# GEH1036/GEK1505 Tutorial 6(week 8)

Tsien Lilong

March 6, 2017

Department of Mathematics,NUS

# Personal Information

- Tsien Lilong
  - Email: qian.lilong@u.nus.edu
  - Phone Number: 90874186
  - Website: tsien.farbox.com
    Go to the website, and scroll down, see the **Work** entry, in the "Tutor" project, click the link for more information. I will provide this slide and the source .tex file on that site.
- Course information
  - EVERY WEEK MONDAY 10:00-11:00 S16-0431

# 1. Finding Euler walk

## Question

For each of the graph, either find an Euler walk (open or closed) or show the reason it does not exist.

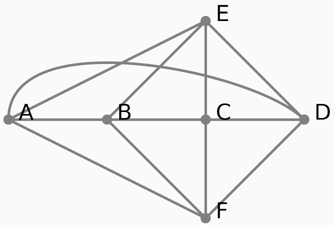
## Theorem (Fleury's Algorithm)

To find an Euler path or an Euler circuit:

1. Make sure the graph has either 0 or 2 odd vertices.
2. If there are 0 odd vertices, start anywhere. If there are 2 odd vertices, start at one of them.
3. Follow edges one at a time. If you have a choice between a bridge and a non-bridge, always choose the non-bridge.
4. Stop when you run out of edges.

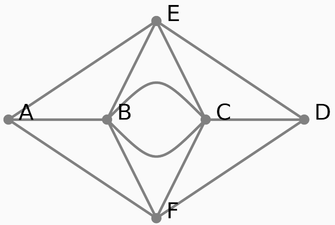
# 1. Finding Euler walk

- The first graph
  - 0 odd vertices, start anywhere.
  - One possible Euler walk is ABCDECFDAFBEA.

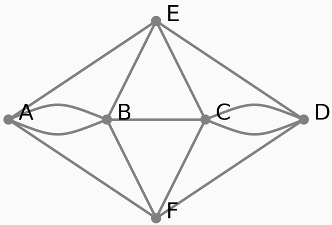


# 1. Finding Euler walk

- The first graph
  - 0 odd vertices, start anywhere.
  - One possible Euler walk is ABCDECFDAFBEA.
- The second graph
  - 4 odd vertices A,B,C,D.
  - Hence no Euler walk exists.



# 1. Finding Euler walk



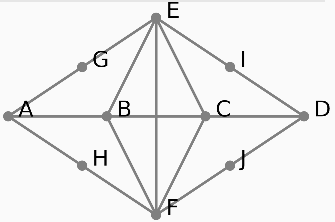

- The first graph
  - 0 odd vertices, start anywhere.
  - One possible Euler walk is ABCDECFDAFBEA.
- The second graph
  - 4 odd vertices A,B,C,D.
  - Hence no Euler walk exists.
- The third graph
  - 2 odd vertices B,C. Start from either B, or C.
  - An example is BEABAFBCDCFDEC.

# 2. Minimal number of edges

# 2. Minimal number of edges

**Question**

For the graph below, determine the minimal number of edges that must be duplicated so that there is a closed Euler walk. Also find such a minimal set of edges.

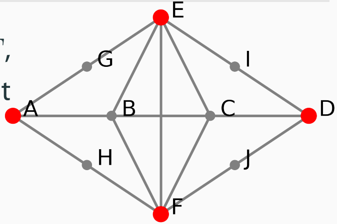


## 2. Minimal number of edges

**Question**

For the graph below, determine the minimal number of edges that must be duplicated so that there is a closed Euler walk. Also find such a minimal set of edges.

- There are 4 odd vertices $A, D, E, F$, we need to duplicate edges so that all vertices are even.

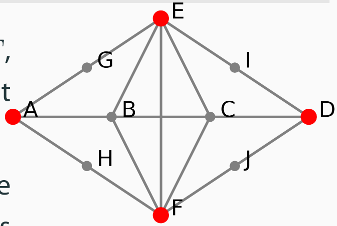


# 2. Minimal number of edges

## Question

For the graph below, determine the minimal number of edges that must be duplicated so that there is a closed Euler walk. Also find such a minimal set of edges.

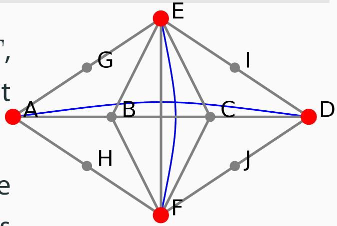
- There are 4 odd vertices $A, D, E, F$, we need to duplicate edges so that all vertices are even.
- the edges to be duplicated must be edges of two paths joining two pairs of odd vertices.



### Question

For the graph below, determine the minimal number of edges that must be duplicated so that there is a closed Euler walk. Also find such a minimal set of edges.

- There are 4 odd vertices $A, D, E, F$, we need to duplicate edges so that all vertices are even.
- the edges to be duplicated must be edges of two paths joining two pairs of odd vertices.



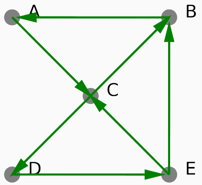

- We can pair the four vertices as (AD,EF) or (AE,DF) or (AF,DE). We need at least 4 edges.

# 3. Direct graph

**Question**

(A) Find an open Euler walk in this graph.

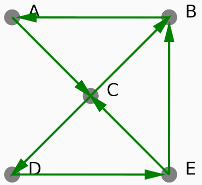
- For a open Euler walk, start at the point whose degree of point out is 1 more then that of point in. i.e. E



**Question**

(A) Find an open Euler walk in this graph.

- For a open Euler walk, start at the point whose degree of point out is 1 more then that of point in. i.e. E
- End at the another odd vertex B

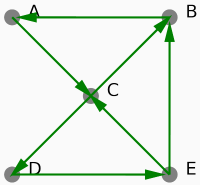


# 3. Direct graph

### Question

(A) Find an open Euler walk in this graph.

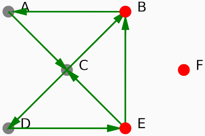
- For a open Euler walk, start at the point whose degree of point out is 1 more then that of point in. i.e. E
- End at the another odd vertex B
- A possible walk is $E \to B \to A \to C \to D \to E \to C \to B$.



# 3. Direct graph (b)

## Question

(b)Add one new vertex and two new directed edges so that the resulting graph has a closed Euler walk.

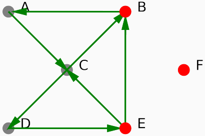


# 3. Direct graph (b)

**Question**

(b)Add one new vertex and two new directed edges so that the resulting graph has a closed Euler walk.

- Consider the two odd vertices B,E and a new vertex F

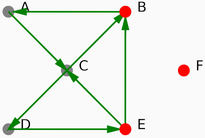


# 3. Direct graph (b)

**Question**

(b)Add one new vertex and two new directed edges so that the resulting graph has a closed Euler walk.

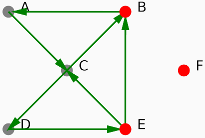
- Consider the two odd vertices B,E and a new vertex F
- To make it a closed walk, we should add a path from B to E.



# 3. Direct graph (b)

**Question**

(b)Add one new vertex and two new directed edges so that the resulting graph has a closed Euler walk.

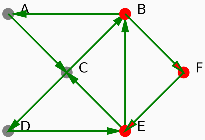



- Consider the two odd vertices B,E and a new vertex F
- To make it a closed walk, we should add a path from B to E.
- And the new vertex F should be connected by at least two edges (one point out one point in).

# 3. Direct graph (b)

**Question**

(b)Add one new vertex and two new directed edges so that the resulting graph has a closed Euler walk.

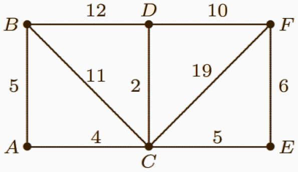



- Consider the two odd vertices B,E and a new vertex F
- To make it a closed walk, we should add a path from B to E.
- And the new vertex F should be connected by at least two edges (one point out one point in).
- Combine the two condition, the two edges must be $B \to F$ and $F \to E$

# 4. Cheapest weight

**Question**

Find the optimal Chinese Postman route starting from A.

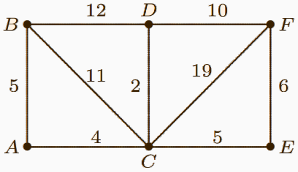


# 4. Cheapest wight

**Question**

Find the optimal Chinese Postman route starting from A.

- The odd vertices are B,C,D,F.

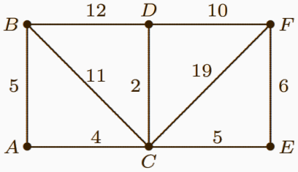


# 4. Cheapest wight

## Question

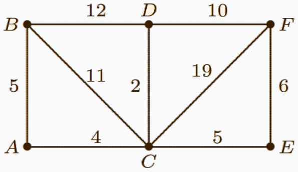
Find the optimal Chinese Postman route starting from A.

- The odd vertices are B,C,D,F.
- Pairing the four vertices as BC, DF. Then add a duplicated path respectively. The weight of added paths is $9(BA, AC) + 10(CF) = 19$.



**Question**

Find the optimal Chinese Postman route starting from A.



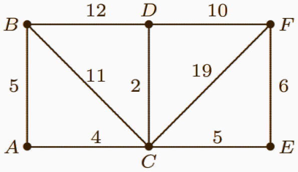

- The odd vertices are B,C,D,F.
- Pairing the four vertices as BC, DF. Then add a duplicated path respectively. The weight of added paths is $9(BA, AC) + 10(CF) = 19$.
- Pairing as BD,CF, the extra weight is $11(BA + AC + CD) + 11(CE + EF) = 22$.

# 4. Cheapest wight

## Question

Find the optimal Chinese Postman route starting from A.

- The odd vertices are B,C,D,F.
- Pairing the four vertices as BC, DF. Then add a duplicated path respectively. The weight of added paths is $9(BA, AC) + 10(CF) = 19$.

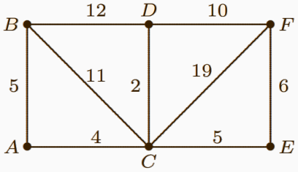



- Pairing as BD,CF, the extra weight is $11(BA + AC + CD) + 11(CE + EF) = 22$.
- Pairing as BF, CD, the extra weight is $20(BA+AC+CE+EF)+2(CD) = 22$.

# 4. Cheapest wight

**Question**

Find the optimal Chinese Postman route starting from A.



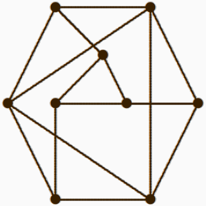

- The odd vertices are B,C,D,F.
- Pairing the four vertices as BC, DF. Then add a duplicated path respectively. The weight of added paths is $9(BA, AC) + 10(CF) = 19$.
- Pairing as BD,CF, the extra weight is $11(BA + AC + CD) + 11(CE + EF) = 22$.
- Pairing as BF, CD, the extra weight is $20(BA+AC+CE+EF)+2(CD) = 22$.
- Choose the minimal case, the shortest time (minimal weight) is $5+4+11+12+2+10+19+5+6+(19) = 74+19 = 93$.

# 5. Minimal proper vertex coloring

# 5. Minimal proper vertex coloring

**Question**

Find a minimal proper vertex coloring for each graph below:

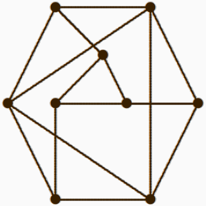


# 5. Minimal proper vertex coloring

**Question**

Find a minimal proper vertex coloring for each graph below:

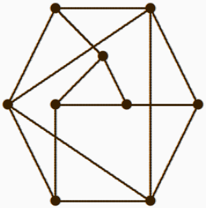
- This graph contains a $K_3$, so by the Vertex Coloring Theorem, the chromatic number is at least 3.



# 5. Minimal proper vertex coloring

**Question**

Find a minimal proper vertex coloring for each graph below:

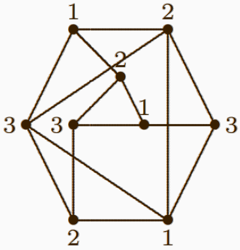


- This graph contains a $K_3$, so by the Vertex Coloring Theorem, the chromatic number is at least 3.
- Start with assigning the $K_3$ three different colors 1,2,3.

# 5. Minimal proper vertex coloring

**Question**

Find a minimal proper vertex coloring for each graph below:

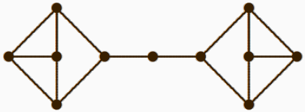



- This graph contains a $K_3$, so by the Vertex Coloring Theorem, the chromatic number is at least 3.
- Start with assigning the $K_3$ three different colors 1,2,3.
- A example of coloring is as shown in the graph. The chromatic number is 4.

# 5. Minimal proper vertex coloring

**Question**

Find a minimal proper vertex coloring for each graph below:

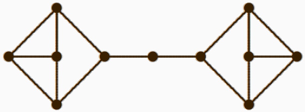


# 5. Minimal proper vertex coloring

**Question**

Find a minimal proper vertex coloring for each graph below:

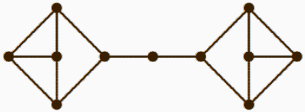
- This graph contains a $K_3$, so by the Vertex Coloring Theorem, the chromatic number is at least 3.



**Question**

Find a minimal proper vertex coloring for each graph below:

- This graph contains a $K_3$, so by the Vertex Coloring Theorem, the chromatic number is at least 3.
- Start with assigning the $K_3$ three different colors 1,2,3.

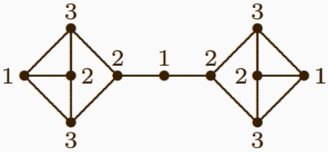


# 5. Minimal proper vertex coloring

**Question**

Find a minimal proper vertex coloring for each graph below:

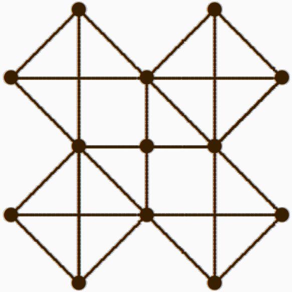
- This graph contains a $K_3$, so by the Vertex Coloring Theorem, the chromatic number is at least 3.
- Start with assigning the $K_3$ three different colors 1,2,3.




- A example of coloring is as shown in the graph. The chromatic number is 3.

**Question**

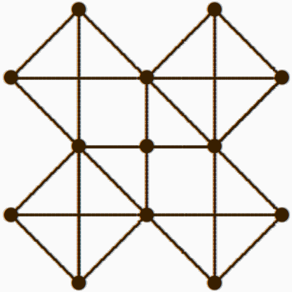
Find a minimal proper vertex coloring for each graph below:



**Question**

Find a minimal proper vertex coloring for each graph below:

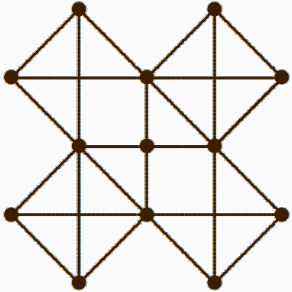
- This graph contains a $K_4$, so by the Vertex Coloring Theorem, the chromatic number is at least 4.



**Question**

Find a minimal proper vertex coloring for each graph below:

- This graph contains a $K_4$, so by the Vertex Coloring Theorem, the chromatic number is at least 4.
- Start with assigning the $K_4$ three different colors 1,2,3,4.

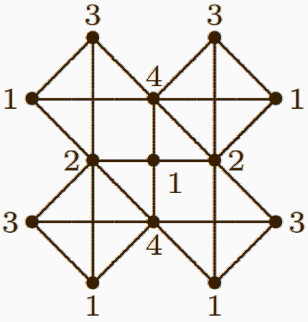


# 5. Minimal proper vertex coloring

**Question**

Find a minimal proper vertex coloring for each graph below:

- This graph contains a $K_4$, so by the Vertex Coloring Theorem, the chromatic number is at least 4.
- Start with assigning the $K_4$ three different colors 1,2,3,4.
- A example of coloring is as shown in the graph. The chromatic number is 4.



# 6. Colors at adjacent vertices

## 6. Colors at adjacent vertices

**Question**

Show that in a minimal proper vertex coloring of a graph, any two colors must occur at adjacent vertices somewhere in the graph.

- Suppose the chromatic number of the graph is k. Consider a proper coloring using k colors, say $1, \ldots, k$.

# 6. Colors at adjacent vertices

**Question**

Show that in a minimal proper vertex coloring of a graph, any two colors must occur at adjacent vertices somewhere in the graph.

- Suppose the chromatic number of the graph is k. Consider a proper coloring using k colors, say $1, \dots, k$.
- Direct solving is not easy, using contradiction instead.

# 6. Colors at adjacent vertices

**Question**

Show that in a minimal proper vertex coloring of a graph, any two colors must occur at adjacent vertices somewhere in the graph.

- Suppose the chromatic number of the graph is k. Consider a proper coloring using k colors, say $1, \dots, k$.
- Direct solving is not easy, using contradiction instead.
- Suppose the claim fails. Then there are at least two color $k, k-1$ which do not appear in the adjacent vertices.

# 6. Colors at adjacent vertices

**Question**

Show that in a minimal proper vertex coloring of a graph, any two colors must occur at adjacent vertices somewhere in the graph.

- Suppose the chromatic number of the graph is k. Consider a proper coloring using k colors, say $1, \ldots, k$.
- Direct solving is not easy, using contradiction instead.
- Suppose the claim fails. Then there are at least two color $k, k-1$ which do not appear in the adjacent vertices.
- Since vertices of colour $k$ are not adjacent to vertices of colour $k-1$, the two sets of vertices can be coloured using the same colour. We obtain a proper coloring using $k-1$ colors.

# 6. Colors at adjacent vertices

**Question**

Show that in a minimal proper vertex coloring of a graph, any two colors must occur at adjacent vertices somewhere in the graph.

- Suppose the chromatic number of the graph is k. Consider a proper coloring using k colors, say $1, \ldots, k$.
- Direct solving is not easy, using contradiction instead.
- Suppose the claim fails. Then there are at least two color $k, k-1$ which do not appear in the adjacent vertices.
- Since vertices of colour $k$ are not adjacent to vertices of colour $k-1$, the two sets of vertices can be coloured using the same colour. We obtain a proper coloring using $k-1$ colors.
- This contradicts with the given condition. Hence claim holds.

# 7. Meeting schedule

# 7. Meeting schedule

## Question

A Student Council has 6 committees. Councilors A, B, C , D, E, F , G, H, I, J are appointed to be members of the committees as shown below:

| | |
|---|---|
| Publicity | A, B, C, D |
| Recreation | A, E, F, G |
| Welfare | G, H, I, J |
| Community | D, E |
| Secretariat | B, F, H |
| Finance | G, I |

If each committee is scheduled to meet for two hours this week, what is the smallest number of two-hour sessions required to schedule all 8 committee meetings so that each of these councilors is able to attend the meetings of the committees he/she is a member of?

## 7. Meeting schedule

- First reformulate the question into graph language.

# 7. Meeting schedule

- First reformulate the question into graph language.
- Vertices: the five committees. Denote them by $P$, $R$, $W$, $C$, $S$ and $F$ respectively.

## 7. Meeting schedule

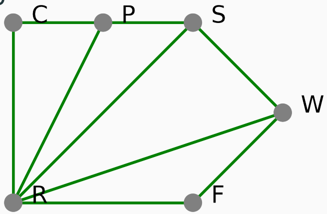
- First reformulate the question into graph language.
- Vertices: the five committees. Denote them by $P$, $R$, $W$, $C$, $S$ and $F$ respectively.
- Edges: If any two committees share some member, connect them with a edge.

# 7. Meeting schedule

- First reformulate the question into graph language.
- Vertices: the five committees. Denote them by $P$, $R$, $W$, $C$, $S$ and $F$ respectively.
- Edges: If any two committees share some member, connect them with a edge.
- Now the graph is generated as following:

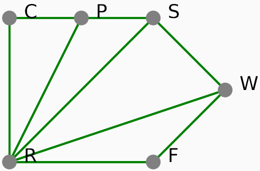
# 7. Meeting schedule

- First reformulate the question into graph language.
- Vertices: the five committees. Denote them by P , R, W , C, S and F respectively.
- Edges: If any two committees share some member, connect them with a edge.
- Now the graph is generated as following:
- Consider what non-adjacent vertices means. It means that non-adjacent committees can meet at the same time, adjacent committees cannot meet at the same time.

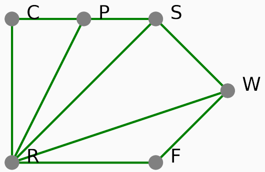


# 7. Meeting schedule (Contd.)

- If we denote the different time slot by different colors. Then condition becomes that non-adjacent vertices can share same color, while adjacent vertices cannot. This is exactly the same with question of proper vertex coloring.

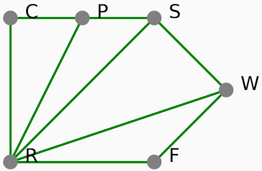


# 7. Meeting schedule (Contd.)



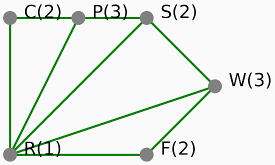

- If we denote the different time slot by different colors. Then condition becomes that non-adjacent vertices can share same color, while adjacent vertices cannot. This is exactly the same with question of proper vertex coloring.
- The question then is find the minimal proper vertex coloring (minimal numbers of different time slots for these committees).

# 7. Meeting schedule (Contd.)




- If we denote the different time slot by different colors. Then condition becomes that non-adjacent vertices can share same color, while adjacent vertices cannot. This is exactly the same with question of proper vertex coloring.
- The question then is find the minimal proper vertex coloring (minimal numbers of different time slots for these committees).
- This graph contains a $K_3$, so the chromatic number is at least 3.

# 7. Meeting schedule (Contd.)




- If we denote the different time slot by different colors. Then condition becomes that non-adjacent vertices can share same color, while adjacent vertices cannot. This is exactly the same with question of proper vertex coloring.
- The question then is find the minimal proper vertex coloring (minimal numbers of different time slots for these committees).
- This graph contains a $K_3$, so the chromatic number is at least 3.
- One way of proper coloring with 3 colors is shown in the picture. So the chromatic number is 3. Back to the original question, the smallest number of two-hour sessions required is 3.

# 8. End

# Thank you for your listening!