1. What are quantifiers?

In **discrete mathematics** and **logic**, *quantifiers* are symbols or words that specify the quantity of specimens in a domain that satisfy a given condition.

They let us make statements about "all" objects or "some" objects without having to list them one by one.

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2. Two main types of quantifiers

1. **Universal Quantifier (∀)**
   - Symbol: ∀
   - Meaning: "For all" or "For every"
   - Example:
     $\forall x \in R,\ x2 \geq 0$∀x \in \mathbb{R}, \; x^2 \geq 0
     → "For all real numbers xx, x2x^2 is nonnegative."
2. **Existential Quantifier (∃)**
   - Symbol: ∃
   - Meaning: "There exists" or "For some"
   - Example:
     $\exists x \in Z,\ x2 = 9$∃x \in \mathbb{Z}, \; x^2 = 9
     → "There exists an integer xx such that x2=9x^2 = 9."
     (True because x=3x = 3 or x=−3x = -3.)

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3. Negation of quantifiers

Negating quantifiers is crucial in proofs (especially with **De Morgan's laws for quantifiers**):

- Negation of universal:
  $\neg(\forall x\, P(x)) \equiv \exists x\, (\neg P(x))$\neg (∀x \, P(x)) \equiv ∃x \, (\neg P(x))
  → "Not all are P" means "Some are not P."
- Negation of existential:
  $\neg(\exists x\, P(x)) \equiv \forall x\, (\neg P(x))$\neg (∃x \, P(x)) \equiv ∀x \, (\neg P(x))
  → "There does not exist an x with P(x)" means "All x do not have P(x)."

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4. Multiple quantifiers

Order matters when quantifiers are nested:

- Example 1:
  ∀x∃y(x+y=0)∀x ∃y \, (x+y=0)
  → "For every x, there exists a y such that x+y=0x+y=0."
  (True: pick y=−xy=-x.)

- Example 2:
  ∃y∀x(x+y=0)∃y ∀x \, (x+y=0)
  → "There exists a single y such that for all x, x+y=0x+y=0."
  (False: no single y works for all x.)

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5. Common phrases in words

- ∀x P(x): "For every x, P(x) is true."
- ∃x P(x): "There exists an x such that P(x) is true."
- ∃!x P(x): "There exists exactly one x such that P(x) is true."