Mission Analysis

1 Introduction

Mission analysis in aircraft design is a critical phase where the intended purpose and operational requirements of the aircraft are thoroughly examined and defined. This process significantly influences the overall design and performance characteristics of the aircraft. Key aspects of mission analysis include:

1. **Defining the Mission Profile**: This involves understanding the specific tasks the aircraft is expected to perform.

2. **Operational Requirements**: Identifying the operational environment and requirements, such as the type of airfields it will operate from, the climatic conditions it will encounter, and the types of missions it will undertake

3. **Performance Goals and Constraints:** Establishing the performance goals like maximum speed, range, fuel efficiency, and service ceiling, along with any constraints imposed by regulations, technology, or cost considerations.

4. **Load and Configuration Analysis:** Understanding the load the aircraft will carry, including passengers, cargo, fuel, and armaments for military aircraft. This also involves analyzing the configuration requirements, such as the need for stealth, agility, or the ability to carry specific equipment.

2 General Approach

2.1 Overview

From the previous section, the initial values of thrust loading $\frac{T_{SL}}{W_{TO}}$ and wing loading $\frac{W_{TO}}{S}$ are determined, now we need to focus on the weight estimation:

$$W_{TO} = W_C + W_P + W_E + W_F \tag{1}$$

Here:

1. W_C: crew weight

2. W_P: payload weight

3. W_E: empty weight

4. W_F: fuel weight

This equation could also be expressed with fuel and empty weight fractions:

$$W_{TO} = W_C + W_P + \frac{W_F}{W_{TO}} W_{TO} + \frac{W_E}{W_{TO}} W_{TO} \tag{2}$$

Among this, the crew and payload weights are usually defined as constants, the **fuel weight ratio** is a function of mission, related to aerodynamics and fuel consumption, and **empty weight fraction** is a function of take-off gross weight from empirical data, the equation could be further expressed as:

$$W_{TO} = \frac{W_C + W_P}{1 - \frac{W_F}{W_{TO}} - \frac{W_E}{W_{TO}}} \tag{3}$$

2.2 Fuel Burn Equation

The generic fuel burn ODE is:

$$\frac{dW}{dt} = \frac{dW_F}{dt} = -\text{TSFC} \times T \tag{4}$$

This could be rewritten as:

$$\frac{dW}{W} = -\text{TSFC} \left(\frac{T}{W} \right) dt \tag{5}$$

Here T is the installed thrust and TSFC is the installed thrust specific fuel consumption.

2.3 Weight Fractions (β)

Instantaneous weight fraction is also used for computing the constraints. β is a fuel or payload correction for different points or segments along the mission. The *beta* value for each segment is essentially an intermediate weight fraction:

$$\beta_n = \frac{W_1}{W_{TO}} \cdot \frac{W_2}{W_1} \cdot \frac{W_3}{W_2} \cdot \ldots \cdot \frac{W_n}{W_{n-1}} \tag{6}$$

After the β values have been estimated or calculated, they must be reapplied to the constraint equations. With new βs, constraint analysis will yield new T_{SL}/W_{TO}, W_{TO}/S values. With these new ratios, mission analysis is performed resulting in new βs. This process is repeated until a convergence on βs is reached.

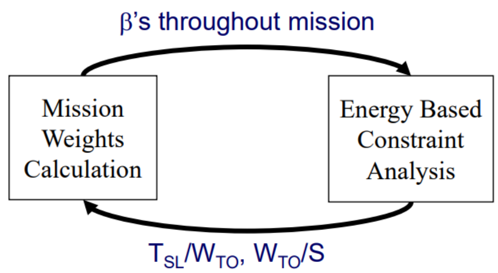

Figure 1: Beta iterations

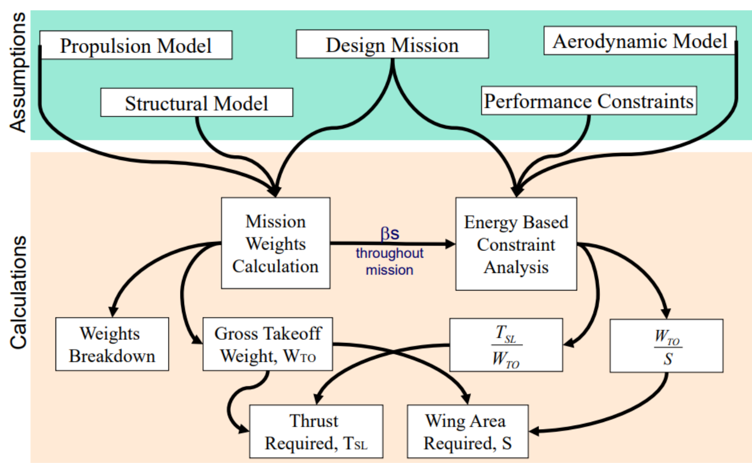

Figure 2: Full Process

Typical β for fighter aircraft:

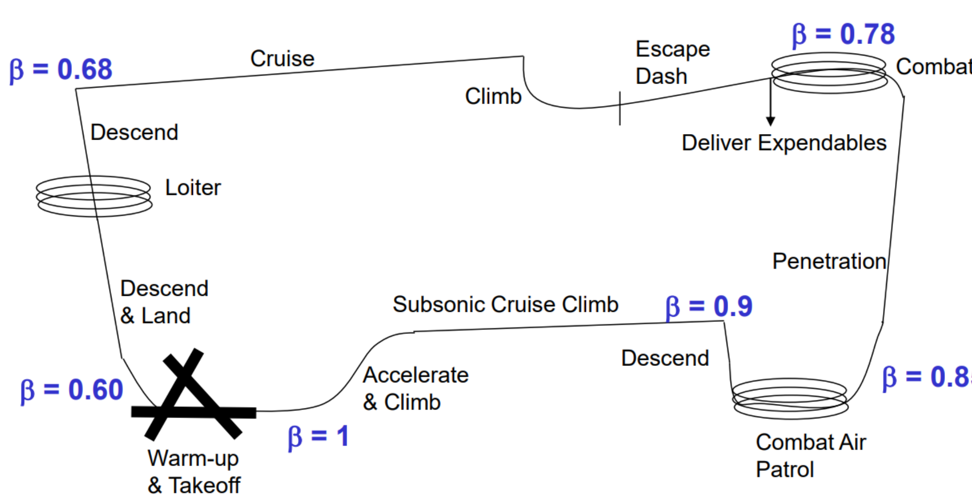

Figure 3: Beta for fighter aircraft

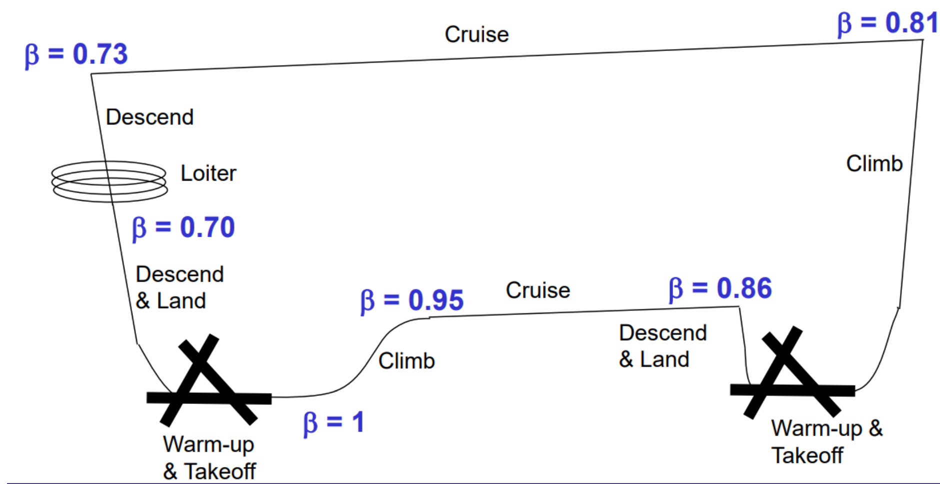

Figure 4: Beta for Cargo/Passenger

This requires knowledge of TSFC behavior and instantaneous thrust loading as a function of time along flight path:

$$\frac{T}{W} = (\frac{\alpha}{\beta})(\frac{T_{SL}}{W_{TO}}) \tag{7}$$

Recall the definition of weight specific excess power:

$$V \cdot \frac{T - (D + R)}{W} = P_s = \frac{d}{dt}(h + \frac{V^2}{2g_0}) = \frac{dz_e}{dt} \tag{8}$$

So the integration could be divided into two classes:

1. $P_s > 0$ (Type A)
2. $P_s = 0$ (Type B)

2.3.1 Type A

Examples of type A including the following cases:

1. Constant speed climb
2. Horizontal acceleration
3. Climb and acceleration
4. Takeoff Acceleration

Rearrange the equation we get:

$$V dt = \frac{d(h + \frac{V^2}{2g_0})}{\frac{T-(D+R)}{W}} \tag{9}$$

$$\frac{T}{W}Vdt = \frac{d(h+\frac{V^2}{2g_0})}{\frac{T-(D+R)}{T}} \tag{10}$$

Here we define:

$$u = \frac{D+R}{T} \tag{11}$$

where u is how total engine thrust work is distributed between mechanical energy and dissipation. Therefore:

$$\frac{T}{W}Vdt = \frac{d(h+\frac{V^2}{2g_0})}{1-u} = \frac{dz_e}{1-u} \tag{12}$$

Combine this with equation 5, we can get:

$$\frac{dW}{W} = -\frac{\text{TSFC}}{V(1-u)}d\left(h+\frac{V^2}{2g_0}\right) = -\frac{\text{TSFC}}{V(1-u)}dz_e \tag{13}$$

After integration, we can get:

$$\frac{W_f}{W_i} = \exp\left(-\frac{\text{TSFC}}{V(1-u)}\Delta\left(h+\frac{V^2}{2g_0}\right)\right) \tag{14}$$

Or in terms of z_e:

$$\frac{W_f}{W_i} = \exp\left(-\frac{\text{TSFC}}{V(1-u)}\Delta z_e\right) \tag{15}$$

One of the important analysis is to show the supposed best way to fly certain legs for minimum fuel used. For type A legs:

$$dW_F = T \times TSFC \times dt = T_{SL}\frac{\alpha TSFC}{P_s}dz_e \tag{16}$$

Define the **fuel consumed specific work (f_s)** as:

$$f_s = \frac{P_s}{\alpha TSFC} \tag{17}$$

Then the integration will be:

$$W_{F_{1-2}} = \int_1^2 dW_F = T_{SL}\int_{z_{e1}}^{z_{e2}}\frac{dz_e}{f_s} \tag{18}$$

From the equation we can see that the **fuel-to-climb** flight corresponds to a flight path that produces the **max thrust work per unit weight of fuel at each energy height level in the climb**, or **max value of f_s at each** z_e.

Similarly, we can express the **min time-to-climb** path:

$$\Delta t = \int_{z_{e1}}^{z_{e2}} \frac{dz_e}{P_s} \tag{19}$$

2.3.2 Type B

When $P_s = 0$, usually the speed and altitude are essentially constant. Examples include:

1. Constant speed cruise
2. Constant speed turn
3. Best cruise Mach number and altitude
4. Loiter
5. Warm-up
6. Takeoff rotation
7. Constant energy height maneuver

For this type of flight, the thrust is **modulated or throttled** so that:

$$T = D + R \tag{20}$$

And the fuel burn equation will be:

$$\frac{dW}{W} = -TSFC(\frac{D+R}{W})dt \tag{21}$$

After integration, we could have:

$$\frac{W_f}{W_i} = exp(-TSFC(\frac{D+R}{W})\Delta t) \tag{22}$$

2.3.3 Summary

TSFC behavior is a complex function of altitude, speed and throttle setting. One of the good way for TSFC approximation is (good for turbojet and turbofan, TSFC is not a strong function of speed and throttle setting):

$$TSFC = C\sqrt{\theta} \tag{23}$$

Here:

1. C: constant
2. θ: represents the usual thermodynamic cycle improvement due to a lower ambient temperature at higher altitude

Then the summary of type A equations will be:

$$\frac{dW}{W} = -\frac{C\sqrt{\theta}}{V(1-u)} d\left(h + \frac{V^2}{2g_0}\right) \tag{24}$$

$$\frac{W_f}{W_i} = \exp\left(-\frac{C\sqrt{\theta}}{V(1-u)} \Delta\left(h + \frac{V^2}{2g_0}\right)\right) \tag{25}$$

The summary to type B equations will be:

$$\frac{dW}{W} = -C\sqrt{\theta}\left(\frac{D+R}{W}\right) dt \tag{26}$$

$$\frac{W_f}{W_i} = \exp\left(-C\sqrt{\theta}\left(\frac{D+R}{W}\right) \Delta t\right) \tag{27}$$

3 Detailed Calculation (Phases)

3.1 Crew and Payload Weight

Assumed known from RFP. The payload weight may comprise of passengers, baggage, cargo and military loads.

3.2 Empty Weight Fraction

The empty weight (W_E) consists of the basic aircraft structure and any permanently attached equipment. The empty weight fractions Γ are mainly determined by empirical correlations.

Cargo aircraft:

$$\Gamma = 1.26 W_{TO}^{-0.08} \tag{28}$$

Passenger aircraft:

$$\Gamma = 1.02 W_{TO}^{-0.06} \tag{29}$$

Fighter aircraft:

$$\Gamma = 2.34 W_{TO}^{-0.13} \tag{30}$$

Twin turboprop aircraft:

$$\Gamma = 1.26 W_{TO}^{-0.08} \tag{31}$$

3.3 Fuel Weight

Fuel weight (W_F) depends on the rate of fuel consumption, which depends on the **thrust specific fuel consumption (TSFC)**. The unit of TSFC is **mass flow rate per unit of thrust**. TSFC depends on the engine cycle, flight conditions.

To find the fuel fraction, we need to break down mission into a number of mission phases and calculate fuel used in each phase.

TSFC is related to the constant C in the last section. Examples include:

1. High bypass turbofan ($M < 0.9$): $C = 1h^{-1}$
2. Low bypass turbofan:
   (a) $C = 1.35h^{-1}$ (M $<$ 1) mil power
   (b) $C = 1.45h^{-1}$ (M $\geq$ 1) mil power
   (c) $C = 2h^{-1}$ max power
3. Turbojet engine:
   (a) $C = 1.45h^{-1}$ (M $<$ 1) mil power
   (b) $C = 1.65h^{-1}$ (M $\geq$ 1) mil power
   (c) $C = 2h^{-1}$ max power
4. Turboprop engine: $C = 0.6h^{-1}$

3.3.1 Phase 1: Engine start and warm up

$\frac{W_1}{W_{TO}}$ is determined using existing data for comparable aircraft.

3.3.2 Phase 2: Taxi

$\frac{W_2}{W_1}$ is determined using existing data for comparable aircraft.

3.3.3 Phase 3: Takeoff

$\frac{W_3}{W_2}$ is determined using existing data for comparable aircraft.

3.3.4 Phase 4: Climb/accelerate to start of cruise

$\frac{W_4}{W_3}$ is determined using existing data for comparable aircraft, or using Breguet's endurance equation, or numerically integrate over a climb profile.

Endurance in aircraft design refers to the duration an aircraft can remain airborne on a single fuel load. The endurance equation for **jet** is:

$$E = \frac{1}{c_t}\frac{L}{D}\ln\left(\frac{W_{\text{final}}}{W_{\text{initial}}}\right) \tag{32}$$

Here, L/D is the lift-drag ratio, which is a measure of the aerodynamic efficiency of the aircraft. c_t represents the **specific fuel consumption (SFC)**, which is a measure of the efficiency of the engine and is defined as **the amount of fuel needed**

to produce a certain amount of thrust for a certain time. For jet, SFC is interchangeable with TSFC, the unit is **fuel mass per unit of thrust per hour**.

The endurance equation for **propeller**:

$$E = \frac{550\eta_{pr}}{cV}\frac{L}{D}\ln\left(\frac{W_{\text{final}}}{W_{\text{initial}}}\right) \tag{33}$$

Here:

1. 550: conversion factor from horsepower to **foot-pounds per second** when using imperial units.
2. η_{pr} is the propeller efficiency, which represents how effectively the propeller converts the engine power into thrust
3. c: the specific fuel consumption of the engine based on power, often given in pounds of fuel per horsepower per hour
4. V: true airspeed of the aircraft in feet per second

3.3.5 Phase 5: Cruise

$\frac{W_5}{W_4}$ is determined using Breguet's range equation, or numerically integrate the generic fuel burn ODE. The Breguet's range equation for jet is:

$$R = \frac{V}{c_t}\frac{L}{D}\ln\left(\frac{W_{\text{final}}}{W_{\text{initial}}}\right) \tag{34}$$

For propeller is:

$$R = \frac{550\eta_{pr}\frac{L}{D}}{c}\ln\left(\frac{W_{\text{final}}}{W_{\text{initial}}}\right) \tag{35}$$

3.3.6 Phase 6: Loiter

$\frac{W_6}{W_5}$ is determined using Breguet's endurance equation, or numerically integrate the generic fuel burn ODE.

3.3.7 Phase 7: Descent

$\frac{W_7}{W_6}$ is determined using existing data for comparable aircraft.

3.3.8 Phase 8: Landing, Taxi and Shutdown

$\frac{W_8}{W_7}$ is determined using existing data for comparable aircraft.

3.4 Weight Calculation

Final to initial weight ratio is expressed as:

$$\frac{W_1}{W_{TO}} \cdot \frac{W_2}{W_1} \cdot \frac{W_3}{W_2} \cdot \ldots \cdot \frac{W_n}{W_{n-1}} = \frac{W_n}{W_{TO}} \tag{36}$$

The mission fuel weight fraction will be:

$$\frac{W_F}{W_{TO}} = 1 - \frac{W_n}{W_{TO}} \tag{37}$$

Trapped fuel in aviation refers to the quantity of fuel that remains unusable in an aircraft's fuel tanks. With this consideration:

$$\frac{W_F}{W_{TO}} = 1.06\left(1 - \frac{W_n}{W_{TO}}\right) \tag{38}$$

The process of estimating takeoff weight is shown below:

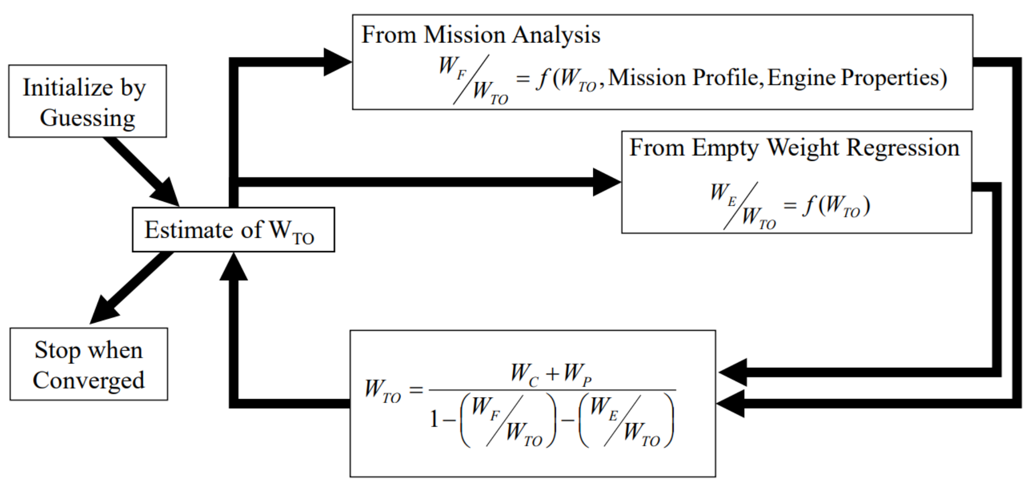

Figure 5: Weight Estimation

4 Case Study

4.1 Case 1: Constant Speed Climb

Variable	Value
dV/dt	0
n	1
R	0
T	αT_{SL}
$h_{initial}, h_{final}, V$	Given
P_s	dh/dt

Table 1: Case 1 Variables

The condition is summarized as:

$$u = \frac{C_D}{C_L}\frac{\beta}{\alpha}\left(\frac{W_{TO}}{T_{SL}}\right) \tag{39}$$

Because V is a constant, so the equation 25 becomes:

$$\frac{W_f}{W_i} = \exp\left\{-\frac{C\sqrt{\theta}}{V}\left[\frac{\Delta h}{1-\left(\frac{C_D}{C_L}\right)\left(\frac{\beta}{\alpha}\right)\left(\frac{W_{TO}}{T_{SL}}\right)}\right]\right\} \tag{40}$$

During the level flight, the relation between drag and lift is:

$$\frac{D}{W} = \frac{D}{L} = \frac{qC_DS}{qC_LS} = \frac{C_D}{C_L} \tag{41}$$

And the change in altitude is:

$$\Delta h = \Delta h_{final} - h_{initial} \tag{42}$$

Typically the variables values are averaged for altitude interval.

4.2 Case 2: Horizontal Acceleration

Variable	Value
dh/dt	0
n	1
R	0
T	αT_{SL}
$h, V_{initial}, V_{final}$	Given
P_s	VdV/g_0dt

Table 2: Case 2 Variables

With constant altitude, the equation becomes:

$$\frac{W_f}{W_i} = \exp\left\{-\frac{C\sqrt{\theta}}{V}\left[\frac{\Delta(V^2/2g_0)}{1-\left(\frac{C_D}{C_L}\right)\left(\frac{\beta}{\alpha}\right)\left(\frac{W_{TO}}{T_{SL}}\right)}\right]\right\} \tag{43}$$

Here:

$$\Delta V^2 = V_{final}^2 - V_{initial}^2 \tag{44}$$

4.3 Case 3: Climb and Acceleration

Just a combination of case 1 and case 2:

$$\frac{W_f}{W_i} = \exp\left\{-\frac{C\sqrt{\theta}}{V}\left[\frac{\Delta(h + V^2/2g_0)}{1 - \left(\frac{C_D}{C_L}\right)\left(\frac{\beta}{\alpha}\right)\left(\frac{W_{TO}}{T_{SL}}\right)}\right]\right\} \tag{45}$$

The fraction of the engine thrust work that is dissipated is expressed as:

$$\left(\frac{C_D}{C_L}\right)\left(\frac{\beta}{\alpha}\right)\left(\frac{W_{TO}}{T_{SL}}\right) \tag{46}$$

And the fraction that is invested in mechanical energy or energy height:

$$1 - \left(\frac{C_D}{C_L}\right)\left(\frac{\beta}{\alpha}\right)\left(\frac{W_{TO}}{T_{SL}}\right) \tag{47}$$

4.4 Case 4: Takeoff Acceleration

Variable	Value
dh/dt	0
n	1
R	Given
T	αT_{SL}
$\rho, D, C_{L_{max}}, k_{TO}$	Given
P_s	VdV/g_0dt

Table 3: Case 4 Variables

The equation is simplified as:

$$\frac{W_f}{W_i} = \exp\left\{-\frac{C\sqrt{\theta}}{g_o}\left[\frac{V_{TO}}{1-u}\right]\right\} \tag{48}$$

Recall the definition of u:

$$u = \frac{D+R}{T} \tag{49}$$

From previous section:

$$\frac{D+R}{\beta W_{TO}} = \xi_{TO}\frac{q}{\beta}\left(\frac{S}{W_{TO}}\right) + \mu_{TO} \tag{50}$$

Therefore after rearrangement:

$$u = \left[\xi_{TO}\left(\frac{qS}{\beta W_{TO}}\right) + \mu_{TO}\right]\frac{\beta}{\alpha}\left(\frac{W_{TO}}{T_{SL}}\right) \tag{51}$$

And:

$$V_{TO} = k_{TO} V_{Stall} \tag{52}$$

$$V_{stall} = \sqrt{\frac{2\beta}{\rho C_{L_{max}}} \left(\frac{W_{TO}}{S} \right)} \tag{53}$$

4.5 Case 5: Constant Altitude/Speed Cruise

Variable	**Value**
dh/dt	0
dV/dt	0
n	1
R	0
$h, V, \Delta s$	Given
P_s	0

Table 4: Case 5 Variables

Recall the definition of cruise range:

$$\Delta s = V \Delta t \tag{54}$$

Therefore the equation 27 will be expressed as:

$$\frac{W_f}{W_i} = \exp\left[-\frac{C\sqrt{\theta}}{V} \left(\frac{C_D}{C_L} \right) \Delta s \right] \tag{55}$$

4.6 Case 6: Constant Altitude/Speed Turn

Variable	**Value**
dh/dt	0
dV/dt	0
n	> 1
R	0
N	Number of turns, Given
P_s	0

Table 5: Case 6 Variables

In level flight, the lift generated by the wings must equal **the weight of the aircraft** plus the **centripetal force needed for the turn**. The level turn could be shown here:

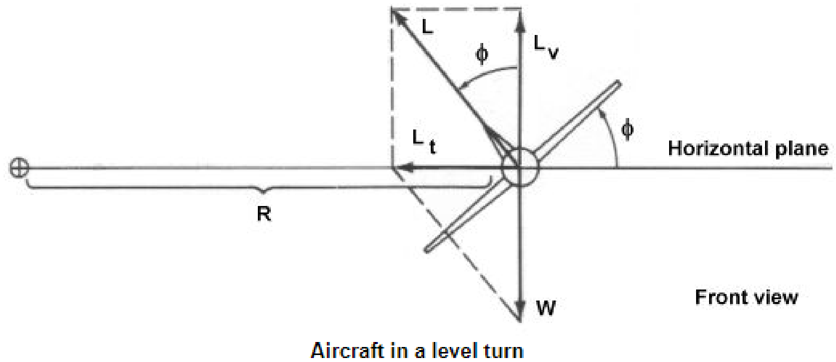

Figure 6: Level turn

Therefore, we have the relationships:

$$L = \frac{W}{\cos(\phi)} \tag{56}$$

And the load factor:

$$n = \frac{L}{W} = \frac{1}{\cos(\phi)} \tag{57}$$

The centripetal force:

$$F_c = L\sin(\phi) = m\frac{V^2}{R_c} = \frac{W}{g}\frac{V^2}{R_c} = \frac{L\cos(\phi)}{g}\frac{V^2}{R_c} \tag{58}$$

Therefore we can express the radius as:

$$R_c = \frac{V^2}{g\tan(\phi)} \tag{59}$$

The turn rate could be expressed as:

$$\omega = \frac{V}{R_c} = \frac{g\tan(\phi)}{V} \tag{60}$$

We can also express this using load factor:

$$n^2 - 1 = \frac{1-\cos^2(\phi)}{\cos^2(\phi)} = \frac{\sin^2(\phi)}{\cos^2(\phi)} = \tan^2(\phi) \tag{61}$$

$$R_c = \frac{V^2}{g\sqrt{n^2-1}} \tag{62}$$

$$\omega = \frac{g\sqrt{n^2-1}}{V} \tag{63}$$

Therefore, the duration of one turn is:

$$\Delta t = \frac{2\pi}{\omega} = \frac{2\pi V}{g\sqrt{n^2 - 1}} \tag{64}$$

With numbers of turn:

$$\Delta t = \frac{2\pi N V}{g\sqrt{n^2 - 1}} \tag{65}$$

And the mass fraction will be:

$$\frac{W_f}{W_i} = \exp\left[-C\sqrt{\theta}\left(\frac{nC_D}{C_L}\right)\frac{2\pi N V}{g_0\sqrt{n^2 - 1}}\right] \tag{66}$$

4.7 Case 7: Best Cruise Mach Number and Altitude (BCM/BCA)

Variable	**Value**
dh/dt	0
dV/dt	0
n	1
R	0
Δs	Value of cruise range, Given
P_s	0

Table 6: Case 7 Variables

Recall the weight change equation for type B:

$$\frac{dW}{W} = -C\sqrt{\theta}\left(\frac{D+R}{W}\right)dt \tag{67}$$

Using Mach number and ds, we can get:

$$\frac{dW}{W} = -C\sqrt{\theta}(\frac{C_d}{C_L})(\frac{ds}{Ma}) \tag{68}$$

Here, a is the speed of sound at specific altitude:

$$a = \sqrt{\gamma R T} \tag{69}$$

For sea level, we have:

$$a_{SL} = \sqrt{\gamma R T_{SL}} \tag{70}$$

Recall the definition of θ:

$$\theta = \frac{T}{T_{SL}} \tag{71}$$

Therefore:

$$\frac{\sqrt{\theta}}{a} = \frac{\sqrt{\frac{T}{T_{SL}}}}{\sqrt{\gamma RT}} = \frac{1}{\sqrt{\gamma RT_{SL}}} = \frac{1}{a_{SL}} \tag{72}$$

Therefore the weight change equation becomes:

$$\frac{dW}{W} = -\left(\frac{C_d}{C_L}\frac{1}{M}\right)\frac{Cds}{a_{SL}} \tag{73}$$

So the weight reduction can be **minimized** by operating at lowest value of ($\left(\frac{C_d}{C_L}\frac{1}{M}\right)$. And this condition will be the **best cruise condition**, denoted using $*$ superscript.

The **critical drag rise** Mach number refers to the Mach number at which the airflow over a substantial part of an aircraft's wing (or other surfaces) transitions from subsonic to supersonic, resulting in a significant increase in drag. When the Mach number below this Mach, we have:

$$\frac{C_D}{C_L} = \frac{K_1 C_L^2 + K_2 C_L + C_{D_0}}{C_L} = K_1 C_L + K_2 + \frac{C_{D_0}}{C_L} \tag{74}$$

Take the derivative of C_L and make it equal to zero:

$$K_1 - \frac{C_{D_0}}{C_L^2} = 0 \tag{75}$$

Therefore:

$$C_L^* = \sqrt{\frac{C_{D_0}}{K_1}} \tag{76}$$

$$C_D^* = 2C_{D_0} + K_2\sqrt{\frac{C_{D_0}}{K_1}} \tag{77}$$

And:

$$(\frac{C_D}{C_L})^* = \sqrt{K_1 C_{D_0}} + K_2 + \sqrt{K_1 C_{D_0}} = \sqrt{4C_{D_0}K_1} + K_2 \tag{78}$$

$$\left(\frac{C_D}{C_L}\frac{1}{M}\right)^* = \frac{\sqrt{4C_{D_0}K_1} + K_2}{M_{CRIT}} \tag{79}$$

Plug in back to weight equation:

$$\frac{dW}{W} = -\left(\frac{\sqrt{4C_{D_0}K_1} + K_2}{M_{CRIT}}\frac{C}{a_{SL}}\right)ds \tag{80}$$

After integration:

$$\frac{W_f}{W_i} = \exp\left[-\frac{\sqrt{4C_{D_0}K_1} + K_2}{M_{CRIT}}\frac{C}{a_{SL}}\Delta s\right] \tag{81}$$

The aircraft must sustain its weight under this condition. The weight could be expressed as:

$$W = qC_LS \tag{82}$$

Recall the dynamic pressure definition:

$$q = \frac{1}{2}\rho V^2 = \frac{1}{2}\rho M^2\gamma RT \tag{83}$$

And the ideal gas law:

$$P = \rho RT \tag{84}$$

Therefore:

$$W = \frac{1}{2}\gamma PM^2C_LS \tag{85}$$

And the weight fraction is:

$$\beta = \frac{W}{W_{TO}} \tag{86}$$

So we can get:

$$\beta W_{TO} = \frac{1}{2}\gamma PM^2C_LS \tag{87}$$

Rearrange:

$$P = \frac{W_{TO}}{S}\frac{2\beta}{\gamma M^2C_L} \tag{88}$$

So the pressure ratio at best cruise condition will be:

$$\delta = \frac{P}{P_{SL}} = \frac{2\beta}{\gamma P_{SL}M_{CRIT}^2}\left(\frac{1}{\sqrt{C_{D_0}/K_1}}\right)\left(\frac{W_{T_0}}{S}\right) \tag{89}$$

Because during the flight, β must gradually diminish as the mission progresses, the altitude must gradually **increase**. If the Mach number is fixed, then the speed will gradually **decrease** (proportional to $\sqrt{\theta}$).

4.8 Case 8: Loiter

Variable	**Value**
dh/dt	0
dV/dt	0
n	1
R	0
Δt	Flight Duration, Given
P_s	0

Table 7: Case 8 Variables

Pretty similar with case 7, the weight change formula now is:

$$\frac{dW}{W} = -C\sqrt{\theta}\left(\frac{C_D}{C_L}\right)dt \tag{90}$$

And the optimum C_D/C_L is still the same, so:

$$\frac{dW}{W} = -C\sqrt{\theta}\left(\sqrt{4C_{D_0}K_1} + K_2\right)dt \tag{91}$$

After integration:

$$\frac{W_f}{W_i} = \exp\left\{-C\sqrt{\theta}\left(\sqrt{4C_{D_0}K_1} + K_2\right)\Delta t\right\} \tag{92}$$

And the pressure variation if the loiter Mach number is given will be:

$$\delta = \frac{2\beta}{\gamma P_{SL}M^2}\left(\frac{1}{\sqrt{C_{D_0}/K_1}}\right)\left(\frac{W_{T_0}}{S}\right) \tag{93}$$

4.9 Case 9: Warm-Up

Variable	**Value**
dh/dt	0
dV/dt	0
n	1
R	$= T = \alpha T_{SL}$
Δt	Warm up time, Given
h	Altitude, Given
P_s	0
V	0, Standing still
D	0, Not moving

Table 8: Case 9 Variables

$$dW = -C\sqrt{\theta}(\alpha T_{SL})dt \tag{94}$$

$$\frac{W_f}{W_i} = 1 - C\sqrt{\theta}\left(\frac{\alpha}{\beta}\left(\frac{T_{SL}}{W_{TO}}\right)\Delta t\right) \tag{95}$$

Where β is evaluated at the beginning of warm-up

4.10 Case 10: Takeoff Rotation

Variable	**Value**
dh/dt	0
dV/dt	0
n	1
$D + R$	$= T = \alpha T_{SL}$
t_R	Warm up time, Given
h	Altitude, Given
P_s	0
V	Given

Table 9: Case 10 Variables

$$dW = -C\sqrt{\theta}(\alpha T_{SL})dt \tag{96}$$

$$\frac{W_f}{W_i} = 1 - C\sqrt{\theta}\left(\frac{\alpha}{\beta}\left(\frac{T_{SL}}{W_{TO}}\right)t_R\right) \tag{97}$$

Where β is evaluated at the beginning of rotation.

4.11 Case 11: Constant Energy Height Maneuver

Variable	**Value**
Δz_e	0
n	1
R	0
$h_{initial}, h_{final}, V_{initial}, V_{final}$	Given

Table 10: Case 11 Variables

In this case potential energy is exchanged for kinetic energy and any loss to aerodynamic drag is balanced by the engine thrust. The weight equation will be:

$$\frac{W_f}{W_i} = \exp\left\{-C\sqrt{\theta}\left(\frac{C_D}{C_L}\right)\Delta t\right\} \tag{98}$$