Important notions

Predicate is a statement or function that can be true or false depending on the values of its variables

- Predicates that describe properties of objects
  - Odd(3) means 3 is an odd number; Odd is a predicate, 3 is an object
  - Equal(5,6) means 5 and 6 are equal; Equal is a predicate, 5,6
- Predicates take arguments and become propositions
- The connectives for propositional logic apply the same way
  - Odd (3) ∧ Prime (3), which means 3 is odd and 3 is prime, true.
  - Even (4) → Prime (4), which means 4 is even then 4 is prime, false.
- Quantifiers make reasoning on multiple objects
- The objects for the quantified statements are chosen from a Domain

Quantifiers allow us to reason about multiple objects

- Existential quantifier, denoted by ∃
  - ∃x "some formula"
  - Means for some x, the statement "some formula" is true.
  - Example: ∃x Odd(x) it means some numbers are odd
  - ∃x Prime(x) it means there exists at least one number that is prime
  - NOTE, it is enough to find one element to make the formula true

Quantifiers allow us to reason about multiple objects

- Universal quantifier, denoted by ∀
  - ∀x "some formula"
  - Means for ALL x, the statement "some formula" is true
  - Example: ∀x (Odd(x) ∨Even(x)) – it means all numbers are either odd or even
  - ∀x (x < x+1) – it means all numbers increase when you add 1
  - NOTE, it is NOT enough to find some elements that make the formula true

Universal Quantifier

- “All Ps are Qs” translates as ∀x (P(x) → Q(x))
- A counter-example proves that a universally quantified statement is false
- Example: ∀x (Prime(x) → Odd(x)), which means all prime numbers are odd
- Let x be 2. But 2 is prime and not odd, so the statement is false
- ∀x (Multiple4(x) → Multiple2(x)), which means all multiples of 4 are multiples of 2. True.
- “No Ps are Qs” Translates as ∀x (P(x) → ¬Q(x))
- If we find one P that is Q then we prove that the statement above is false
- Example: ∀x (Prime(x) → ¬Square(x)), which means no prime number is square number. True.
- ∀x (Prime(x) → ¬Even(x)), which means no prime number is even. False.

Existential Quantifier

- “Some Ps are Qs” translates as ∃x (P(x) ∧ Q(x))
- Existentially quantified statements are true if an evidence example exists
- Example: ∃x (Prime(x)∧Even(x)) – some prime numbers are even, true
- Example: ∃x (Professor(x)∧ Under2 (x)) – some professors are under 2. Not true!
- “Some Ps are not Qs” translates as ∃x (P(x) ∧ ¬ Q(x))
- Example: ∃x (Prime(x)∧ ¬Even(x)) – some prime numbers are not even. True

Quantifiers to connectives

- $\exists x, P(x)$ and domain is D = $\{x_1, x_2, .., x_n\}$ means

$$P(x_1) \vee P(x_2) \vee \cdots \vee P(x_n)$$

- $\forall x, P(x)$ and domain is D = $\{x_1, x_2, .., x_n\}$ means

$$P(x_1) \wedge P(x_2) \wedge \cdots \wedge P(x_n)$$

De Morgan's Law

- $\exists x, P(x)$ and domain is D = $\{x_1, x_2, \ldots, x_n\}$ means
$$P(x_1) \vee P(x_2) \vee \cdots \vee P(x_n)$$

- $\neg\exists x, P(x) \equiv \neg((P(x_1) \vee P(x_2) \vee \cdots \vee P(x_n))$

$$\equiv \neg(P(x_1) \wedge \neg(P(x_2) \wedge \ldots \wedge \neg(P(x_n))$$

$$\equiv \forall x, \neg P(x)$$

De Morgan's Law

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$$\equiv \neg(P(x_1) \vee \neg(P(x_2) \vee \ldots \vee \neg(P(x_n))$$

$$\equiv \exists x, \neg P(x)$$

Negate the following

De Morgan's Law
$\neg(p \vee q) \equiv \neg p \wedge \neg q$

- $\forall x(p(x) \to q(x))$

- $\neg\forall x(p(x) \to q(x)) \equiv \exists x \neg(p(x) \to q(x))$

$$\equiv \exists x \neg(\neg p(x) \vee q(x))$$

$$\equiv \exists x\big(\neg\neg p(x) \wedge \neg q(x)\big)$$