

GEH1036/GEK1505 Tutorial 6(week 8)

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National University
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Go to the website, and scroll down, see the **Work** entry, in the “Tutor” project, click the link for more information. I will provide this slide and the source [.tex](#) file on that site.

◆ Course information

- ▶ EVERY WEEK MONDAY 10:00-11:00 S16-0431

Question

For each of the graph, either find an Euler walk (open or closed) or show the reason it does not exist.

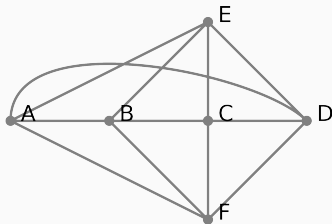
Theorem (Fleury's Algorithm)

To find an Euler path or an Euler circuit:

1. Make sure the graph has either 0 or 2 odd vertices.
2. If there are 0 odd vertices, start anywhere. If there are 2 odd vertices, start at one of them.
3. Follow edges one at a time. If you have a choice between a bridge and a non-bridge, **always choose the non-bridge**.
4. Stop when you run out of edges.

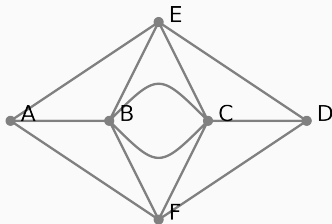
1. Finding Euler walk

- ◆ The first graph
 - ▶ 0 odd vertices, start anywhere.
 - ▶ One possible Euler walk is ABCDECFDAFBEA.



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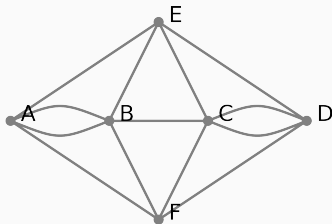
- ◆ The first graph
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- ◆ The second graph
 - ▶ 4 odd vertices A,B,C,D.
 - ▶ Hence no Euler walk exists.



1. Finding Euler walk

◆ The first graph

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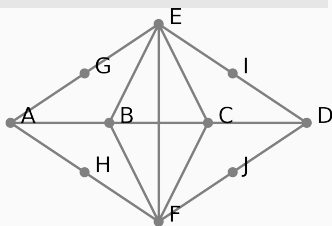
◆ The third graph

- ▶ 2 odd vertices B,C. Start from either B, or C.
- ▶ An example is BEABAFBCDCFDEC.

2. Minimal number of edges

Question

For the graph below, determine the minimal number of edges that must be duplicated so that there is a closed Euler walk. Also find such a minimal set of edges.

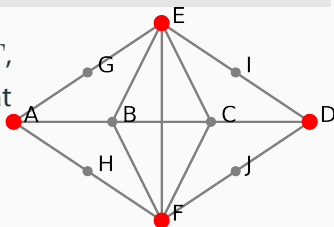


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- ◆ There are 4 odd vertices A, D, E, F , we need to duplicate edges so that all vertices are even.

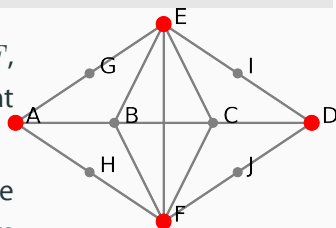


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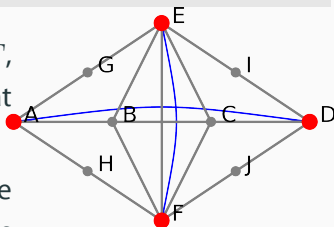


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- ◆ the edges to be duplicated must be edges of two paths joining two pairs of odd vertices.
- ◆ We can pair the four vertices as (AD, EF) or (AE, DF) or (AF, DE) . We need at least 4 edges.

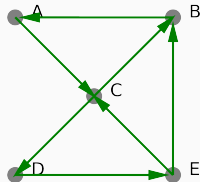


3. Direct graph

Question

(A) Find an open Euler walk in this graph.

- ♦ For a open Euler walk, start at the point whose degree of point out is 1 more the that of point in. i.e. E

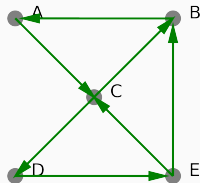


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- ♦ End at the another odd vertex B

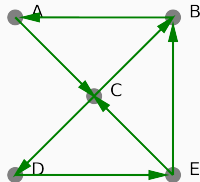


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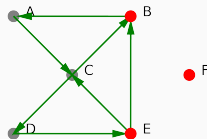
- ♦ For a open Euler walk, start at the point whose degree of point out is 1 more the that of point in. i.e. E
- ♦ End at the another odd vertex B
- ♦ A possible walk is $E \rightarrow B \rightarrow A \rightarrow C \rightarrow D \rightarrow E \rightarrow C \rightarrow B$.



3. Direct graph (b)

Question

(b) Add one new vertex and two new directed edges so that the resulting graph has a closed Euler walk.

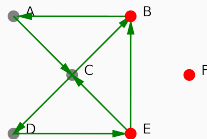


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- ◆ Consider the two odd vertices B,E and a new vertex F

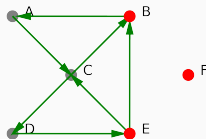


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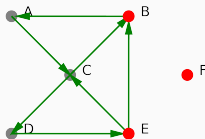


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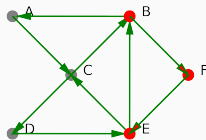


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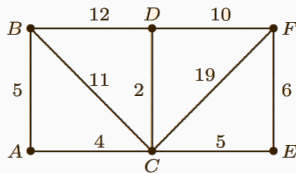
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- ◆ And the new vertex F should be connected by at least two edges (one point out one point in).
- ◆ Combine the two condition, the two edges must be $B \rightarrow F$ and $F \rightarrow E$



4. Cheapest wight

Question

Find the optimal Chinese Postman route starting from A.

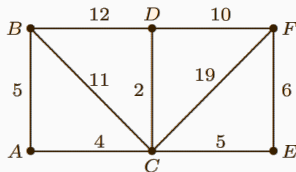


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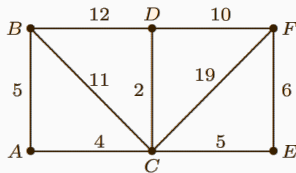


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- ♦ Pairing the four vertices as BC, DF. Then add a duplicated path respectively. The weight of added paths is $9(BA, AC) + 10(CF) = 19$.

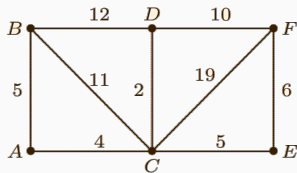


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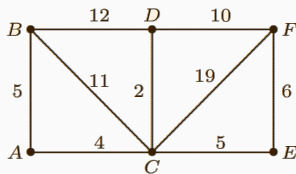


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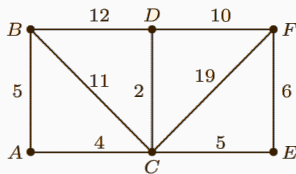


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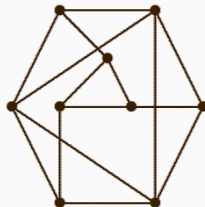
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- ◆ Choose the minimal case, the shortest time (minimal weight) is $5 + 4 + 11 + 12 + 2 + 10 + 19 + 5 + 6 = 74$.



5. Minimal proper vertex coloring

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Find a minimal proper vertex coloring for each graph below:

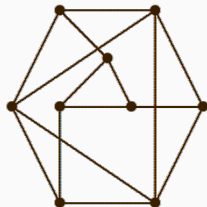


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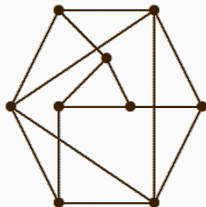


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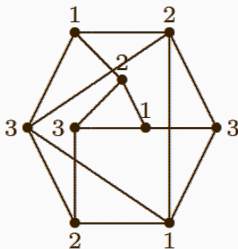


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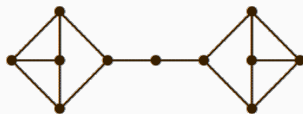


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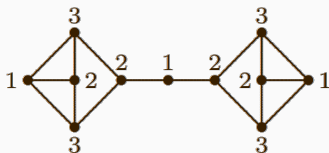


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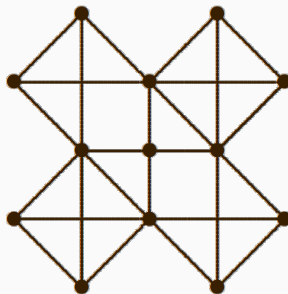


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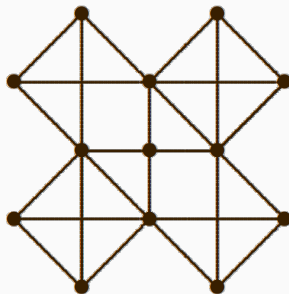


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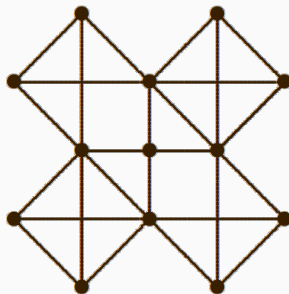


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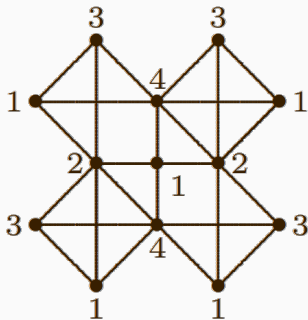


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6. Colors at adjacent vertices

Question

Show that in a minimal proper vertex coloring of a graph, any two colors must occur at adjacent vertices somewhere in the graph.

- ◆ Suppose the chromatic number of the graph is k . Consider a proper coloring using k colors, say $1, \dots, k$.

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- ◆ Since vertices of colour k are not adjacent to vertices of colour $k-1$, the two sets of vertices can be coloured using the same colour. We obtain a proper coloring using $k-1$ colors.
- ◆ This contradicts with the given condition. Hence claim holds.

7. Meeting schedule

Question

A Student Council has 6 committees. Councilors A, B, C, D, E, F, G, H, I, J are appointed to be members of the committees as shown below:

Publicity	A, B, C, D
Recreation	A, E, F, G
Welfare	G, H, I, J
Community	D, E
Secretariat	B, F, H
Finance	G, I

If each committee is scheduled to meet for two hours this week, what is the smallest number of two-hour sessions required to schedule all 8 committee meetings so that each of these councilors is able to attend the meetings of the committees he/she is a member of?

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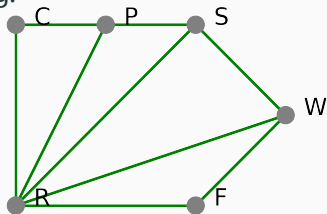
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- ◆ Now the graph is generated as following:

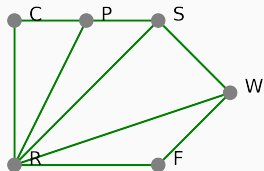
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- ◆ Edges: If any two committees share some member, connect them with an edge.
- ◆ Now the graph is generated as following:
- ◆ Consider what non-adjacent vertices means. It means that non-adjacent committees can meet at the same time, adjacent committees cannot meet at the same time.



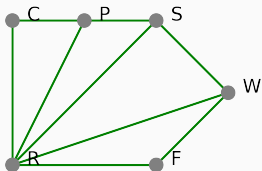
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- ♦ If we denote the different time slot by different colors. Then condition becomes that non-adjacent vertices can share same color, while adjacent vertices cannot. This is exactly the same with question of proper vertex coloring.



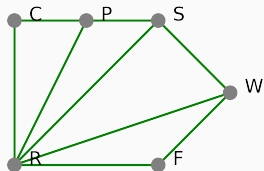
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- ◆ The question then is find the minimal proper vertex coloring (minimal numbers of different time slots for these committees).
- ◆ This graph contains a K_3 , so the chromatic number is at least 3.
- ◆ One way of proper coloring with 3 colors is shown in the picture. So the chromatic number is 3. Back to the original question, the smallest number of two-hour sessions required is 3.

