

Propositional Equivalences

- ▲ Formula A and B are **equivalent** if they have the same truth tables
- ▲ We denote the equivalence by \equiv
- ▲ $A \equiv B$ means that A and B always have the same truth values, regardless of how the variables are assigned.
- ▲ Note that \equiv is NOT a connective

De Morgan's Laws

$$1. \neg(p \wedge q) \equiv \neg p \vee \neg q$$

Build the truth tables for these formulae!

$$2. \neg(p \vee q) \equiv \neg p \wedge \neg q$$

Example: negate the following

- It is Wednesday and it is not sunny
- **NOT** (It is Wednesday **and** it is not sunny)
- **NOT** (It is Wednesday) **or** **NOT** (it is not sunny)
- It is NOT Wednesday or it is sunny

Truth table for $\neg(p \wedge q) \equiv \neg p \vee \neg q$

p	q	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
1	1	0	0	0	0
1	0	1	0	1	1
0	1	1	1	0	1
0	0	1	1	1	1

Important equivalence

- ▲ $(p \rightarrow q) \equiv (\neg p \vee q)$
- ▲ Can you write the right hand side using conjunction?
- ▲ Use De Morgan's Law
- ▲ $(\neg p \vee q) \equiv \neg(\neg \neg p \wedge \neg q) \equiv \neg(p \wedge \neg q)$
- ▲ So $(p \rightarrow q) \equiv \neg(p \wedge \neg q)$

Another Equivalence: Contrapositive

- ▲ $p \rightarrow q \equiv \neg q \rightarrow \neg p$
- ▲ Why is it true?
- ▲ $p \rightarrow q \equiv \neg p \vee q$
- ▲ $\neg p \vee q \equiv q \vee \neg p \equiv \neg q \rightarrow \neg p$


$$a \rightarrow b \equiv \neg a \vee b$$

Truth table for

$$2. \neg(p \vee q) \equiv \neg p \wedge \neg q$$

p	q	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
1	1	0	0	0	0
1	0	0	0	1	0
0	1	0	1	0	0
0	0	1	1	1	1

Truth Table for $\neg(p \vee q) \equiv \neg p \wedge \neg q$

Equivalency with conjunction and negation

- Is it possible to convert all operators to conjunction and negation?
- For example, $p \vee q$ based on the **De Morgan's Law**, can be written as $\neg(\neg p \wedge \neg q)$
- Or, $p \rightarrow q$, can be written as its equivalent $\neg p \vee q$, and based on the **De Morgan's Law**, can be written as $\neg(p \wedge \neg q)$
- Since we can write the equivalence of disjunction, using negation and conjunction, it means we can rewrite all logical formulae using these two operators

Rewrite $p \vee (q \rightarrow r)$ only using conjunction and negation

- $p \vee (q \rightarrow r) \equiv$
- $\neg(\neg p \wedge \neg(q \rightarrow r)) \equiv$
- $\neg(\neg p \wedge \neg(\neg q \vee r)) \equiv$
- $\neg(\neg p \wedge (q \wedge \neg r))$
- We can also convert each conjunction to its equivalent using disjunction and negation!

