L'Aquila Osc. Paper, version 2

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1 General formula for 2×2

For a symmetric matrix M:

$$M = \begin{pmatrix} A & B \\ B & D \end{pmatrix} \tag{1}$$

We can diagonalize this Hamiltonian with a unitary matrix S:

$$S = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{2}$$

we get:

$$\begin{gathered} H_D = S^T H S = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} A & B \\ B & D \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} A\cos\theta - B\sin\theta & A\sin\theta + B\cos\theta \\ B\cos\theta - D\sin\theta & B\sin\theta + D\cos\theta \end{pmatrix} = \\ = \begin{pmatrix} A\cos^2\theta - 2B\sin\theta\cos\theta - D\sin^2\theta & A\sin\theta\cos\theta + B(\cos^2\theta - \sin^2\theta) - D\sin\theta\cos\theta \\ A\sin\theta\cos\theta + B(\cos^2\theta - \sin^2\theta) - D\sin\theta\cos\theta & A\sin^2\theta + 2B\sin\theta\cos\theta + D\cos^2\theta \end{pmatrix} = \\ = \begin{pmatrix} A\cos^2\theta - B\sin(2\theta) - D\sin^2\theta & (A-D)\frac{\sin(2\theta)}{2} + B\cos(2\theta) \\ (A-D)\frac{\sin(2\theta)}{2} + B\cos(2\theta) & A\sin^2\theta + B\sin(2\theta) + D\cos^2\theta \end{pmatrix} \end{gathered} \tag{3}$$

If we want H_D to be diagonal, its nondiagonal terms need to be 0, what constraints the angle θ:

$$\begin{gathered} (D-A)\frac{\sin(2\theta)}{2} + B\cos(2\theta) = 0 \\ \tan(2\theta) = \frac{2B}{D-A} \end{gathered} \tag{4}$$

Then:

$$\begin{gathered} \cos(2\theta) = \sqrt{\frac{1}{1+\tan^2(2\theta)}} = \frac{1}{1+\frac{4B^2}{(D-A)^2}} = \frac{D-A}{\sqrt{(D-A)^2+4B^2}} \\ \sin(2\theta) = \sqrt{1-\cos^2(2\theta)} = \frac{2B}{\sqrt{(D-A)^2+4B^2}} \end{gathered} \tag{5}$$

Then we get diagonal matrix:

$$H_D^2 = \begin{pmatrix} A\frac{1+\cos(2\theta)}{2} - B\sin(2\theta) + D\frac{1-\cos(2\theta)}{2} & 0 \\ 0 & A\frac{1-\cos(2\theta)}{2} + B\sin(2\theta) + D\frac{1+\cos(2\theta)}{2} \end{pmatrix} \tag{6}$$

Then matrix H_D has the form:

$$H_D = \begin{pmatrix} A\left(\frac{1}{2}+\frac{D-A}{2\sqrt{(D-A)^2+4B^2}}\right)-\frac{2B^2}{\sqrt{(D-A)^2+4B^2}}+D\left(\frac{1}{2}-\frac{D-A}{2\sqrt{(D-A)^2+4B^2}}\right) & 0 \\ 0 & A\left(\frac{1}{2}-\frac{D-A}{2\sqrt{(D-A)^2+4B^2}}\right)+\frac{2B^2}{\sqrt{(D-A)^2+4B^2}}+D\left(\frac{1}{2}+\frac{D-A}{2\sqrt{(D-A)^2+4B^2}}\right) \end{pmatrix}$$

$$= \begin{pmatrix} \frac{A+D}{2}+\frac{-(D-A)^2-4B^2}{2\sqrt{(D-A)^2+4B^2}} & 0 \\ 0 & \frac{A+D}{2}+\frac{(D-A)^2+4B^2}{2\sqrt{(D-A)^2+4B^2}} \end{pmatrix} = \begin{pmatrix} \frac{A+D}{2}-\sqrt{(D-A)^2+4B^2} & \\ 0 & \frac{A+D}{2}+\sqrt{(D-A)^2+4B^2} \end{pmatrix} \tag{7}$$

2 2 Level system (precession)

For interaction Hamiltonian we have:

$$H = \begin{pmatrix} 2\omega_3 & 2\omega_1 \\ 2\omega_1 & -2\omega_3 \end{pmatrix} \tag{8}$$

where:

$$\begin{aligned} \omega_1 &= \omega \sin\alpha \\ \omega_3 &= \omega \cos\alpha \end{aligned} \tag{9}$$

We can diagonalize it with matrix:

$$S = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}, \tag{10}$$

where

$$\tan 2\theta = \frac{\omega_1}{\omega_3} \tag{11}$$

Then we have:

$$\begin{aligned} \sin(2\theta) &= \frac{\omega_1}{\omega_1+\omega_3} \\ \cos(2\theta) &= \frac{\omega_3}{\omega_1+\omega_3} \end{aligned} \tag{12}$$

If we diagonalize, we get:

$$H_D = S^T H S = \begin{pmatrix} 2\sqrt{\omega_1^2+\omega_3^2} & 0 \\ 0 & -2\sqrt{\omega_1^2+\omega_3^2} \end{pmatrix} \tag{13}$$

Matrix, which shows the osc. amplitudes is:

$$\begin{aligned} S_{diag} &= S e^{-iHt} S^T = \\ &= \begin{pmatrix} e^{2it\sqrt{\omega_1^2+\omega_3^2}}\sin^2(\theta)+e^{-2it\sqrt{\omega_1^2+\omega_3^2}}\cos^2(\theta) & -i\sin(2\theta)\sin(2t\sqrt{\omega_1^2+\omega_3^2}) \\ -i\sin(2\theta)\sin(2t\sqrt{\omega_1^2+\omega_3^2}) & e^{2it\sqrt{\omega_1^2+\omega_3^2}}\cos^2(\theta)+e^{-2it\sqrt{\omega_1^2+\omega_3^2}}\sin^2(\theta) \end{pmatrix} \end{aligned} \tag{14}$$

Then we can calculate transition probability from state 1 to state 2 and also wise versa, squaring the amplitude moduls. Better to show in matrix form (also we consider decay rate):

So, we have:

$$P = e^{-\Gamma t} \begin{pmatrix} 1-\sin^2(2\theta)\sin^2(2t\sqrt{\omega_1^2+\omega_3^2}) & \sin^2(2\theta)\sin^2(2t\sqrt{\omega_1^2+\omega_3^2}) \\ \sin^2(2\theta)\sin^2(2t\sqrt{\omega_1^2+\omega_3^2}) & 1-\sin^2(2\theta)\sin^2(2t\sqrt{\omega_1^2+\omega_3^2}) \end{pmatrix} \tag{15}$$

Simplifying:

$$P = e^{-\Gamma t} \begin{pmatrix} 1-\sin^2(2\theta)\sin^2(2\omega t) & \sin^2(2\theta)\sin^2(2\omega t) \\ \sin^2(2\theta)\sin^2(2\omega t) & 1-\sin^2(2\theta)\sin^2(2\omega t) \end{pmatrix} \tag{16}$$

Now let us define the osc. frequncy ω as:

$$\Omega = 2\omega \tag{17}$$

And using (56)

$$\sin^2(2\theta) = \frac{\omega^2 \sin^2\alpha}{\omega^2(\sin\alpha + \cos\alpha)^2} = \frac{\sin^2\alpha}{(\sin\alpha + \cos\alpha)^2} \tag{18}$$

Then Osc. Probability has the form:

$$P = e^{-\Gamma t}\begin{pmatrix} 1 - \frac{\sin^2\alpha}{(\sin\alpha+\cos\alpha)^2}\sin^2(\Omega t) & \frac{\sin^2\alpha}{(\sin\alpha+\cos\alpha)^2}\sin^2(\Omega t) \\ \frac{\sin^2\alpha}{(\sin\alpha+\cos\alpha)^2})\sin^2(\Omega t) & 1 - \frac{\sin^2\alpha}{(\sin\alpha+\cos\alpha)^2}\sin^2(\Omega t) \end{pmatrix} \tag{19}$$

Now for the limiting case $\Omega t \ll 1$ for probabilities for state 1 to stay in state 1 or go to state 2, we get:

$$\begin{aligned} P_{1-1} &= e^{-\Gamma t}\left(1 - \Omega^2 t^2 \frac{\sin^2\alpha}{(\sin\alpha + \cos\alpha)^2}\right) \\ P_{1-2} &= e^{-\Gamma t}\Omega^2 t^2 \frac{\sin^2\alpha}{(\sin\alpha + \cos\alpha)^2} \end{aligned} \tag{20}$$

3 New form

$$H = \begin{pmatrix} 2\boldsymbol{\omega}\cdot\boldsymbol{\sigma} & \varepsilon \\ \varepsilon & 2\boldsymbol{\omega'}\cdot\boldsymbol{\sigma} \end{pmatrix} \tag{21}$$

$$H^2 = \begin{pmatrix} 2\boldsymbol{\omega}\cdot\boldsymbol{\sigma} & \varepsilon\cdot\sigma_0 \\ \varepsilon\cdot\sigma_0 & 2\boldsymbol{\omega'}\cdot\boldsymbol{\sigma} \end{pmatrix}\begin{pmatrix} 2\boldsymbol{\omega}\cdot\boldsymbol{\sigma} & \varepsilon\cdot\sigma_0 \\ \varepsilon\cdot\sigma_0 & 2\boldsymbol{\omega'}\cdot\boldsymbol{\sigma} \end{pmatrix} \tag{22}$$

$$H^2 = \begin{pmatrix} 4\omega^2+\varepsilon^2 & 0 & 2\varepsilon(\omega_3+\omega'_3) & 2\varepsilon(\omega_1+\omega'_1) \\ 0 & 4\omega^2+\varepsilon^2 & 2\varepsilon(\omega_1+\omega'_1) & -2\varepsilon(\omega_3+\omega'_3) \\ 2\varepsilon(\omega_3+\omega'_3) & 2\varepsilon(\omega_1+\omega'_1) & 4\omega'^2+\varepsilon^2 & 0 \\ 2\varepsilon(\omega_1+\omega'_1) & -2\varepsilon(\omega_3+\omega'_3) & 0 & 4\omega'^2+\varepsilon^2 \end{pmatrix} \tag{23}$$

where:

$$\begin{aligned} \omega_1 &= \omega\sin(\alpha) \\ \omega_3 &= \omega\cos(\alpha) \\ \omega'_1 &= \omega'\sin(\alpha') \\ \omega'_3 &= \omega'\cos(\alpha') \end{aligned} \tag{24}$$

Now if we rotate H^2 with the matrix S_1:

$$S_1 = \begin{pmatrix} \cos(\phi) & \sin(\phi) & 0 & 0 \\ -\sin(\phi) & \cos(\phi) & 0 & 0 \\ 0 & 0 & \cos(\phi) & \sin(\phi) \\ 0 & 0 & -\sin(\phi) & \cos(\phi) \end{pmatrix} \tag{25}$$

We get:

$$\begin{pmatrix} \varepsilon^2+4\omega^2 & 0 & 2\varepsilon(-\sin(2\phi)(\omega_1+\omega'_1)+\cos(2\phi)(\omega_3+\omega'_3)) & 2\varepsilon(\cos(2\phi)(\omega_1+\omega'_1)+\sin(2\phi)(\omega_3+\omega'_3)) \\ 0 & \varepsilon^2+4\omega^2 & 2\varepsilon(\cos(2\phi)(\omega_1+\omega'_1)+\sin(2\phi)(\omega_3+\omega'_3)) & 2\varepsilon(\sin(2\phi)(\omega_1+\omega'_1)-\cos(2\phi)(\omega_3+\omega'_3)) \\ 2\varepsilon(-\sin(2\phi)(\omega_1+\omega'_1)+\cos(2\phi)(\omega_3+\omega'_3)) & 2\varepsilon(\cos(2\phi)(\omega_1+\omega'_1)+\sin(2\phi)(\omega_3+\omega'_3)) & \varepsilon^2+4\omega'^2 & 0 \\ 2\varepsilon(\cos(2\phi)(\omega_1+\omega'_1)+\sin(2\phi)(\omega_3+\omega'_3)) & 2\varepsilon(\sin(2\phi)(\omega_1+\omega'_1)-\cos(2\phi)(\omega_3+\omega'_3)) & 0 & \varepsilon^2+4\omega'^2 \end{pmatrix} \tag{26}$$

From here, components $(1,4),(2,3),(3,2),(4,1)$ to be 0, we should have:

$$\tan(2\phi) = -\frac{\omega_1+\omega_1'}{\omega_3+\omega_3'} \tag{27}$$

And the Matrix (25) becomes blockdiagonal:

$$H_1^2 = S_1^T H^2 S_1 =$$
$$= \begin{pmatrix} \varepsilon^2+4\omega^2 & 0 & 2\varepsilon\frac{(\omega_1+\omega_1')^2+(\omega_3+\omega_3')^2}{\sqrt{(\omega_1+\omega_1')^2+(\omega_3+\omega_3')^2}} & 0 \\ 0 & \varepsilon^2+4\omega^2 & 0 & -2\varepsilon\frac{(\omega_1+\omega_1')^2+(\omega_3+\omega_3')^2}{\sqrt{(\omega_1+\omega_1')^2+(\omega_3+\omega_3')^2}} \\ 2\varepsilon\frac{(\omega_1+\omega_1')^2+(\omega_3+\omega_3')^2}{\sqrt{(\omega_1+\omega_1')^2+(\omega_3+\omega_3')^2}} & 0 & \varepsilon^2+4{\omega'}^2 & 0 \\ 0 & -2\varepsilon\frac{(\omega_1+\omega_1')^2+(\omega_3+\omega_3')^2}{\sqrt{(\omega_1+\omega_1')^2+(\omega_3+\omega_3')^2}} & 0 & \varepsilon^2+4{\omega'}^2 \end{pmatrix} \tag{28}$$

Which can be written:

$$H_1^2 = \begin{pmatrix} \varepsilon^2+4\omega^2 & 0 & 2\varepsilon\sqrt{(\omega_1+\omega_1')^2+(\omega_3+\omega_3')^2} & 0 \\ 0 & \varepsilon^2+4\omega^2 & 0 & -2\varepsilon\sqrt{(\omega_1+\omega_1')^2+(\omega_3+\omega_3')^2} \\ 2\varepsilon\sqrt{(\omega_1+\omega_1')^2+(\omega_3+\omega_3')^2} & 0 & \varepsilon^2+4\omega^2 & 0 \\ 0 & -2\varepsilon\sqrt{(\omega_1+\omega_1')^2+(\omega_3+\omega_3')^2} & 0 & \varepsilon^2+4\omega^2 \end{pmatrix} \tag{29}$$

Then, we rotate H_3^2 with the matrix S_1:

$$S_2 = \begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & \cos\theta & 0 & -\sin\theta \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & \sin\theta & 0 & \cos\theta \end{pmatrix} \tag{30}$$

We get (for simplicity, I will write d_1, d_2, d_3, d_4 instead of diagonal terms for here and then simplify them:

$$H_2^2 = S_2^T H_1^2 S_2 = \begin{pmatrix} d_1 & 0 & a & 0 \\ 0 & d_2 & 0 & -a \\ a & 0 & d_3 & 0 \\ 0 & -a & 0 & d_4 \end{pmatrix} \tag{31}$$

where

$$\begin{aligned} &a = -2\varepsilon[\sin(2\phi)\cos(2\theta)(\omega_1+\omega_1') + 2\varepsilon\cos(2\theta)\cos(2\phi)(\omega_3+\omega_3')] + 2\sin(2\theta)(\omega_1^2+\omega_3^2-{\omega_1'}^2-{\omega_3'}^2) \\ &d_1, d_2, d_3, d_4 \text{ (given later)} \end{aligned} \tag{32}$$

Now if we simplify a, we get:

$$\begin{aligned} a =& -2\varepsilon[\sin(2\phi)\cos(2\theta)(\omega_1+\omega_1') + 2\varepsilon\cos(2\theta)\cos(2\phi)(\omega_3+\omega_3')] + 2\sin(2\theta)(\omega_1^2+\omega_3^2-{\omega_1'}^2-{\omega_3'}^2) = \\ =& \, 2\varepsilon\cos(2\theta)[-\sin(2\phi)(\omega_1+\omega_1') + \cos(2\phi)(\omega_3+\omega_3')] + 2\tan(2\theta)\cos(2\theta)(\omega_1^2+\omega_3^2-{\omega_1'}^2-{\omega_3'}^2) \end{aligned} \tag{33}$$

But now if we want (30) to be diagonal, we mast constrain θ:

$$\tan(2\theta) = \frac{4\varepsilon[-\sin(2\phi)(\omega_1+\omega_1') + \cos(2\phi)(\omega_3+\omega_3')]}{4{\omega'}^2 - 4\omega^2} \tag{34}$$

Then a becomes:

$$a = \cos(2\theta)\tan(2\theta)\frac{4{\omega'}^2-4\omega^2}{2} + 2\tan(2\theta)\cos(2\theta)(\omega^2+{\omega'}^2) = 0 \tag{35}$$

Then we have diagonalized H_2:

$$H_D^2 = \begin{pmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 \\ 0 & 0 & d_3 & 0 \\ 0 & 0 & 0 & d_4 \end{pmatrix} \tag{36}$$

where:

$$\begin{aligned} d_1 = d_2 &= \varepsilon^2 + \frac{2(\omega^2 - {\omega'}^2)}{\cos(2\theta)} + 2(\omega^2 + {\omega'}^2) \\ d_3 = d_4 &= \varepsilon^2 + \frac{2({\omega'}^2 - \omega^2)}{\cos(2\theta)} + 2(\omega^2 + {\omega'}^2) \end{aligned} \tag{37}$$

To sum up, the matrix which diagonalizes H^2 is:

$$S = S_1 \cdot S_2 = \begin{pmatrix} \cos(\theta)\cos(\phi) & \sin(\phi)\cos(\theta) & \sin(\theta)\cos(\phi) & -\sin(\theta)\sin(\phi) \\ -\sin(\phi)\cos(\theta) & \cos(\theta)\cos(\phi) & -\sin(\theta)\sin(\phi) & -\sin(\theta)\cos(\phi) \\ -\sin(\theta)\cos(\phi) & \sin(\theta)\sin(\phi) & \cos(\theta)\cos(\phi) & \sin(\phi)\cos(\theta) \\ \sin(\theta)\sin(\phi) & \sin(\theta)\cos(\phi) & -\sin(\phi)\cos(\theta) & \cos(\theta)\cos(\phi) \end{pmatrix} \tag{38}$$

And:

$$H_D^2 = S^T H^2 S = \begin{pmatrix} \varepsilon^2 + \frac{2(\omega^2 - {\omega'}^2)}{\cos(2\theta)} + 2(\omega^2 + {\omega'}^2) & 0 & 0 & 0 \\ 0 & \varepsilon^2 + \frac{2(\omega^2 - {\omega'}^2)}{\cos(2\theta)} + 2(\omega^2 + {\omega'}^2) & 0 & 0 \\ 0 & 0 & \varepsilon^2 + \frac{2({\omega'}^2 - \omega^2)}{\cos(2\theta)} + 2(\omega^2 + {\omega'}^2) & 0 \\ 0 & 0 & 0 & \varepsilon^2 + \frac{2({\omega'}^2 - \omega^2)}{\cos(2\theta)} + 2(\omega^2 + {\omega'}^2) \end{pmatrix} \tag{39}$$

Where d_1 and d_2 correspond to $(2\tilde{\omega})^2$, and d_3 and d_4 – $(2\tilde{\omega}')^2$.

$$\begin{aligned} \tan^2(2\theta) &= \frac{16\varepsilon^2[(\omega_1 + \omega_1')^2 + (\omega_3 + \omega_3')^2]^2}{(4{\omega'}^2 - 4\omega^2)^2} \\ 1 + \tan^2(2\theta) &= \frac{(4{\omega'}^2 - 4\omega^2)^2 + 16\varepsilon^2[(\omega_1 + \omega_1')^2 + (\omega_3 + \omega_3')^2]^2}{(4{\omega'}^2 - \omega^2)^2} \\ \cos(2\theta) &= \sqrt{\frac{1}{1 + \tan^2(2\theta)}} = \frac{4{\omega'}^2 - 4\omega^2}{\sqrt{(4{\omega'}^2 - 4\omega^2)^2 + 16\varepsilon^2[(\omega_1 + \omega_1')^2 + (\omega_3 + \omega_3')^2]^2}} \\ \sin(2\theta) &= \sqrt{1 - \cos^2(2\theta)} = \frac{16\varepsilon^2[(\omega_1 + \omega_1')^2 + (\omega_3 + \omega_3')^2]}{\sqrt{(4{\omega'}^2 - 4\omega^2)^2 + 16\varepsilon^2[(\omega_1 + \omega_1')^2 + (\omega_3 + \omega_3')^2]^2}} \end{aligned} \tag{40}$$

Then the first two diagonal terms of H_D ($H_D^2[1,1]$ and $H_D[2,2]$) have the form:

$$\begin{aligned} H_D^2[1,1] = H_D^2[2,2] = \varepsilon^2 + 2(\omega^2 + {\omega'}^2) + \frac{2(\omega^2 - {\omega'}^2)\sqrt{(4{\omega'}^2 - 4\omega^2)^2 + 16\varepsilon^2[(\omega_1 + \omega_1')^2 + (\omega_3 + \omega_3')^2]^2}}{4({\omega'}^2 - \omega^2)} = \\ = \varepsilon^2 + 2(\omega^2 + {\omega'}^2) - \sqrt{(\omega^2 - {\omega'}^2)^2 + 4\varepsilon^2[(\omega_1 + \omega_1')^2 + (\omega_3 + \omega_3')^2]^2} \end{aligned} \tag{41}$$

And:

$$\begin{aligned} H_D^2[3,3] = H_D^2[4,4] = \varepsilon^2 + 2(\omega^2 + {\omega'}^2) + \frac{2({\omega'}^2 - \omega^2)\sqrt{(4{\omega'}^2 - 4\omega^2)^2 + 16\varepsilon^2[(\omega_1 + \omega_1')^2 + (\omega_3 + \omega_3')^2]^2}}{4({\omega'}^2 - \omega^2)} = \\ = \varepsilon^2 + 2(\omega^2 + {\omega'}^2) + \sqrt{(\omega^2 - {\omega'}^2)^2 + 4\varepsilon^2[(\omega_1 + \omega_1')^2 + (\omega_3 + \omega_3')^2]^2} \end{aligned} \tag{42}$$

So, for H_D^2 we have:

$$H_D^2 = \begin{pmatrix} \varepsilon^2 + 2(\omega^2+\omega'^2) - \sqrt{(\omega^2-\omega'^2)^2 + 4\varepsilon^2[(\omega_1+\omega_1')^2+(\omega_3+\omega_3')^2]^2} & 0 & 0 & 0 \\ 0 & \varepsilon^2 + 2(\omega^2+\omega'^2) - \sqrt{(\omega^2-\omega'^2)^2 + 4\varepsilon^2[(\omega_1+\omega_1')^2+(\omega_3+\omega_3')^2]^2} & 0 & 0 \\ 0 & 0 & \varepsilon^2 + 2(\omega^2+\omega'^2) + \sqrt{(\omega^2-\omega'^2)^2 + 4\varepsilon^2[(\omega_1+\omega_1')^2+(\omega_3+\omega_3')^2]^2} & 0 \\ 0 & 0 & 0 & \varepsilon^2 + 2(\omega^2+\omega'^2) + \sqrt{(\omega^2-\omega'^2)^2 + 4\varepsilon^2[(\omega_1+\omega_1')^2+(\omega_3+\omega_3')^2]^2} \end{pmatrix} \tag{43}$$

Generaly we have:

$$H = \begin{pmatrix} 2\boldsymbol{\omega}\cdot\boldsymbol{\sigma} & \varepsilon\cdot\sigma_0 \\ \varepsilon\cdot\sigma_0 & 2\boldsymbol{\omega'}\cdot\boldsymbol{\sigma} \end{pmatrix} = \begin{pmatrix} 2\omega_3 & 2\omega_1 & \varepsilon & 0 \\ 2\omega_1 & -2\omega_3 & 0 & \varepsilon \\ \varepsilon & 0 & 2\omega_3' & 2\omega_1' \\ 0 & \varepsilon & 2\omega_1' & -2\omega_3' \end{pmatrix} \tag{44}$$

Then rotating with S_1 we have:

$$H_1 = S_1^T H S_1 = \begin{pmatrix} -2\omega\sin(\alpha)\sin(2\phi) + 2\omega\cos(\alpha)\cos(2\phi) & 2\omega\sin(\alpha)\cos(2\phi) + 2\omega\cos(\alpha)\sin(2\phi) & \varepsilon & 0 \\ 2\omega\sin(\alpha)\cos(2\phi) + 2\omega\cos(\alpha)\sin(2\phi) & 2\omega\sin(\alpha)\sin(2\phi) - 2\omega\cos(\alpha)\cos(2\phi) & 0 & \varepsilon \\ \varepsilon & 0 & -2\omega'\sin(\alpha')\sin(2\phi) + 2\omega'\cos(\alpha')\cos(2\phi) & 2\omega'\sin(\alpha')\cos(2\phi) + 2\omega'\cos(\alpha'\sin(2\phi) \\ 0 & \varepsilon & 2\omega'\sin(\alpha')\cos(2\phi) + 2\omega'\cos(\alpha'\sin(2\phi) & 2\omega'\sin(\alpha')\sin(2\phi) - 2\omega'\cos(\alpha')\cos(2\phi) \end{pmatrix} \tag{45}$$

Using here (58), (55) and considering $\sin(2\phi) < 0$ because we need $\tan(2\phi)$ with $-$, we get:

$$H_1 = \begin{pmatrix} 2\omega_1\frac{(\omega_1+\omega_1')}{\sqrt{(\omega_3+\omega_3')^2+(\omega_1+\omega_1')^2}} + 2\omega_3\frac{(\omega_3+\omega_3')}{\sqrt{(\omega_3+\omega_3')^2+(\omega_1+\omega_1')^2}} & 2\omega_1\frac{(\omega_3+\omega_3')}{\sqrt{(\omega_3+\omega_3')^2+(\omega_1+\omega_1')^2}} - 2\omega_3\frac{(\omega_1+\omega_1')}{\sqrt{(\omega_3+\omega_3')^2+(\omega_1+\omega_1')^2}} & \varepsilon & 0 \\ 2\omega_1\frac{(\omega_3+\omega_3')}{\sqrt{(\omega_3+\omega_3')^2+(\omega_1+\omega_1')^2}} - 2\omega_3\frac{(\omega_1+\omega_1')}{\sqrt{(\omega_3+\omega_3')^2+(\omega_1+\omega_1')^2}} & -2\omega_1\frac{(\omega_1+\omega_1')}{\sqrt{(\omega_3+\omega_3')^2+(\omega_1+\omega_1')^2}} - 2\omega_3\frac{(\omega_3+\omega_3')}{\sqrt{(\omega_3+\omega_3')^2+(\omega_1+\omega_1')^2}} & 0 & \varepsilon \\ \varepsilon & 0 & 2\omega_1'\frac{(\omega_1+\omega_1')}{\sqrt{(\omega_3+\omega_3')^2+(\omega_1+\omega_1')^2}} + 2\omega_3'\frac{(\omega_3+\omega_3')}{\sqrt{(\omega_3+\omega_3')^2+(\omega_1+\omega_1')^2}} & 2\omega_1'\frac{(\omega_3+\omega_3')}{\sqrt{(\omega_3+\omega_3')^2+(\omega_1+\omega_1')^2}} - 2\omega_3'\frac{(\omega_1+\omega_1')}{\sqrt{(\omega_3+\omega_3')^2+(\omega_1+\omega_1')^2}} \\ 0 & \varepsilon & 2\omega_1'\frac{(\omega_3+\omega_3')}{\sqrt{(\omega_3+\omega_3')^2+(\omega_1+\omega_1')^2}} - 2\omega_3'\frac{(\omega_1+\omega_1')}{\sqrt{(\omega_3+\omega_3')^2+(\omega_1+\omega_1')^2}} & -2\omega_1'\frac{(\omega_1+\omega_1')}{\sqrt{(\omega_3+\omega_3')^2+(\omega_1+\omega_1')^2}} - 2\omega_3'\frac{(\omega_3+\omega_3')}{\sqrt{(\omega_3+\omega_3')^2+(\omega_1+\omega_1')^2}} \end{pmatrix} \tag{46}$$

Simplification gives:

$$H_1 = \begin{pmatrix} \frac{2\omega^2+2(\omega_1\omega_1'+\omega_3\omega_3')}{\sqrt{(\omega_3+\omega_3')^2+(\omega_1+\omega_1')^2}} & \frac{2(\omega_1\omega_3'-\omega_3\omega_1')}{\sqrt{(\omega_3+\omega_3')^2+(\omega_1+\omega_1')^2}} & \varepsilon & 0 \\ \frac{2(\omega_1\omega_3'-\omega_3\omega_1')}{\sqrt{(\omega_3+\omega_3')^2+(\omega_1+\omega_1')^2}} & -\frac{2\omega^2+2(\omega_1\omega_1'+\omega_3\omega_3')}{\sqrt{(\omega_3+\omega_3')^2+(\omega_1+\omega_1')^2}} & 0 & \varepsilon \\ \varepsilon & 0 & \frac{2\omega'^2+2(\omega_1\omega_1'+\omega_3\omega_3')}{\sqrt{(\omega_3+\omega_3')^2+(\omega_1+\omega_1')^2}} & \frac{2(\omega_1'\omega_3-\omega_3'\omega_1)}{\sqrt{(\omega_3+\omega_3')^2+(\omega_1+\omega_1')^2}} \\ 0 & \varepsilon & \frac{2(\omega_1'\omega_3-\omega_3'\omega_1)}{\sqrt{(\omega_3+\omega_3')^2+(\omega_1+\omega_1')^2}} & -\frac{2\omega'^2+2(\omega_1\omega_1'+\omega_3\omega_3')}{\sqrt{(\omega_3+\omega_3')^2+(\omega_1+\omega_1')^2}} \end{pmatrix} \tag{47}$$

This is now in form of the first Hamiltonian given in paper which has $\boldsymbol{\omega}+\boldsymbol{\omega'}$ in z direction.

4 Matrix 8x8

Now the interaction Hamiltonian is:

$$H_8 = \begin{pmatrix} 2\boldsymbol{\omega}\cdot\boldsymbol{\sigma} & 0 & \varepsilon & \eta \\ 0 & -2\boldsymbol{\omega}\cdot\boldsymbol{\sigma} & \eta & \varepsilon \\ \varepsilon & \eta & 2\boldsymbol{\omega'}\cdot\boldsymbol{\sigma} & 0 \\ \eta & \varepsilon & 0 & -2\boldsymbol{\omega'}\cdot\boldsymbol{\sigma} \end{pmatrix} \tag{48}$$

$$H_8 = \begin{pmatrix} 2\omega_3 & 2\omega_1 & 0 & 0 & \varepsilon & 0 & \eta & 0 \\ 2\omega 1 & -2\omega_3 & 0 & 0 & 0 & \varepsilon & 0 & \eta \\ 0 & 0 & -2\omega_3 & -2\omega_1 & \eta & 0 & \varepsilon & 0 \\ 0 & 0 & -2\omega_1 & 2\omega_3 & 0 & \eta & 0 & \varepsilon \\ \varepsilon & 0 & \eta & 0 & 2\omega_3' & 2\omega_1' & 0 & 0 \\ 0 & \varepsilon & 0 & \eta & 2\omega_1' & -2\omega_3' & 0 & 0 \\ \eta & 0 & \varepsilon & 0 & 0 & 0 & -2\omega_3' & -2\omega_1' \\ 0 & \eta & 0 & \varepsilon & 0 & 0 & -2\omega_1' & 2\omega_3' \end{pmatrix} \tag{49}$$

5 Different number sectors squared H

- 2×2: Hamiltonian:

$$H_2 = \begin{pmatrix} \boldsymbol{\omega} \cdot \boldsymbol{\sigma} & \varepsilon \\ \varepsilon & -\boldsymbol{\omega'} \cdot \boldsymbol{\sigma} \end{pmatrix} \tag{50}$$

$$H_2^2 = \begin{pmatrix} \varepsilon^2 + \sigma^2\omega^2 & \varepsilon\boldsymbol{\sigma} \cdot (\boldsymbol{\omega} - \boldsymbol{\omega'}) \\ \varepsilon\boldsymbol{\sigma} \cdot (\boldsymbol{\omega} - \boldsymbol{\omega'}) & \varepsilon^2 + \sigma^2\omega^2 \end{pmatrix} = \begin{pmatrix} \varepsilon^2 + \omega^2 & \varepsilon\boldsymbol{\sigma} \cdot (\boldsymbol{\omega} - \boldsymbol{\omega'}) \\ \varepsilon\boldsymbol{\sigma} \cdot (\boldsymbol{\omega} - \boldsymbol{\omega'}) & \varepsilon^2 + \omega^2 \end{pmatrix} \tag{51}$$

- 4x4
  Hamiltonian:

$$H_{4eps} = \begin{pmatrix} \boldsymbol{\omega} \cdot \boldsymbol{\sigma} & 0 & 0 & \varepsilon \\ 0 & -\boldsymbol{\omega} \cdot \boldsymbol{\sigma} & \varepsilon & 0 \\ 0 & \varepsilon & \boldsymbol{\omega'} \cdot \boldsymbol{\sigma} & 0 \\ \varepsilon & 0 & 0 & -\boldsymbol{\omega'} \cdot \boldsymbol{\sigma} \end{pmatrix} \tag{52}$$

$$\begin{aligned} H_{4eps}^2 &= \begin{pmatrix} \varepsilon^2 + \sigma^2\omega^2 & 0 & 0 & \varepsilon\boldsymbol{\sigma} \cdot (\boldsymbol{\omega} - \boldsymbol{\omega'}) \\ 0 & \varepsilon^2 + \sigma^2\omega^2 & \varepsilon\boldsymbol{\sigma} \cdot (-\boldsymbol{\omega} + \boldsymbol{\omega'}) & 0 \\ 0 & \varepsilon\boldsymbol{\sigma} \cdot (-\boldsymbol{\omega} + \boldsymbol{\omega'}) & \varepsilon^2 + \sigma^2\omega^2 & 0 \\ \varepsilon\boldsymbol{\sigma} \cdot (\boldsymbol{\omega} - \boldsymbol{\omega'}) & 0 & 0 & \varepsilon^2 + \sigma^2\omega^2 \end{pmatrix} \\ &= \begin{pmatrix} \varepsilon^2 + \omega^2 & 0 & 0 & \varepsilon\boldsymbol{\sigma} \cdot (\boldsymbol{\omega} - \boldsymbol{\omega'}) \\ 0 & \varepsilon^2 + \omega^2 & \varepsilon\boldsymbol{\sigma} \cdot (-\boldsymbol{\omega} + \boldsymbol{\omega'}) & 0 \\ 0 & \varepsilon\boldsymbol{\sigma} \cdot (-\boldsymbol{\omega} + \boldsymbol{\omega'}) & \varepsilon^2 + \omega^2 & 0 \\ \varepsilon\boldsymbol{\sigma} \cdot (\boldsymbol{\omega} - \boldsymbol{\omega'}) & 0 & 0 & \varepsilon^2 + \omega^2 \end{pmatrix} \end{aligned} \tag{53}$$

  Hamiltonian:

$$H_{4eteps} = \begin{pmatrix} \boldsymbol{\omega} \cdot \boldsymbol{\sigma} & 0 & \eta & \varepsilon \\ 0 & -\boldsymbol{\omega} \cdot \boldsymbol{\sigma} & \varepsilon & \eta \\ \eta & \varepsilon & \boldsymbol{\omega'} \cdot \boldsymbol{\sigma} & 0 \\ \varepsilon & \eta & 0 & -\boldsymbol{\omega'} \cdot \boldsymbol{\sigma} \end{pmatrix} \tag{54}$$

$$\begin{aligned} H_{4eteps}^2 &= \begin{pmatrix} \varepsilon^2 + \eta^2 + \sigma^2\omega^2 & 2\eta\varepsilon & \eta\boldsymbol{\sigma} \cdot (\boldsymbol{\omega} + \boldsymbol{\omega'}) & \varepsilon\boldsymbol{\sigma} \cdot (\boldsymbol{\omega} - \boldsymbol{\omega'}) \\ 2\eta\varepsilon & \varepsilon^2 + \eta^2 + \sigma^2\omega^2 & \varepsilon\boldsymbol{\sigma} \cdot (-\boldsymbol{\omega} + \boldsymbol{\omega'}) & \eta\boldsymbol{\sigma} \cdot (-\boldsymbol{\omega} - \boldsymbol{\omega'}) \\ \eta\boldsymbol{\sigma} \cdot (\boldsymbol{\omega} + \boldsymbol{\omega'}) & \varepsilon\boldsymbol{\sigma} \cdot (-\boldsymbol{\omega} + \boldsymbol{\omega'}) & \varepsilon^2 + \eta^2 + \sigma^2\omega^2 & 2\eta\varepsilon \\ \varepsilon\boldsymbol{\sigma} \cdot (\boldsymbol{\omega} - \boldsymbol{\omega'}) & \eta\boldsymbol{\sigma} \cdot (-\boldsymbol{\omega} - \boldsymbol{\omega'}) & 2\eta\varepsilon & \varepsilon^2 + \eta^2 + \sigma^2\omega^2 \end{pmatrix} \\ &= \begin{pmatrix} \varepsilon^2 + \eta^2 + \omega^2 & 2\eta\varepsilon & \eta\boldsymbol{\sigma} \cdot (\boldsymbol{\omega} + \boldsymbol{\omega'}) & \varepsilon\boldsymbol{\sigma} \cdot (\boldsymbol{\omega} - \boldsymbol{\omega'}) \\ 2\eta\varepsilon & \varepsilon^2 + \eta^2 + \omega^2 & \varepsilon\boldsymbol{\sigma} \cdot (-\boldsymbol{\omega} + \boldsymbol{\omega'}) & \eta\boldsymbol{\sigma} \cdot (-\boldsymbol{\omega} - \boldsymbol{\omega'}) \\ \eta\boldsymbol{\sigma} \cdot (\boldsymbol{\omega} + \boldsymbol{\omega'}) & \varepsilon\boldsymbol{\sigma} \cdot (-\boldsymbol{\omega} + \boldsymbol{\omega'}) & \varepsilon^2 + \eta^2 + \omega^2 & 2\eta\varepsilon \\ \varepsilon\boldsymbol{\sigma} \cdot (\boldsymbol{\omega} - \boldsymbol{\omega'}) & \eta\boldsymbol{\sigma} \cdot (-\boldsymbol{\omega} - \boldsymbol{\omega'}) & 2\eta\varepsilon & \varepsilon^2 + \eta^2 + \omega^2 \end{pmatrix} \end{aligned} \tag{55}$$

- 6x6
  Hamiltonian:

$$H_6 = \begin{pmatrix} \boldsymbol{\omega}\cdot\boldsymbol{\sigma} & 0 & 0 & \varepsilon & 0 & \varepsilon \\ 0 & -\boldsymbol{\omega}\cdot\boldsymbol{\sigma} & \varepsilon & 0 & \varepsilon & 0 \\ 0 & \varepsilon & \boldsymbol{\omega_1}\cdot\boldsymbol{\sigma} & 0 & 0 & \varepsilon \\ \varepsilon & 0 & 0 & -\boldsymbol{\omega_1}\cdot\boldsymbol{\sigma} & \varepsilon & 0 \\ 0 & \varepsilon & 0 & \varepsilon & \boldsymbol{\omega_2}\cdot\boldsymbol{\sigma} & 0 \\ \varepsilon & 0 & \varepsilon & 0 & 0 & -\boldsymbol{\omega_2}\cdot\boldsymbol{\sigma} \end{pmatrix} \tag{56}$$

$$H_6^2 = \begin{pmatrix} 2\varepsilon^2+\sigma^2\omega^2 & 0 & \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(\boldsymbol{\omega}-\boldsymbol{\omega_1}) & \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(\boldsymbol{\omega}-\boldsymbol{\omega_2}) \\ 0 & 2\varepsilon^2+\sigma^2\omega^2 & \varepsilon\boldsymbol{\sigma}(-\boldsymbol{\omega}+\boldsymbol{\omega_1}) & \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(-\boldsymbol{\omega}+\boldsymbol{\omega_2}) & \varepsilon^2 \\ \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(-\boldsymbol{\omega}+\boldsymbol{\omega_1}) & 2\varepsilon^2+\sigma^2\omega_1^2 & 0 & \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(\boldsymbol{\omega_1}-\boldsymbol{\omega_2}) \\ \varepsilon\boldsymbol{\sigma}(\boldsymbol{\omega}-\boldsymbol{\omega_1}) & \varepsilon^2 & 0 & 2\varepsilon^2+\sigma^2\omega_1^2 & \varepsilon\boldsymbol{\sigma}(-\boldsymbol{\omega_1}+\boldsymbol{\omega_2}) & \varepsilon^2 \\ \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(-\boldsymbol{\omega}+\boldsymbol{\omega_2}) & \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(-\boldsymbol{\omega_1}+\boldsymbol{\omega_2}) & 2\varepsilon^2+\sigma^2\omega_2^2 & 0 \\ \varepsilon\boldsymbol{\sigma}(\boldsymbol{\omega}-\boldsymbol{\omega_2}) & \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(\boldsymbol{\omega_1}-\boldsymbol{\omega_2}) & \varepsilon^2 & 0 & 2\varepsilon^2+\sigma^2\omega_2^2 \end{pmatrix} =$$

$$= \begin{pmatrix} 2\varepsilon^2+\omega^2 & 0 & \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(\boldsymbol{\omega}-\boldsymbol{\omega_1}) & \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(\boldsymbol{\omega}-\boldsymbol{\omega_2}) \\ 0 & 2\varepsilon^2+\omega^2 & \varepsilon\boldsymbol{\sigma}(-\boldsymbol{\omega}+\boldsymbol{\omega_1}) & \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(-\boldsymbol{\omega}+\boldsymbol{\omega_2}) & \varepsilon^2 \\ \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(-\boldsymbol{\omega}+\boldsymbol{\omega_1}) & 2\varepsilon^2+\omega_1^2 & 0 & \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(\boldsymbol{\omega_1}-\boldsymbol{\omega_2}) \\ \varepsilon\boldsymbol{\sigma}(\boldsymbol{\omega}-\boldsymbol{\omega_1}) & \varepsilon^2 & 0 & 2\varepsilon^2+\omega_1^2 & \varepsilon\boldsymbol{\sigma}(-\boldsymbol{\omega_1}+\boldsymbol{\omega_2}) & \varepsilon^2 \\ \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(-\boldsymbol{\omega}+\boldsymbol{\omega_2}) & \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(-\boldsymbol{\omega_1}+\boldsymbol{\omega_2}) & 2\varepsilon^2+\omega_2^2 & 0 \\ \varepsilon\boldsymbol{\sigma}(\boldsymbol{\omega}-\boldsymbol{\omega_2}) & \varepsilon^2 & \varepsilon\boldsymbol{\sigma}(\boldsymbol{\omega_1}-\boldsymbol{\omega_2}) & \varepsilon^2 & 0 & 2\varepsilon^2+\omega_2^2 \end{pmatrix} \tag{57}$$

6 Results

6.1 3 sectors

Hamiltonian:

$$H^{\mathrm{Fl}} = \left(\begin{array}{cc|cc|cc} m & 0 & 0 & \varepsilon & 0 & \varepsilon \\ 0 & m & \varepsilon & 0 & \varepsilon & 0 \\ \hline 0 & \varepsilon & m & 0 & 0 & \varepsilon \\ \varepsilon & 0 & 0 & m & \varepsilon & 0 \\ \hline 0 & \varepsilon & 0 & \varepsilon & m & 0 \\ \varepsilon & 0 & \varepsilon & 0 & 0 & m \end{array}\right) \tag{58}$$

Diagonalising matrices:

1.

$$U_{n\bar{n}} = \left(\begin{array}{cc|cc|cc} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{array}\right), \tag{59}$$

2.

$$U_{1324} = \left(\begin{array}{cc|cc|cc} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} & 0 & 0 \\ \hline -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right), \tag{60}$$

3.

$$U_{3526} = \left(\begin{array}{cc|ccc|c} 1 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & \frac{\sqrt{3}}{3} & 0 & 0 & 0 & \frac{\sqrt{6}}{3} \\ 0 & 0 & \frac{\sqrt{3}}{3} & 0 & \frac{\sqrt{6}}{3} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{6}}{3} & 0 & \frac{\sqrt{3}}{3} & 0 \\ \hline 0 & -\frac{\sqrt{6}}{3} & 0 & 0 & 0 & \frac{\sqrt{3}}{3} \end{array}\right) \tag{61}$$

Diagonalizing matrix:

$$S = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{6} & \frac{1}{2} & \frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{6} \\ \frac{1}{2} & \frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{6} & -\frac{1}{2} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{6} & -\frac{1}{2} & \frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{6} \\ -\frac{1}{2} & \frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{6} & \frac{1}{2} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} \\ 0 & \frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} & 0 & \frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{6} \\ 0 & -\frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} & 0 & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} \end{pmatrix} \tag{62}$$

Diagonalized Hamiltonian:

$$H_D = S^{\dagger} H S = \begin{pmatrix} m-\varepsilon & 0 & 0 & 0 & 0 & 0 \\ 0 & m+\varepsilon & 0 & 0 & 0 & 0 \\ 0 & 0 & m-\varepsilon & 0 & 0 & 0 \\ 0 & 0 & 0 & m+\varepsilon & 0 & \\ 0 & 0 & 0 & 0 & m-2\varepsilon & 0 \\ 0 & 0 & 0 & 0 & 0 & m+2\varepsilon \end{pmatrix} \tag{63}$$

For simplicity, we can substruct from H_D masses, we get:

$$H_{D_R} = H_D - I * m = \begin{pmatrix} -\varepsilon & 0 & 0 & 0 & 0 & 0 \\ 0 & \varepsilon & 0 & 0 & 0 & 0 \\ 0 & 0 & -\varepsilon & 0 & 0 & 0 \\ 0 & 0 & 0 & \varepsilon & 0 & \\ 0 & 0 & 0 & 0 & -2\varepsilon & 0 \\ 0 & 0 & 0 & 0 & 0 & 2\varepsilon \end{pmatrix} \tag{64}$$

Probabilities of a neutron going in different states:

- P_{00} - Neutron staying as neutron;
- P_{01} - Neutron osc in ordinary antineutron;
- P_{02} Neutron osc. in Mirror neutron one;
- P_{04} - Neutron osc. in mirror anti-nutron one;
- P_{05} - Neutron osc. in mirror neutron two;
- P_{06} - Neutro osc. mirror anti-neutron two.

Calculated directly from:

$$S_{diag} = e^{-iSH_{D_R}S^\dagger t} = Se^{-iH_{D_R}t}S^\dagger \tag{65}$$

$$P = S_{diag}S_{diag}^\dagger \tag{66}$$

$$P_{6_{00}} = 1 - \frac{4}{9}\sin^2\left(\frac{t\varepsilon}{2}\right) - \frac{4}{9}\sin^2(t\varepsilon) - \frac{4}{9}\sin^2\left(\frac{3t\varepsilon}{2}\right) - \frac{1}{9}\sin^2(2t\varepsilon) \tag{33}$$

$$P_{6_{01}} = \frac{4}{9}\sin^2\left(\frac{t\varepsilon}{2}\right) - \frac{4}{9}\sin^4(t\varepsilon) + \frac{8}{9}\sin^2(t\varepsilon) - \frac{4}{9}\sin^2\left(\frac{3t\varepsilon}{2}\right) \tag{34}$$

$$P_{6_{02}} = \frac{4}{9}\sin^2\left(\frac{t\varepsilon}{2}\right)\sin^2\left(\frac{3t\varepsilon}{2}\right) \tag{35}$$

$$P_{603} = -\frac{2}{9}\sin^2\left(\frac{t\varepsilon}{2}\right) - \frac{4}{9}\sin^4(t\varepsilon) + \frac{5}{9}\sin^2(t\varepsilon) + \frac{2}{9}\sin^2\left(\frac{3t\varepsilon}{2}\right) \tag{36}$$

$$P_{6_{04}} = \frac{4}{9}\sin^2\left(\frac{t\varepsilon}{2}\right)\sin^2\left(\frac{3t\varepsilon}{2}\right) \tag{67}$$

$$P_{6_{05}} = -\frac{2}{9}\sin^2\left(\frac{t\varepsilon}{2}\right) - \frac{4}{9}\sin^4(t\varepsilon) + \frac{5}{9}\sin^2(t\varepsilon) + \frac{2}{9}\sin^2\left(\frac{3t\varepsilon}{2}\right) \tag{38}$$

If we expand to better see:

$$\begin{aligned}
P_{00} &= 1 - 2t^2\varepsilon^2 + \frac{3}{2}t^4\varepsilon^4 - \frac{101}{180}t^6\varepsilon^6 + \frac{1289}{10080}t^8\varepsilon^8, \\
P_{01} &= \frac{1}{9}t^6\varepsilon^6 - \frac{1}{18}t^8\varepsilon^8, \\
P_{02} &= \frac{1}{4}t^4\varepsilon^4 - \frac{5}{24}t^6\varepsilon^6 + \frac{209}{2880}t^8\varepsilon^8, \\
P_{03} &= t^2\varepsilon^2 - t^4\varepsilon^4 + \frac{13}{30}t^6\varepsilon^6 - \frac{137}{1260}t^8\varepsilon^8, \\
P_{04} &= \frac{1}{4}t^4\varepsilon^4 - \frac{5}{24}t^6\varepsilon^6 + \frac{209}{2880}t^8\varepsilon^8, \\
P_{05} &= t^2\varepsilon^2 - t^4\varepsilon^4 + \frac{13}{30}t^6\varepsilon^6 - \frac{137}{1260}t^8\varepsilon^8.
\end{aligned} \tag{68}$$

Calculated approximately from expansion:

$$S_{exp} = e^{-iHt} = I - i\,H\,t - \frac{1}{2}H^2t^2 + \frac{i}{6}H^3t^3 + \frac{1}{24}H^4t^4 \tag{69}$$

$$P = S_{exp}S_{exp}^\dagger \tag{70}$$

So, we have:

$$
\begin{aligned}
P_{00} &= 1 - 2t^2\varepsilon^2 + \frac{3}{2}t^4\varepsilon^4 - \frac{1}{2}t^6\varepsilon^6 + \frac{1}{16}t^8\varepsilon^8, \\
P_{01} &= \frac{1}{9}t^6\varepsilon^6, \\
P_{02} &= \frac{1}{4}t^4\varepsilon^4 - \frac{5}{24}t^6\varepsilon^6 + \frac{25}{576}t^8\varepsilon^8, \\
P_{03} &= t^2\varepsilon^2 - t^4\varepsilon^4 + \frac{1}{4}t^6\varepsilon^6, \\
P_{04} &= \frac{1}{4}t^4\varepsilon^4 - \frac{5}{24}t^6\varepsilon^6 + \frac{25}{576}t^8\varepsilon^8, \\
P_{05} &= t^2\varepsilon^2 - t^4\varepsilon^4 + \frac{1}{4}t^6\varepsilon^6.
\end{aligned}
\tag{71}
$$

Probability plots look like:

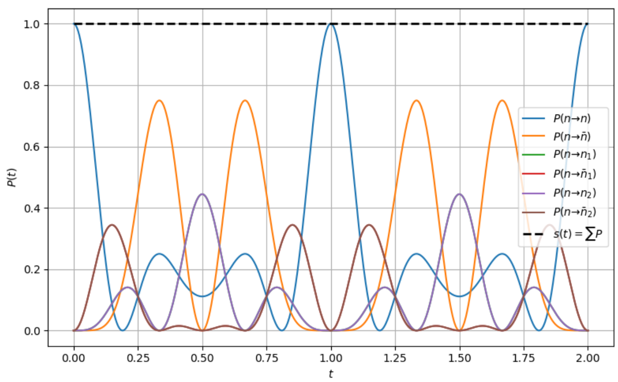

Figure 1: Caption

From (71) we can calculate probability of neutron going into the mirror sector:

$$
P_{\text{mirror}} = P_{02} + P_{03} + P_{04} + P_{05} = \frac{4}{9}\left(1 - \cos(3\varepsilon t)\right) = \frac{4}{9}\cdot 2\sin^2\left(\frac{3\varepsilon t}{2}\right) = \frac{8}{9}\sin^2\left(\frac{3\varepsilon t}{2}\right) \tag{72}
$$

If we expand for small time:

$$
P_{\text{mirror}} = \frac{8}{9}\left(\frac{9}{4}\varepsilon^2 t^2 - \frac{27}{16}\varepsilon^4 t^4 + \frac{81}{160}\varepsilon^6 t^6 + \ldots\right) = \left(2\varepsilon^2 t^2 - \frac{3}{2}\varepsilon^4 t^4 + \frac{9}{20}\varepsilon^6 t^6 + \ldots\right) \tag{73}
$$

6.2 4 sectors

6.2.1 Only ε mixing

Hamiltonian:

$$
H_4 = \begin{pmatrix}
m & 0 & 0 & \varepsilon & 0 & \varepsilon & 0 & \varepsilon \\
0 & m & \varepsilon & 0 & \varepsilon & 0 & \varepsilon & 0 \\
0 & \varepsilon & m & 0 & 0 & \varepsilon & 0 & \varepsilon \\
\varepsilon & 0 & 0 & m & \varepsilon & 0 & \varepsilon & 0 \\
0 & \varepsilon & 0 & \varepsilon & m & 0 & 0 & \varepsilon \\
\varepsilon & 0 & \varepsilon & 0 & 0 & m & \varepsilon & 0 \\
0 & \varepsilon & 0 & \varepsilon & 0 & \varepsilon & m & 0 \\
\varepsilon & 0 & \varepsilon & 0 & \varepsilon & 0 & 0 & m
\end{pmatrix} \tag{74}
$$

Reduced Hamiltonian is:

$$H_{4_R} = H_4 - I \cdot m = \begin{pmatrix} 0 & 0 & 0 & \varepsilon & 0 & \varepsilon & 0 & \varepsilon \\ 0 & 0 & \varepsilon & 0 & \varepsilon & 0 & \varepsilon & 0 \\ 0 & \varepsilon & 0 & 0 & 0 & \varepsilon & 0 & \varepsilon \\ \varepsilon & 0 & 0 & 0 & \varepsilon & 0 & \varepsilon & 0 \\ 0 & \varepsilon & 0 & \varepsilon & 0 & 0 & 0 & \varepsilon \\ \varepsilon & 0 & \varepsilon & 0 & 0 & 0 & \varepsilon & 0 \\ 0 & \varepsilon & 0 & \varepsilon & 0 & \varepsilon & 0 & 0 \\ \varepsilon & 0 & \varepsilon & 0 & \varepsilon & 0 & 0 & 0 \end{pmatrix} \tag{75}$$

We can getprobabilities directly from:

$$S_4 = e^{-iH_{4_R}t} \tag{76}$$

$$P = S_4 S_4^{\dagger} \tag{77}$$

So, we get:

$$\begin{aligned}
P_{00} &= 1 - 12\sin^2\left(\frac{t\varepsilon}{2}\right) + 60\sin^4\left(\frac{t\varepsilon}{2}\right) - 160\sin^6\left(\frac{t\varepsilon}{2}\right) + 240\sin^8\left(\frac{t\varepsilon}{2}\right) - 192\sin^{10}\left(\frac{t\varepsilon}{2}\right) + 64\sin^{12}\left(\frac{t\varepsilon}{2}\right), \\
P_{01} &= \sin^6(t\varepsilon), \\
P_{02} &= -\frac{1}{2}\sin^4(t\varepsilon) + \frac{1}{4}\sin^2(t\varepsilon)\sin^2(3t\varepsilon) + \frac{7}{16}\sin^2(t\varepsilon) + \frac{1}{16}\sin^2(2t\varepsilon) - \frac{3}{16}\sin^2(3t\varepsilon) + \frac{1}{16}\sin^2(4t\varepsilon), \\
P_{03} &= -\frac{1}{4}\sin^2(t\varepsilon)\sin^2(3t\varepsilon) + \frac{1}{16}\sin^2(t\varepsilon) + \frac{1}{16}\sin^2(2t\varepsilon) + \frac{3}{16}\sin^2(3t\varepsilon) - \frac{1}{16}\sin^2(4t\varepsilon), \\
P_{04} &= -\frac{1}{2}\sin^4(t\varepsilon) + \frac{1}{4}\sin^2(t\varepsilon)\sin^2(3t\varepsilon) + \frac{7}{16}\sin^2(t\varepsilon) + \frac{1}{16}\sin^2(2t\varepsilon) - \frac{3}{16}\sin^2(3t\varepsilon) + \frac{1}{16}\sin^2(4t\varepsilon), \\
P_{05} &= -\frac{1}{4}\sin^2(t\varepsilon)\sin^2(3t\varepsilon) + \frac{1}{16}\sin^2(t\varepsilon) + \frac{1}{16}\sin^2(2t\varepsilon) + \frac{3}{16}\sin^2(3t\varepsilon) - \frac{1}{16}\sin^2(4t\varepsilon), \\
P_{06} &= -\frac{1}{2}\sin^4(t\varepsilon) + \frac{1}{4}\sin^2(t\varepsilon)\sin^2(3t\varepsilon) + \frac{7}{16}\sin^2(t\varepsilon) + \frac{1}{16}\sin^2(2t\varepsilon) - \frac{3}{16}\sin^2(3t\varepsilon) + \frac{1}{16}\sin^2(4t\varepsilon), \\
P_{07} &= -\frac{1}{4}\sin^2(t\varepsilon)\sin^2(3t\varepsilon) + \frac{1}{16}\sin^2(t\varepsilon) + \frac{1}{16}\sin^2(2t\varepsilon) + \frac{3}{16}\sin^2(3t\varepsilon) - \frac{1}{16}\sin^2(4t\varepsilon).
\end{aligned} \tag{78}$$

If we expand we get:

$$\begin{aligned}
P_{00} &= 1 - 3t^2\varepsilon^2 + 4t^4\varepsilon^4 - \frac{47}{15}t^6\varepsilon^6 + \frac{169}{105}t^8\varepsilon^8, \\
P_{01} &= t^6\varepsilon^6 - t^8\varepsilon^8, \\
P_{02} &= t^4\varepsilon^4 - \frac{5}{3}t^6\varepsilon^6 + \frac{6}{5}t^8\varepsilon^8, \\
P_{03} &= t^2\varepsilon^2 - \frac{7}{3}t^4\varepsilon^4 + \frac{107}{45}t^6\varepsilon^6 - \frac{442}{315}t^8\varepsilon^8, \\
P_{04} &= t^4\varepsilon^4 - \frac{5}{3}t^6\varepsilon^6 + \frac{6}{5}t^8\varepsilon^8, \\
P_{05} &= t^2\varepsilon^2 - \frac{7}{3}t^4\varepsilon^4 + \frac{107}{45}t^6\varepsilon^6 - \frac{442}{315}t^8\varepsilon^8, \\
P_{06} &= t^4\varepsilon^4 - \frac{5}{3}t^6\varepsilon^6 + \frac{6}{5}t^8\varepsilon^8, \\
P_{07} &= t^2\varepsilon^2 - \frac{7}{3}t^4\varepsilon^4 + \frac{107}{45}t^6\varepsilon^6 - \frac{442}{315}t^8\varepsilon^8.
\end{aligned} \tag{79}$$

Probabilities look like:

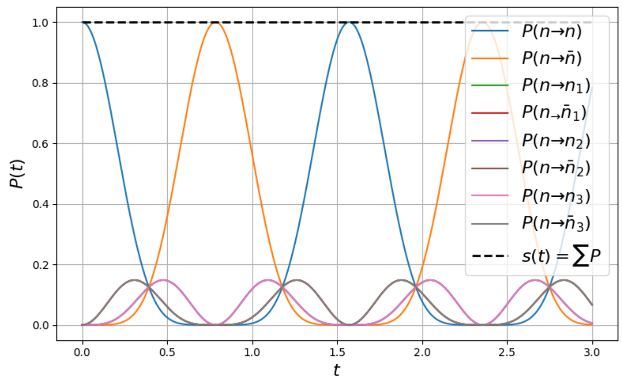

Figure 2: Caption

Probability for neutron to go in mirror sector is:

$$
\begin{aligned}
P_{\text{mirror}} &= \frac{3}{8}\left(1-\cos(4\varepsilon t)\right) = \frac{3}{8}\left(2\sin^2(2\varepsilon t)\right) = \\
&= \frac{6}{8}\left(4\varepsilon^2 t^2 - \frac{16\varepsilon^4 t^4}{3} + \frac{128\varepsilon^6 t^6}{45} + \ldots\right) = \\
&= 3\varepsilon^2 t^2 - 4\varepsilon^4 t^4 + \frac{32}{15}\varepsilon^6 t^6 + \ldots
\end{aligned}
\tag{80}
$$

But if we want to see eigenvalues, we can diagonalize aadn get Diagonalized Hamiltonian:

$$
H_D = \begin{pmatrix}
-\varepsilon & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -3\varepsilon & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\varepsilon & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \varepsilon & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\varepsilon & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\varepsilon & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 3\varepsilon & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon
\end{pmatrix}
\tag{81}
$$

In thiiscase we have 4 differenteigenvalues.

6.2.2 ε and η mixing:

$$
H_{4_{\varepsilon\eta}} = \begin{pmatrix}
m & 0 & 0 & \varepsilon & 0 & \eta & 0 & \eta \\
0 & m & \varepsilon & 0 & \eta & 0 & \eta & 0 \\
0 & \varepsilon & m & 0 & 0 & \eta & 0 & \eta \\
\varepsilon & 0 & 0 & m & \eta & 0 & \eta & 0 \\
0 & \eta & 0 & \eta & m & 0 & 0 & \varepsilon \\
\eta & 0 & \eta & 0 & 0 & m & \varepsilon & 0 \\
0 & \eta & 0 & \eta & 0 & \varepsilon & m & 0 \\
\eta & 0 & \eta & 0 & \varepsilon & 0 & 0 & m
\end{pmatrix}
\tag{82}
$$

Reduced Hamiltonian is:

$$H_{4r_{\varepsilon\eta}} = \begin{pmatrix} 0 & 0 & 0 & \varepsilon & 0 & \eta & 0 & \eta \\ 0 & 0 & \varepsilon & 0 & \eta & 0 & \eta & 0 \\ 0 & \varepsilon & 0 & 0 & 0 & \eta & 0 & \eta \\ \varepsilon & 0 & 0 & 0 & \eta & 0 & \eta & 0 \\ 0 & \eta & 0 & \eta & 0 & 0 & 0 & \varepsilon \\ \eta & 0 & \eta & 0 & 0 & 0 & \varepsilon & 0 \\ 0 & \eta & 0 & \eta & 0 & \varepsilon & 0 & 0 \\ \eta & 0 & \eta & 0 & \varepsilon & 0 & 0 & 0 \end{pmatrix} \tag{83}$$

The diagonalising matrices:

1.

$$S_{88}^{(1)} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{84}$$

2.

$$S_{88}^{(2)} = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{85}$$

3.

$$S_{88}^{(3)} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{86}$$

The diagonalizing matrix is:

$$S_{88} = S_{88}^{(1)} S_{88}^{(2)} S_{88}^{(3)} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \tag{87}$$

Diagonalized Hamiltonian is:

$$H_D = \begin{pmatrix} -\varepsilon & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\varepsilon - 2\eta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \varepsilon - 2\eta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \varepsilon & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\varepsilon & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\varepsilon + 2\eta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon + 2\eta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon \end{pmatrix} \tag{88}$$

In ths case we have 6 different eigenvalues.

Probabilities of a neutron going in different states calculated directly from:

$$S_{diag} = S e^{iHt} S^{\dagger} \tag{89}$$

$$P = S_{diag} S_{diag}^{\dagger} \tag{90}$$

We have P-s:

$$
\begin{aligned}
P_{00} &= 1 + t^2\left(-\varepsilon^2 - 2\eta^2\right) + t^4\left(\frac{\varepsilon^4}{3} + 2\varepsilon^2\eta^2 + \frac{5\eta^4}{3}\right) \\
&\quad + t^6\left(-\frac{2\varepsilon^6}{45} - \frac{2\varepsilon^4\eta^2}{3} - \frac{5\varepsilon^2\eta^4}{3} - \frac{34\eta^6}{45}\right) + t^8\left(\frac{\varepsilon^8}{315} + \frac{4\varepsilon^6\eta^2}{45} + \frac{5\varepsilon^4\eta^4}{9} + \frac{34\varepsilon^2\eta^6}{45} + \frac{13\eta^8}{63}\right), \\
P_{01} &= t^6\varepsilon^2\eta^4 + t^8\left(-\frac{\varepsilon^4\eta^4}{3} - \frac{2\varepsilon^2\eta^6}{3}\right), \\
P_{02} &= t^4\eta^4 + t^6\left(-\varepsilon^2\eta^4 - \frac{2\eta^6}{3}\right) + t^8\left(\frac{\varepsilon^4\eta^4}{3} + \frac{2\varepsilon^2\eta^6}{3} + \frac{\eta^8}{5}\right), \\
P_{03} &= t^2\varepsilon^2 + t^4\left(-\frac{\varepsilon^4}{3} - 2\varepsilon^2\eta^2\right) + t^6\left(\frac{2\varepsilon^6}{45} + \frac{2\varepsilon^4\eta^2}{3} + \frac{5\varepsilon^2\eta^4}{3}\right) \\
&\quad + t^8\left(-\frac{\varepsilon^8}{315} - \frac{4\varepsilon^6\eta^2}{45} - \frac{5\varepsilon^4\eta^4}{9} - \frac{34\varepsilon^2\eta^6}{45}\right), \\
P_{04} &= t^4\varepsilon^2\eta^2 + t^6\left(-\frac{\varepsilon^4\eta^2}{3} - \frac{4\varepsilon^2\eta^4}{3}\right) + t^8\left(\frac{2\varepsilon^6\eta^2}{45} + \frac{4\varepsilon^4\eta^4}{9} + \frac{32\varepsilon^2\eta^6}{45}\right), \\
P_{05} &= t^2\eta^2 + t^4\left(-\varepsilon^2\eta^2 - \frac{4\eta^4}{3}\right) + t^6\left(\frac{\varepsilon^4\eta^2}{3} + \frac{4\varepsilon^2\eta^4}{3} + \frac{32\eta^6}{45}\right) \\
&\quad + t^8\left(-\frac{2\varepsilon^6\eta^2}{45} - \frac{4\varepsilon^4\eta^4}{9} - \frac{32\varepsilon^2\eta^6}{45} - \frac{64\eta^8}{315}\right), \\
P_{06} &= t^4\varepsilon^2\eta^2 + t^6\left(-\frac{\varepsilon^4\eta^2}{3} - \frac{4\varepsilon^2\eta^4}{3}\right) + t^8\left(\frac{2\varepsilon^6\eta^2}{45} + \frac{4\varepsilon^4\eta^4}{9} + \frac{32\varepsilon^2\eta^6}{45}\right), \\
P_{07} &= t^2\eta^2 + t^4\left(-\varepsilon^2\eta^2 - \frac{4\eta^4}{3}\right) + t^6\left(\frac{\varepsilon^4\eta^2}{3} + \frac{4\varepsilon^2\eta^4}{3} + \frac{32\eta^6}{45}\right) \\
&\quad + t^8\left(-\frac{2\varepsilon^6\eta^2}{45} - \frac{4\varepsilon^4\eta^4}{9} - \frac{32\varepsilon^2\eta^6}{45} - \frac{64\eta^8}{315}\right).
\end{aligned} \tag{91}
$$

Visualisation: parameters:

- $\varepsilon = 2$;
- $\eta = 1$,

We get:

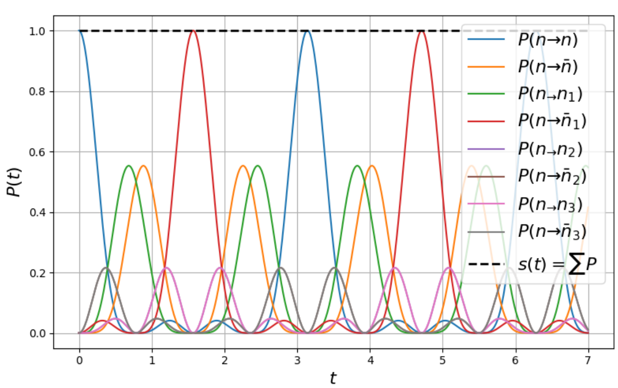

Figure 3: 1 Ordinary and 3 Mirror sectors neutron osc. probabilities with different mixing with different mirror sectors.

6.2.3 Numerical

Hamiltonian:

$$H_{num} = \begin{pmatrix} a\cos\alpha & a\sin\alpha & \eta & \varepsilon \\ a\sin\alpha & -a\cos\alpha & \varepsilon & \eta \\ \eta & \varepsilon & c - b\cos\beta & b\sin\beta \\ \varepsilon & \eta & b\sin\beta & c + b\cos\beta \end{pmatrix} \tag{92}$$

Here we produce 2 set of parameters:

1. Parameters:

- $a = 50$;
- $b = 50$;
- $\varepsilon = 1$;
- $\eta = 0$;
- $\alpha = 0.7$;
- $\beta = 0.5$;
- $c = 0$.

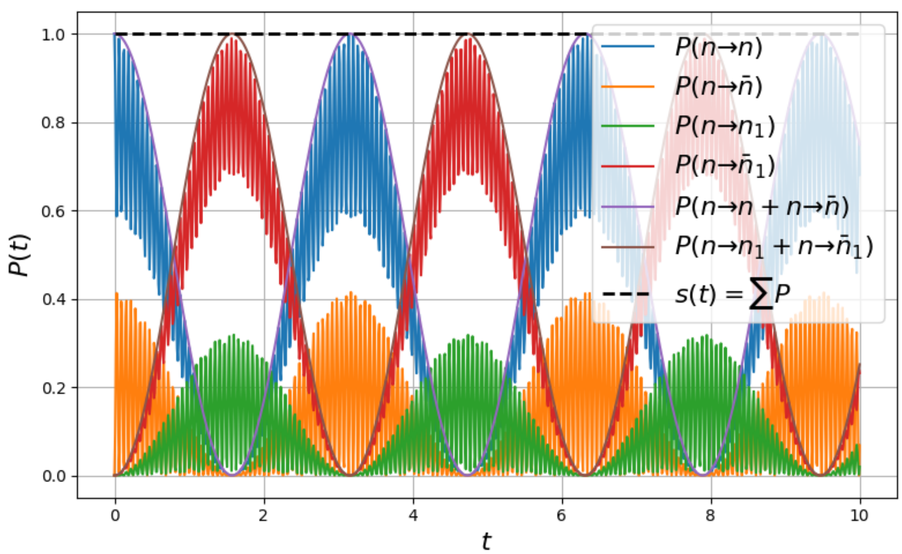

Figure 4: 1 Ordinary and 1 Mirror sectors neutron osc. probabilities with different mixings.

2. Parameters:

   - $a = 50$;
   - $b = 50$;
   - $\varepsilon = 1$;
   - $\eta = 0.306$;
   - $\alpha = 0.905$;
   - $\beta = 0.45$;
   - $c = 0.45$

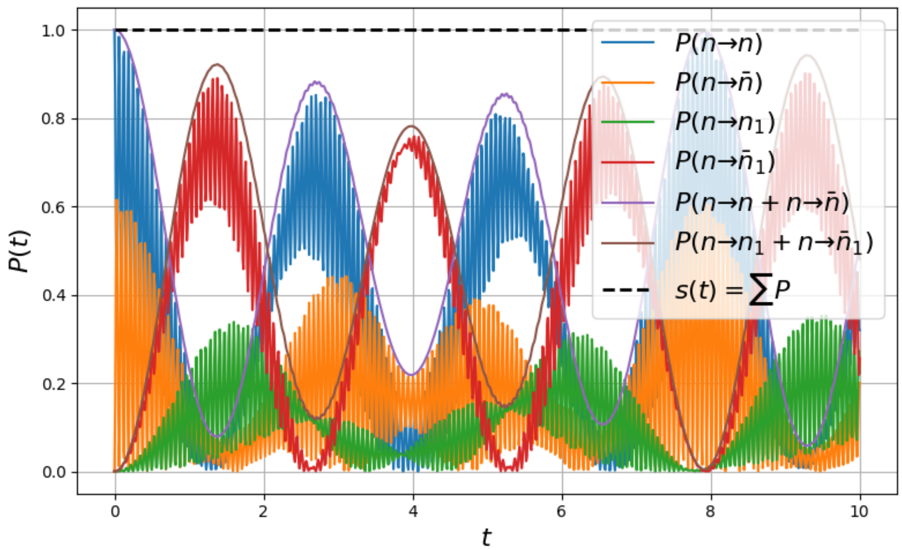

Figure 5: 1 Ordinary and 1 Mirror sectors neutron osc. probabilities with different mixings and different parameters.