CM1020

BSc EXAMINATION

COMPUTER SCIENCE

Discrete Mathematics

Release date: Monday 6 March 2023 at 12:00 midday Greenwich Mean Time

Submission date: Tuesday 7 March 2023 by 12:00 midday Greenwich Mean Time

Time allowed: 24 hours to submit

INSTRUCTIONS TO CANDIDATES:

Section A of this assessment consists of a set of **TEN** Multiple Choice Questions (MCQs) which you will take separately from this paper. You should attempt to answer **ALL** the questions in Section A. The maximum mark for Section A is **40**.

Section A will be completed online on the VLE. You may choose to access the MCQs at any time following the release of the paper, but once you have accessed the MCQs you must submit your answers before the deadline or within **4 hours** of starting whichever occurs first.

Section B of this assessment is an online assessment to be completed within the same 24-hour window as Section A. We anticipate that approximately **1 hour** is sufficient for you to answer Section B. Candidates must answer **TWO** out of the THREE questions in Section B. The maximum mark for Section B is **60**.

Calculators are not permitted in this examination. Credit will only be given if all workings are shown.

You should complete Section B of this paper and submit your answers as **one document**, if possible, in Microsoft Word or a PDF to the appropriate area on the VLE. Each file uploaded must be accompanied by a coversheet containing your **candidate number**. In addition, your answers must have your candidate number written clearly at the top of the page before you upload your work. Do not write your name anywhere in your answers.

UL23/0207

SECTION A

Candidates should answer the **TEN** Multiple Choice Questions (MCQs) quiz, **Question 1** in Section A on the VLE.

SECTION B

Candidates should answer any **TWO** questions from Section B.

Question 1

(a) List the elements of the following sets:

i. $\{x | x \in Z \wedge (x^2 = 6)\}$

ii. $\{x | x \in \mathbb{Z} \wedge (x^2 = 9)\}$

iii. $\{x | x \in \mathbb{N} \wedge (x \bmod 2 = 1) \wedge (x < 10)\}$ [3]

(b) Let A and B be two sets such that $|A| = |B| = n$ and $|A \cap B| = 1$. Find

i. $|A \cup B|$

ii. $|\mathcal{P}(A \cup B)|$

where n is a positive integer and $\mathcal{P}(S)$ represents the power set of a set S. Show your working. [4]

(c) Prove the following set identities, using either Venn Diagrams or the rules of sets. Show your working.

i. $(A \cap B) \cup (A \cap B) = A$

ii. $(A - B) - C \subseteq A - \overline{C}$

iii. $(A - C) \cap (C - B) = \emptyset$ [6]

(d) Let p, q and r be three propositions for which p and q are true, and r is false. Determine the truth value of for each of the following: [3]

i. $p \rightarrow (r \rightarrow q)$

ii. $(p \oplus r) \rightarrow \neg q$

iii. $p \wedge (r \rightarrow q)$

(e) The universe of discourse is the set of all positive integers, $\mathbb{Z}$^$^+$.
What are the truth values for each of the following:

i. $\exists x \forall y (x \leq y)$

ii. $\exists x \exists y (x + y = 0) \vee (x * y = 0)$

iii. $\forall x \forall y (x * y \geq x + y)$ [3]

(f) Re-write the following statements without any negations on quantifiers:

i. $\neg \exists x P(x)$

ii. $\neg \exists x \neg \exists y P(x, y)$

iii. $\neg \exists x \forall y P(x, y)$ [3]

(g) Decide whether the following arguments are valid or not. State the Rule of Inference or fallacy used.

i. Let A, B and C be three sets. Prove by contradiction that if $A \cap B \subseteq C$ and $x \in B$, then $x \not\in A - C$. [4]

ii. Suppose that I want to purchase a tablet computer. I can choose either a large or a small screen; a 64GB, 128GB, or 256GB storage capacity, and a black, white, gold, or silver cover. How many different options do I have?

[4]

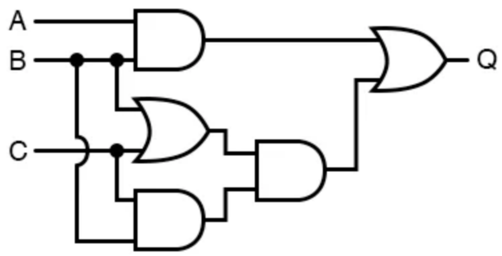
Question 2

(a) Minimise the following logic function using the Karnaugh maps method:

$$f(a,b,c) = a'b + bc' + bc + ab'c'$$

[6]

(b) Given the following logical circuit with three inputs A, B and C:



i. Use the boolean algebra notation and write down the boolean expression of the output, Q of this circuit. [4]

ii. Simplify the logical expression in (i). Explain your answer. [5]

(c) Let f be a function $\mathbb{R} - \{-3\} \to \mathbb{R} - \{1\}$ with $f(x) = \frac{x}{x+3}$.

i. Show that f is a bijective function [4]

ii. Find the inverse function f^{-1} [2]

iii. Plot the curves of both function f and f^{-1} on the same graph. [2]

iv. suppose we change the co-domain of the function f to be $\mathbb{R}$:

$$f:\ \mathbb{R} - \{-3\} \to \mathbb{R}$$

v. Is f still a bijective function? Explain your answer. [3]

(d) How many binary sequences of length 8 start with a 1 and end with a 0? [4]

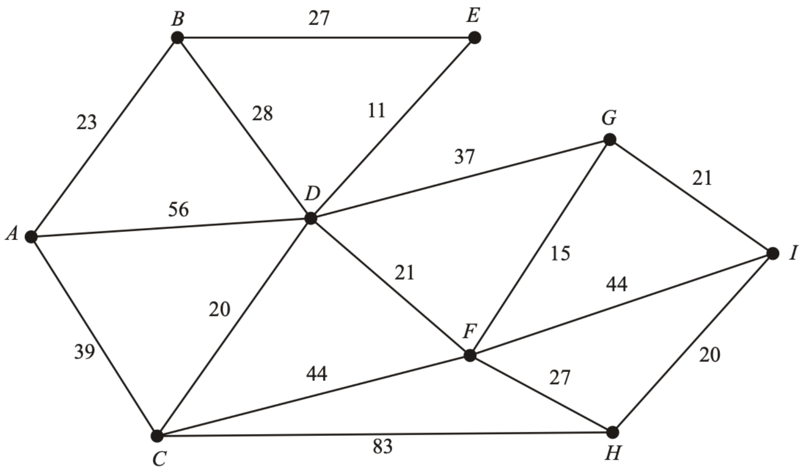
Question 3

(a) Explain the difference between an Euler path and an Euler cycle. [3]

(b) Find the maximum number of comparisons to be made to find any record in a binary search tree which holds 3000 records. [3]

(c) Explain what is meant by the term 'path'.

(d) The figure shows a network of cycle tracks. The number on each edge represents the length, in miles, of that track. Jay wishes to cycle from A to I as part of a cycling holiday. She wishes to minimise the distance she travels.



Use Dijkstra's algorithm to find the shortest path from A to I. Show your working.

[6]

(e) Given S is the set of integers $\{2, 3, 4, 5, 6, 7, 8\}$. Let $\mathcal{R}$ be a relation defined on S by the following condition such that,

$$\forall x, y \in S, xRy \iff x \mod 2 = y \mod 2$$

i. Draw the digraph of $\mathcal{R}$. [2]

ii. Show that $\mathcal{R}$ is an equivalence relation. [6]

iii. Find the equivalence classes for $\mathcal{R}$ [2]
item is $\mathcal{R}$ a partial order? Explain your answer [2]

(f) Let$f : A \to B$ and $g : B \to C$ be functions. Prove that if $g \ o \ f$ is one-to-one , then f is one-to-one. [6]

END OF PAPER