UNIVERSITY OF LONDON

CM1020

BSc EXAMINATION

COMPUTER SCIENCE

Discrete Mathematics

Release date: Tuesday 10 September 2024 at 12:00 midday British Summer Time
Close date: Wednesday 11 September 2024 by 12:00 midday British Summer Time
Time allowed: 4 hours to submit

INSTRUCTIONS TO CANDIDATES:
Part A of this assessment consists of a set of **TEN** Multiple Choice Questions (MCQs). You should attempt to answer **ALL** the questions in **Part A.** The maximum mark for Part A is **40**.

Candidates must answer **TWO** out of the **THREE** questions in **Part B**. The maximum mark for Part B is **60**.
Part A and Part B will be completed online together on the Inspera exam platform. You may choose to access either part first upon entering the test area but must complete both parts within **4 hours** of doing so.

A handheld non-programmable calculator may be used when answering questions on this paper, but it must not be able to display graphics, text, or algebraic equations. Please hold your calculator to the camera at the start of the examination to clearly show the make and type.

You may use **ONE** A4 page of previously prepared notes in this examination. Please hold up your notes to the camera at the start of the examination.

File upload is permitted in this examination.

Do not write your name anywhere in your answers.

PART A

Question 1

Candidates should answer the **TEN** Multiple Choice Questions (MCQs) in Part A.

PART B

Candidates should answer any **TWO** questions from Part B.

Question 2

(a) Define the set of all integers that are multiples of 5 using set-builder notation. [2 marks]

(b) Given two sets A and B with $A = \{1, 3, 5, 7\}$ and $B = \{3, 5, 8, 9\}$, list the symmetric difference of sets A and B, $A \oplus B$. [3 marks]

(c) Given the universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and three sets $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, and $C = \{5, 6, 7, 8\}$.

i. Draw a Venn diagram representing A, B and C.

ii. Calculate $\overline{A} \cap \overline{B} \cap \overline{C}$.

iii. Illustrate $\overline{A} \cap \overline{B} \cap \overline{C}$ on Venn diagram.

[6 marks]

(d) Let A and B be two sets. Prove that: If $A \subseteq B$, then $A \cap \overline{B} = \emptyset$.

[4 marks]

(e) Prove whether each of the following set identities is correct or not:

i. $(A - B) - C \subseteq A - C$

ii. $(A - B) \cap C = (C - B) \cap A$

iii. $(A - B) \cup C = (C - B) \cup A$

[6 marks]

(f) Let p and q be two propositions. Is $(p \rightarrow q) \vee (q \rightarrow q)$ a tautology? Explain your answer. [5 marks]

(g) The domain of discourse consists of all positive integers, $\mathbb{Z}^+$. What are the truth values for each of the following:

i. $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}(y > x)$

ii. $\exists! x \in \mathbb{Z}, (x^2 = 4)$

iii. $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}(x < y)$

iv. $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, x = 2y$

[4 marks]

Question 3

(a) What is the scope of a quantifier? [2 marks]

(b) What is the difference between free and bound variables in a quantified statement? [4 marks]

(c) Write the negation of each of the following quantified statements:

i. $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, \forall z \in \mathbb{R}, x + y = z$

ii. $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, \exists z \in \mathbb{R}, x.y + z = 0$

[4 marks]

(d) Given the following truth table for a four-input logic circuit:

x	y	z	F(x, y, x)
0	o	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

i. Write the sum-of-products expression for this truth table.

[2 marks]

ii. Draw the circuit for the boolean expression $f(x, y, z)$.

iii. Translate the truth table into its corresponding Karnaugh map.

[2 marks]

iv. Use the Karnaugh map in (iii) to minimise the sum-of-products expression in (i). [2 marks]

v. Use the results in (iv) and give a more simplified circuit for $f(x, y, z)$.

[2 marks]

(e) Let $f : (1, \infty) \to \mathbb{R}$ defined by $f(x) = \ln(x^2 - 1)$. Determine if f is injective, surjective, or bijective. Explain your answer. [6 marks]

(f) Let A and B be two sets and let $f : A \to B$ be a function. Prove that if f is injective then for every $A_1, A_2 \subseteq A$, $f(A_1 \cap A_2) = f(A_1) \cap f(A_2)$.

[6 marks]

Question 4

(a) Define an Eulerian path and an Eulerian circuit. [4 marks]

(b) Either draw a graph with the following specified properties, or explain why no such graph exists:

i. A 3-regular graph with five vertices.

[2 marks]

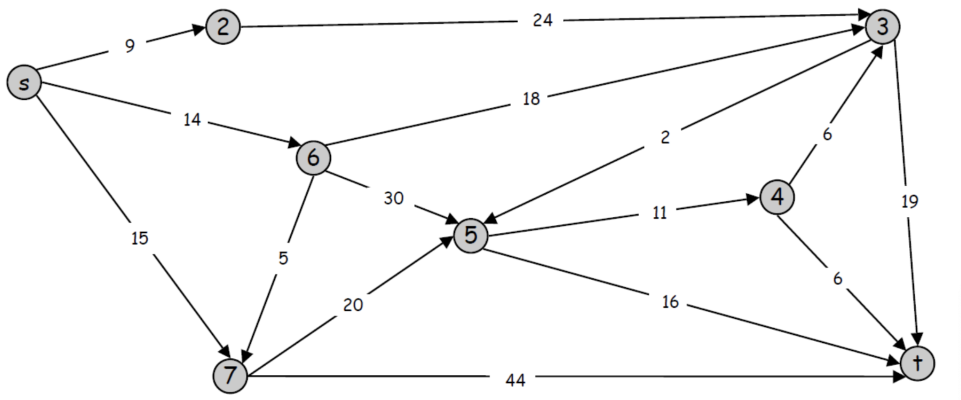
ii. A simple graph with five vertices with degrees 2, 2, 3, 4, and 5.

[3 marks]

iii. A simple graph in which each vertex has degree 3 and which has exactly six vertices.

[3 marks]

(c) Given the following weighted directed graph:



Use Dijkstra's algorithm and find the shortest path from s to t. [4 marks]

(d) Given S is the set of integers $\{2, 3, 4, 5, 6, 7, 8\}$. Let $\mathcal{R}$ be a relation defined on S by the following condition such that,

$$\forall x, y \in S, xRy \iff x + y \mod 2 = 0$$

i. Draw the digraph of $\mathcal{R}$. [2 marks]

ii. Show that $\mathcal{R}$ is an equivalence relation. [6 marks]

iii. Find the equivalence classes for $\mathcal{R}$.

[2 marks]

(e) Let A and B. be two finite sets, with $|A| = m$. and $|B| = n$.

How many distinct functions (mappings), $f : A \to B$, can you define from set A to set B?

[4 marks]

END OF PAPER

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