CM1020

BSc EXAMINATION

COMPUTER SCIENCE

Discrete Mathematics

Release date: Thursday 28 September 2023 at 12:00 midday British Summer Time

Close date: Friday 29 September 2023 by 12:00 midday British Summer Time

Time allowed: 4 hours to submit.

INSTRUCTIONS TO CANDIDATES:

Part A of this assessment consists of a set of **TEN** Multiple Choice Questions (MCQs). You should attempt to answer **ALL** the questions in **Part A.** The maximum mark for Part A is **40**.

Candidates must answer **TWO** out of the **THREE** questions in **Part B**. The maximum mark for Part B is **60**.

Part A and Part B will be completed online together on the Inspera exam platform. You may choose to access either part first upon entering the test area but must complete both parts within **4 hours** of doing so.

Calculators are **not** permitted in this examination. Credit will only be given if all workings are shown.

Do not write your name anywhere in your answers.

PART A

Candidates should answer the **TEN** Multiple Choice Questions (MCQs) quiz, **Question 1** in Part A of the test area.

PART B

Candidates should answer any **TWO** questions from Part B.

Question 1

(a) List the elements of the following sets:

i. $\{x|x \in \mathbb{Z} \land (x^2 = -x)\}$
ii. $\{x|x \in \mathbb{N} \land (x \mod 3 = 1) \land (x < 12)\}$

[2 marks]

(b) Let A and B be two sets such $|A| = |B| = n$ and $|A \cap B| = 2$. Find the following:

i. $|A \cup B|$
ii. $|\mathcal{P}(A \cup B)|$

where n is a positive integer and $\mathcal{P}(S)$ represents the power set of a set S.

[4 marks]

(c) Suppose A, B, and C are sets. Explain whether the following are true or false:

i. $(A - B) \cap C = (C - B) \cap A$
ii. $(A - B) \cup C = (C - B) \cup A$
iii. $(A - C) \cap (C - B) = \emptyset$

[6 marks]

(d) Let p, q and r be three propositions. Answer the following questions: [3 marks]

i. Construct a truth table for the compound proposition $(p \land \neg q) \rightarrow (r \lor q)$.
ii. Is this compound proposition a tautology? explain your answer.

[5 marks]

(e) The universe of discourse consists of all positive intergers, $\mathbb{Z}^+$. What are the truth values for each of the following:

i. $\exists x \exists y (x + y = 0) \lor (x * y = 0)$
ii. $\forall x \forall y (x * y \geq x + y)$

[2 marks]

(f) Consider the statements: $\forall a \in \mathbb{Z}$ and $b \in \mathbb{N}$, $\exists c \in \mathbb{N}$ such that $ac > ab$.

i. Write down the negation of the statement.

ii. Is the original statement true or false? Explain your answer.

[4 marks]

(g) Let A, B and C be three sets. Prove by contradiction that if $A \cap B \subseteq C$ and $x \in B$, then $x \notin A - C$. [4 marks]

Question 2

(a) Consider selecting 4 objects from the set $A = \{1, 2, 3, 4, 5, 6, 9, 10, 12, 14, 15\}$.

i. How many ordered sequences without repetition can be chosen from A ?

ii. How many ordered sequences with repetition can be chosen from A ?

iii. How many unordered sequences without repetition can be chosen from A ?

iv. How many unordered sequences with repetition can be chosen from A ?

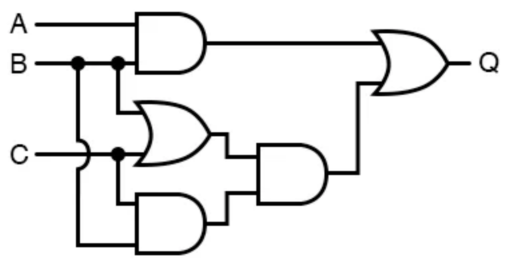
[8 marks]

(b) Minimise the following logic function using the Karnaugh maps method:

$$f(a,b,c) = a'b + bc' + bc + ab'c'$$

[4 marks]

(c) Given the following logical circuit with three inputs A, B and C:



i. Use the boolean algebra notation and write down the boolean expression of the output, Q of this circuit. [4 marks]

ii. Simplify the logical expression in (i). Explain your answer. [5 marks]

(d) Let f be a function $\mathbb{R} - \{-5\} \to \mathbb{R} - \{1\}$ with $f(x) = \frac{x}{x+5}$.

i. Show that f is a bijective function [2 marks]

ii. Find the inverse function f^{-1} [2 marks]

iii. Plot the curves of both function f and f^{-1} on the same graph. [2 marks]

iv. Suppose we change the co-domain of the function f to be $\mathbb{R}$:

$$f : \mathbb{R} - \{-5\} \to \mathbb{R}$$

Is f still a bijective function? Explain your answer. [3 marks]

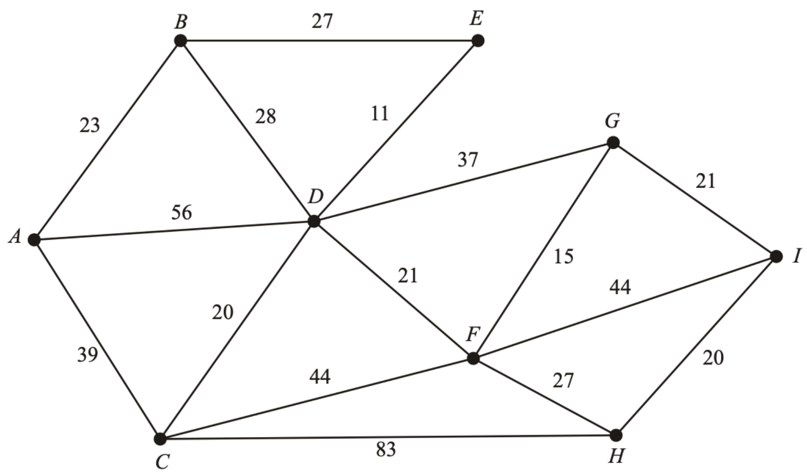
Question 3

(a) Explain the difference between an Euler path and an Euler cycle. [2 marks]

(b) Find the maximum number of comparisons to be made to find any record in a binary search tree which holds 3000 records. [3 marks]

(c) i. Explain what is meant by the term ‘path’.

ii. The figure shows a network of cycle tracks. The number on each edge represents the length, in miles, of that track. Jay wishes to cycle from A to I as part of a cycling holiday. She wishes to minimise the distance she travels.



Use Dijkstra’s algorithm to find the shortest path from A to I. Show all necessary working.

[6 marks]

(d) Let $A = \{a, b, c, d\}$ and let $\mathcal{R} = \{(a, b), (b, c), (c, d), (d, b)\}$ be a relation on A.

i. Draw the digraph representing $\mathcal{R}$ [2 marks]

ii. is $\mathcal{R}$ symmetric, anti-symmetric or transitive? Explain your answer [3 marks]

iii. Determine the transitive closure $\mathcal{R}^*$ of $\mathcal{R}$ [2 marks]

iv. Determine a matrix $M_{\mathcal{R}^*}$ representing $\mathcal{R}$ [2 marks]

(e) Let $S(n) = \sum_{i=1}^{n} 2^{i-1}$ with n is positive integer.

i. Evaluate $S(2)$

ii. Prove by induction that $S(n) = 2^n - 1$, for all integer $n \geq 1$

[6 marks]

(f) Simplify the following using the the laws of boolean algebra:

$$(x.x.x.y.y + \overline{x}.y.y)\overline{(x.x + x.\overline{y}.\overline{y}.\overline{y})}$$

Show all working. [4 marks]

END OF PAPER

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