

L'Aquila Osc. Paper, version 2

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1 General formula for 2×2

For a symmetric matrix M :

$$M = \begin{pmatrix} A & B \\ B & D \end{pmatrix} \quad (1)$$

We can diagonalize this Hamiltonian with a unitary matrix S :

$$S = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \quad (2)$$

we get:

$$\begin{aligned} H_D = S^T H S &= \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} A & B \\ B & D \end{pmatrix} \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} A \cos \theta - B \sin \theta & A \sin \theta + B \cos \theta \\ B \cos \theta - D \sin \theta & B \sin \theta + D \cos \theta \end{pmatrix} = \\ &= \begin{pmatrix} A \cos^2 \theta - 2B \sin \theta \cos \theta - D \sin^2 \theta & A \sin \theta \cos \theta + B(\cos^2 \theta - \sin^2 \theta) - D \sin \theta \cos \theta \\ A \sin \theta \cos \theta + B(\cos^2 \theta - \sin^2 \theta) - D \sin \theta \cos \theta & A \sin^2 \theta + 2B \sin \theta \cos \theta + D \cos^2 \theta \end{pmatrix} = \\ &= \begin{pmatrix} A \cos^2 \theta - B \sin(2\theta) - D \sin^2 \theta & (A - D) \frac{\sin(2\theta)}{2} + B \cos(2\theta) \\ (A - D) \frac{\sin(2\theta)}{2} + B \cos(2\theta) & A \sin^2 \theta + B \sin(2\theta) + D \cos^2 \theta \end{pmatrix} \end{aligned} \quad (3)$$

If we want H_D to be diagonal, its nondiagonal terms need to be 0, what constraints the angle θ :

$$(D - A) \frac{\sin(2\theta)}{2} + B \cos(2\theta) = 0 \quad (4)$$

$$\tan(2\theta) = \frac{2B}{D - A}$$

Then:

$$\cos(2\theta) = \sqrt{\frac{1}{1 + \tan^2(2\theta)}} = \frac{1}{1 + \frac{4B^2}{(D-A)^2}} = \frac{D - A}{\sqrt{(D - A)^2 + 4B^2}} \quad (5)$$

$$\sin(2\theta) = \sqrt{1 - \cos^2(2\theta)} = \frac{2B}{\sqrt{(D - A)^2 + 4B^2}}$$

Then we get diagonal matrix:

$$H_D^2 = \begin{pmatrix} A \frac{1+\cos(2\theta)}{2} - B \sin(2\theta) + D \frac{1-\cos(2\theta)}{2} & 0 \\ 0 & A \frac{1-\cos(2\theta)}{2} + B \sin(2\theta) + D \frac{1+\cos(2\theta)}{2} \end{pmatrix} \quad (6)$$

Then matrix H_D has the form:

$$\begin{aligned}
H_D &= \begin{pmatrix} A \left(\frac{1}{2} + \frac{D-A}{2\sqrt{(D-A)^2+4B^2}} \right) - \frac{2B^2}{\sqrt{(D-A)^2+4B^2}} + D \left(\frac{1}{2} - \frac{D-A}{2\sqrt{(D-A)^2+4B^2}} \right) & 0 \\ 0 & A \left(\frac{1}{2} - \frac{D-A}{2\sqrt{(D-A)^2+4B^2}} \right) + \frac{2B^2}{\sqrt{(D-A)^2+4B^2}} + D \left(\frac{1}{2} + \frac{D-A}{2\sqrt{(D-A)^2+4B^2}} \right) \end{pmatrix} \\
&= \begin{pmatrix} \frac{A+D}{2} + \frac{-(D-A)^2-4B^2}{2\sqrt{(D-A)^2+4B^2}} & 0 \\ 0 & \frac{A+D}{2} + \frac{(D-A)^2+4B^2}{2\sqrt{(D-A)^2+4B^2}} \end{pmatrix} = \begin{pmatrix} \frac{A+D}{2} - \sqrt{(D-A)^2+4B^2} & \frac{A+D}{2} + \sqrt{(D-A)^2+4B^2} \\ 0 & 0 \end{pmatrix}
\end{aligned} \tag{7}$$

2 2 Level system (precession)

For interaction Hamiltonian we have:

$$H = \begin{pmatrix} 2\omega_3 & 2\omega_1 \\ 2\omega_1 & -2\omega_3 \end{pmatrix} \tag{8}$$

where:

$$\begin{aligned}
\omega_1 &= \omega \sin \alpha \\
\omega_3 &= \omega \cos \alpha
\end{aligned} \tag{9}$$

We can diagonalize it with matrix:

$$S = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}, \tag{10}$$

where

$$\tan 2\theta = \frac{\omega_1}{\omega_3} \tag{11}$$

Then we have:

$$\begin{aligned}
\sin(2\theta) &= \frac{\omega_1}{\omega_1 + \omega_3} \\
\cos(2\theta) &= \frac{\omega_3}{\omega_1 + \omega_3}
\end{aligned} \tag{12}$$

If we diagonalize, we get:

$$H_D = S^T H S = \begin{pmatrix} 2\sqrt{\omega_1^2 + \omega_3^2} & 0 \\ 0 & -2\sqrt{\omega_1^2 + \omega_3^2} \end{pmatrix} \tag{13}$$

Matrix, which shows the osc. amplitudes is:

$$\begin{aligned}
S_{diag} &= S e^{-iHt} S^T = \\
&= \begin{pmatrix} e^{2it\sqrt{\omega_1^2 + \omega_3^2}} \sin^2(\theta) + e^{-2it\sqrt{\omega_1^2 + \omega_3^2}} \cos^2(\theta) & -i \sin(2\theta) \sin(2t\sqrt{\omega_1^2 + \omega_3^2}) \\ -i \sin(2\theta) \sin(2t\sqrt{\omega_1^2 + \omega_3^2}) & e^{2it\sqrt{\omega_1^2 + \omega_3^2}} \cos^2(\theta) + e^{-2it\sqrt{\omega_1^2 + \omega_3^2}} \sin^2(\theta) \end{pmatrix}
\end{aligned} \tag{14}$$

Then we can calculate transition probability from state 1 to state 2 and also wise versa, squaring the amplitude moduls. Better to show in matrix form (also we consider decay rate):

So, we have:

$$P = e^{-\Gamma t} \begin{pmatrix} 1 - \sin^2(2\theta) \sin^2(2t\sqrt{\omega_1^2 + \omega_3^2}) & \sin^2(2\theta) \sin^2(2t\sqrt{\omega_1^2 + \omega_3^2}) \\ \sin^2(2\theta) \sin^2(2t\sqrt{\omega_1^2 + \omega_3^2}) & 1 - \sin^2(2\theta) \sin^2(2t\sqrt{\omega_1^2 + \omega_3^2}) \end{pmatrix} \tag{15}$$

Simplifying:

$$P = e^{-\Gamma t} \begin{pmatrix} 1 - \sin^2(2\theta) \sin^2(2\omega t) & \sin^2(2\theta) \sin^2(2\omega t) \\ \sin^2(2\theta) \sin^2(2\omega t) & 1 - \sin^2(2\theta) \sin^2(2\omega t) \end{pmatrix} \tag{16}$$

Now let us define the osc. frequency ω as:

$$\Omega = 2\omega \quad (17)$$

And using (56)

$$\sin^2(2\theta) = \frac{\omega^2 \sin^2 \alpha}{\omega^2(\sin \alpha + \cos \alpha)^2} = \frac{\sin^2 \alpha}{(\sin \alpha + \cos \alpha)^2} \quad (18)$$

Then Osc. Probability has the form:

$$P = e^{-\Gamma t} \begin{pmatrix} 1 - \frac{\sin^2 \alpha}{(\sin \alpha + \cos \alpha)^2} \sin^2(\Omega t) & \frac{\sin^2 \alpha}{(\sin \alpha + \cos \alpha)^2} \sin^2(\Omega t) \\ \frac{\sin^2 \alpha}{(\sin \alpha + \cos \alpha)^2} \sin^2(\Omega t) & 1 - \frac{\sin^2 \alpha}{(\sin \alpha + \cos \alpha)^2} \sin^2(\Omega t) \end{pmatrix} \quad (19)$$

Now for the limiting case $\Omega t \ll 1$ for probabilities for state 1 to stay in state 1 or go to state 2, we get:

$$P_{1-1} = e^{-\Gamma t} \left(1 - \Omega^2 t^2 \frac{\sin^2 \alpha}{(\sin \alpha + \cos \alpha)^2} \right) \quad (20)$$

$$P_{1-2} = e^{-\Gamma t} \Omega^2 t^2 \frac{\sin^2 \alpha}{(\sin \alpha + \cos \alpha)^2}$$

3 New form

$$H = \begin{pmatrix} 2\omega \cdot \sigma & \varepsilon \\ \varepsilon & 2\omega' \cdot \sigma \end{pmatrix} \quad (21)$$

$$H^2 = \begin{pmatrix} 2\omega \cdot \sigma & \varepsilon \cdot \sigma_0 \\ \varepsilon \cdot \sigma_0 & 2\omega' \cdot \sigma \end{pmatrix} \begin{pmatrix} 2\omega \cdot \sigma & \varepsilon \cdot \sigma_0 \\ \varepsilon \cdot \sigma_0 & 2\omega' \cdot \sigma \end{pmatrix} \quad (22)$$

$$H^2 = \begin{pmatrix} 4\omega^2 + \varepsilon^2 & 0 & 2\varepsilon(\omega_3 + \omega'_3) & 2\varepsilon(\omega_1 + \omega'_1) \\ 0 & 4\omega^2 + \varepsilon^2 & 2\varepsilon(\omega_1 + \omega'_1) & -2\varepsilon(\omega_3 + \omega'_3) \\ 2\varepsilon(\omega_3 + \omega'_3) & 2\varepsilon(\omega_1 + \omega'_1) & 4\omega'^2 + \varepsilon^2 & 0 \\ 2\varepsilon(\omega_1 + \omega'_1) & -2\varepsilon(\omega_3 + \omega'_3) & 0 & 4\omega'^2 + \varepsilon^2 \end{pmatrix} \quad (23)$$

where:

$$\begin{aligned} \omega_1 &= \omega \sin(\alpha) \\ \omega_3 &= \omega \cos(\alpha) \\ \omega'_1 &= \omega' \sin(\alpha') \\ \omega'_3 &= \omega' \cos(\alpha') \end{aligned} \quad (24)$$

Now if we rotate H^2 with the matrix S_1 :

$$S_1 = \begin{pmatrix} \cos(\phi) & \sin(\phi) & 0 & 0 \\ -\sin(\phi) & \cos(\phi) & 0 & 0 \\ 0 & 0 & \cos(\phi) & \sin(\phi) \\ 0 & 0 & -\sin(\phi) & \cos(\phi) \end{pmatrix} \quad (25)$$

We get:

$$\begin{pmatrix} \varepsilon^2 + 4\omega^2 & 0 & 2\varepsilon(-\sin(2\phi)(\omega_1 + \omega'_1) + \cos(2\phi)(\omega_3 + \omega'_3)) & 2\varepsilon(\cos(2\phi)(\omega_1 + \omega'_1) + \sin(2\phi)(\omega_3 + \omega'_3)) \\ 0 & \varepsilon^2 + 4\omega'^2 & 2\varepsilon(\cos(2\phi)(\omega_1 + \omega'_1) + \sin(2\phi)(\omega_3 + \omega'_3)) & 2\varepsilon(\sin(2\phi)(\omega_1 + \omega'_1) - \cos(2\phi)(\omega_3 + \omega'_3)) \\ 2\varepsilon(-\sin(2\phi)(\omega_1 + \omega'_1) + \cos(2\phi)(\omega_3 + \omega'_3)) & 2\varepsilon(\cos(2\phi)(\omega_1 + \omega'_1) + \sin(2\phi)(\omega_3 + \omega'_3)) & \varepsilon^2 + 4\omega'^2 & 0 \\ 2\varepsilon(\cos(2\phi)(\omega_1 + \omega'_1) + \sin(2\phi)(\omega_3 + \omega'_3)) & 2\varepsilon(\sin(2\phi)(\omega_1 + \omega'_1) - \cos(2\phi)(\omega_3 + \omega'_3)) & 0 & \varepsilon^2 + 4\omega'^2 \end{pmatrix} \quad (26)$$

From here, components $(1, 4), (2, 3), (3, 2), (4, 1)$ to be 0, we should have:

$$\tan(2\phi) = -\frac{\omega_1 + \omega'_1}{\omega_3 + \omega'_3} \quad (27)$$

And the Matrix (25) becomes blockdiagonal:

$$H_1^2 = S_1^T H^2 S_1 =$$

$$= \begin{pmatrix} \varepsilon^2 + 4\omega^2 & 0 & 2\varepsilon \frac{(\omega_1 + \omega'_1)^2 + (\omega_3 + \omega'_3)^2}{\sqrt{(\omega_1 + \omega'_1)^2 + (\omega_3 + \omega'_3)^2}} & 0 \\ 0 & \varepsilon^2 + 4\omega^2 & 0 & -2\varepsilon \frac{(\omega_1 + \omega'_1)^2 + (\omega_3 + \omega'_3)^2}{\sqrt{(\omega_1 + \omega'_1)^2 + (\omega_3 + \omega'_3)^2}} \\ 2\varepsilon \frac{(\omega_1 + \omega'_1)^2 + (\omega_3 + \omega'_3)^2}{\sqrt{(\omega_1 + \omega'_1)^2 + (\omega_3 + \omega'_3)^2}} & 0 & \varepsilon^2 + 4\omega'^2 & 0 \\ 0 & -2\varepsilon \frac{(\omega_1 + \omega'_1)^2 + (\omega_3 + \omega'_3)^2}{\sqrt{(\omega_1 + \omega'_1)^2 + (\omega_3 + \omega'_3)^2}} & 0 & \varepsilon^2 + 4\omega'^2 \end{pmatrix} \quad (28)$$

Which can be written:

$$H_1^2 = \begin{pmatrix} \varepsilon^2 + 4\omega^2 & 0 & 2\varepsilon \sqrt{(\omega_1 + \omega'_1)^2 + (\omega_3 + \omega'_3)^2} & 0 \\ 0 & \varepsilon^2 + 4\omega^2 & 0 & -2\varepsilon \sqrt{(\omega_1 + \omega'_1)^2 + (\omega_3 + \omega'_3)^2} \\ 2\varepsilon \sqrt{(\omega_1 + \omega'_1)^2 + (\omega_3 + \omega'_3)^2} & 0 & \varepsilon^2 + 4\omega^2 & 0 \\ 0 & -2\varepsilon \sqrt{(\omega_1 + \omega'_1)^2 + (\omega_3 + \omega'_3)^2} & 0 & \varepsilon^2 + 4\omega^2 \end{pmatrix} \quad (29)$$

Then, we rotate H_3^2 with the matrix S_1 :

$$S_2 = \begin{pmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & \cos \theta & 0 & -\sin \theta \\ -\sin \theta & 0 & \cos \theta & 0 \\ 0 & \sin \theta & 0 & \cos \theta \end{pmatrix} \quad (30)$$

We get (for simplicity, I will write d_1, d_2, d_3, d_4 instead of diagonal terms for here and then simplify them:

$$H_2^2 = S_2^T H_1^2 S_2 = \begin{pmatrix} d_1 & 0 & a & 0 \\ 0 & d_2 & 0 & -a \\ a & 0 & d_3 & 0 \\ 0 & -a & 0 & d_4 \end{pmatrix} \quad (31)$$

where

$$a = -2\varepsilon[\sin(2\phi)\cos(2\theta)(\omega_1 + \omega'_1) + 2\varepsilon\cos(2\theta)\cos(2\phi)(\omega_3 + \omega'_3)] + 2\sin(2\theta)(\omega_1^2 + \omega_3^2 - \omega_1'^2 - \omega_3'^2) \quad (32)$$

d_1, d_2, d_3, d_4 (given later)

Now if we simplify a , we get:

$$a = -2\varepsilon[\sin(2\phi)\cos(2\theta)(\omega_1 + \omega'_1) + 2\varepsilon\cos(2\theta)\cos(2\phi)(\omega_3 + \omega'_3)] + 2\sin(2\theta)(\omega_1^2 + \omega_3^2 - \omega_1'^2 - \omega_3'^2) = \quad (33)$$

$$= 2\varepsilon\cos(2\theta)[-\sin(2\phi)(\omega_1 + \omega'_1) + \cos(2\phi)(\omega_3 + \omega'_3)] + 2\tan(2\theta)\cos(2\theta)(\omega_1^2 + \omega_3^2 - \omega_1'^2 - \omega_3'^2)$$

But now if we want (30) to be diagonal, we must constrain θ :

$$\tan(2\theta) = \frac{4\varepsilon[-\sin(2\phi)(\omega_1 + \omega'_1) + \cos(2\phi)(\omega_3 + \omega'_3)]}{4\omega'^2 - 4\omega^2} \quad (34)$$

Then a becomes:

$$a = \cos(2\theta)\tan(2\theta)\frac{4\omega'^2 - 4\omega^2}{2} + 2\tan(2\theta)\cos(2\theta)(\omega^2 + \omega'^2) = 0 \quad (35)$$

Then we have diagonalized H_2 :

$$H_D^2 = \begin{pmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 \\ 0 & 0 & d_3 & 0 \\ 0 & 0 & 0 & d_4 \end{pmatrix} \quad (36)$$

where:

$$\begin{aligned} d_1 = d_2 &= \varepsilon^2 + \frac{2(\omega^2 - \omega'^2)}{\cos(2\theta)} + 2(\omega^2 + \omega'^2) \\ d_3 = d_4 &= \varepsilon^2 + \frac{2(\omega'^2 - \omega^2)}{\cos(2\theta)} + 2(\omega^2 + \omega'^2) \end{aligned} \quad (37)$$

To sum up, the matrix which diagonalizes H^2 is:

$$S = S_1 \cdot S_2 = \begin{pmatrix} \cos(\theta) \cos(\phi) & \sin(\phi) \cos(\theta) & \sin(\theta) \cos(\phi) & -\sin(\theta) \sin(\phi) \\ -\sin(\phi) \cos(\theta) & \cos(\theta) \cos(\phi) & -\sin(\theta) \sin(\phi) & -\sin(\theta) \cos(\phi) \\ -\sin(\theta) \cos(\phi) & \sin(\theta) \sin(\phi) & \cos(\theta) \cos(\phi) & \sin(\phi) \cos(\theta) \\ \sin(\theta) \sin(\phi) & \sin(\theta) \cos(\phi) & -\sin(\phi) \cos(\theta) & \cos(\theta) \cos(\phi) \end{pmatrix} \quad (38)$$

And:

$$H_D^2 = S^T H^2 S = \begin{pmatrix} \varepsilon^2 + \frac{2(\omega^2 - \omega'^2)}{\cos(2\theta)} + 2(\omega^2 + \omega'^2) & 0 & 0 & 0 \\ 0 & \varepsilon^2 + \frac{2(\omega^2 - \omega'^2)}{\cos(2\theta)} + 2(\omega^2 + \omega'^2) & 0 & 0 \\ 0 & 0 & \varepsilon^2 + \frac{2(\omega'^2 - \omega^2)}{\cos(2\theta)} + 2(\omega^2 + \omega'^2) & 0 \\ 0 & 0 & 0 & \varepsilon^2 + \frac{2(\omega'^2 - \omega^2)}{\cos(2\theta)} + 2(\omega^2 + \omega'^2) \end{pmatrix} \quad (39)$$

Where d_1 and d_2 correspond to $(2\tilde{\omega})^2$, and d_3 and $d_4 - (2\tilde{\omega}')^2$.

$$\begin{aligned} \tan^2(2\theta) &= \frac{16\varepsilon^2[(\omega_1 + \omega'_1)^2 + (\omega_3 + \omega'_3)^2]^2}{(4\omega'^2 - 4\omega^2)^2} \\ 1 + \tan^2(2\theta) &= \frac{(4\omega'^2 - 4\omega^2)^2 + 16\varepsilon^2[(\omega_1 + \omega'_1)^2 + (\omega_3 + \omega'_3)^2]^2}{(4\omega'^2 - \omega^2)^2} \\ \cos(2\theta) &= \sqrt{\frac{1}{1 + \tan^2(2\theta)}} = \frac{4\omega'^2 - 4\omega^2}{\sqrt{(4\omega'^2 - 4\omega^2)^2 + 16\varepsilon^2[(\omega_1 + \omega'_1)^2 + (\omega_3 + \omega'_3)^2]^2}} \\ \sin(2\theta) &= \sqrt{1 - \cos^2(2\theta)} = \frac{16\varepsilon^2[(\omega_1 + \omega'_1)^2 + (\omega_3 + \omega'_3)^2]}{\sqrt{(4\omega'^2 - 4\omega^2)^2 + 16\varepsilon^2[(\omega_1 + \omega'_1)^2 + (\omega_3 + \omega'_3)^2]^2}} \end{aligned} \quad (40)$$

Then the first two diagonal terms of H_D ($H_D^2[1, 1]$ and $H_D^2[2, 2]$) have the form:

$$\begin{aligned} H_D^2[1, 1] = H_D^2[2, 2] &= \varepsilon^2 + 2(\omega^2 + \omega'^2) + \frac{2(\omega^2 - \omega'^2)\sqrt{(4\omega'^2 - 4\omega^2)^2 + 16\varepsilon^2[(\omega_1 + \omega'_1)^2 + (\omega_3 + \omega'_3)^2]^2}}{4(\omega'^2 - \omega^2)} = \\ &= \varepsilon^2 + 2(\omega^2 + \omega'^2) - \sqrt{(\omega^2 - \omega'^2)^2 + 4\varepsilon^2[(\omega_1 + \omega'_1)^2 + (\omega_3 + \omega'_3)^2]^2} \end{aligned} \quad (41)$$

And:

$$\begin{aligned} H_D^2[3, 3] = H_D^2[4, 4] &= \varepsilon^2 + 2(\omega^2 + \omega'^2) + \frac{2(\omega'^2 - \omega^2)\sqrt{(4\omega'^2 - 4\omega^2)^2 + 16\varepsilon^2[(\omega_1 + \omega'_1)^2 + (\omega_3 + \omega'_3)^2]^2}}{4(\omega'^2 - \omega^2)} = \\ &= \varepsilon^2 + 2(\omega^2 + \omega'^2) + \sqrt{(\omega^2 - \omega'^2)^2 + 4\varepsilon^2[(\omega_1 + \omega'_1)^2 + (\omega_3 + \omega'_3)^2]^2} \end{aligned} \quad (42)$$

So, for H_D^2 we have:

$$H_D^2 = \begin{pmatrix} \varepsilon^2 + 2(\omega^2 + \omega'^2) - \sqrt{(\omega^2 - \omega'^2)^2 + 4\varepsilon^2[(\omega_1 + \omega'_1)^2 + (\omega_3 + \omega'_3)^2]^2} & 0 & 0 & 0 \\ 0 & \varepsilon^2 + 2(\omega^2 + \omega'^2) - \sqrt{(\omega^2 - \omega'^2)^2 + 4\varepsilon^2[(\omega_1 + \omega'_1)^2 + (\omega_3 + \omega'_3)^2]^2} & 0 & 0 \\ 0 & 0 & \varepsilon^2 + 2(\omega^2 + \omega'^2) + \sqrt{(\omega^2 - \omega'^2)^2 + 4\varepsilon^2[(\omega_1 + \omega'_1)^2 + (\omega_3 + \omega'_3)^2]^2} & 0 \\ 0 & 0 & 0 & \varepsilon^2 + 2(\omega^2 + \omega'^2) + \sqrt{(\omega^2 - \omega'^2)^2 + 4\varepsilon^2[(\omega_1 + \omega'_1)^2 + (\omega_3 + \omega'_3)^2]^2} \end{pmatrix} \quad (43)$$

Generally we have:

$$H = \begin{pmatrix} 2\boldsymbol{\omega} \cdot \boldsymbol{\sigma} & \varepsilon \cdot \boldsymbol{\sigma}_0 \\ \varepsilon \cdot \boldsymbol{\sigma}_0 & 2\boldsymbol{\omega}' \cdot \boldsymbol{\sigma} \end{pmatrix} = \begin{pmatrix} 2\omega_3 & 2\omega_1 & \varepsilon & 0 \\ 2\omega_1 & -2\omega_3 & 0 & \varepsilon \\ \varepsilon & 0 & 2\omega'_3 & 2\omega'_1 \\ 0 & \varepsilon & 2\omega'_1 & -2\omega'_3 \end{pmatrix} \quad (44)$$

Then rotating with S_1 we have:

$$H_1 = S_1^T H S_1 = \begin{pmatrix} -2\omega \sin(\alpha) \sin(2\phi) + 2\omega \cos(\alpha) \cos(2\phi) & 2\omega \sin(\alpha) \cos(2\phi) + 2\omega \cos(\alpha) \sin(2\phi) & \varepsilon & 0 \\ 2\omega \sin(\alpha) \cos(2\phi) + 2\omega \cos(\alpha) \sin(2\phi) & 2\omega \sin(\alpha) \sin(2\phi) - 2\omega \cos(\alpha) \cos(2\phi) & 0 & \varepsilon \\ \varepsilon & 0 & -2\omega' \sin(\alpha') \sin(2\phi) + 2\omega' \cos(\alpha') \cos(2\phi) & 2\omega' \sin(\alpha') \cos(2\phi) + 2\omega' \cos(\alpha') \sin(2\phi) \\ 0 & \varepsilon & 2\omega' \sin(\alpha') \cos(2\phi) + 2\omega' \cos(\alpha') \sin(2\phi) & 2\omega' \sin(\alpha') \sin(2\phi) - 2\omega' \cos(\alpha') \cos(2\phi) \end{pmatrix} \quad (45)$$

Using here (58), (55) and considering $\sin(2\phi) < 0$ because we need $\tan(2\phi)$ with $-$, we get:

$$H_1 = \begin{pmatrix} 2\omega_1 \frac{(\omega_1 + \omega'_1)}{\sqrt{(\omega_3 + \omega'_3)^2 + (\omega_1 + \omega'_1)^2}} + 2\omega_3 \frac{(\omega_3 + \omega'_3)}{\sqrt{(\omega_3 + \omega'_3)^2 + (\omega_1 + \omega'_1)^2}} & 2\omega_1 \frac{(\omega_3 + \omega'_3)}{\sqrt{(\omega_3 + \omega'_3)^2 + (\omega_1 + \omega'_1)^2}} - 2\omega_3 \frac{(\omega_1 + \omega'_1)}{\sqrt{(\omega_3 + \omega'_3)^2 + (\omega_1 + \omega'_1)^2}} & \varepsilon & 0 \\ 2\omega_1 \frac{(\omega_3 + \omega'_3)}{\sqrt{(\omega_3 + \omega'_3)^2 + (\omega_1 + \omega'_1)^2}} - 2\omega_3 \frac{(\omega_1 + \omega'_1)}{\sqrt{(\omega_3 + \omega'_3)^2 + (\omega_1 + \omega'_1)^2}} & -2\omega_1 \frac{(\omega_1 + \omega'_1)}{\sqrt{(\omega_3 + \omega'_3)^2 + (\omega_1 + \omega'_1)^2}} - 2\omega_3 \frac{(\omega_3 + \omega'_3)}{\sqrt{(\omega_3 + \omega'_3)^2 + (\omega_1 + \omega'_1)^2}} & 0 & \varepsilon \\ \varepsilon & 0 & 2\omega'_1 \frac{(\omega_1 + \omega'_1)}{\sqrt{(\omega_3 + \omega'_3)^2 + (\omega_1 + \omega'_1)^2}} + 2\omega'_3 \frac{(\omega_3 + \omega'_3)}{\sqrt{(\omega_3 + \omega'_3)^2 + (\omega_1 + \omega'_1)^2}} & 2\omega'_1 \frac{(\omega_3 + \omega'_3)}{\sqrt{(\omega_3 + \omega'_3)^2 + (\omega_1 + \omega'_1)^2}} - 2\omega'_3 \frac{(\omega_1 + \omega'_1)}{\sqrt{(\omega_3 + \omega'_3)^2 + (\omega_1 + \omega'_1)^2}} \\ 0 & \varepsilon & 2\omega'_1 \frac{(\omega_3 + \omega'_3)}{\sqrt{(\omega_3 + \omega'_3)^2 + (\omega_1 + \omega'_1)^2}} - 2\omega'_3 \frac{(\omega_1 + \omega'_1)}{\sqrt{(\omega_3 + \omega'_3)^2 + (\omega_1 + \omega'_1)^2}} & -2\omega'_1 \frac{(\omega_1 + \omega'_1)}{\sqrt{(\omega_3 + \omega'_3)^2 + (\omega_1 + \omega'_1)^2}} - 2\omega'_3 \frac{(\omega_3 + \omega'_3)}{\sqrt{(\omega_3 + \omega'_3)^2 + (\omega_1 + \omega'_1)^2}} \end{pmatrix} \quad (46)$$

Simplification gives:

$$H_1 = \begin{pmatrix} \frac{2\omega^2 + 2(\omega_1 \omega'_1 + \omega_3 \omega'_3)}{\sqrt{(\omega_3 + \omega'_3)^2 + (\omega_1 + \omega'_1)^2}} & \frac{2(\omega_1 \omega'_3 - \omega_3 \omega'_1)}{\sqrt{(\omega_3 + \omega'_3)^2 + (\omega_1 + \omega'_1)^2}} & \varepsilon & 0 \\ \frac{2(\omega_1 \omega'_3 - \omega_3 \omega'_1)}{\sqrt{(\omega_3 + \omega'_3)^2 + (\omega_1 + \omega'_1)^2}} & -\frac{2\omega^2 + 2(\omega_1 \omega'_1 + \omega_3 \omega'_3)}{\sqrt{(\omega_3 + \omega'_3)^2 + (\omega_1 + \omega'_1)^2}} & 0 & \varepsilon \\ \varepsilon & 0 & \frac{2\omega'^2 + 2(\omega_1 \omega'_1 + \omega_3 \omega'_3)}{\sqrt{(\omega_3 + \omega'_3)^2 + (\omega_1 + \omega'_1)^2}} & \frac{2(\omega'_1 \omega_3 - \omega'_3 \omega_1)}{\sqrt{(\omega_3 + \omega'_3)^2 + (\omega_1 + \omega'_1)^2}} \\ 0 & \varepsilon & \frac{2(\omega'_1 \omega_3 - \omega'_3 \omega_1)}{\sqrt{(\omega_3 + \omega'_3)^2 + (\omega_1 + \omega'_1)^2}} & -\frac{2\omega'^2 + 2(\omega_1 \omega'_1 + \omega_3 \omega'_3)}{\sqrt{(\omega_3 + \omega'_3)^2 + (\omega_1 + \omega'_1)^2}} \end{pmatrix} \quad (47)$$

This is now in form of the first Hamiltonian given in paper which has $\boldsymbol{\omega} + \boldsymbol{\omega}'$ in z direction.

4 Matrix 8x8

Now the interaction Hamiltonian is:

$$H_8 = \begin{pmatrix} 2\boldsymbol{\omega} \cdot \boldsymbol{\sigma} & 0 & \varepsilon & \eta \\ 0 & -2\boldsymbol{\omega} \cdot \boldsymbol{\sigma} & \eta & \varepsilon \\ \varepsilon & \eta & 2\boldsymbol{\omega}' \cdot \boldsymbol{\sigma} & 0 \\ \eta & \varepsilon & 0 & -2\boldsymbol{\omega}' \cdot \boldsymbol{\sigma} \end{pmatrix} \quad (48)$$

$$H_8 = \begin{pmatrix} 2\omega_3 & 2\omega_1 & 0 & 0 & \varepsilon & 0 & \eta & 0 \\ 2\omega_1 & -2\omega_3 & 0 & 0 & 0 & \varepsilon & 0 & \eta \\ 0 & 0 & -2\omega_3 & -2\omega_1 & \eta & 0 & \varepsilon & 0 \\ 0 & 0 & -2\omega_1 & 2\omega_3 & 0 & \eta & 0 & \varepsilon \\ \varepsilon & 0 & \eta & 0 & 2\omega'_3 & 2\omega'_1 & 0 & 0 \\ 0 & \varepsilon & 0 & \eta & 2\omega'_1 & -2\omega'_3 & 0 & 0 \\ \eta & 0 & \varepsilon & 0 & 0 & 0 & -2\omega'_3 & -2\omega'_1 \\ 0 & \eta & 0 & \varepsilon & 0 & 0 & -2\omega'_1 & 2\omega'_3 \end{pmatrix} \quad (49)$$

5 Different number sectors squared H

- 2×2 : Hamiltonian:

$$H_2 = \begin{pmatrix} \boldsymbol{\omega} \cdot \boldsymbol{\sigma} & \varepsilon \\ \varepsilon & -\boldsymbol{\omega}' \cdot \boldsymbol{\sigma} \end{pmatrix} \quad (50)$$

$$H_2^2 = \begin{pmatrix} \varepsilon^2 + \sigma^2 \omega^2 & \varepsilon \boldsymbol{\sigma} \cdot (\boldsymbol{\omega} - \boldsymbol{\omega}') \\ \varepsilon \boldsymbol{\sigma} \cdot (\boldsymbol{\omega} - \boldsymbol{\omega}') & \varepsilon^2 + \sigma^2 \omega'^2 \end{pmatrix} = \begin{pmatrix} \varepsilon^2 + \omega^2 & \varepsilon \boldsymbol{\sigma} \cdot (\boldsymbol{\omega} - \boldsymbol{\omega}') \\ \varepsilon \boldsymbol{\sigma} \cdot (\boldsymbol{\omega} - \boldsymbol{\omega}') & \varepsilon^2 + \omega'^2 \end{pmatrix} \quad (51)$$

- 4×4

Hamiltonian:

$$H_{4eps} = \begin{pmatrix} \boldsymbol{\omega} \cdot \boldsymbol{\sigma} & 0 & 0 & \varepsilon \\ 0 & -\boldsymbol{\omega} \cdot \boldsymbol{\sigma} & \varepsilon & 0 \\ 0 & \varepsilon & \boldsymbol{\omega}' \cdot \boldsymbol{\sigma} & 0 \\ \varepsilon & 0 & 0 & -\boldsymbol{\omega}' \cdot \boldsymbol{\sigma} \end{pmatrix} \quad (52)$$

$$H_{4eps}^2 = \begin{pmatrix} \varepsilon^2 + \sigma^2 \omega^2 & 0 & 0 & \varepsilon \boldsymbol{\sigma} \cdot (\boldsymbol{\omega} - \boldsymbol{\omega}') \\ 0 & \varepsilon^2 + \sigma^2 \omega^2 & \varepsilon \boldsymbol{\sigma} \cdot (-\boldsymbol{\omega} + \boldsymbol{\omega}') & 0 \\ 0 & \varepsilon \boldsymbol{\sigma} \cdot (-\boldsymbol{\omega} + \boldsymbol{\omega}') & \varepsilon^2 + \sigma^2 \omega^2 & 0 \\ \varepsilon \boldsymbol{\sigma} \cdot (\boldsymbol{\omega} - \boldsymbol{\omega}') & 0 & 0 & \varepsilon^2 + \sigma^2 \omega^2 \end{pmatrix} \quad (53)$$

$$= \begin{pmatrix} \varepsilon^2 + \omega^2 & 0 & 0 & \varepsilon \boldsymbol{\sigma} \cdot (\boldsymbol{\omega} - \boldsymbol{\omega}') \\ 0 & \varepsilon^2 + \omega^2 & \varepsilon \boldsymbol{\sigma} \cdot (-\boldsymbol{\omega} + \boldsymbol{\omega}') & 0 \\ 0 & \varepsilon \boldsymbol{\sigma} \cdot (-\boldsymbol{\omega} + \boldsymbol{\omega}') & \varepsilon^2 + \omega^2 & 0 \\ \varepsilon \boldsymbol{\sigma} \cdot (\boldsymbol{\omega} - \boldsymbol{\omega}') & 0 & 0 & \varepsilon^2 + \omega^2 \end{pmatrix}$$

Hamiltonian:

$$H_{4eteps} = \begin{pmatrix} \boldsymbol{\omega} \cdot \boldsymbol{\sigma} & 0 & \eta & \varepsilon \\ 0 & -\boldsymbol{\omega} \cdot \boldsymbol{\sigma} & \varepsilon & \eta \\ \eta & \varepsilon & \boldsymbol{\omega}' \cdot \boldsymbol{\sigma} & 0 \\ \varepsilon & \eta & 0 & -\boldsymbol{\omega}' \cdot \boldsymbol{\sigma} \end{pmatrix} \quad (54)$$

$$H_{4eteps}^2 = \begin{pmatrix} \varepsilon^2 + \eta^2 + \sigma^2 \omega^2 & 2\eta\varepsilon & \eta \boldsymbol{\sigma} \cdot (\boldsymbol{\omega} + \boldsymbol{\omega}') & \varepsilon \boldsymbol{\sigma} \cdot (\boldsymbol{\omega} - \boldsymbol{\omega}') \\ 2\eta\varepsilon & \varepsilon^2 + \eta^2 + \sigma^2 \omega^2 & \varepsilon \boldsymbol{\sigma} \cdot (-\boldsymbol{\omega} + \boldsymbol{\omega}') & \eta \boldsymbol{\sigma} \cdot (-\boldsymbol{\omega} - \boldsymbol{\omega}') \\ \eta \boldsymbol{\sigma} \cdot (\boldsymbol{\omega} + \boldsymbol{\omega}') & \varepsilon \boldsymbol{\sigma} \cdot (-\boldsymbol{\omega} + \boldsymbol{\omega}') & \varepsilon^2 + \eta^2 + \sigma^2 \omega^2 & 2\eta\varepsilon \\ \varepsilon \boldsymbol{\sigma} \cdot (\boldsymbol{\omega} - \boldsymbol{\omega}') & \eta \boldsymbol{\sigma} \cdot (-\boldsymbol{\omega} - \boldsymbol{\omega}') & 2\eta\varepsilon & \varepsilon^2 + \eta^2 + \sigma^2 \omega^2 \end{pmatrix} \quad (55)$$

$$= \begin{pmatrix} \varepsilon^2 + \eta^2 + \omega^2 & 2\eta\varepsilon & \eta \boldsymbol{\sigma} \cdot (\boldsymbol{\omega} + \boldsymbol{\omega}') & \varepsilon \boldsymbol{\sigma} \cdot (\boldsymbol{\omega} - \boldsymbol{\omega}') \\ 2\eta\varepsilon & \varepsilon^2 + \eta^2 + \omega^2 & \varepsilon \boldsymbol{\sigma} \cdot (-\boldsymbol{\omega} + \boldsymbol{\omega}') & \eta \boldsymbol{\sigma} \cdot (-\boldsymbol{\omega} - \boldsymbol{\omega}') \\ \eta \boldsymbol{\sigma} \cdot (\boldsymbol{\omega} + \boldsymbol{\omega}') & \varepsilon \boldsymbol{\sigma} \cdot (-\boldsymbol{\omega} + \boldsymbol{\omega}') & \varepsilon^2 + \eta^2 + \omega^2 & 2\eta\varepsilon \\ \varepsilon \boldsymbol{\sigma} \cdot (\boldsymbol{\omega} - \boldsymbol{\omega}') & \eta \boldsymbol{\sigma} \cdot (-\boldsymbol{\omega} - \boldsymbol{\omega}') & 2\eta\varepsilon & \varepsilon^2 + \eta^2 + \omega^2 \end{pmatrix}$$

- 6x6

Hamiltonian:

$$H_6 = \begin{pmatrix} \boldsymbol{\omega} \cdot \boldsymbol{\sigma} & 0 & 0 & \varepsilon & 0 & \varepsilon \\ 0 & -\boldsymbol{\omega} \cdot \boldsymbol{\sigma} & \varepsilon & 0 & \varepsilon & 0 \\ 0 & \varepsilon & \boldsymbol{\omega}_1 \cdot \boldsymbol{\sigma} & 0 & 0 & \varepsilon \\ \varepsilon & 0 & 0 & -\boldsymbol{\omega}_1 \cdot \boldsymbol{\sigma} & \varepsilon & 0 \\ 0 & \varepsilon & 0 & \varepsilon & \boldsymbol{\omega}_2 \cdot \boldsymbol{\sigma} & 0 \\ \varepsilon & 0 & \varepsilon & 0 & 0 & -\boldsymbol{\omega}_2 \cdot \boldsymbol{\sigma} \end{pmatrix} \quad (56)$$

$$H_6^2 = \begin{pmatrix} 2\varepsilon^2 + \sigma^2\omega^2 & 0 & \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(\boldsymbol{\omega} - \boldsymbol{\omega}_1) & \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(\boldsymbol{\omega} + \boldsymbol{\omega}_2) \\ 0 & 2\varepsilon^2 + \sigma^2\omega^2 & \varepsilon\boldsymbol{\sigma}(-\boldsymbol{\omega} + \boldsymbol{\omega}_1) & \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(-\boldsymbol{\omega} + \boldsymbol{\omega}_2) & \varepsilon^2 \\ \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(-\boldsymbol{\omega} + \boldsymbol{\omega}_1) & 2\varepsilon^2 + \sigma^2\omega_1^2 & 0 & \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(\boldsymbol{\omega}_1 + \boldsymbol{\omega}_2) \\ \varepsilon\boldsymbol{\sigma}(\boldsymbol{\omega} - \boldsymbol{\omega}_1) & \varepsilon^2 & 0 & 2\varepsilon^2 + \sigma^2\omega_1^2 & \varepsilon\boldsymbol{\sigma}(-\boldsymbol{\omega}_1 + \boldsymbol{\omega}_2) & \varepsilon^2 \\ \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(-\boldsymbol{\omega} + \boldsymbol{\omega}_2) & \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(-\boldsymbol{\omega}_1 + \boldsymbol{\omega}_2) & 2\varepsilon^2 + \sigma^2\omega_2^2 & 0 \\ \varepsilon\boldsymbol{\sigma}(\boldsymbol{\omega} + \boldsymbol{\omega}_2) & \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(\boldsymbol{\omega}_1 + \boldsymbol{\omega}_2) & \varepsilon^2 & 0 & 2\varepsilon^2 + \sigma^2\omega_2^2 \end{pmatrix} =$$

$$= \begin{pmatrix} 2\varepsilon^2 + \omega^2 & 0 & \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(\boldsymbol{\omega} - \boldsymbol{\omega}_1) & \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(\boldsymbol{\omega} + \boldsymbol{\omega}_2) \\ 0 & 2\varepsilon^2 + \omega^2 & \varepsilon\boldsymbol{\sigma}(-\boldsymbol{\omega} + \boldsymbol{\omega}_1) & \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(-\boldsymbol{\omega} + \boldsymbol{\omega}_2) & \varepsilon^2 \\ \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(-\boldsymbol{\omega} + \boldsymbol{\omega}_1) & 2\varepsilon^2 + \omega_1^2 & 0 & \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(\boldsymbol{\omega}_1 + \boldsymbol{\omega}_2) \\ \varepsilon\boldsymbol{\sigma}(\boldsymbol{\omega} - \boldsymbol{\omega}_1) & \varepsilon^2 & 0 & 2\varepsilon^2 + \omega_1^2 & \varepsilon\boldsymbol{\sigma}(-\boldsymbol{\omega}_1 + \boldsymbol{\omega}_2) & \varepsilon^2 \\ \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(-\boldsymbol{\omega} + \boldsymbol{\omega}_2) & \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(-\boldsymbol{\omega}_1 + \boldsymbol{\omega}_2) & 2\varepsilon^2 + \omega_2^2 & 0 \\ \varepsilon\boldsymbol{\sigma}(\boldsymbol{\omega} + \boldsymbol{\omega}_2) & \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(\boldsymbol{\omega}_1 + \boldsymbol{\omega}_2) & \varepsilon^2 & 0 & 2\varepsilon^2 + \omega_2^2 \end{pmatrix} \quad (57)$$

6 Results

6.1 3 sectors

Hamiltonian:

$$H^{\text{Fl}} = \begin{pmatrix} m & 0 & 0 & \varepsilon & 0 & \varepsilon \\ 0 & m & \varepsilon & 0 & \varepsilon & 0 \\ 0 & \varepsilon & m & 0 & 0 & \varepsilon \\ \varepsilon & 0 & 0 & m & \varepsilon & 0 \\ \hline 0 & \varepsilon & 0 & \varepsilon & m & 0 \\ \varepsilon & 0 & \varepsilon & 0 & 0 & m \end{pmatrix} \quad (58)$$

Diagonalising matrices:

1.

$$U_{n\bar{n}} = \left(\begin{array}{cc|cc|cc} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{array} \right), \quad (59)$$

2.

$$U_{1324} = \left(\begin{array}{cc|cc|cc} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} & 0 & 0 \\ \hline -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right), \quad (60)$$

3.

$$U_{3526} = \left(\begin{array}{cc|cc|cc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{3} & 0 & 0 & 0 & \frac{\sqrt{6}}{3} \\ 0 & 0 & \frac{\sqrt{3}}{3} & 0 & \frac{\sqrt{6}}{3} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{6}}{3} & 0 & \frac{\sqrt{3}}{3} & 0 \\ \hline 0 & -\frac{\sqrt{6}}{3} & 0 & 0 & 0 & \frac{\sqrt{3}}{3} \end{array} \right) \quad (61)$$

Diagonalizing matrix:

$$S = \left(\begin{array}{cccccc} \frac{1}{2} & -\frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{6} & \frac{1}{2} & \frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{6} \\ \frac{1}{2} & \frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{6} & -\frac{1}{2} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{6} & -\frac{1}{2} & \frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{6} \\ -\frac{1}{2} & \frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{6} & \frac{1}{2} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} \\ 0 & \frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} & 0 & \frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{6} \\ 0 & -\frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} & 0 & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} \end{array} \right) \quad (62)$$

Diagonalized Hamiltonian:

$$H_D = S^\dagger H S = \left(\begin{array}{cccccc} m - \varepsilon & 0 & 0 & 0 & 0 & 0 \\ 0 & m + \varepsilon & 0 & 0 & 0 & 0 \\ 0 & 0 & m - \varepsilon & 0 & 0 & 0 \\ 0 & 0 & 0 & m + \varepsilon & 0 & 0 \\ 0 & 0 & 0 & 0 & m - 2\varepsilon & 0 \\ 0 & 0 & 0 & 0 & 0 & m + 2\varepsilon \end{array} \right) \quad (63)$$

For simplicity, we can subtract from H_D masses, we get:

$$H_{D_R} = H_D - I * m = \left(\begin{array}{cccccc} -\varepsilon & 0 & 0 & 0 & 0 & 0 \\ 0 & \varepsilon & 0 & 0 & 0 & 0 \\ 0 & 0 & -\varepsilon & 0 & 0 & 0 \\ 0 & 0 & 0 & \varepsilon & 0 & 0 \\ 0 & 0 & 0 & 0 & -2\varepsilon & 0 \\ 0 & 0 & 0 & 0 & 0 & 2\varepsilon \end{array} \right) \quad (64)$$

Probabilities of a neutron going in different states:

- P_{00} - Neutron staying as neutron;
- P_{01} - Neutron osc in ordinary antineutron;
- P_{02} Neutron osc. in Mirror neutron one;
- P_{04} - Neutron osc. in mirror anti-nutron one;
- P_{05} - Neutron osc. in mirror neutron two;
- P_{06} - Neutro osc. mirror anti-neutron two.

Calculated directly from:

$$S_{diag} = e^{-iSH_{DR}S^\dagger t} = Se^{-iH_{DR}t}S^\dagger \quad (65)$$

$$P = S_{diag}S_{diag}^\dagger \quad (66)$$

$$P_{6_{00}} = 1 - \frac{4}{9} \sin^2\left(\frac{t\varepsilon}{2}\right) - \frac{4}{9} \sin^2(t\varepsilon) - \frac{4}{9} \sin^2\left(\frac{3t\varepsilon}{2}\right) - \frac{1}{9} \sin^2(2t\varepsilon) \quad (33)$$

$$P_{6_{01}} = \frac{4}{9} \sin^2\left(\frac{t\varepsilon}{2}\right) - \frac{4}{9} \sin^4(t\varepsilon) + \frac{8}{9} \sin^2(t\varepsilon) - \frac{4}{9} \sin^2\left(\frac{3t\varepsilon}{2}\right) \quad (34)$$

$$P_{6_{02}} = \frac{4}{9} \sin^2\left(\frac{t\varepsilon}{2}\right) \sin^2\left(\frac{3t\varepsilon}{2}\right) \quad (35)$$

$$P_{6_{03}} = -\frac{2}{9} \sin^2\left(\frac{t\varepsilon}{2}\right) - \frac{4}{9} \sin^4(t\varepsilon) + \frac{5}{9} \sin^2(t\varepsilon) + \frac{2}{9} \sin^2\left(\frac{3t\varepsilon}{2}\right) \quad (36)$$

$$P_{6_{04}} = \frac{4}{9} \sin^2\left(\frac{t\varepsilon}{2}\right) \sin^2\left(\frac{3t\varepsilon}{2}\right) \quad (67)$$

$$P_{6_{05}} = -\frac{2}{9} \sin^2\left(\frac{t\varepsilon}{2}\right) - \frac{4}{9} \sin^4(t\varepsilon) + \frac{5}{9} \sin^2(t\varepsilon) + \frac{2}{9} \sin^2\left(\frac{3t\varepsilon}{2}\right) \quad (38)$$

If we expand to better see:

$$\begin{aligned} P_{00} &= 1 - 2t^2\varepsilon^2 + \frac{3}{2}t^4\varepsilon^4 - \frac{101}{180}t^6\varepsilon^6 + \frac{1289}{10080}t^8\varepsilon^8, \\ P_{01} &= \frac{1}{9}t^6\varepsilon^6 - \frac{1}{18}t^8\varepsilon^8, \\ P_{02} &= \frac{1}{4}t^4\varepsilon^4 - \frac{5}{24}t^6\varepsilon^6 + \frac{209}{2880}t^8\varepsilon^8, \\ P_{03} &= t^2\varepsilon^2 - t^4\varepsilon^4 + \frac{13}{30}t^6\varepsilon^6 - \frac{137}{1260}t^8\varepsilon^8, \\ P_{04} &= \frac{1}{4}t^4\varepsilon^4 - \frac{5}{24}t^6\varepsilon^6 + \frac{209}{2880}t^8\varepsilon^8, \\ P_{05} &= t^2\varepsilon^2 - t^4\varepsilon^4 + \frac{13}{30}t^6\varepsilon^6 - \frac{137}{1260}t^8\varepsilon^8. \end{aligned} \quad (68)$$

Calculated approximately from expansion:

$$S_{exp} = e^{-iHt} = I - iHt - \frac{1}{2}H^2t^2 + \frac{i}{6}H^3t^3 + \frac{1}{24}H^4t^4 \quad (69)$$

$$P = S_{exp}S_{exp}^\dagger \quad (70)$$

So, we have:

$$\begin{aligned}
P_{00} &= 1 - 2t^2\varepsilon^2 + \frac{3}{2}t^4\varepsilon^4 - \frac{1}{2}t^6\varepsilon^6 + \frac{1}{16}t^8\varepsilon^8, \\
P_{01} &= \frac{1}{9}t^6\varepsilon^6, \\
P_{02} &= \frac{1}{4}t^4\varepsilon^4 - \frac{5}{24}t^6\varepsilon^6 + \frac{25}{576}t^8\varepsilon^8, \\
P_{03} &= t^2\varepsilon^2 - t^4\varepsilon^4 + \frac{1}{4}t^6\varepsilon^6, \\
P_{04} &= \frac{1}{4}t^4\varepsilon^4 - \frac{5}{24}t^6\varepsilon^6 + \frac{25}{576}t^8\varepsilon^8, \\
P_{05} &= t^2\varepsilon^2 - t^4\varepsilon^4 + \frac{1}{4}t^6\varepsilon^6.
\end{aligned} \tag{71}$$

Probability plots look like:

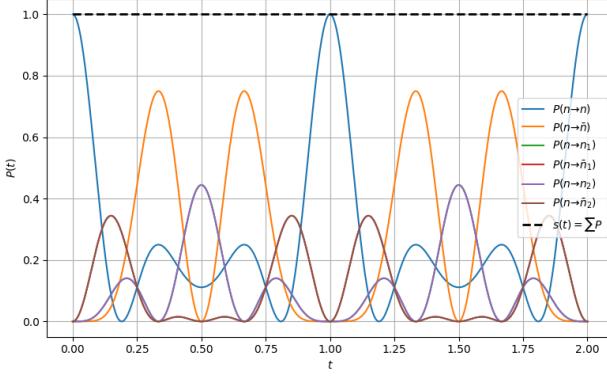


Figure 1: Caption

From (71) we can calculate probability of neutron going into the mirror sector:

$$P_{\text{mirror}} = P_{02} + P_{03} + P_{04} + P_{05} = \frac{4}{9}(1 - \cos(3\varepsilon t)) = \frac{4}{9} \cdot 2 \sin^2\left(\frac{3\varepsilon t}{2}\right) = \frac{8}{9} \sin^2\left(\frac{3\varepsilon t}{2}\right) \tag{72}$$

If we expand for small time:

$$P_{\text{mirror}} = \frac{8}{9} \left(\frac{9}{4}\varepsilon^2 t^2 - \frac{27}{16}\varepsilon^4 t^4 + \frac{81}{160}\varepsilon^6 t^6 + \dots \right) = \left(2\varepsilon^2 t^2 - \frac{3}{2}\varepsilon^4 t^4 + \frac{9}{20}\varepsilon^6 t^6 + \dots \right) \tag{73}$$

6.2 4 sectors

6.2.1 Only ε mixing

Hamiltonian:

$$H_4 = \begin{pmatrix} m & 0 & 0 & \varepsilon & 0 & \varepsilon & 0 & \varepsilon \\ 0 & m & \varepsilon & 0 & \varepsilon & 0 & \varepsilon & 0 \\ 0 & \varepsilon & m & 0 & 0 & \varepsilon & 0 & \varepsilon \\ \varepsilon & 0 & 0 & m & \varepsilon & 0 & \varepsilon & 0 \\ 0 & \varepsilon & 0 & \varepsilon & m & 0 & 0 & \varepsilon \\ \varepsilon & 0 & \varepsilon & 0 & 0 & m & \varepsilon & 0 \\ 0 & \varepsilon & 0 & \varepsilon & 0 & \varepsilon & m & 0 \\ \varepsilon & 0 & \varepsilon & 0 & \varepsilon & 0 & 0 & m \end{pmatrix} \tag{74}$$

Reduced Hamiltonian is:

$$H_{4R} = H_4 - I \cdot m = \begin{pmatrix} 0 & 0 & 0 & \varepsilon & 0 & \varepsilon & 0 & \varepsilon \\ 0 & 0 & \varepsilon & 0 & \varepsilon & 0 & \varepsilon & 0 \\ 0 & \varepsilon & 0 & 0 & 0 & \varepsilon & 0 & \varepsilon \\ \varepsilon & 0 & 0 & 0 & \varepsilon & 0 & \varepsilon & 0 \\ 0 & \varepsilon & 0 & \varepsilon & 0 & 0 & 0 & \varepsilon \\ \varepsilon & 0 & \varepsilon & 0 & 0 & 0 & \varepsilon & 0 \\ 0 & \varepsilon & 0 & \varepsilon & 0 & \varepsilon & 0 & 0 \\ \varepsilon & 0 & \varepsilon & 0 & \varepsilon & 0 & 0 & 0 \end{pmatrix} \quad (75)$$

We can get probabilities directly from:

$$S_4 = e^{-iH_{4R}t} \quad (76)$$

$$P = S_4 S_4^\dagger \quad (77)$$

So, we get:

$$P_{00} = 1 - 12 \sin^2\left(\frac{t\varepsilon}{2}\right) + 60 \sin^4\left(\frac{t\varepsilon}{2}\right) - 160 \sin^6\left(\frac{t\varepsilon}{2}\right) + 240 \sin^8\left(\frac{t\varepsilon}{2}\right) - 192 \sin^{10}\left(\frac{t\varepsilon}{2}\right) + 64 \sin^{12}\left(\frac{t\varepsilon}{2}\right),$$

$$P_{01} = \sin^6(t\varepsilon),$$

$$P_{02} = -\frac{1}{2} \sin^4(t\varepsilon) + \frac{1}{4} \sin^2(t\varepsilon) \sin^2(3t\varepsilon) + \frac{7}{16} \sin^2(t\varepsilon) + \frac{1}{16} \sin^2(2t\varepsilon) - \frac{3}{16} \sin^2(3t\varepsilon) + \frac{1}{16} \sin^2(4t\varepsilon),$$

$$P_{03} = -\frac{1}{4} \sin^2(t\varepsilon) \sin^2(3t\varepsilon) + \frac{1}{16} \sin^2(t\varepsilon) + \frac{1}{16} \sin^2(2t\varepsilon) + \frac{3}{16} \sin^2(3t\varepsilon) - \frac{1}{16} \sin^2(4t\varepsilon),$$

$$P_{04} = -\frac{1}{2} \sin^4(t\varepsilon) + \frac{1}{4} \sin^2(t\varepsilon) \sin^2(3t\varepsilon) + \frac{7}{16} \sin^2(t\varepsilon) + \frac{1}{16} \sin^2(2t\varepsilon) - \frac{3}{16} \sin^2(3t\varepsilon) + \frac{1}{16} \sin^2(4t\varepsilon),$$

$$P_{05} = -\frac{1}{4} \sin^2(t\varepsilon) \sin^2(3t\varepsilon) + \frac{1}{16} \sin^2(t\varepsilon) + \frac{1}{16} \sin^2(2t\varepsilon) + \frac{3}{16} \sin^2(3t\varepsilon) - \frac{1}{16} \sin^2(4t\varepsilon),$$

$$P_{06} = -\frac{1}{2} \sin^4(t\varepsilon) + \frac{1}{4} \sin^2(t\varepsilon) \sin^2(3t\varepsilon) + \frac{7}{16} \sin^2(t\varepsilon) + \frac{1}{16} \sin^2(2t\varepsilon) - \frac{3}{16} \sin^2(3t\varepsilon) + \frac{1}{16} \sin^2(4t\varepsilon),$$

$$P_{07} = -\frac{1}{4} \sin^2(t\varepsilon) \sin^2(3t\varepsilon) + \frac{1}{16} \sin^2(t\varepsilon) + \frac{1}{16} \sin^2(2t\varepsilon) + \frac{3}{16} \sin^2(3t\varepsilon) - \frac{1}{16} \sin^2(4t\varepsilon). \quad (78)$$

If we expand we get:

$$\begin{aligned} P_{00} &= 1 - 3t^2\varepsilon^2 + 4t^4\varepsilon^4 - \frac{47}{15}t^6\varepsilon^6 + \frac{169}{105}t^8\varepsilon^8, \\ P_{01} &= t^6\varepsilon^6 - t^8\varepsilon^8, \\ P_{02} &= t^4\varepsilon^4 - \frac{5}{3}t^6\varepsilon^6 + \frac{6}{5}t^8\varepsilon^8, \\ P_{03} &= t^2\varepsilon^2 - \frac{7}{3}t^4\varepsilon^4 + \frac{107}{45}t^6\varepsilon^6 - \frac{442}{315}t^8\varepsilon^8, \\ P_{04} &= t^4\varepsilon^4 - \frac{5}{3}t^6\varepsilon^6 + \frac{6}{5}t^8\varepsilon^8, \\ P_{05} &= t^2\varepsilon^2 - \frac{7}{3}t^4\varepsilon^4 + \frac{107}{45}t^6\varepsilon^6 - \frac{442}{315}t^8\varepsilon^8, \\ P_{06} &= t^4\varepsilon^4 - \frac{5}{3}t^6\varepsilon^6 + \frac{6}{5}t^8\varepsilon^8, \\ P_{07} &= t^2\varepsilon^2 - \frac{7}{3}t^4\varepsilon^4 + \frac{107}{45}t^6\varepsilon^6 - \frac{442}{315}t^8\varepsilon^8. \end{aligned} \quad (79)$$

Probabilities look like:

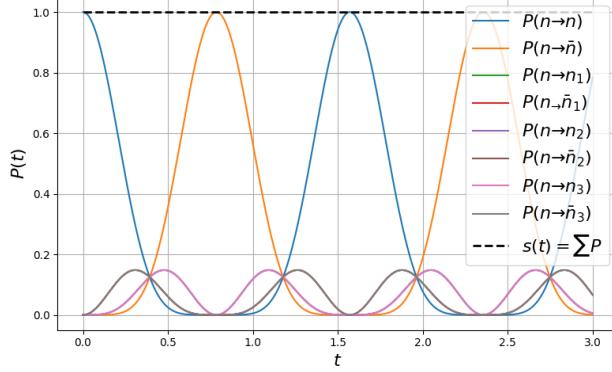


Figure 2: Caption

Probability for neutron to go in mirror sector is:

$$\begin{aligned}
 P_{\text{mirror}} &= \frac{3}{8} (1 - \cos(4\varepsilon t)) = \frac{3}{8} (2 \sin^2(2\varepsilon t)) = \\
 &= \frac{6}{8} \left(4\varepsilon^2 t^2 - \frac{16\varepsilon^4 t^4}{3} + \frac{128\varepsilon^6 t^6}{45} + \dots \right) = \\
 &= 3\varepsilon^2 t^2 - 4\varepsilon^4 t^4 + \frac{32}{15}\varepsilon^6 t^6 + \dots
 \end{aligned} \tag{80}$$

But if we want to see eigenvalues, we can diagonalize and get Diagonalized Hamiltonian:

$$H_D = \begin{pmatrix} -\varepsilon & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -3\varepsilon & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\varepsilon & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \varepsilon & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\varepsilon & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\varepsilon & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3\varepsilon & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon \end{pmatrix} \tag{81}$$

In thiiscase we have 4 different eigenvalues.

6.2.2 ε and η mixing:

$$H_{4_{\varepsilon\eta}} = \begin{pmatrix} m & 0 & 0 & \varepsilon & 0 & \eta & 0 & \eta \\ 0 & m & \varepsilon & 0 & \eta & 0 & \eta & 0 \\ 0 & \varepsilon & m & 0 & 0 & \eta & 0 & \eta \\ \varepsilon & 0 & 0 & m & \eta & 0 & \eta & 0 \\ 0 & \eta & 0 & \eta & m & 0 & 0 & \varepsilon \\ \eta & 0 & \eta & 0 & 0 & m & \varepsilon & 0 \\ 0 & \eta & 0 & \eta & 0 & \varepsilon & m & 0 \\ \eta & 0 & \eta & 0 & \varepsilon & 0 & 0 & m \end{pmatrix} \tag{82}$$

Reduced Hamiltonian is:

$$H_{4r_{\varepsilon\eta}} = \begin{pmatrix} 0 & 0 & 0 & \varepsilon & 0 & \eta & 0 & \eta \\ 0 & 0 & \varepsilon & 0 & \eta & 0 & \eta & 0 \\ 0 & \varepsilon & 0 & 0 & 0 & \eta & 0 & \eta \\ \varepsilon & 0 & 0 & 0 & \eta & 0 & \eta & 0 \\ 0 & \eta & 0 & \eta & 0 & 0 & 0 & \varepsilon \\ \eta & 0 & \eta & 0 & 0 & 0 & \varepsilon & 0 \\ 0 & \eta & 0 & \eta & 0 & \varepsilon & 0 & 0 \\ \eta & 0 & \eta & 0 & \varepsilon & 0 & 0 & 0 \end{pmatrix} \quad (83)$$

The diagonalising matrices:

1.

$$S_{88}^{(1)} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad (84)$$

2.

$$S_{88}^{(2)} = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix} \quad (85)$$

3.

$$S_{88}^{(3)} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (86)$$

The diagonalizing matrix is:

$$S_{88} = S_{88}^{(1)} S_{88}^{(2)} S_{88}^{(3)} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad (87)$$

Diagonalized Hamiltonian is:

$$H_D = \begin{pmatrix} -\varepsilon & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\varepsilon - 2\eta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \varepsilon - 2\eta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \varepsilon & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\varepsilon & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\varepsilon + 2\eta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon + 2\eta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon \end{pmatrix} \quad (88)$$

In this case we have 6 different eigenvalues.

Probabilities of a neutron going in different states calculated directly from:

$$S_{diag} = S e^{i H t} S^\dagger \quad (89)$$

$$P = S_{diag} S_{diag}^\dagger \quad (90)$$

We have P -s:

$$\begin{aligned}
P_{00} &= 1 + t^2(-\varepsilon^2 - 2\eta^2) + t^4 \left(\frac{\varepsilon^4}{3} + 2\varepsilon^2\eta^2 + \frac{5\eta^4}{3} \right) \\
&\quad + t^6 \left(-\frac{2\varepsilon^6}{45} - \frac{2\varepsilon^4\eta^2}{3} - \frac{5\varepsilon^2\eta^4}{3} - \frac{34\eta^6}{45} \right) + t^8 \left(\frac{\varepsilon^8}{315} + \frac{4\varepsilon^6\eta^2}{45} + \frac{5\varepsilon^4\eta^4}{9} + \frac{34\varepsilon^2\eta^6}{45} + \frac{13\eta^8}{63} \right), \\
P_{01} &= t^6 \varepsilon^2 \eta^4 + t^8 \left(-\frac{\varepsilon^4\eta^4}{3} - \frac{2\varepsilon^2\eta^6}{3} \right), \\
P_{02} &= t^4 \eta^4 + t^6 \left(-\varepsilon^2 \eta^4 - \frac{2\eta^6}{3} \right) + t^8 \left(\frac{\varepsilon^4\eta^4}{3} + \frac{2\varepsilon^2\eta^6}{3} + \frac{\eta^8}{5} \right), \\
P_{03} &= t^2 \varepsilon^2 + t^4 \left(-\frac{\varepsilon^4}{3} - 2\varepsilon^2\eta^2 \right) + t^6 \left(\frac{2\varepsilon^6}{45} + \frac{2\varepsilon^4\eta^2}{3} + \frac{5\varepsilon^2\eta^4}{3} \right) \\
&\quad + t^8 \left(-\frac{\varepsilon^8}{315} - \frac{4\varepsilon^6\eta^2}{45} - \frac{5\varepsilon^4\eta^4}{9} - \frac{34\varepsilon^2\eta^6}{45} \right), \\
P_{04} &= t^4 \varepsilon^2 \eta^2 + t^6 \left(-\frac{\varepsilon^4\eta^2}{3} - \frac{4\varepsilon^2\eta^4}{3} \right) + t^8 \left(\frac{2\varepsilon^6\eta^2}{45} + \frac{4\varepsilon^4\eta^4}{9} + \frac{32\varepsilon^2\eta^6}{45} \right), \\
P_{05} &= t^2 \eta^2 + t^4 \left(-\varepsilon^2 \eta^2 - \frac{4\eta^4}{3} \right) + t^6 \left(\frac{\varepsilon^4\eta^2}{3} + \frac{4\varepsilon^2\eta^4}{3} + \frac{32\eta^6}{45} \right) \\
&\quad + t^8 \left(-\frac{2\varepsilon^6\eta^2}{45} - \frac{4\varepsilon^4\eta^4}{9} - \frac{32\varepsilon^2\eta^6}{45} - \frac{64\eta^8}{315} \right), \\
P_{06} &= t^4 \varepsilon^2 \eta^2 + t^6 \left(-\frac{\varepsilon^4\eta^2}{3} - \frac{4\varepsilon^2\eta^4}{3} \right) + t^8 \left(\frac{2\varepsilon^6\eta^2}{45} + \frac{4\varepsilon^4\eta^4}{9} + \frac{32\varepsilon^2\eta^6}{45} \right), \\
P_{07} &= t^2 \eta^2 + t^4 \left(-\varepsilon^2 \eta^2 - \frac{4\eta^4}{3} \right) + t^6 \left(\frac{\varepsilon^4\eta^2}{3} + \frac{4\varepsilon^2\eta^4}{3} + \frac{32\eta^6}{45} \right) \\
&\quad + t^8 \left(-\frac{2\varepsilon^6\eta^2}{45} - \frac{4\varepsilon^4\eta^4}{9} - \frac{32\varepsilon^2\eta^6}{45} - \frac{64\eta^8}{315} \right).
\end{aligned} \tag{91}$$

Visualisation: parameters:

- $\varepsilon = 2$;
- $\eta = 1$,

We get:

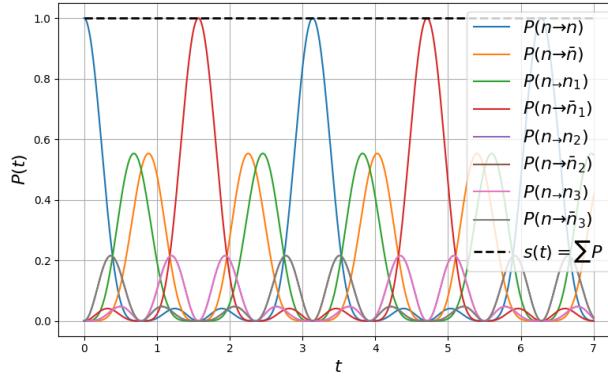


Figure 3: 1 Ordinary and 3 Mirror sectors neutron osc. probabilities with different mixing with different mirror sectors.

6.2.3 Numerical

Hamiltonian:

$$H_{num} = \begin{pmatrix} a \cos \alpha & a \sin \alpha & \eta & \varepsilon \\ a \sin \alpha & -a \cos \alpha & \varepsilon & \eta \\ \eta & \varepsilon & c - b \cos \beta & b \sin \beta \\ \varepsilon & \eta & b \sin \beta & c + b \cos \beta \end{pmatrix} \quad (92)$$

Here we produce 2 set of parameters:

1. Parameters:

- $a = 50$;
- $b = 50$;
- $\varepsilon = 1$;
- $\eta = 0$;
- $\alpha = 0.7$;
- $\beta = 0.5$;
- $c = 0$.

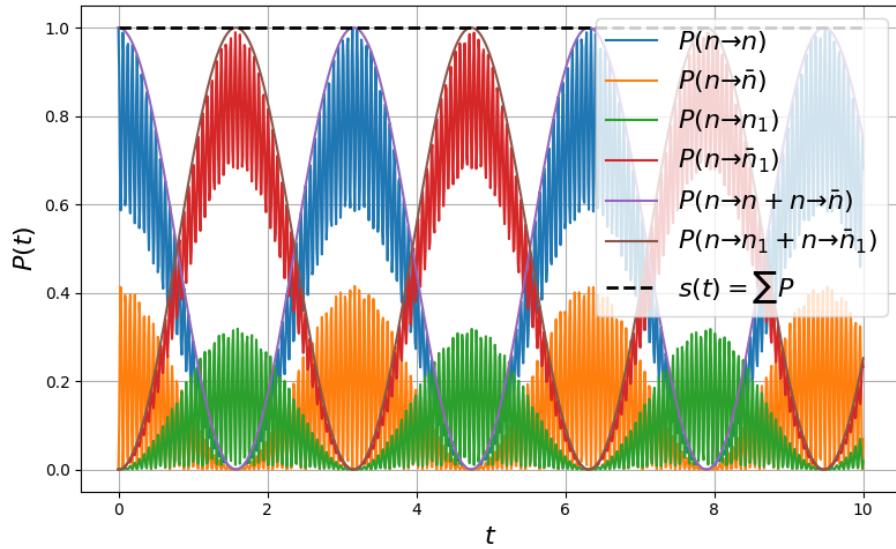


Figure 4: 1 Ordinary and 1 Mirror sectors neutron osc. probabilities with different mixings.

2. Parameters:

- $a = 50$;
- $b = 50$;
- $\varepsilon = 1$;
- $\eta = 0.306$;
- $\alpha = 0.905$;
- $\beta = 0.45$;
- $c = 0.45$

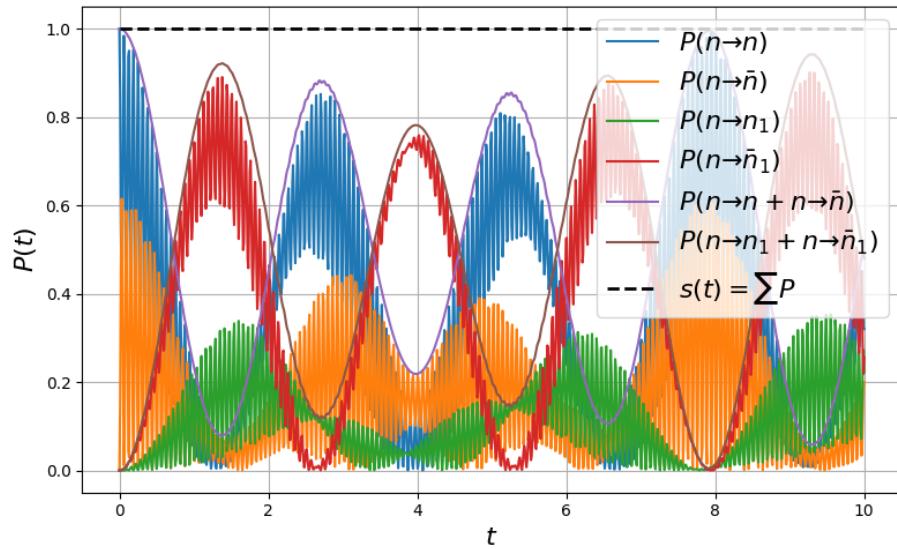


Figure 5: 1 Ordinary and 1 Mirror sectors neutron osc. probabilities with different mixings and different parameters.