Stagnation

1 Stagnation Properties

In this chapter, we rewrite all the expressions in terms of Mach number. Recall the content in Thermodynamics, for **stagnation temperature**:

$$\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2}\frac{u^2}{\gamma RT} = 1 + \frac{\gamma - 1}{2}M^2 \tag{1}$$

Also we know:

$$\frac{p_o}{p} = (\frac{T_o}{T})^{\frac{\gamma}{\gamma-1}} \tag{2}$$

So we have **stagnation pressure**:

$$\frac{p_o}{p} = (1 + \frac{\gamma - 1}{2}\frac{u^2}{\gamma RT})^{\frac{\gamma}{\gamma-1}} = (1 + \frac{\gamma - 1}{2}M^2)^{\frac{\gamma}{\gamma-1}} \tag{3}$$

In addition:

$$\frac{\rho_o}{\rho} = (\frac{T_o}{T})^{\frac{1}{\gamma-1}} \tag{4}$$

So we have **stagnation density**:

$$\frac{\rho_o}{\rho} = (1 + \frac{\gamma - 1}{2}M^2)^{\frac{1}{\gamma-1}} \tag{5}$$

Some remarks:

1. If actual flow has **no external work and adiabatic**, then h_o, T_o are constant
2. If the flow is also **reversible**, then p_o, ρ_o, s are also constant

2 Stagnation vs Bernoulli

Recall the Bernoulli equation, which can also give us a 'stagnation' pressure:

$$p_o = p + \frac{1}{2}\rho u^2 \tag{6}$$

What is the relationship between the real stagnation and fake stagnation? Here we introduce the **Taylor Expansion**:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + ... \tag{7}$$

Now if we assume:

$$\frac{\gamma-1}{2}M^2 = x \tag{8}$$

Then we have:

$$\begin{aligned}\frac{p_o}{p} &= (1+\frac{\gamma-1}{2}M^2)^{\frac{\gamma}{\gamma-1}} \\ &= 1 + \frac{\gamma}{\gamma-1}\frac{\gamma-1}{2}M^2 + \frac{1}{2}\frac{\gamma}{\gamma-1}(\frac{\gamma}{\gamma-1}-1)(\frac{\gamma-1}{2}M^2)^2 + ... \\ &= 1 + \frac{\gamma}{2}M^2 + \frac{\gamma}{2}(\frac{M^2}{2})^2 + ...\end{aligned} \tag{9}$$

Recall that:

$$M^2 = \frac{v^2}{a^2} = \frac{v^2}{\gamma p/\rho} \tag{10}$$

Finally:

$$p_0 = p + \frac{1}{2}\rho v^2 + \frac{1}{2}\rho v^2\frac{M^2}{4} + ... \tag{11}$$

Therefore we can see that high terms are negligible for small M, which means Bernoulli only valid under low M (incompressible) condition.

3 Stagnation vs Static

For the **static properties**:

1. Represent the properties you would measure if you were **moving with the flow at the local flow velocity**

2. Defined in **flow's reference frame**

For the **stagnation properties:**

1. Always defined by conditions at a point

2. Represent the **static** properties you would measure if **you bring the fluid to stop with respect to a chosen observer.**

3. Depends on observer's reference frame