\ 4 & 2\end{bmatrix}$ ✗

antisimm d) $\begin{pmatrix}0 & 2\\ -2 & 0\end{pmatrix} + \begin{bmatrix}0 & -2\\ 2 & 0\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ ✓

(7) $2x_1^2 + 2x_1x_2 + 2x_3^2$

	x_1	x_2	x_3
x_1	2	1	0
x_2	1	0	0
x_3	0	0	2

(a)

8) $\cos(\theta) = \dfrac{\langle v, w\rangle}{\|v\| \cdot \|w\|}$ $\quad v = \overset{p(x)}{\begin{bmatrix}0\\1\\0\end{bmatrix}}$ $\quad w = \overset{q(x)}{\begin{bmatrix}1\\0\\0\end{bmatrix}}$ $\quad \cos(\theta) = 0$ $\quad \cos(\frac{\pi}{2}) = 0$

$= 0$ (d) $\qquad = \frac{\pi}{2}$

- $\|v\| = 0 \cdot 0 + 1 \cdot 1 + (-1)\cdot(-1)$
  $= 0 + 1 + 1 = 2$
  $= \sqrt{2}$
- $\|w\| = 0^2 \cdot 0^2 + (1)^2 \cdot (1)^2 + (-1)^2 \cdot (-1)^2$
  $= 0 + 1 + 1$
  $= \sqrt{2}$

$\langle v, w\rangle$

$= 0 \cdot 0 + 1 \cdot (1)^2 + (-1) \cdot \overset{1}{\cancel{(-1)^2}}$

$= 0 + 1 - 1$

$= 0$

Nome & Cognome: ______________________________

9.(1 pt.) Quale delle seguenti applicazioni lineari $T : \mathbb{C}^2 \to \mathbb{C}^2$ **non** è autoaggiunto rispetto al prodottto hermitiano Euclideo?

~~(a)~~ $T(x, y) = (2x - y, -x + 2\sqrt{2}y)$.

(b) $T(x, y) = (x - (i + 2)y, (i - 2)x + y)$.

(c) $T(x, y) = (x + 2y, 2x)$.

(d) $T(x, y) = (x - 2iy, 2ix + y)$.

(e) $T(x, y) = (2ix - (1 + i)y, -(1 - i)x + 2y)$.

10.(1 pt.) La distanza tra le rette $r_1 = (2, 0, 0) + \mathrm{Span}(1, 1, 1)$ e $r_2 = (1, 2, 0) + \mathrm{Span}(-1, 1, -1)$ è:

(a) $2\sqrt{2}$. ~~(b)~~ $\frac{1}{\sqrt{2}}$. (c) 2. (d) $\frac{4}{\sqrt{2}}$. (e) 8.

10) $r_1 = \begin{pmatrix} 2\\0\\0 \end{pmatrix} + m\begin{pmatrix} 1\\1\\1 \end{pmatrix}$ $\quad r_2 = \begin{pmatrix} 1\\2\\0 \end{pmatrix} + t\begin{pmatrix} -1\\1\\-1 \end{pmatrix}$

$$\begin{cases} 2+m = 1-t \\ m = 2+t \\ m = -t \end{cases} \Rightarrow \begin{cases} 2-t = 1-t \\ -t = 2+t \\ // \end{cases} \Rightarrow \begin{cases} 2 = 1 \quad \times \end{cases}$$

$$\begin{cases} \lambda_1 - \lambda_2 = 0 \\ \lambda_1 + \lambda_2 = 0 \\ \lambda_1 - \lambda_2 = 0 \end{cases} \Rightarrow \begin{cases} \lambda_1 = \lambda_2 \\ \lambda_2 + \lambda_2 = 0 \\ // \end{cases} \Rightarrow \begin{cases} \lambda_1 = 0 \\ \lambda_2 = 0 \\ 0 = 0 \end{cases} \quad \text{Lin ind.}$$

$$d(r_1, r_2) = \frac{\det(t_1 | t_2 | \vec{v})}{\| t_1 \times t_2 \|} = \frac{\cancel{2}}{2\sqrt{2}} = \frac{1}{\sqrt{2}} \quad (b)$$

$$\begin{bmatrix} 1\\2\\0 \end{bmatrix} - \begin{bmatrix} 2\\0\\0 \end{bmatrix} = \begin{bmatrix} -1\\2\\0 \end{bmatrix}$$

$$t_1 \times t_2 = \begin{bmatrix} -2\\0\\2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$d_1 = \det\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = -1-1 = -2$$

$$d_2 = \det\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} = -1+1 = 0$$

$$d_3 = \det\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = 1-(-1) = 2$$

$$\| t_1 \times t_2 \| = \cancel{(-2)\cdot(-2)}^{4} + \cancel{2\cdot 2}^{4} = \sqrt{8}$$
$$2\sqrt{2}$$

$$\det\begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 2 \\ 1 & -1 & 0 \end{bmatrix}$$

$A^2 \to A^2 + A^1$
$A^3 \to A^3 + A^1$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 3 \\ 1 & 0 & 1 \end{bmatrix}$$

$$(-1)^{1+1} \cdot 1 \cdot \det\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} = 2$$

$$\sqrt{8} = \sqrt{2^2 \cdot 2} = 2\sqrt{2}$$

9) $\langle T(v), w \rangle = \langle v, T(w) \rangle$ $\quad v = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ $\quad w = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$

(a)

$= T(v) \Rightarrow (2x_1 - y_1, -x_1 + 2\sqrt{2}y_1)$

$T(w) \Rightarrow (2x_2 - y_2, -x_2 + 2\sqrt{2}y_2)$

$$\langle T(v), w \rangle = (2x_1 - y_1)\overline{x_2} + (-x_1 + 2\sqrt{2}y_1)\cdot\overline{y_2}$$
$$= 2x_1\overline{x_2} - y_1\overline{x_2} - x_1\overline{y_2} + 2\sqrt{2}y_1\overline{y_2}$$

$$\langle v, T(w) \rangle = (2x_2 - y_2)\overline{x_1} + (-x_2 + 2\sqrt{2}y_2)\overline{y_1}$$
$$= 2\overline{x_1}x_2 - \overline{x_1}y_2 - x_2\overline{y_1} + 2\sqrt{2}y_2\overline{y_1}$$

Nome & Cognome: ______________________________

Risposta aperta
Per ricevere punteggio parziale, dovete mostrare il vostro lavoro!

11.(11 pts.) Data la matrice

$$A = \begin{pmatrix} 2k & 0 & 0 \\ 1 & 0 & 1 \\ 0 & k^2 & 0 \end{pmatrix}$$

con un parametro $k \in \mathbb{C}$.

(1) Al variare di k, trovare gli autovalori di A. ✓

(2) Per quali valori di k la matrice A è diagonalizzabile? ✓

(3) Per $k = i$, trovare una base di autovettori di A.

$\lambda_1 = 2k$
$\lambda_2 = k$
$\lambda_3 = -k$

1) $p_\lambda(A) = \det \begin{bmatrix} 2k-\lambda & 0 & 0 \\ 1 & 0-\lambda & 1 \\ 0 & k^2 & 0-\lambda \end{bmatrix}$

$\cdot\ (0-\lambda+k) = (k-\lambda)$
$\cdot\ (0-\lambda-k)\quad (-k-\lambda)$

Laplace su 1,1

$\cancel{(-1)^{1+1}} \cdot 2k-\lambda \cdot \det \begin{bmatrix} 0-\lambda & 1 \\ k^2 & 0-\lambda \end{bmatrix} = (2k-\lambda)\cdot[(0-\lambda)^2 - k^2] = (2k-\lambda)(k-\lambda)(-k-\lambda)$

2) $\det(A) \neq 0$

$\det \begin{bmatrix} 2k & 0 & 0 \\ 1 & 0 & 1 \\ 0 & k^2 & 0 \end{bmatrix} = (-1)^{2+3} \cdot 1 \cdot \det \begin{bmatrix} 2k & 0 \\ 0 & k^2 \end{bmatrix}$

Laplace su (2,3)

$= -2k^3 \Rightarrow \quad -2k^3 = 0 \Rightarrow k^3 = 0 \qquad k \neq 0$

3) $\begin{bmatrix} 2i & 0 & 0 \\ 1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \Rightarrow p_\lambda(A) = \det \begin{bmatrix} 2i-\lambda & 0 & 0 \\ 1 & 0-\lambda & 1 \\ 0 & -1 & 0-\lambda \end{bmatrix}$

$\lambda_1 = 2i$
$\lambda_2 = 1$
$\lambda_3 = -1$

$= 2i-\lambda \cdot \det \begin{pmatrix} 0-\lambda & 1 \\ -1 & 0-\lambda \end{pmatrix}$

$= (2i-\lambda)\cdot[(0-\lambda)^2 - 1] = (2i-\lambda)(1-\lambda)(-1-\lambda)$

$\begin{cases} (2i-1)x = 0 \\ x \quad -y + z = 0 \\ \quad -y - z = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \\ -y + z = 0 \\ -y - z = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = t \\ z = t \in \mathbb{R} \end{cases} \Rightarrow t \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

$\begin{cases} -1 + 1 = 0 \\ -1 - 1 \end{cases}$

Non viene

mi sa che dovevo semplicemente sostituire K dagli auto-valori di (1)

(3)

$$\begin{bmatrix} 2k-\lambda & 0 & 0 \\ 1 & 0-\lambda & 1 \\ 0 & k^2 & 0-\lambda \end{bmatrix}$$

$\lambda_1 = 2i$

$\lambda_2 = i$

$\lambda_3 = -i$

$$\begin{cases} (2i-2i)\,x = 0 \\ x - 2iy + z = 0 \\ i^2 y - 2iz = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \\ -2iy + z = 0 \\ -y - 2iz = 0 \end{cases} \Rightarrow \begin{cases} // \\ -y - 2iy = 0 \\ z = 2iy \end{cases}$$

$$\Rightarrow \begin{cases} y(-1-2i) = 0 \\ z = 2iy \end{cases} \Rightarrow \Big|$$

Nome & Cognome: ______________________________

12.(11 pts.) Sia $T : \mathbb{R}^3 \to \mathbb{R}^3$ l'endomorfismo dato da

$$T({}^t(x, y, z)) = {}^t(x + 2y + 3z, 2x + 4y + 6z, 3x + 6y + 8z).$$

(1) Determinare la matrice A associata a T rispetto alla base canonica di $\mathbb{R}^3$, e calcolare il rango di T. ✓

(2) Calcolare **tutti** i vettori ${}^t(x, y, z)$ tali che $T({}^t(x, y, z)) = {}^t(5, 10, 5)$. ✓

(3) Determinare **tutti** i valori reali di k tali che il vettore ${}^t(k, k^2, 0)$ appartiene all'immagine di T. ✓

1) $e = \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}$

$T(e_1) = (1, 2, 3)$ $\begin{cases}\lambda_1 = 1\\ \lambda_2 = 2\\ \lambda_3 = 3\end{cases}$

$T(e_2) = (2, 4, 6)$ $\begin{cases}\lambda_1 = 2\\ \lambda_2 = 4\\ \lambda_3 = 6\end{cases}$

$T(e_3) = (3, 6, 8)$ $\begin{cases}\lambda_1 = 3\\ \lambda_2 = 6\\ \lambda_3 = 8\end{cases}$

$[T]_e = \begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 6\\ 3 & 6 & 8\end{bmatrix}$

$A_2 \to A_2 - 2A_1$

$\begin{array}{ccc}2 & 4 & 6 & -\\ 2 & 4 & 6\\ \hline 0 & 0 & 0\end{array}$ $= \begin{bmatrix}1 & 2 & 3\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}$ $\text{rnk}(A) = 2$

$A_3 \to A_3 - 3A_1$

$\begin{array}{ccc}3 & 6 & 8 & -\\ 3 & 6 & 9\\ \hline 0 & 0 & -1\\ \cdot -1 \; 0 & 0 & 1\end{array}$

2) $\begin{cases}x + 2y + 3z = 5\\ 2x + 4y + 6z = 10\\ 3x + 6y + 8z = 5\end{cases} \Rightarrow \left[\begin{array}{ccc|c}1 & 2 & 3 & 5\\ 2 & 4 & 6 & 10\\ 3 & 6 & 8 & 5\end{array}\right]$

$A_2 \to A_2 - 2A_1$

$\begin{array}{cccc}2 & 4 & 6 & 10 & -\\ 2 & 4 & 6 & 10\\ \hline 0 & 0 & 0 & 0\end{array}$

$= \left[\begin{array}{ccc|c}1 & 2 & 3 & 5\\ 0 & 0 & 1 & 10\\ 0 & 0 & 0 & 0\end{array}\right]$

$A_3 \to A_3 - 3A_1$

$\begin{array}{cccc}3 & 6 & 8 & 5 & -\\ 3 & 6 & 9 & 15 & =\\ \hline 0 & 0 & \cancel{-}1 & -10\\ 0 & 0 & 1 & 10\end{array}$

$\begin{cases}x + 2y + 30 = 5\\ z = 10\end{cases}$

$\begin{cases}x = -25 - 2t\\ y = t\\ z = 10\end{cases}$ ${}^t\begin{bmatrix}-27\\ 1\\ 10\end{bmatrix}$

$$\begin{array}{r} \overset{1}{-27}\cdot \\ 2 \\ \hline -54 \end{array} \qquad \begin{array}{r} \overset{2}{-27}\cdot \\ 3 \\ \hline -81 \end{array}$$

Nome & Cognome: ______________________________

$$t\begin{bmatrix} -25-2t \\ t \\ 10 \end{bmatrix} \quad \underset{\Rightarrow}{t=1} \quad \begin{cases} -27+2+30 = 5 \checkmark \\ -54+4+60 = 64-54 = 10 \checkmark \\ -81+6+80 = -1+6 = 5 \checkmark \end{cases}$$

3) $$\left[\begin{array}{ccc|c} 1 & 2 & 3 & k \\ 2 & 4 & 6 & k^2 \\ 3 & 6 & 8 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & k \\ 0 & 0 & 0 & k^2-2k \\ 0 & 0 & 1 & 3k \end{array}\right] \Rightarrow k^2-2k=0$$

b^2-4ac

$(-2)^2-\cancel{4\cdot 0} \Rightarrow 4$

$$\frac{-b\pm\sqrt{\Delta}}{2a} = \frac{+2\pm\sqrt{4}}{2} = \begin{cases} \frac{2+2}{2}=2 \\ \frac{2-2}{2}=0 \end{cases}$$

$A_2 \to A_2-2A_1$

$$\begin{array}{cccc} 2 & 4 & 6 & k^2 & - \\ 2 & 4 & 6 & 2k \\ \hline 0 & 0 & 0 & k^2-2k \end{array}$$

$A_3 \to A_3-3A_1$

$$\begin{array}{cccc} 3 & 6 & 8 & 0 & - \\ 3 & 6 & 9 & 3k \\ \hline 0 & 0 & -1 & -3k \\ 0 & 0 & 1 & 3k \end{array}$$

$k=2$

$$\begin{cases} x+2y+3z=2 \\ z=6 \end{cases} \Rightarrow \begin{cases} x+2y+18=2 \\ z=6 \end{cases} \Rightarrow \begin{cases} x=-16-2t \\ y=t\in\mathbb{R} \\ z=6 \end{cases} \begin{bmatrix} -16-2t \\ t \\ 6 \end{bmatrix}$$

$$\begin{cases} -18+2+18=2 \\ -36+4+36=4 \\ -54+6+48 \Rightarrow -54+54=0 \end{cases} \Rightarrow \begin{array}{cc} 2 & k \\ 4 & k^2 \\ 0 & 0 \checkmark \end{array} \Rightarrow t\begin{bmatrix} -18 \\ t \\ 6 \end{bmatrix}$$

$$\begin{array}{r} \overset{1}{-18}\cdot \\ 2 \\ \hline -36 \end{array} \qquad \begin{array}{r} \overset{2}{-18}\cdot \\ 3 \\ \hline -54 \end{array}$$

$$\begin{array}{r} 42+ \\ 6 \\ \hline 48 \end{array} \qquad \begin{array}{r} \overset{1}{48}+ \\ 6 \\ \hline 54 \end{array}$$

$k=0$

$$\begin{cases} x+2y+3z=0 \\ z=0 \end{cases} \Rightarrow \begin{cases} x=-2t \\ y=t \\ z=0 \end{cases} \Rightarrow \begin{bmatrix} -2t \\ t \\ 0 \end{bmatrix} \Rightarrow t\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{cases} -2+2=0 & k \quad k=0 \\ -4+4=0 & k^2 \\ -6+6=0 & 0 \end{cases}$$