UNIVERSITY OF LONDON

CM1020

BSc EXAMINATION

COMPUTER SCIENCE

Discrete Mathematics

Release date: Monday 10 March 2025 at 12:00 midday Greenwich Mean Time

Close date: Tuesday 11 March 2025 by 12:00 midday Greenwich Mean Time

Time allowed: 4 hours to submit

INSTRUCTIONS TO CANDIDATES:

Part A of this assessment consists of a set of **TEN** Multiple Choice Questions (MCQs). You should attempt to answer **ALL** the questions in **Part A.** The maximum mark for Part A is **40**.

Candidates must answer **TWO** out of the **THREE** questions in **Part B**. The maximum mark for Part B is **60**.

Part A and Part B will be completed online together on the Inspera exam platform. You may choose to access either part first upon entering the test area but must complete both parts within **4 hours** of doing so.

A handheld non-programmable calculator may be used when answering questions on this paper, but it must not be able to display graphics, text, or algebraic equations. Please hold your calculator to the camera at the start of the examination to clearly show the make and type.

You may use **ONE** A4 page of previously prepared notes in this examination. Please hold up your notes to the camera at the start of the examination.

You will need to submit your answers by file upload.

Write your answers clearly and make sure that each answer is numbered and is separated from the next one by leaving an empty space or by drawing a horizontal line after the end of each answer.

Do not write your name anywhere in your answers.

PART A

Question 1

Candidates should answer the **TEN** Multiple Choice Questions (MCQs) in Part A.

PART B

Candidates should answer any **TWO** questions from Part B.

Question 2

(a) Define the set of all integers that are multiples of 3 and greater than 10 using set-builder notation.

[2 marks]

(b) Given two sets P and Q with $P = \{2, 4, 6, 8\}$ and $Q = \{4, 5, 6, 7\}$, list the symmetric difference of sets P and Q, $P \oplus Q$.

[3 marks]

(c) Given the universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and three sets $X = \{2, 4, 6\}$, $Y = \{4, 5, 6, 7\}$, and $Z = \{6, 7, 8, 9\}$:

i. Draw a Venn diagram representing X, Y and Z.

ii. Calculate $\overline{X} \cap \overline{Y} \cap \overline{Z}$.

iii. Illustrate $\overline{X} \cap \overline{Y} \cap \overline{Z}$ on Venn diagram.

[6 marks]

(d) Let A, B, and C be subsets of a universal set U. Prove the following statement:

If $A \subseteq B$ and $C \subseteq \overline{B}$, then $A \cap C = \emptyset$.

[4 marks]

(e) Prove whether each of the following set identities is correct or not:

i. $(A \cap B) \cup C = A \cap (B \cup C)$

ii. $(A \cup C) - B = (A - B) \cup (C - B)$

[6 marks]

(f) Let p and q be two propositions. Show that $[p \wedge (p \rightarrow q)] \rightarrow q$ is a tautology. Explain your answer. [5 marks]

(g) What are the truth values for each of the following:

i. $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}\,(y^2 > x)$

ii. $\exists! x \in \mathbb{R}, (x^3 = 8)$

iii. $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}\,(x + y = x)$

iv. $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, (x = 2y + 1)$

[4 marks]

Question 3

(a) What is the difference between the scope of a quantifier and the binding of a variable? Provide examples to illustrate your explanation.

[6 marks]

(b) Write the negation of each of the following quantified statements:

i. $\forall x \exists y P(x, y)$

ii. $\exists x \forall y P(x, y)$

iii. $\forall x(P(x) \to Q(x))$

iv. $\exists x \in \mathbb{Z}\ \forall y \in \mathbb{Z}(x > y)$

[4 marks]

(c) Given the following truth table for a three-input logic circuit:

x	y	z	$f(x, y, z)$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

i. Write the canonical sum-of-products (SOP) expression for the function $f(x, y, z)$. [2 marks]

ii. Draw the circuit for the Boolean expression obtained in (i).

[2 marks]

iii. Construct the corresponding Karnaugh map for $f(x, y, z)$.

[2 marks]

iv. Use the Karnaugh map to simplify the Boolean expression to its minimized form.

[2 marks]

v. Draw the simplified circuit based on the minimized Boolean expression.

[2 marks]

(d) The function f is given by:

$f(x) = \ln(4x - 2)$ where $x \in \mathbb{R}$ and $x > \frac{1}{2}$

i. Find an expression for $f^{-1}(x)$ in its simplest form. [2 marks]

ii. State the range of $f^{-1}(x)$. [2 marks]

iii. Solve the equation $f(x) = 1$.

[2 marks]

(e) How many binary sequences of length 10 start with a 10 and end with a 01?

[4 marks]

Question 4

(a) Define an Eulerian path and an Eulerian circuit. [4 marks]

(b) Draw a graph with each of the following specified properties, or explain why no such graph exists:

i. A 3-regular graph with five vertices.

[2 marks]

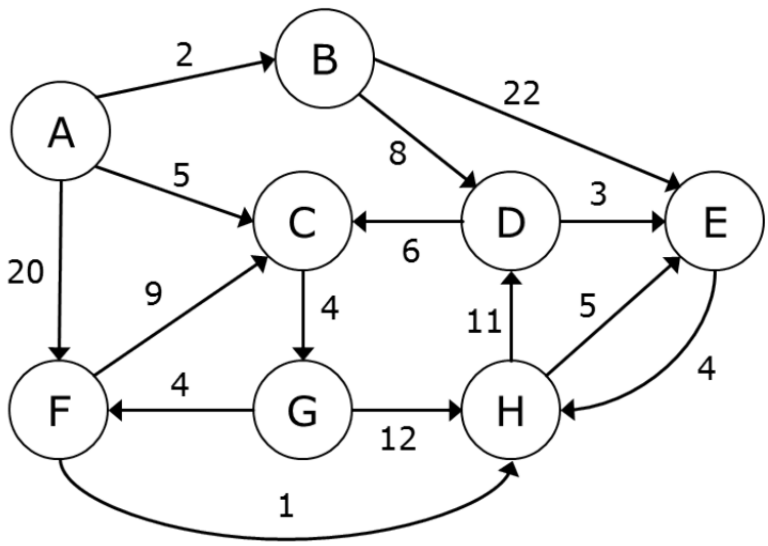
ii. A simple graph with five vertices having degrees 2, 2, 3, 4, and 5.

[3 marks]

iii. A simple graph with six vertices in which each vertex has degree 3.

[3 marks]

(c) Given the following weighted directed graph:



i. Use Dijkstra's algorithm to calculate the single-source shortest paths from vertex A to every other vertex. As the algorithm proceeds, cross out old values and write in new ones, from left to right in each cell. If during your algorithm two unvisited vertices have the same distance, use alphabetical order to determine which one is selected first. Show your steps in the table below:

$Vertex$	$Distance$	$Path$
A		
B		
C		
D		
E		
F		
G		
H		

[4 marks]

ii. What is the lowest-cost path from A to H in the graph, as computed above?

[2 marks]

(d) Let $\mathcal{R}$ be a relation defined on $\mathbb{R}$, the set of all real numbers, by the following condition such that,

$$\forall x, y \in \mathbb{R}, x\mathcal{R}y \iff x \leq y$$

Prove or disprove that $\mathcal{R}$ is reflexive, symmetric, anti-symmetric, and transitive.

[8 marks]

(e) Let A and B be two finite sets, with $|A| = m$. and $|B| = n$.

How many distinct functions (mappings), $f : A \to B$, can you define from set A to set B?

[4 marks]

END OF PAPER

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