Propositional Equivalences

- Formula A and B are equivalent if they have the same truth tables
- We denote the equivalence by $\equiv$
- $A \equiv B$ means that A and B always have the same truth values, regardless of how the variables are assigned.
- Note that $\equiv$ is NOT a connective

De Morgan's Laws

1. $\neg(p \wedge q) \equiv \neg p \vee \neg q$

2. $\neg(p \vee q) \equiv \neg p \wedge \neg q$

Build the truth tables for these formulae!

Example: negate the following

- It is Wednesday and it is not sunny
- NOT (It is Wednesday and it is not sunny)
- NOT (It is Wednesday) or NOT (it is not sunny)
- It is NOT Wednesday or it is sunny

Truth table for $\neg(p \wedge q) \equiv \neg p \vee \neg q$

p	q	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
1	1	0	0	0	0
1	0	1	0	1	1
0	1	1	1	0	1
0	0	1	1	1	1

Important equivalence

- $(p \rightarrow q) \equiv (\neg p \vee q)$
- Can you write the right hand side using conjunction?
- Use De Morgan's Law
- $(\neg p \vee q) \equiv \neg(\neg\neg p \wedge \neg q) \equiv \neg(p \wedge \neg q)$

- So $(p \rightarrow q) \equiv \neg(p \wedge \neg q)$

Another Equivalence: Contrapositive

- $p \rightarrow q \equiv \neg q \rightarrow \neg p$
- Why is it true?
- $p \rightarrow q \equiv \neg p \vee q$
- $\neg p \vee q \equiv q \vee \neg p \equiv \neg q \rightarrow \neg p$

$$a \rightarrow b \equiv \neg a \vee b$$

Equivalency with conjunction and negation

- **Is it possible to convert all operators to conjunction and negation?**
- **For example, $p \vee q$ based on the De Morgan's Law, can be written as $\neg(\neg p \wedge \neg q)$**
- **Or, $p \rightarrow q$, can be written as its equivalent $\neg p \vee q$, and based on the De Morgan's Law, can be written as $\neg(p \wedge \neg q)$**
- **Since we can write the equivalence of disjunction, using negation and conjunction, it means we can rewrite all logical formulae using these two operators**

Rewrite $p \vee (q \rightarrow r)$ only using conjunction and negation

- $p \vee (q \rightarrow r) \equiv$
- $\neg\big(\neg p \wedge \neg(q \rightarrow r)\big) \equiv$
- $\neg\big(\neg p \wedge \neg(\neg q \vee r)\big) \equiv$
- $\neg\big(\neg p \wedge (q \wedge \neg r)\big)$

- **We can also convert each conjunction to its equivalent using disjunction and negation!**

Example

- **Without using the truth table, show that the following statement is true:** $\neg(\exists x[P(x) \wedge Q(x)]) \equiv \forall x[P(x) \rightarrow \neg Q(x)]$
- $\neg(\exists x[P(x) \wedge Q(x)]) \equiv$
- $\forall x \neg[P(x) \wedge Q(x)] \equiv$
- $\forall x[\neg P(x) \vee \neg Q(x)] \equiv$
- $\forall x[P(x) \rightarrow \neg Q(x)]$
- $\neg(\exists x[P(x) \wedge Q(x)]) \equiv \forall x[P(x) \rightarrow \neg Q(x)]$