Propositional Equivalences

- ▲ Formula A and B are equivalent if they have the same truth tables
- \triangle We denote the equivalence by \equiv
- $A \equiv B$ means that A and B always have the same truth values, regardless of how the variables are assigned.
- ▲ Note that \equiv is NOT a connective

De Morgan's Laws

$$\mathbf{1.} \neg (p \land q) \equiv \neg p \lor \neg q$$

Build the truth tables for these formulae!

2.
$$\neg (p \lor q) \equiv \neg p \land \neg q$$

Example: negate the following

- It is Wednesday and it is not sunny
- NOT (It is Wednesday and it is not sunny)
- NOT (It is Wednesday) or NOT (it is not sunny)
- It is NOT Wednesday or it is sunny

Truth table for $\neg(p \land q) \equiv \neg p \lor \neg q$

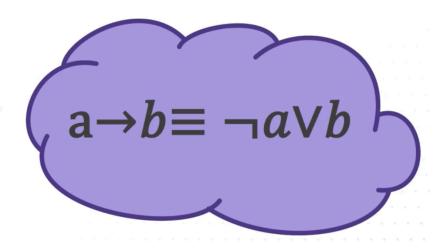
р	q	$\neg(p \land q)$	$\neg p$	$\neg q$	$\neg p \lor \neg q$
1	1	0	0	0	0
1	0	1	0	1	1
0	1	1	1	0	1
0	0	1	1	1	1

Important equivalence

- \triangle $(p \rightarrow q) \equiv (\neg p \lor q)$
- ▲ Can you write the right hand side using conjunction?
- ▲ Use De Morgan's Law
- \blacktriangle So $(p \rightarrow q) \equiv \neg (p \land \neg q)$

Another Equivalence: Contrapositive

- ▲ Why is it true?



Equivalency with conjunction and negation

- Is it possible to convert all operators to conjunction and negation?
- For example, $p \lor q$ based on the De Morgan's Law, can be written as $\neg(\neg p \land \neg q)$
- Or, $p \to q$, can be written as its equivalent $\neg p \lor q$, and based on the De Morgan's Law, can be written as $\neg (p \land \neg q)$
- Since we can write the equivalence of disjunction, using negation and conjunction, it means we can rewrite all logical formulae using these two operators

Rewrite $p \lor (q \rightarrow r)$ only using conjunction and negation

- $p \lor (q \rightarrow r) \equiv$
- $\neg (\neg p \land \neg (q \rightarrow r)) \equiv$
- $\neg (\neg p \land \neg (\neg q \lor r)) \equiv$
- $\neg (\neg p \land (q \land \neg r))$
- We can also convert each conjunction to its equivalent using disjunction and negation!

Example

- Without using the truth table, show that the following statement is true: $\neg(\exists x[P(x)\land Q(x)])\equiv \forall x[P(x)\rightarrow \neg Q(x)]$
- $\neg(\exists x[P(x)\land Q(x)]) \equiv$
- $\forall x \neg [P(x) \land Q(x)] \equiv$
- $\forall x [\neg P(x) \lor \neg Q(x)] \equiv$
- $\forall x[P(x) \rightarrow \neg Q(x)]$
- $\neg(\exists x[P(x)\land Q(x)])\equiv \forall x[P(x)\rightarrow \neg Q(x)]$