GEH1036/GEK1505 Tutorial 4(week 6)

Tsien Lilong

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Department of Mathematics,NUS

Personal Information

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    Go to the website, and scroll down, see the **Work** entry, in the "Tutor" project, click the link for more information. I will provide this slide and the source .tex file on that site.
- Course information
  - EVERY WEEK MONDAY 10:00-11:00 S16-0431

1. Cycles in Complete Graph

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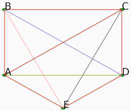
- Choose any 4 vertices from the 5, having $\binom{5}{4}$ ways.



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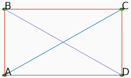

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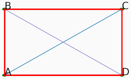

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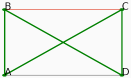

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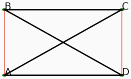

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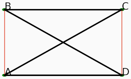
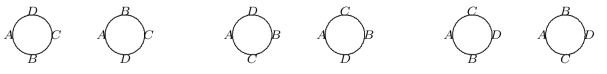


- Another way, given 4 vertices, each cycle corresponds two circular arrangement (why two? because clockwise and anti-clockwise cycles are the same)



2. Cycles in a Cube

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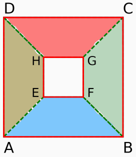
Counting the cycles of length 4 in the following picture.

- There are 6 of them as showing in this picture (4 colored regions, one inner small square and one outer big square). ABCDA, EFGHE, ABFEA, BCGFB, CDHGC, DAEHA



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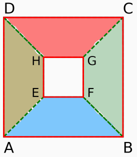
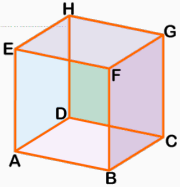


- We can also think in another way. The graph is "equivalent" with a "cube". Each face corresponds a cycle in the original graph.



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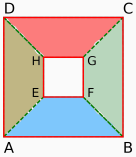

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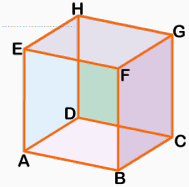

3. Vertices Number

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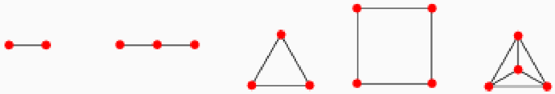
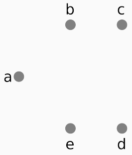
- Degree of a vertex: a vertex has degree 5 means that that point is connected by 5 edges.
- Then the total number of edges is $5n/2 = 100$, i.e. $n = 40$.
- Why divide by "2"? It is because every two vertices share one edge. Consider the following case:



4. Graph Drawing

Question

Draw a graph with 5 vertices, 4 of which have degree 3 and one has degree 2.



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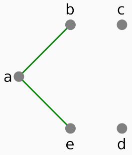
- Draw 5 vertices and first choose randomly one (a, for example) to be 2-degree.



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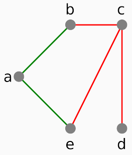
- Consider c, d respectively, each have degree 3 means connected by other 3 vertices. But a is impossible (why?), then must be b,e,d for c (b,c,e for d).



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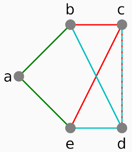

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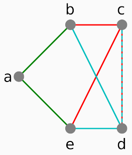
- Consider c, d respectively, each have degree 3 means connected by other 3 vertices. But a is impossible (why?), then must be b,e,d for c (b,c,e for d).
- Then consider b and e, all the conditions satisfied. That is exactly the final answer. Only one way!



5. Existence of degree greater or equal 6

Question

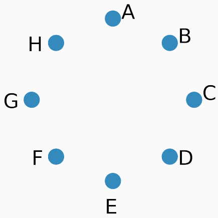
A graph containing 8 vertices and 21 edges. Show that there is at least one vertex with degree $\geqslant 6$

By the Degree Theorem, the sum of the degrees is $2 \times 21 = 42$. Since there are 8 vertices, the average degree is $42/8 = 5.25$.
Therefore there must be one vertex whose degree is greater than or equal to 6. It is same using Pigeon-holes principles. (Think as tutorial 3)

6. Nonexistence of Graph

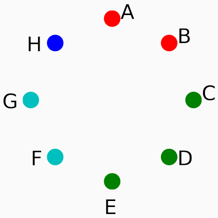
Question

Explain that there is not a graph with 8 vertices such that: 2 vertices have degree 7; 2 vertices have degree 4; one vertex has degree 2.



6. Existence of a Graph

- Suppose $d(A)=d(B)=7, d(C)=d(D)=d(E)=6,$ $d(F)=d(G)=4, d(H)=2$.



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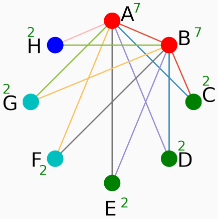

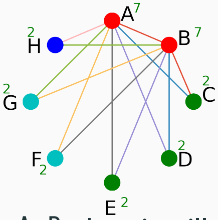

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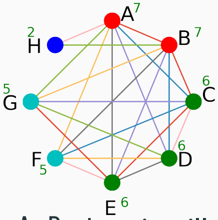

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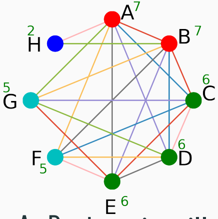

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- Consider F (or G), it has already be connected to A,B,C,D,E, which implies that the degree of F is $\geqslant 5$. However impossible.

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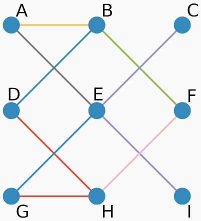

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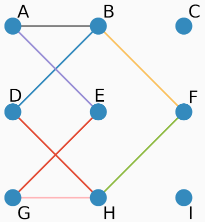

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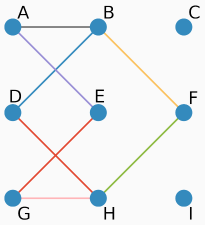

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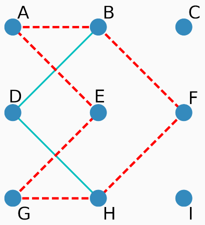

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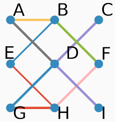

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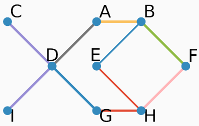

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Ends

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