Midterm

`Question 1 (A) i`

Step 1: $X \cap Y \cap Z$

- $X \cap Y = \{4,6,8,10\}$
- $(X \cap Y) \cap Z = \{ 10 \}$

The Answer is: $X \cap Y \cap Z = \{10\}$

Step 2: $Y \setminus Z$ (elements inY but not in Z)

- $Y = \{4,6,8,10,12\}$
- $Z = \{2,10,12,14,16\}$

So:

$Y \setminus Z=\{4,6,8\}$

Now: $X \cup (Y \setminus Z)$

- $X = \{2,4,6,8,10\}$
- $Y \setminus Z=\{4,6,8\}$

So the union is:

$X \cup (Y \setminus Z) = \{2,4,6,8,10\}$

`Question 1 (A) ii`

- $X = \{2,4,6,8,10\}$
- $X \cap Y \cap Z = \{10\}$

Clearly, $X \not\subseteq \{10\}$

The anser is No, $X \not\subseteq X \cap Y \cap Z$ because elements like 2, 4, 6, and 8 are in X but not in the intersection.

`Question 1 (A) iii`

Step 1: $Y \cap Z = \{10,12\}$ Step 2: $X \cup (Y \cap Z) = \{2,4,6,8,10,12\}$

This set has 6 elements.

So:$| P (X \cup (Y \cap Z))|$= 2^6 =64

`Question 1 (B)`

The only sets A satisfying $P (A) \subseteq \{\varnothing,\{\varnothing\},\{\{\varnothing\}\}\}$ are:

- $A=\varnothing$,
- $A=\{\varnothing\}$
- $A=\{\{\varnothing\}\}$

`Question 1 (C)`

- (⇒) If $A \subseteq B$, then any element in $A \cap C$ is also in $B \cap C$. So the inclusion holds.
- (⇐) If $A \cap C \subseteq B \cap C$ for all C, choose C = A. Then: $A \cap A = A \subseteq B \cap A \subseteq B$ $A \subseteq B$ must hold.

The statement is correct

`Question 1 (D)`

$A \subseteq B$ and $C \subseteq B$ only mean both sets are inside B, but they can still share elements.

Being subsets of the same set does not imply disjointness.

`Question 2 (A) i`

f(x) is not defined for all $x \in R$

So f is not a function from R to R

`Question 2 (A) ii`

Well defined for all integers, one output per input

Yes, this is a function from Z to Z

`Question 2 (A) iii`

f(x) is not defined for negative values of x

So f is not a function from R to R

`Question 2 (B)`

- Compute both sides

(f o g)(x) = f(g(x)) = f $(\sqrt{x} + 3)^2$ + b = c + $\sqrt{x}$ + 9 + b = (g o f) (x) + g (f(x)) = g(${x}^2$ + b) = $\sqrt{x^2 + b + 3}$

- Set them equal

x + 6 $\sqrt{x}$ + 9 + b = $\sqrt{x^2 + b + 3}$

- There is no value of b for which (f o g)(x) = (g o f)(x)

`Question 2 (C)`

- 2 $\log_4$(x) = $\log_4$(x^2) => $\log_4$(x^2) - $\log_4$(3x-2) = 0 => $\log_4$ ($\frac{x^2}{3x-2}$) = 0

- $\frac{x^2}{3x-2}$ = 1 => x^2 = 3x + 2 = 0 => (x-1)(x-2) = 0

- x = 1 or x = 2

`Qestion 2 (D) i`

f'(x) = $\frac{d}{dx}$ (e^x + x) = e^x + 1 > 0

ffor all $x \in R^*$

Yes, f is one to one

Qestion 2 (D) ii

f(x) never reaches values less than some bound

So no, f is not onto.

Qestion 2 (E)

Let a_1, a_1 $\in$ A and suppose:

(g o f)(a_1) = (g o f)(a_2) => g(f(a_1)) = g(f(a_2))

Since g is one-to one:

f(a_1) = f(a_2)

And since f is also one-to-one:

a_1 = a_2

This proves that g o f maps distinct inputs to disctinct outputs

Question 3 (A) i

p	**q**	**r**	**$p \oplus q$**	**$(p \oplus q) \rightarrow r$**	**$p \vee q$**	**$p \wedge r$**	**$(p \vee q) \rightarrow (p \wedge r)$**
F	F	F	F	T	F	F	T
F	F	T	F	T	F	F	T
F	T	F	T	F	T	F	F
F	T	T	T	T	T	F	F
T	F	F	T	F	T	F	F
T	F	T	T	T	T	T	T
T	T	F	F	T	T	F	F
T	T	T	F	T	T	T	T

Question 3 (A) ii

- $(p \oplus q)$ -> r: not a tautology
- (p V q) -> $(p \wedge r)$: not a tautology

Question 3 (B)

We are given:

- p = False
- q = True
- r = False
- s = True

The truth value needs to be evaluated

$((p \vee \neg r) \wedge (q \rightarrow s)) \leftrightarrow ((\neg p \wedge q) \vee (r \rightarrow s))$

- First the left side

$(p \vee \neg r) = (F \vee \neg F) = (F \vee T) = T$

$(q \rightarrow s) = (T \rightarrow T) = T$

$T \wedge T = T$

- Right Side

$(\neg p \wedge q) = (T \wedge T) = T$

$(r \rightarrow s) = (F \rightarrow T) = T$

$T \vee T = T$

The conclusion is the compound proposition is true

`Question 3 (C) i`

$r \rightarrow (p \wedge q)$

`Question 3 (C) ii`

$r \rightarrow (p \oplus q)$

`Question 3 (C) iii`

$r \leftrightarrow (p \wedge q)$

`Question 3 (D)`

- Conrapositive

∀ x ∈ R, if 1 ≤ x ≤ 2 then x^2 − 3 x + 2 ≤ 0

- Converse

∀ x ∈ R, if x > 2 or x < 1 then x^2 − 3x + 2 > 0

- Inverse

∀ x ∈ R, if x^2 − 3x + 2 ≤ 0 then 1 ≤ x ≤ 2

`Question 3 (E)`

Left Side

$(p \wedge q) \vee (r \rightarrow s) \equiv (p \wedge q) \vee (\neg r \vee s)$

Right Side

$((p \vee r) \rightarrow s) \wedge ((q \vee r) \rightarrow s)$

The conclusion is the expression is not a tautology.

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Question 4 (A) i
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$\forall x (D(x) \rightarrow M(x))$

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Question 4 (A) ii
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$\forall x (P(x) \wedge W(x) \rightarrow H(x))$

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Question 4 (A) iii
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$\exists x \forall y(x \neq y \rightarrow L(x,y))$

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Question 4 (A) iv
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$\forall x ((A(x) \wedge W(x)) \rightarrow F(x))$

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Question 4 (B) i
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Try x = 5 -> take y = 0.1:$5 \cdot 0.1 = 0.5 < 1$

Try x = -3 -> take y = -0.1:$-3 \cdot 0.1 = 0.3 < 1$

Try x = 0 -> take y = 1 => $0 \cdot 1 = 0 < 1$

we can always choose small enough $xy < 1$

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Question 4 (B) ii
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There is no real $x \neq 0$, and integer y such tha $xy \leq 1$

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Question 4 (B) iii
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For any $x \neq 0$, let y = $\frac{2}{x}$ $\in$ R -> xy = x * $\frac{2}{x}$ = 2

So , it true such a y always exists

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Question 4 (C)
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Using logical identity:

$\neg (P \vee Q) \equiv \neg P \wedge \neg Q$

So:

$\neg[\forall x \exists y(M(x) \wedge N(y)) \vee \forall z(K(z) \rightarrow L(z))] \Rightarrow \neg\forall x\exists y(M(x) \wedge N(y)) \wedge \neg\forall z(K(z) \rightarrow L(z))$

$\exists x \forall y\neg(M(x) \wedge N(y)) \wedge \exists z(K(z) \wedge \neg L(z))$

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Question 5 (A) i
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- Simplify:

$(p \cdot q \cdot r)+(r \cdot s)$

- Negate:

$\neg[(p \cdot q \cdot r)+(r \cdot s)] = \neg(p \cdot q \cdot r) \cdot \neg(r \cdot s)$

- Now apply again

$= (\neg p+\neg q+\neg r) \cdot (\neg r+\neg s)$

`Question 5 (A) ii`

$(x+y) \cdot (x+y) \cdot (y+z)=(x+y) \cdot (y+z)$

`Question 5 (C)`

$(a+b)(c+d)=a \cdot c+a \cdot d+b \cdot c+b \cdot d$

`Question 5 (D) ii`

	D=0	**D=1**
C=0	1	1
C=1	1	1

`Question 5 (D) iii`

$F(A,B,C,D) = A \cdot B$