Propositional Equivalences

- Formula A and B are equivalent if they have the same truth tables
- We denote the equivalence by ≡
- A ≡ B means that A and B always have the same truth values, regardless of how the variables are assigned.
- Note that ≡ is NOT a connective

De Morgan's Laws

1. $\neg(p \wedge q) \equiv \neg p \vee \neg q$

2. $\neg(p \vee q) \equiv \neg p \wedge \neg q$

Build the truth tables for these formulae!

Example: negate the following

- It is Wednesday and it is not sunny
- NOT (It is Wednesday and it is not sunny)
- NOT (It is Wednesday) or NOT (it is not sunny)
- It is NOT Wednesday or it is sunny

Truth table for $\neg(p \wedge q) \equiv \neg p \vee \neg q$

p	q	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
1	1	0	0	0	0
1	0	1	0	1	1
0	1	1	1	0	1
0	0	1	1	1	1

Important equivalence

- $(p \to q) \equiv (\neg p \vee q)$
- Can you write the right hand side using conjunction?
- Use De Morgan's Law
- $(\neg p \vee q) \equiv \neg(\neg\neg p \wedge \neg q) \equiv \neg(p \wedge \neg q)$

- So $(p \to q) \equiv \neg(p \wedge \neg q)$

Another Equivalence: Contrapositive

- $p \to q \equiv \neg q \to \neg p$
- Why is it true?
- $p \to q \equiv \neg p \vee q$
- $\neg p \vee q \equiv q \vee \neg p \equiv \neg q \to \neg p$

$$a \to b \equiv \neg a \vee b$$

Truth table for

$$2.\ \neg(p \vee q) \equiv \neg p \wedge \neg q$$

p	q	p ∨ q	¬(p ∨ q)	¬p	¬q	¬p ∧ ¬q
True	True	True	False	False	False	False
True	False	True	False	False	True	False
False	True	True	False	True	False	False
False	False	False	True	True	True	True

Truth Table for ¬(p ∨ q) ≡ ¬p ∧ ¬q

Equivalency with conjunction and negation

- **Is it possible to convert all operators to conjunction and negation?**
- **For example,** $p \vee q$ **based on the De Morgan's Law, can be written as** $\neg(\neg p \wedge \neg q)$
- **Or,** $p \rightarrow q$**, can be written as its equivalent** $\neg p \vee q$**, and based on the De Morgan's Law, can be written as** $\neg(p \wedge \neg q)$
- **Since we can write the equivalence of disjunction, using negation and conjunction, it means we can rewrite all logical formulae using these two operators**

Rewrite $\mathrm{p} \vee (\mathrm{q} \rightarrow \mathrm{r})$ only using conjunction and negation

- $\mathrm{p} \vee (\mathrm{q} \rightarrow \mathrm{r}) \equiv$
- $\neg\big(\neg p \wedge \neg(q \rightarrow r)\big) \equiv$
- $\neg\big(\neg p \wedge \neg(\neg q \vee r)\big) \equiv$
- $\neg\big(\neg p \wedge (q \wedge \neg r)\big)$

- **We can also convert each conjunction to its equivalent using disjunction and negation!**