GEH1036/GEK1505 Tutorial 5(week 7)

Tsien Lilong

February 27, 2017

Department of Mathematics,NUS

Personal Information

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    Go to the website, and scroll down, see the **Work** entry, in the "Tutor" project, click the link for more information. I will provide this slide and the source .tex file on that site.
- Course information
  - EVERY WEEK MONDAY 10:00-11:00 S16-0431

1. Trees with 7 vertices

Question

Draw all the trees with 7 unlabelled vertices.

We draw them according to the maximal degree (denote by Δ) among all the vertices. Note that the number of the edges is 6, then the sum of the degrees is $2 \times 6 = 12$.

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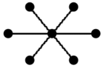

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$\Delta = 5$: the distribution of degrees of the 7 vertices is 5+2+1+1+1+1+1.

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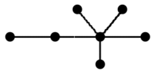

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$\Delta = 4$: the distribution of degrees of the 7 vertices is 4+2+2+1+1+1+1 or 4+3+1+1+1+1.

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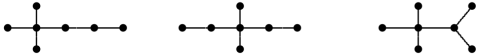

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$\Delta = 3$: the distribution of degrees of the 7 vertices is 3+3+2+1+1+1+1 or 3+2+2+2+1+1+1.

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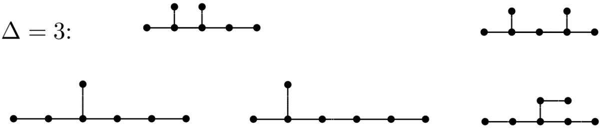

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$\Delta = 2$: the distribution of degrees of the 7 vertices is 2+2+2+2+2+1+1.

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Let n be the number of vertices. Since a tree must have n – 1 edges, by the Degree Theorem,

$$5 + 4 \times 2 + (n - 5) = 2(n - 1) \Rightarrow n = 10.$$

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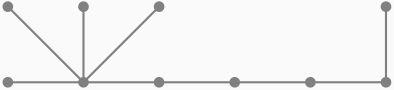
An example of this graph is,



3. Drawing trees

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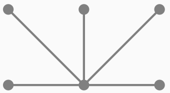

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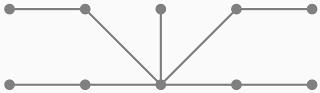

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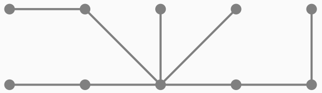
- Or choose 3.



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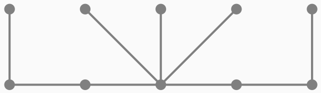

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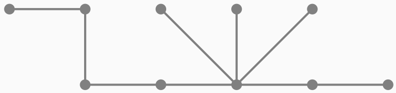

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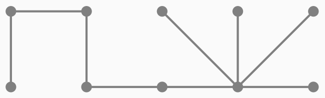

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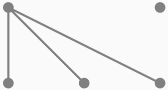
Is it possible to find a tree with 5 vertices and at least two vertices of degree 3? Justify your answer.



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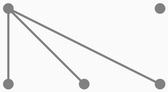

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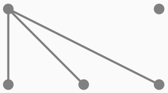
$$2 \times 4 = \sum_{i \in V(G)} d(v) \geqslant 2 \times 3 + 3 = 9$$

(every vertex in a tree must has at least 1 degree), which is impossible.



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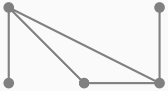
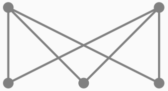


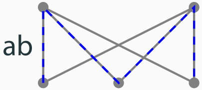


5. Spanning Tree

Question

Let T be a spanning tree in a graph G. Suppose G contains a cycle C and that ab is an edge of C that is also in T. Show that there is always at least one edge uv in C that can replace ab in T so that the resulting graph is still a spanning tree of G.

- First consider a example.



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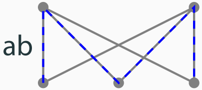

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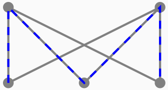
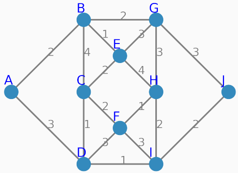


- First consider a example.
- Delete ab from T, the resulting graph, denoted by $T-ab$, consists of two disjoint trees T_1 and T_2.
- Since C is a cycle containing the edge ab, there will be an edge uv of C different from ab that joins one vertex of T_1 and one vertex of T_2.

6. Minimal spanning tree

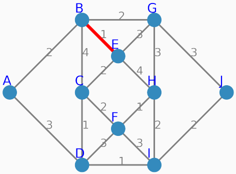
Question

Find a minimal spanning tree and its weight in the folowing graph with Prim's Algorithm.



6. Minimal spanning tree

- Using the Prim's algorithm, chose a minimal weighted edge,BE for example,$S = \{B, E\}$.



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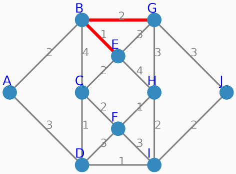

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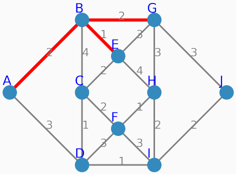
- Then AB, $S = \{A, B, E, G\}$.



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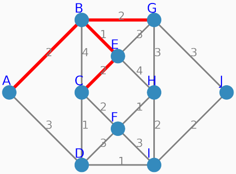

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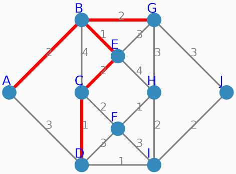

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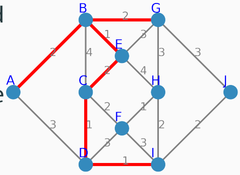

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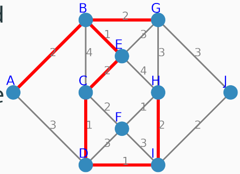

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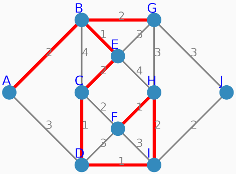

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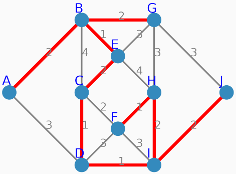

- Using the Prim's algorithm, chose a minimal weighted edge,BE for example,$S = \{B, E\}$.
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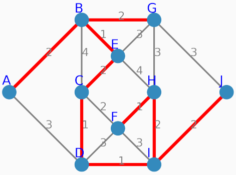

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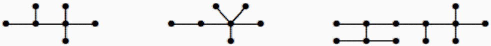
- The final tree is $AB, BE, BG, EC, CD, DI, IH, , HF, IJ$. The weight is therefore 2 + 1 + 2 + 2 + 1 + 1 + 2 + 1 + 2 = 14.

7. Vulnerable tree

Question

Find a minimal set of edges that can be added to each of the following trees so that in the resulting graph, every edge belongs to some cycle.



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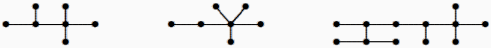

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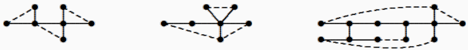

  Note the number of them as n, then we have at least $[\frac{n}{2}]$ edges to connect them.
- The case with minimal edges is to link those one-degree vertices pairly. And can be checked that every edge is contained in a cycle.

8. End

Thank you for listening!