

# CM1015 Computational Mathematics

## Course Description

This module gives you the mathematical foundations you need to learn how to think abstractly and introduces you to many of the standard mathematical tools and models necessary to understand and design computational systems and algorithms. By taking this module you will learn a wide range of the mathematical concepts and techniques that underpin Computer Science. In particular, you will study number systems, sequences and series, special functions, graphing, linear algebra and basic concept of combinatorics and probability theory.

Learners get to practice the above in multiple, weekly exercises and their knowledge is tested through engaging assignments and quizzes.

## Course Goals and Objectives

Upon successful completion of this course, you will be able to:

1. Transform numbers between number bases and perform arithmetic in number bases
2. Use trigonometric definitions and identities to solve triangles and trigonometric equations, and to compute with vectors
3. Describe, represent, analyse and discover relationships between quantities by using functions, graphs, limits and differentiation, with functions including trigonometric, exponential and logarithmic
4. Translate between geometric and algebraic representations of shapes and spaces, including points, lines, vectors, matrices and linear transformations
5. Use combinatorial techniques to describe, represent and count sample spaces and events, and calculate probabilities

## Textbook and Readings

Specific essential readings for each topic from the following list are included in the Readings page for each topic:

- **Foundation Maths** Croft, A. and R. Davison Foundation maths. (Harlow: Pearson, 2016) 6th edition
- **Number Theory for Computing** Song Y. Yan (2002): Springer
- **Precalculus with Limits** Larson, R. (2017): Cengage Learning
- **A Level Physics for OCR A: Year 1 and AS** Bone, G., G. Chadha, N. Saunders (2015): OUP Oxford - PDF available via Coursera
- **Mathematics for Computer Graphics** Vince, J. (2010): Springer
- **Linear Algebra: Step by Step** Singh, K. (2013): Oxford Press

## Course Outline

The course consists of ten topics divided into 20 weeks that focus on key concepts.

Topic 1: Number bases	<p><b>Key concepts:</b> Number bases, conversion and operations.</p> <p><b>Learning outcomes:</b></p> <ul style="list-style-type: none"> <li>• Represent numbers in different bases</li> <li>• Convert from one number base to another</li> <li>• Perform basic operations with binary numbers.</li> </ul>
Topic 2: Sequences and series	<p><b>Key concepts:</b> Sequences and series, recursion, arithmetic and geometric sequences.</p> <p><b>Learning outcomes:</b></p> <ul style="list-style-type: none"> <li>• Explain the notion of a number sequence and of convergence/divergence of a sequence</li> <li>• Use the recursion and the induction principle to define the elements of a sequence with application to arithmetic and geometric sequences</li> <li>• Introduce the notion of series and perform summation of geometric and arithmetic series</li> </ul>
Topic 3: Modular arithmetic	<p><b>Key concepts:</b> Modular arithmetic, congruent integers.</p> <p><b>Learning outcomes:</b></p> <ul style="list-style-type: none"> <li>• Define congruence modulo an integer</li> <li>• Classify two integers as congruent</li> <li>• Perform operations with congruent numbers</li> </ul>

<p>Topic 4: Angles, triangles and trigonometry</p>	<p><b>Key concepts:</b> Angles, triangles and trigonometric relations.</p> <p><b>Learning outcomes:</b></p> <ul style="list-style-type: none"> <li>• Work with angles and angle units, convert between degrees and radians</li> <li>• Derive and apply basic properties and trigonometric relations in triangles</li> <li>• Solve basic problems with triangle elements using sine and cosine rules</li> </ul>
<p>Topic 5: Graph sketching and kinematics</p>	<p><b>Key concepts:</b> Functions, Cartesian coordinates, graphs, kinematics.</p> <p><b>Learning outcomes:</b></p> <ul style="list-style-type: none"> <li>• Define a function, its domain, codomain and type</li> <li>• Locate points on the plane using Cartesian coordinates</li> <li>• Plot a function in Cartesian coordinates</li> <li>• Derive and plot speed and distance travelled for uniform and uniformly accelerated motion</li> </ul>
<p>Topic 6: Trigonometric functions</p>	<p><b>Key concepts:</b> Trigonometric functions.</p> <p><b>Learning outcomes:</b></p> <ul style="list-style-type: none"> <li>• Extend definition of Sin, Cos and Tan to any angle and derive their properties using the unit circumference in Cartesian coordinates</li> <li>• Define and use inverse of trigonometric functions (Arc functions)</li> <li>• Plot trigonometric functions in Cartesian Coordinates</li> </ul>

<p>Topic 7: Exponential and Logarithmic functions</p>	<p><b>Key concepts:</b> Exponential and logarithmic functions.</p> <p><b>Learning outcomes:</b></p> <ul style="list-style-type: none"> <li>• Define exponential function from extension of integer powers to powers of any real number and derive its basic properties</li> <li>• Define logarithm as inverse of exponential function and derive its basic properties</li> <li>• Plot exponential and logarithmic functions in the Cartesian plane</li> </ul>
<p>Topic 8: Limits and differentiation</p>	<p><b>Key concepts:</b> Limits and differentiation.</p> <p><b>Learning outcomes:</b></p> <ul style="list-style-type: none"> <li>• Define and calculate the limit of a function at a point</li> <li>• Use limits to identify asymptotic behavior of a function</li> <li>• Formulate and calculate the derivative of a function from first principles</li> <li>• Use differentiation to characterize the behavior of a function, to individuate local minima and maxima and turning points in its plot</li> </ul>
<p>Topic 9: Algebra, Vectors and Matrices</p>	<p><b>Key concepts:</b> Algebra, vector and matrices.</p> <p><b>Learning outcomes:</b></p> <ul style="list-style-type: none"> <li>• Define vector spaces and linear transformations</li> <li>• Represent linear transformations as matrices and define composition of transformations as product of matrices</li> <li>• Solve a matrix equation of the form <math>Mv=w</math> where <math>M</math> is a square matrix and <math>v</math> and <math>w</math> two column vectors</li> </ul>

<p>Topic 10: Combinatorics and Probabilities</p>	<p><b>Key concepts:</b> Combinatorics and Probability.</p> <p><b>Learning outcomes:</b></p> <ul style="list-style-type: none"> <li>• Define combinations and permutations and learn how to apply them to counting problems</li> <li>• Describe the sample space of a given experiment and compute the probability <math>P(x)</math> of an outcome <math>x</math> of the experiment</li> <li>• Compute the joint probability of two events and determine if they are dependent or independent</li> </ul>
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## Learning Activities of This Course

The course is comprised of the following elements:

- **Lecture videos.** In each week the concepts you need to know will be presented through a collection of short video lectures. You may stream these videos for playback within the browser by clicking on their titles or download the videos.
- **Practice Quizzes.** Topics include practice quizzes, intended for you to assess your understanding of the content. You will be allowed unlimited attempts at each practice quiz. There is no time limit on how long you take to complete each attempt at the quiz. These quizzes do not contribute toward your final score in the class.
- **Graded Assignments.** There are two graded assignments, each is worth 50% of the final module grade. Each of these assignments is comprised of multiple parts which learners work on during earlier weeks. All assignments will be graded by the project tutors.
- **Discussion Prompt.** Topics also include discussion prompts. You will see the discussion prompt alongside other items in the lesson. Each prompt provides a space for you to respond. After responding, you can see and comment on your peers' responses. All prompts and responses are also accessible from the general discussion forum and the topic discussion forum.
- **Readings.** Topics may include several suggested readings. They are good supplementary materials for you to further understand the course topics.

## How to Pass This Course

The module will contain a range of summative and formative assessments. Summative assessments are assessments which contribute directly towards your final grade. Formative assessments do not count directly towards your final grade. Instead, they provide you with opportunities for low stakes practice and will often provide some sort of feedback about your progress. For example, a practice quiz might provide you with feedback about why a particular answer was wrong.

The module has two assessments: a midterm assessment and a final exam. The midterm assessment comprises five quizzes based on the first five topics and a question sheet. The exam comprises a quiz section and a written section. The midterm and exam each carry 50% of the grade.

There are also several activities that are graded but have 0 weight. That means that they will not count towards your final grade, but they are a key part of your learning, and you need to do them.

This is a detailed breakdown of all the marks:

<b>Activity</b>	<b>Required? (Summative)</b>	<b>Deadline week</b>	<b>Estimated time per course</b>	<b>% of final grade</b>
End of topic quizzes for topics 1-5	Yes	Approximately weeks 1-10	1-2 hours	25%
Written, staff graded coursework	Yes	Approximately week 13	20 hours	25%
Written examination	Yes	Approximately week 22	3-4 hours	50%