# Introduction, PCA, SVD + Assignment 0

(Neural Networks Implementation and Application Tutorial)

Vilém Zouhar, Noon Pokaratsiri Goldstein

10th November 2021

#### Overview

- Introduction
- Requirements, Materials, Assignments
- PCA, SVD
- Current assignment
- QA

### Hello

Who am I?

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Who are you?



#### Introduction

#### Choose and answer at least two questions:

- On scale from 1-10 how proficient are you in programming and mathematics?
- What topics of Neural Networks excite you the most?
- What topics of Neural Networks excite you the least?
- What programming languages do you know?
- How best can the tutorial sessions be helpful to your needs?

#### Also the following:

- Who is your groupmate?
- Will you be attending Vilém's or Noon's tutorials?

## Requirements

### Tutorial Requirements (exam admission)

- 60% of mandatory points (~10 assignments, 10 points each)
- Tutorial points only for exam admission (no final grade influence)

#### **Tutorial Bonus Points**

- ~2pts for extra exercises in the assignments
- 1pt for answering a question in a tutorial
- ??pt for fixing errors in tutorial presentations
  - github.com/zouharvi/uds-nnia-tutorial

#### Final Project

None

#### Transfer from last year

- Maybe possible (tbd)
- Assignments recommended (because of the exam)

#### What's available

- Lectures by Prof. Klakow (recorded)
- Tutorials (not recorded, but allowed for private sharing)
- Corrected homework
- Consultations
  - Only in specific cases
  - By default no email and no personal chat
  - Ask questions during the lecture / tutorials
- Public forum (please use Piazza)
  - Ask questions
  - ▶ Other students will also benefit from the answers
  - ▶ You can answer someone else's issue

### Assignments

- Mandatory groups of 2
- Usually 2 exercises per assignment + a possible bonus question
- Jupyter notebook templates
  - Assignment + solution in the same notebook
  - Can use Google Colab or local runtime
  - Write solutions in Python files and import them
  - Submitted notebook must only contain your analysis and outputs
- Only one submission per group
  - Submit through Teams

### Dates / Times

- Lecture:
  - ► Tuesday 14:15-15:45
- Tutorials:
  - ▶ Vilém: Wednesday 16:00-18:00
  - ▶ Noon: Thursday 08:30-10:00
- Assignments
  - ► Released (usually) by Wednesday 08:00 (available in Teams)
  - ▶ Deadline (next) by Wednesday 08:00 (submit in Teams)
- Exam: TBD

#### **Tutorial Content**

- Review of the topics covered in class
- Presentation of the past assignment
- Discussing the current assignment

## Organization

Questions?

### Assignment 0

- Questions?
- Did it work?
- How long did it take?

#### Feedback:

- Change TODO to Solution.
- Don't forget to write amount of work.
  - Useful for our estimates of difficulty.

Few definitions (+how are they implemented in Python/Numpy/PyTorch)

- Scalars
- Vectors
- Matrices
- Tensors

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Identify the following objects (Python lists):

- [5.0, 3.0]
- 5.0
- [True]
- [[5, 1], [0, 4]]
- [[True, False], [False, True]]
- [ [[0,1], [0,1], [0,1]], [[0,1], [0,1], [0,1]] ]

#### A few operations and properties involving matrices:

- Transpose
- Inverse
- Dot product (i.e. matrix multiplication)
  - $C = AB, C_{i,j} = \sum_k A_{i,k} B_{k,j}$

#### Common Properties:

- A(B+C) = AB + AC
- A(BC) = (AB)C
- AB ≠ BA
- $\bullet$   $(AB)^T = B^T A^T$

#### **Definitions:**

• Eigenvector, Eigenvalue

- Every real matrix has an eigenvalue decomposition (in  $\mathbb{R}$ ).
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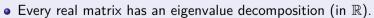
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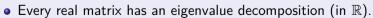
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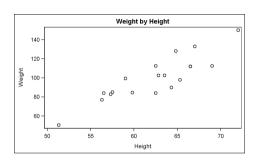
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  - Project to *k* dimensions:  $A_k = AQ_k$

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## Linear Algebra Basics - True or False? 🧐

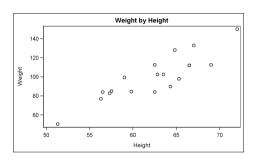
$$A = \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}$$

- Is  $v_1 = (1, -1)$  an eigenvector of A?
- 2 Is  $v_2 = (2,1)$  an eigenvector of A?
- **1** Is  $v_3 = (2,2)$  an eigenvector of A?



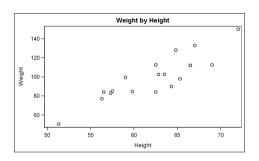


• What will be the first principal component?



# Questions (9)

- What will be the first principal component?
- Does anyone know how PCA works?

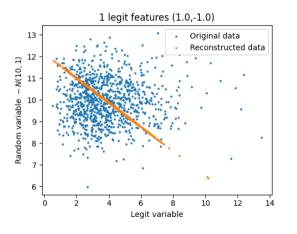


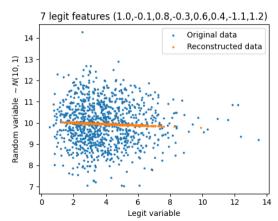
# Questions (9)

- What will be the first principal component?
- Does anyone know how PCA works?
- What does it mean that we take only k largest principal components?

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#### **Standardization**

- Is not normalization!  $(x' = \frac{x}{|x|})$
- $X = \frac{X \text{mean}(X)}{\text{std}(X)}$
- Compute either:
  - With Numpy: X = (X-X.mean())/np.std(X)
  - With Scikit: StandardScaler().fit\_transform(X)
- Why do we need standardization for PCA?



### Assignment 1

• Any questions?

### Typesetting Tips

- Do **not** write A \* B, use \cdot or \times:  $A \cdot B$ ,  $A \times B$ .
- Use LaTeX functions when available, e.g.  $\log$ ,  $\sin$ :  $\log(x)$ ,  $\sin(x)$ , **not**  $\log(x)$ ,  $\sin(x)$ .
- Do **not** write plain text in math mode, use \$\text{ComputeEigenvalues}(X)\$

#### Resources

- Course Website: lsv.uni-saarland.de/neural-networks-implementation-and-application-winter-2021-2022-2
- Piazza: https://piazza.com/class/kvc3vzhsvh55rt
- Tutorial repository github.com/zouharvi/uds-nnia-tutorial
- Lecture & tutorial teams channels