

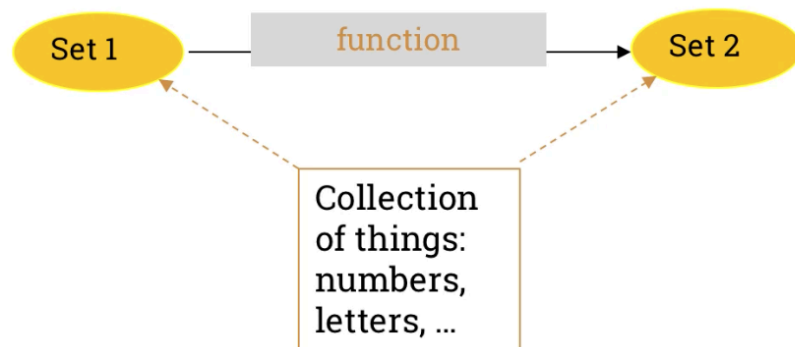
# Introduction

A function is a relation between a set of inputs and a set of outputs so that each input maps to exactly one output.

The concept of a function is central to computer programming.

Most of what a programmer writes consists of 'functions' that do parts of the work of the program.

## Definition of a function



A **function** is a well-behaved relation, that is, given a starting point we know exactly where to go.

# Definition of a function

**Definition:** A function  $f$  from a set  $A$  to a set  $B$  is an assignment of exactly one element of  $B$  to each element of  $A$ .

If  $f$  is a function from  $A$  to  $B$ , we write:

$$f: A \rightarrow B$$

This can be read as  $f$  maps  $A$  to  $B$ .

$$x \in A: x \rightarrow f(x) = y \quad (y \in B)$$

## Terminology

Given a function  $f: A \rightarrow B$

$$x \in A \rightarrow f(x) = y \in B$$

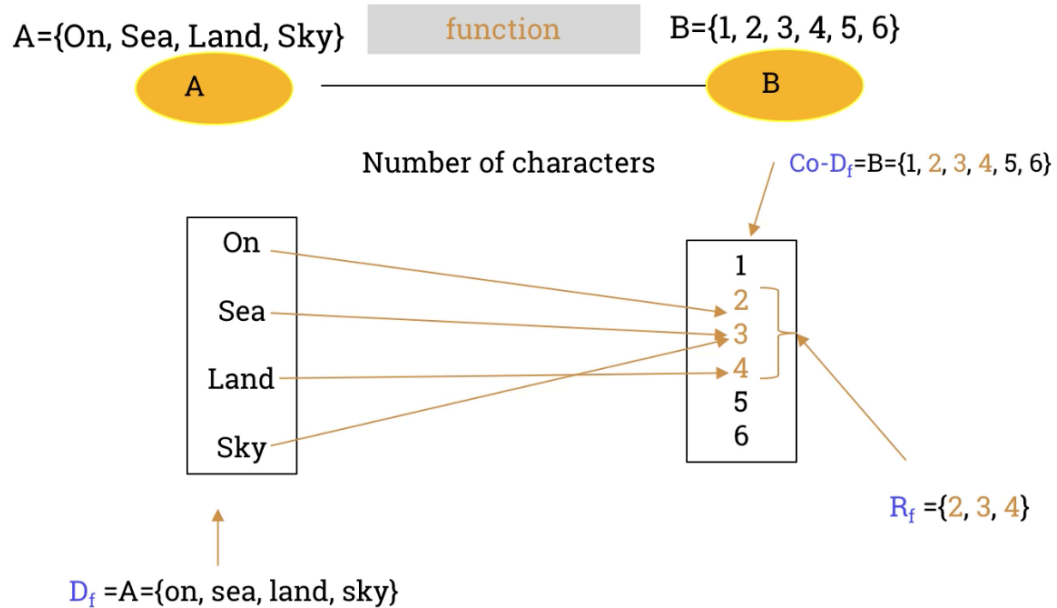
$A$  is the set of inputs and is called the domain of  $f$ . We write  $D_f = A$ .

$B$  is the set containing the outputs and is called the co-domain of  $f$ . We write  $\text{co-}D_f = B$ .

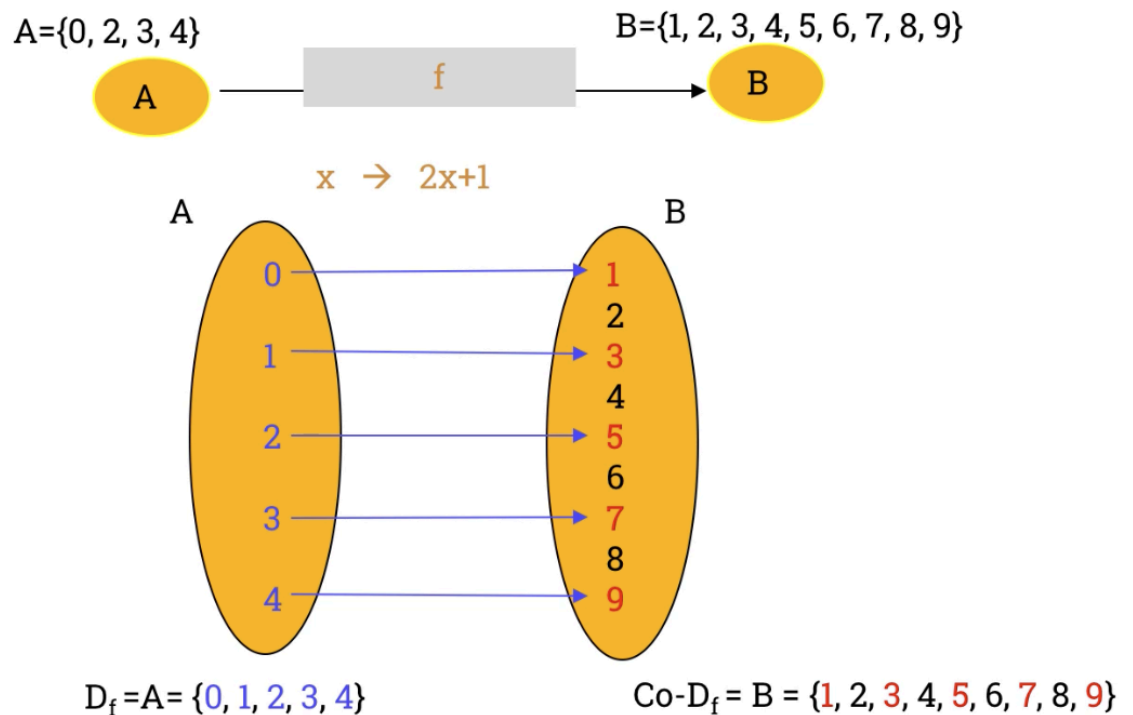
The set of all outputs is called the range of  $f$  and it is written as  $R_f$ .

$y$  is called the image of  $x$ , whereas  $x$  is called the pre-image of  $y$ . We write  $f(x) = y$ .

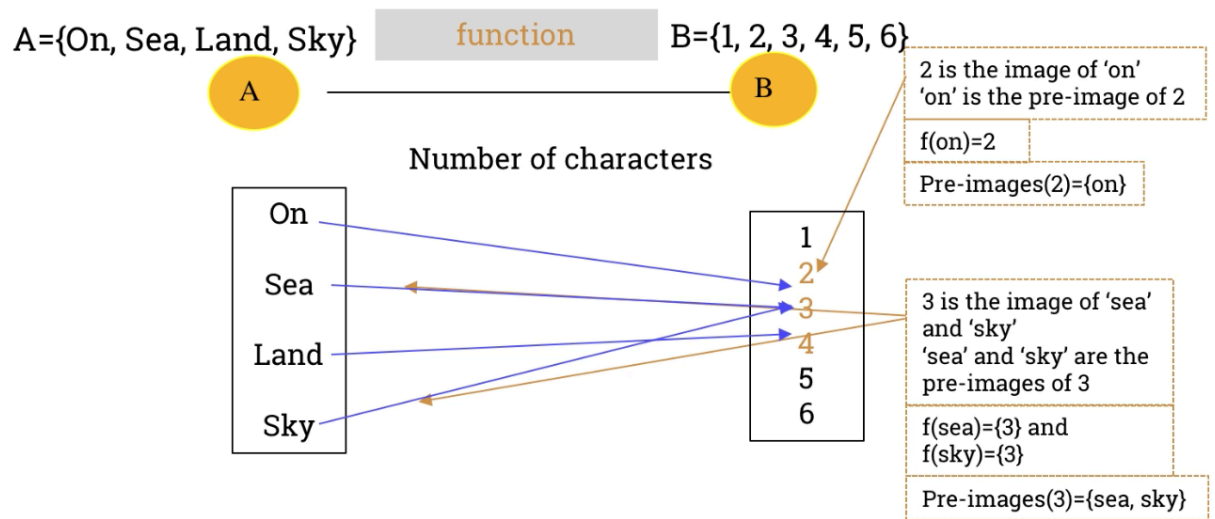
# Domain, co-domain and range



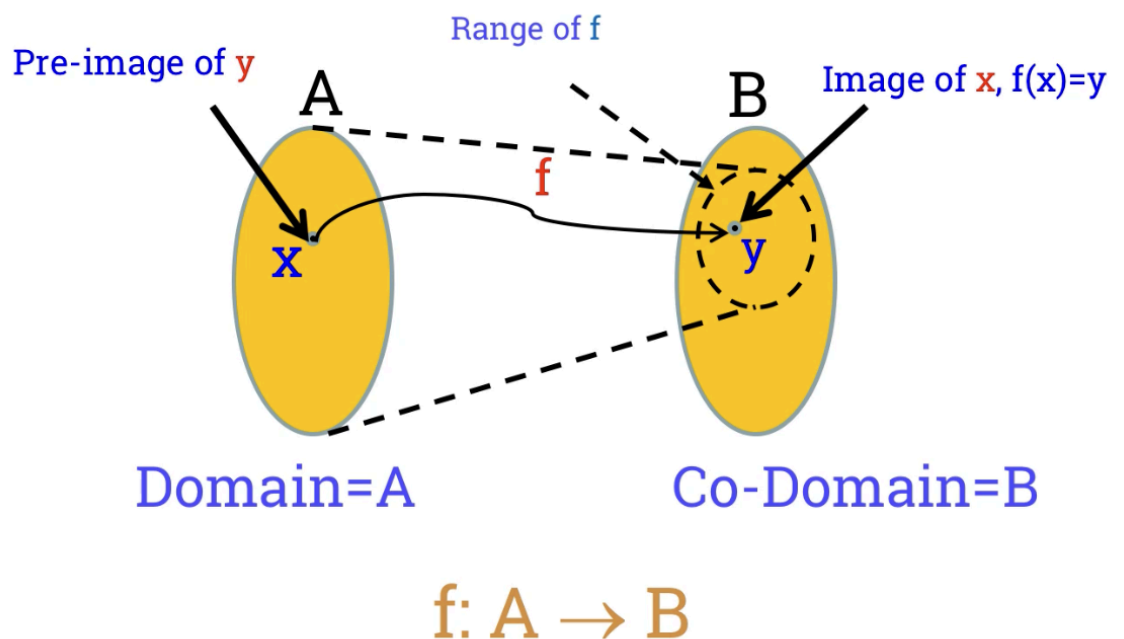
## Example $f: A \rightarrow B$



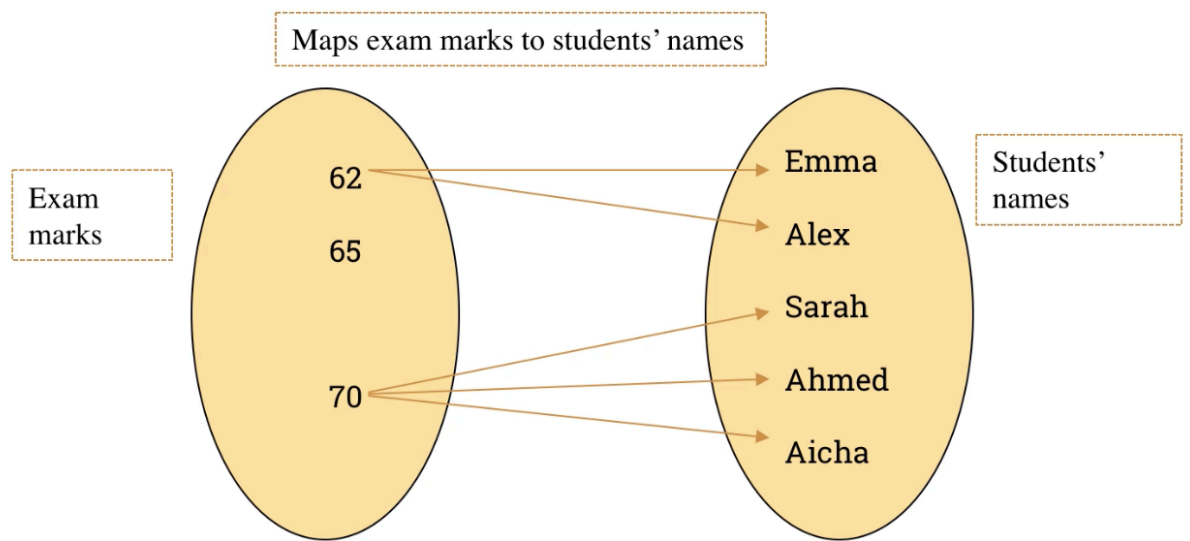
# Images and pre-images



## Visualization



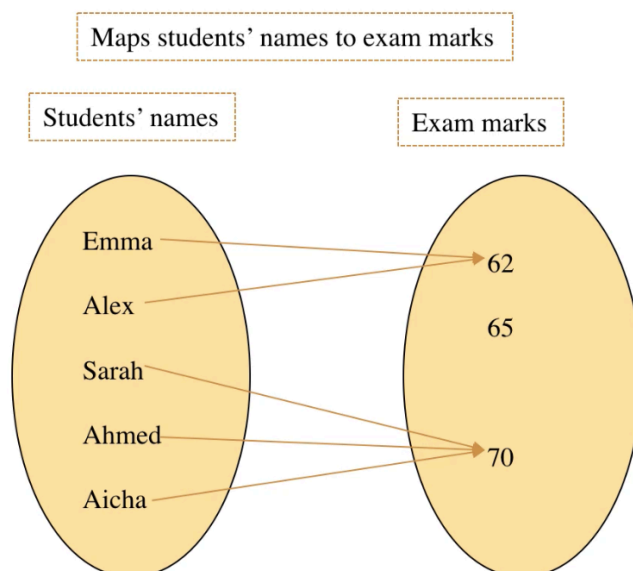
# Example: this relation is not a function



This relation is **not a function** for 2 reasons:

1. 65 has no image
2. 62 and 70 has more than one image.

# Example: this relation is a function



This relation is **a function** as each student's name is mapped to one mark.

# Exercise 1

Given the following function:

$$f: \mathbb{Z} \rightarrow \mathbb{Z} \text{ with } f(x) = |x|$$

Find the domain, co-domain and range of the function  $f$ .

Find the set of pre-images(1).

## Solution 1

$$f: \mathbb{Z} \rightarrow \mathbb{Z} \text{ with } f(x) = |x|$$

$$D_f = \mathbb{Z}$$

$$\text{Co-}D_f = \mathbb{Z}$$

$$R_f = \mathbb{Z}^+ \cup \{0\} = [0, +\infty[$$

$$f(-1) = f(1) = 1 \text{ hence, Pre-images of } 1 = \{-1, 1\}.$$

# Exercise 2

Given the following function:

$$g: \mathbb{R} \rightarrow \mathbb{R} \text{ with } g(x) = x^2 + 1$$

Find the domain, co-domain and range of the function  $g$ .

Find the set of pre-images(5).

## Solution 2

$$g: \mathbb{R} \rightarrow \mathbb{R} \text{ with } g(x) = x^2 + 1$$

$$\begin{aligned} D_g &= \mathbb{R} \\ \text{Co-}D_g &= \mathbb{R} \end{aligned}$$

$$R_g = [1, +\infty[$$

$$g(-2) = g(2) = 5 \text{ hence, pre-images of } 5 = \{-2, 2\}$$

# Summary

- Definition of a function
- Domain, co-domain and range of a function
- Image, pre-image (antecedent) of an element