GEH1036 GEK1505 Tutorial 3 week 5

Tsien Lilong

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Department of Mathematics,NUS

Personal Information

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    Go to the website, and scroll down, see the **Work** entry, in the "Tutor" project, click the link for more information. I will provide this slide and the source .tex file on that site.
- Course information
  - EVERY WEEK MONDAY 10:00-11:00 S16-0431

1. Bus Routes

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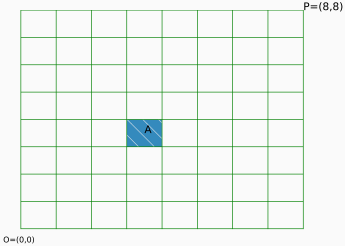
Question

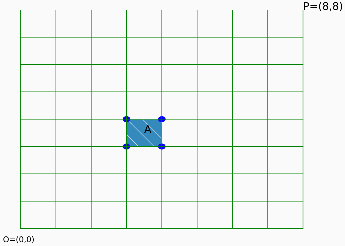
Count the possible routs from O to P, suppose the bus can only go up or right, no traffic on the boundary of A



Question

Count the possible routs from O to P, suppose the bus can only go up or right, no traffic on the boundary of A

1. The four corners of A are (3,3),(3,4),(4,4),(4,3). Note that if bus pass through (3,3), it must pass through either (3,4),(4,3), same for (4,4).



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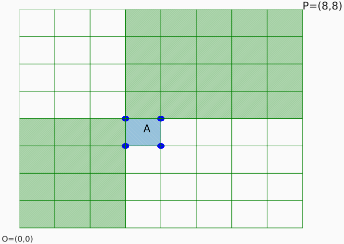

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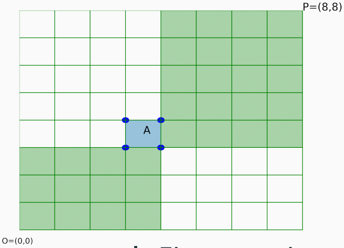
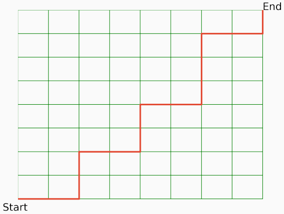


2. Consider the two circumstances separately.First passing through (3,4).
3. Second, passing through (4,3).

1. Bus Routes

- Counting the possibles routs in $m \times n$ rectangular grid are equal to counting different choices of 0-1 strings of length $m+n$ with n 0s, m 1s. Having $\binom{m+n}{n}$ different choices.(see lecture notes).

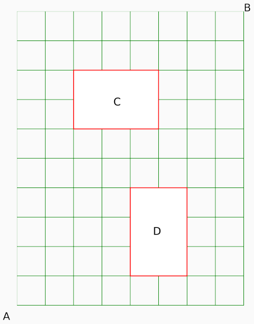


- In this problem, we have $\binom{16}{8}$ routes from O to P. Among which we have $\binom{7}{3}\binom{9}{4} + \binom{7}{3}\binom{9}{4}$ routes passing through border of A. The total number of possible routes is $\binom{16}{8} - \binom{7}{3}\binom{9}{4} + \binom{7}{3}\binom{9}{4} = 4050$.

2. Shortest Routes

Question

Finding number of shortest routes to travel from A to B along the routes shown in the figure.



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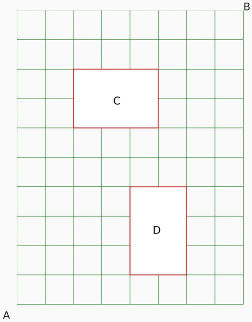
Finding number of shortest routes to travel from A to B along the routes shown in the figure.

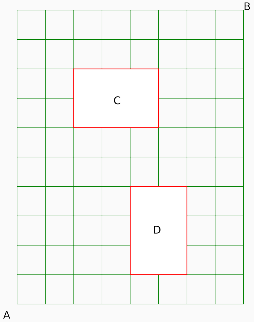
- Shortest way means: only turn right or up.



Question

Finding number of shortest routes to travel from A to B along the routes shown in the figure.

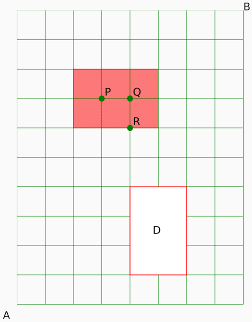
- Shortest way means: only turn right or up.
- Number of routes from A to B without restriction is $\binom{18}{8} = 43758$.



Question

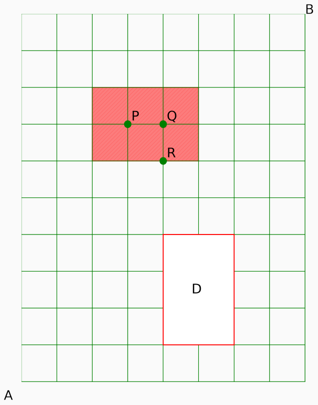
Finding number of shortest routes to travel from A to B along the routes shown in the figure.

- Shortest way means: only turn right or up.
- Number of routes from A to B without restriction is $\binom{18}{8} = 43758$.
- Number of routes go through region A. (Can pass the boundary). Two possible ways.



Number of routes passing through C

There are two possible ways: A→P →B and A→R →Q →B

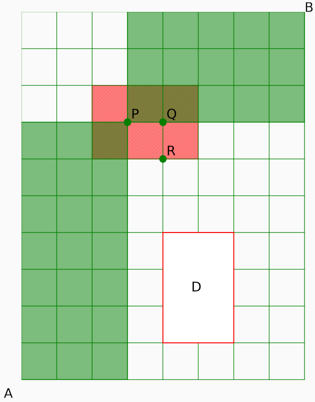


Number of routes passing through C

There are two possible ways: A $\to$ P $\to$ B and A $\to$ R $\to$ Q $\to$ B

- A $\to$ P $\to$ B The total number is

$$\underbrace{\binom{10}{3}}_{\text{A to P}}\ \underbrace{\binom{8}{3}}_{\text{P to B}} = 6720$$



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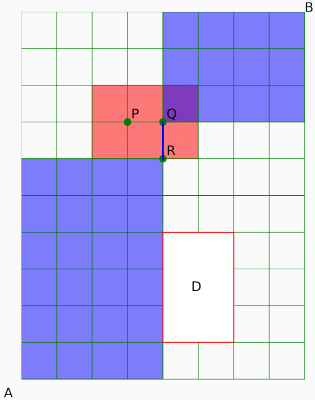
- A→R →Q →B

$$\underbrace{\binom{10}{4}}_{\text{A to R}}\ \underbrace{\binom{7}{3}}_{\text{Q to B}} = 7350$$



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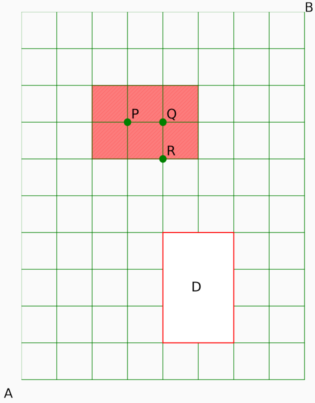
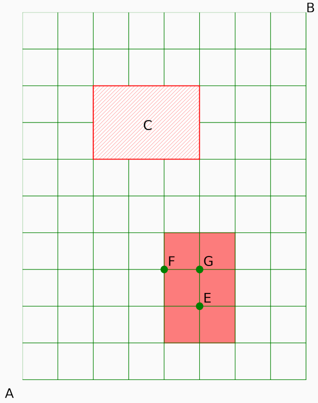
- A→P →BThe total number is
  $$\underbrace{\binom{10}{3}}_{\text{A to P}}\underbrace{\binom{8}{3}}_{\text{P to B}} = 6720$$
- A→R →Q →B
  $$\underbrace{\binom{10}{4}}_{\text{A to R}}\underbrace{\binom{7}{3}}_{\text{Q to B}} = 7350$$
- Total: 6270+7350=14070.



Number of Shortest routes

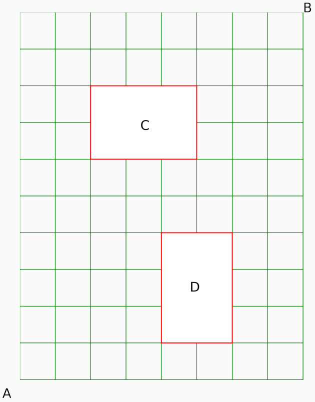
- Similar for region D , which is $\binom{7}{2}\binom{11}{3} + \binom{7}{3}\binom{10}{3} = 7665$



Number of Shortest routes

- Similar for region D , which is
$\binom{7}{2}\binom{11}{3}+\binom{7}{3}\binom{10}{3}=7665$

- The required number of routes is
$43758 - 14070 - 7665 = 22023.$



3. Coefficient of $a^5b^2c^8$

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$$(a+b+c)^{15} = \underbrace{(a+b+c)\cdot(a+b+c)\cdot\ldots\cdot(a+b+c)}_{15}.$$

- First, a can come from any 5 of the 15 factors, i.e. $\binom{15}{5}$.

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- First, a can come from any 5 of the 15 factors, i.e. $\binom{15}{5}$.
- Second, b can come from any 2 of the remanding 10 factors, i.e. $\binom{10}{2}$.

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- Last, Only 8 terms left undetermined, they are all c s. Only one choice.

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- Last, Only 8 terms left undetermined, they are all c s. Only one choice.
- Total number of choices $\binom{15}{5}\binom{10}{2} = 135135$, i.e. coefficient of $a^5b^2c^8$ is 135135.

4. Panel Painting

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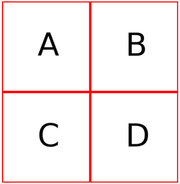
Question

Painting the 4 rectangle panels with adjacent ones painted with different color. 10 colours in total, how many choices?



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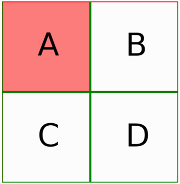
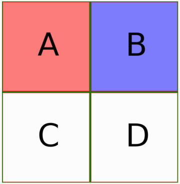


4. Panel Painting

Question

Painting the 4 rectangle panels with adjacent ones painted with different color. 10 colours in total, how many choices?

- Coloring A , having 10 choices.
- Coloring B , having 9 colours left.



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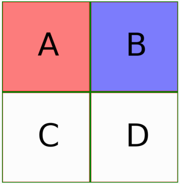
- Coloring A , having 10 choices.
- Coloring B , having 9 colours left.
- Coloring C , having two cases:



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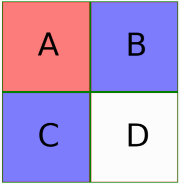
  - Choose same color with B , one choice. Then D has 9 choices.



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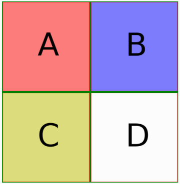

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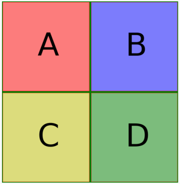


- Coloring A , having 10 choices.
- Coloring B , having 9 colours left.
- Coloring C , having two cases:
  - Choose same color with B , one choice. Then D has 9 choices.
  - Choose different color with A and B , 8 choices left. Then D has 8 choices.
- Totally we have $10 \times 9 \times (1 \times 9 + 8 \times 8) = 6570$ different choices.

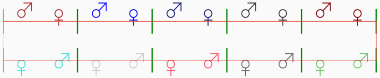
5. Ways of Seating

Question

Find the number of ways seating them in each of the following cases.

(a) In each row no person is next to another person of the same gender.

(b) Each person is seated opposite another person of the same gender.



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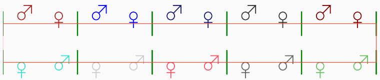
- 10 couples in total, 5 couples each row.



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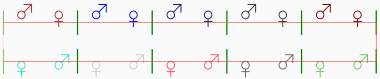
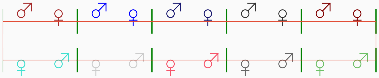


- First we can choose 5 couples for the row 1. $\binom{10}{5}$ ways.

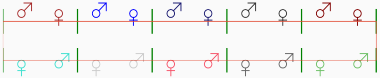
5. Ways of Seating case a

In each row no person is next to another person of the same gender.



In each row no person is next to another person of the same gender.

- 5! ways to put 5 couples in the seats for each row.



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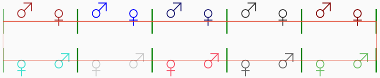

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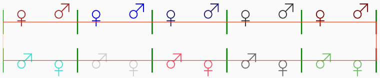

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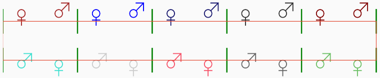

- In each row, males and females must alternate. When the first couple is given (2 ways), the rest of the row are fixed.
- Totally having $\binom{10}{5}(5! \times 2)^2 = 14515200$

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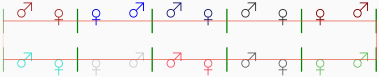
Each person is seated opposite another person of the same gender.



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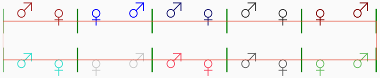

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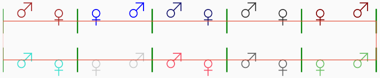

- 5! ways to put 5 couples in the seats for each row.
- In each row, males and female must alternate. Between each couple, the order of gender can be exchanged, 2^5 ways. When way of first row are given, the gender order has only one choice for the condition.

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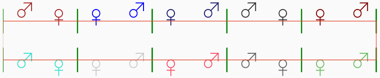

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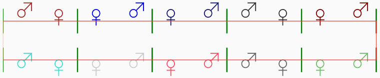

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- In each row, males and female must alternate. Between each couple, the order of gender can be exchanged, 2^5 ways. When way of first row are given, the gender order has only one choice for the condition.
- Totally having $\binom{10}{5}(5!)^2 \times 2^5 = 116121600$

6. Penny picking

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Question

12 "1967"s, 7 "1968"s and 11 "1971"s, how many you pick can make sure to get 5 pennies from the same year?

Pigeon hole Principle

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Pigeon hole Principle

- 3 pigeon holes: 1967,1968 and 1971

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Pigeon hole Principle

- 3 pigeon holes: 1967,1968 and 1971
- Required number N is the smallest integer such that $\lceil \frac{N}{3} \rceil = 5$, in which $N = 13$.

Question

12 "1967"s, 7 "1968"s and 11 "1971"s, how many you pick can make sure to get 5 pennies from the same year?

Pigeon hole Principle

- 3 pigeon holes: 1967,1968 and 1971
- Required number N is the smallest integer such that $[\frac{N}{3}] = 5$, in which $N = 13$.
- Thinking in another way, maximally we can have 4 pennies for each year, that is 12. Additional one will guarantees that at least five pennies are from the same year.

7. Cards picking

Question

52 cards, ♣, ♦, ♥, ♠ each have 13 cards .

(a) Pigeon holes: ♣, ♦, ♥, ♠

[1] $|a|$,for $a \in \mathbb{R}$, means the smallest number that is larger than a. e.g. $|1.1| = 2$

7. Cards Picking

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52 cards, ♣, ♦, ♥, ♠ each have 13 cards .

(a) Pigeon holes: ♣, ♦, ♥, ♠

(a) The required number N is the smallest number$\lceil \frac{N}{4} \rceil = 3$[1], which N=9.

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(a) Directly, maximally we can have $2♣ + 2♦ + 2♥ + 2♠$. Any one more guarantees picking 3 cards of same suit. i.e. N= 9.

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(b) We are considering the specific pigeon hole . Pigeon hole principle cannot be used here.

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7. Cards Picking

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52 cards, $\clubsuit$, $\diamondsuit$, $\heartsuit$, $\spadesuit$ each have 13 cards .

(a) Pigeon holes: $\clubsuit$, $\diamondsuit$, $\heartsuit$, $\spadesuit$

(a) The required number N is the smallest number $\lceil \frac{N}{4} \rceil = 3$[1], which N=9.

(a) Directly, maximally we can have $2\clubsuit + 2\diamondsuit + 2\heartsuit + 2\spadesuit$. Any one more guarantees picking 3 cards of same suit. i.e. N= 9.

(b) We are considering the specific pigeon hole . Pigeon hole principle cannot be used here.

(b) Consider the extreme case: $13\clubsuit + 13\diamondsuit + 13\spadesuit$. Adding three 3 , i.e. 42 cards, will make sure at least pick 3 hearts.

[1] $\lceil a \rceil$,for $a \in \mathbb{R}$, means the smallest number that is larger than a. e.g. $\lceil 1.1 \rceil = 2$

8. Intergers Picking

Question

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- By pigeon-holes principle, at least $[210/185] = 2$ subsets with equal sum.

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Ends

Thank you for your listening