Droplet

1 Evaporation

1.1 Overview

Liquids are composed of molecules that are in close contact with each other, held together by intermolecular forces (like van der Waals forces or hydrogen bonding). These forces make the molecules stick together and give the liquid its characteristic properties. However, these forces are not as strong as the covalent or ionic bonds that hold atoms together within individual molecules.

In a liquid, molecules are in constant motion, colliding with each other and exchanging energy. The kinetic energy of a molecule is related to its temperature. Even in a body of liquid at a uniform temperature, not all molecules have the same energy. The energy distribution is described by the Maxwell-Boltzmann distribution.

At the surface of the liquid, some molecules will have enough kinetic energy to overcome the intermolecular forces that are pulling them into the liquid. When this happens, these molecules can escape from the liquid into the air above. **This is evaporation.**

When a molecule evaporates, it takes some energy from the liquid with it. This energy, known as latent heat, is required to change the phase of the substance. As a result, evaporation has a cooling effect on the liquid left behind, as the average energy of the remaining molecules decreases.

1.2 Evaporation in Vacuum?

Water can evaporate in a vacuum container. In fact, water will evaporate faster in a vacuum compared to normal atmospheric conditions.

Under normal atmospheric pressure, water molecules need to overcome both intermolecular forces (i.e., the forces holding the water molecules together in the liquid state) and atmospheric pressure to enter the gas phase.

In a vacuum, the atmospheric pressure is effectively zero. So, the water molecules only need to overcome the intermolecular forces to transition from a liquid to a gas. This makes the evaporation process faster.

Furthermore, in a vacuum, the boiling point of water lowers significantly due to the lack of atmospheric pressure. Water may boil at room temperature or even lower, depending on the strength of the vacuum. Boiling, like evaporation, **is a phase transition from liquid to gas, but it occurs throughout the volume of the liquid rather than just at the surface.**

However, it should be noted that while the evaporation or boiling may be faster in a vacuum, it also means that the water will cool down rapidly (due to the evaporation cooling effect), which could **freeze the remaining water** if the process happens fast enough and there is no additional heat source. This phenomenon is called "freeze-drying" and is used in some food preservation processes.
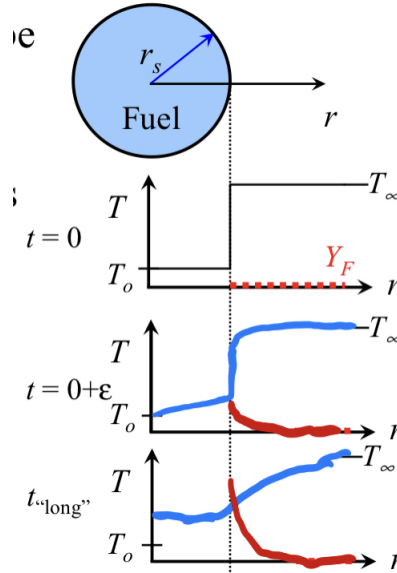
1.3   Evaporation Process

Figure 1: Evaporation Process

Steps:

1. Instantly place drop in hot gas

2. After short time, heating and evaporation

3. Eventually reach quasi-steady state

4. Mass loss, evaporation must be balanced by heat transfer to drop

1.4   Evaporation Rate

Now we want to calculate how fast does droplet lose mass. First we have assumptions:

1. Quiescent, infinite medium

2. Quasi-steady

3. Uniform droplet

4. Single component liquid

5. Binary diffusion

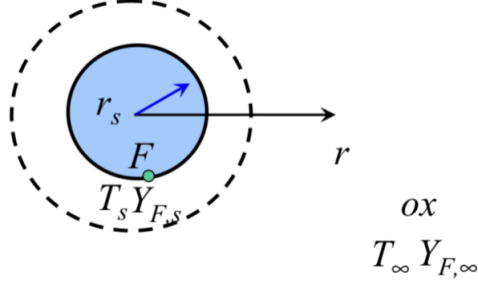6. Constant TD properties

7. No viscous dissipation, buoyancy

Figure 2: Droplet Parameters

After analysis in mass, species and energy conservation, we get:

$$\dot{m} = -4\pi\rho D r_s ln(1 + B_y) \tag{1}$$

Here B_y is the spalding transfer number for mass:

$$B_y = \frac{Y_{F,s} - Y_{F,\infty}}{1 - Y_{F,s}} \tag{2}$$

Also:

$$\dot{m} = -4\pi\rho\alpha r_s ln(1 + B_h) \tag{3}$$

Here B_h is the spalding number for energy:

$$B_h = \frac{c_p(T_\infty - T_s)}{h_{fg}} \tag{4}$$

Based on the unity Lewis number assumption:

$$B_y = B_h \tag{5}$$

$$\frac{Y_{F,s} - Y_{F,\infty}}{1 - Y_{F,s}} = \frac{c_p(T_\infty - T_s)}{h_{fg}} \tag{6}$$

So now we only need to find the relation between $Y_{F,s}$ and T_s.

1.5   Wet Bulb Temperature

The wet bulb temperature is the lowest temperature that can be reached by the evaporation of water only. It is the temperature one feels when wet skin is exposed to moving air. It's called "wet bulb" temperature because it is measured by using a thermometer covered in a water-soaked cloth over the bulb. As water on the cloth evaporates, it reduces the temperature reading, thus indicating the wet bulb temperature.

Assume phase equilibrium between liquid and vapor at droplet surface. Using **Clausius Clapeyron Equation**, we can determine the vapor pressure of interface:

$$p_{vap} \approx Ce^{-h_{fg}(T_{wb})/RT_{wb}} \tag{7}$$

This relation can also expressed by mass fraction:

$$Y_{F,s} = [1 + (\frac{p}{Ce^{-h_{fg}(T_s)/RT_s}} - 1)\frac{\bar{W}_{ox}}{\bar{W}_F}]^{-1} \tag{8}$$

1.6   Final Solution

With the relationship between $Y_{F,s}$ and T_s, we can calculate the evaporation rate:

$$\frac{\dot{m}}{r_s} = -4\pi\rho D ln(1 + B_y) = -4\pi\rho\alpha ln(1 + B_h) \tag{9}$$

If we have constant ρD, $\rho\alpha$, T_s, $Y_{F,s}$, then we have:

$$\frac{\dot{m}}{r_s} = const \tag{10}$$

1.7   Droplet Lifetime

Now we know how fast the droplet will evaporate, we also want to know how long until droplet evaporated. Assume the diameter of the droplet as d_s, then the droplet mass:

$$m_d = \rho_l \pi d_s^3/6 \tag{11}$$

Then, the mass loss rate is just the derivative:

$$\dot{m}_d = \frac{1}{2}\rho_l \pi d_s^2 \dot{d}_s \tag{12}$$

Rearrange:

$$\frac{\dot{m}_d}{d_s} = \rho_l \frac{\pi}{2} d_s \dot{d}_s = \rho_l \frac{\pi}{4}\frac{d(d_s^2)}{dt} \tag{13}$$

Recall that:

$$\frac{\dot{m}}{r_s} = -4\pi\rho_g\alpha ln(1 + B_h) \tag{14}$$

Therefore:

$$\frac{\dot{m}_d}{d_s} = -2\pi\rho_g \alpha ln(1 + B_h) \tag{15}$$

$$\frac{d(d_s^2)}{dt} = \frac{4}{\rho_l \pi}\frac{\dot{m}_d}{d_s} = -8\frac{\rho_g}{\rho_l}\alpha ln(1 + B_h) = K \tag{16}$$

We define K as the evaporation constant.
Because the droplet's size keeps decreasing, we can define D^2 Law as:

$$\frac{d(d_s^2)}{dt} = -K \tag{17}$$

$$d_s^2(t) = d_s^2(0) - Kt \tag{18}$$

And the life time is defined as:

$$t_d = \frac{d_s^2(0)}{K} \tag{19}$$

Some remarks:

1. Small droplets evaporate much faster

2. Lighter liquids evaporate faster

3. Volatile liquids evaporate faster

2 Droplet Burning

2.1 Description

Besides the assumptions for droplet evaporation, the reacting case has additional assumptions:

1. Isobaric, negligible radiation transfer

2. Fuel and oxidizer completely reacted in stoichiometric proportions

$$\frac{\dot{m}_F}{\dot{m}_O} = \left(\frac{\dot{m}_F}{\dot{m}_O}\right)_{stoich} \tag{20}$$
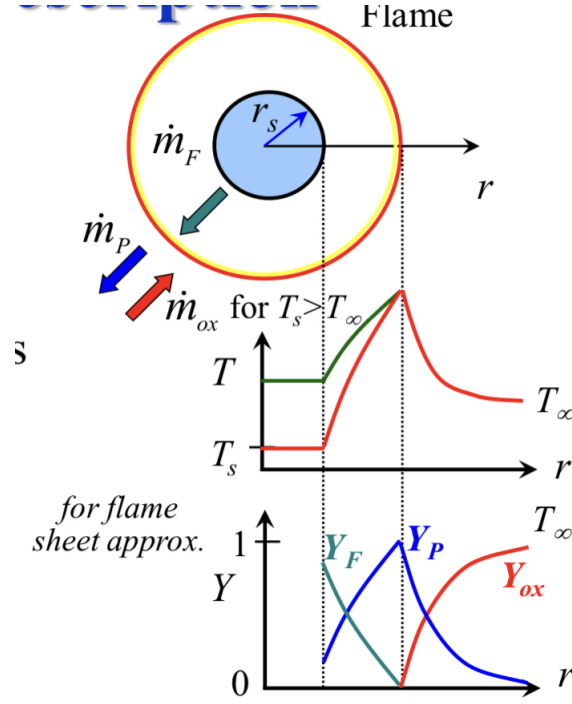
3. Le = 1

Figure 3: Droplet Burning

Some remarks:

1. Only fuel flow inside flame

2. Maximum temperature in flame

3. finite amount of products next to droplet

2.2 Governing Equations

Based on **Spalding Type Approach**, we can define combinations of scalars to produce quantities that have no sources/sinks beyond surface, similar B.C. and similar transport equations.

First, we define 3 intermediate variable:

$$b_y = \frac{Y_F - Y_o(m_F/m_o)_{stoich}}{(Y_{F,s}-1) - Y_{o,s}(m_F/m_o)_{stoich}} \longleftarrow 0 \; for \; complete \; combustion \; in \; flame$$

evap.
case
$$b_{ho} = \frac{c_pT + \Delta h_R Y_o(m_F/m_o)_{stoich}}{\Delta h_{vap} + \Delta h_R Y_{o,s}(m_F/m_o)_{stoich}} \qquad b_{hF} = \frac{c_pT + \Delta h_R Y_F}{\Delta h_{vap} + \Delta h_R(Y_{F,s}-1)}$$

With the assumption of unity Lewis number, all of them are the same, so the conservation equation becomes:

$$r^2 \dot{m}'' \frac{db}{dr} = \frac{d}{dr}(\rho D r^2 \frac{db}{dr}) \tag{21}$$

Integrate this equation twice and rearrange:

$$\frac{\dot{m}}{4\pi\rho Dr} = ln(\frac{b_\infty - b_s + 1}{b(r) - b_s + 1}) \tag{22}$$

If we define $B = b_\infty - b_s$, then at $r = r_s$:

$$\dot{m} = 4\pi\rho Dr_s ln(1 + B) \tag{23}$$

Recall the evaporating case:

$$\dot{m} = -4\pi\rho Dr_s ln(1 + B_y) \tag{24}$$

Some remarks:

1. Same form as in evaporating case, with different **Spalding Transfer Numbers**

2. Can still use the relation between $Y_{F,s}$ and T_s

3. ρD constant assumption may not be valid, because there is a large temperature change
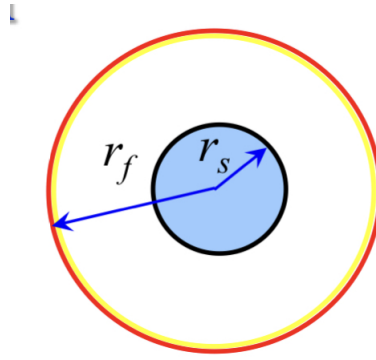
2.3   Flame Location

Figure 4: Flame Location

Based on calculation:

$$\frac{r_f}{r_s} = \frac{ln(1 + B)}{ln(1 + Y_{o,\infty}(m_F/m_o)_{stoich}} \tag{25}$$

2.4   Droplet Life Time

D^2 Law still applies to this case, with some modification:

$$K = 8\frac{k}{c_p\rho_l}ln(1 + B) \tag{26}$$

Because B is larger for burning, K is larger, so the burning droplet lifetime is shorter.