## Markov Chain

### 1 Introduction

A Markov Chain is a mathematical model that describes a sequence of possible events in which the probability of each event **depends only on the state attained** in the previous event. It is a conceptual model for discrete states and time, and **stochastic** (outcomes based upon random probability) evolution.

## 2 Elements

#### 2.1 States

These are the possible situations or conditions the system can be in. Each state is mutually exclusive, meaning the system can only be in one state at a given time. Notice that the state of the system include **states of all nodes**.

#### 2.2 Transitions

These represent the movement or changes between states. Each transition has an associated probability.

#### 2.3 Transition Probabilities

These are the probabilities that the system moves from one state to another during a transition. They form the core of a Markov Chain. The probabilities associated with all possible transitions from a particular state should sum to 1.

#### 2.4 Initial Distribution

This describes the probability distribution of the system's state at the start (or time t=0). It indicates the likelihood that the system begins in any particular state.

# 3 Case Study

Assume we want to model the spread of an infectious disease.

## 3.1 Case Setup

We have the following parameters:

- 1. k: Duration of infection. In this case, k=2.
- 2.  $\tau$ : Infection probability
- 3. Node label 0: Susceptible
- 4. Node label -1: Recovered
- 5. Node label 1 and 2: Infection date 1 and 2.

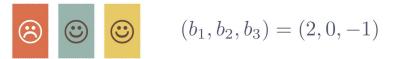


Figure 1: State Setup.

### 3.2 Transition

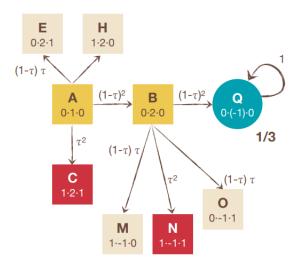


Figure 2: State Transition.

We define the transition from x to y as:

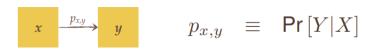


Figure 3: Transition from X to Y.

Then we could have the transition matrix:

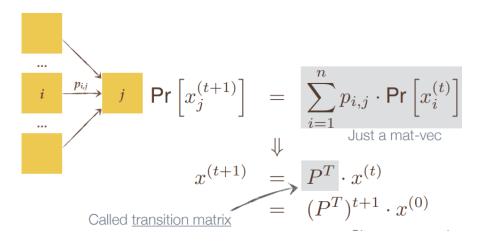


Figure 4: Transition Matrix.

The total number of the states could be expressed as:

$$(k+2)^n \tag{1}$$

Where:

- 1. n: the number of patients
- 2. 2: the number of safe states, which is susceptible and recovered.