Pressure Recovery

1 Overview

In high speed flight, an air inlet is a form of **compressor**: it accepts air initially at free stream Mach number and pressure and converts to **lower Mach number and higher static pressure (but lower stagnation/total pressure)**, as required by the engine.

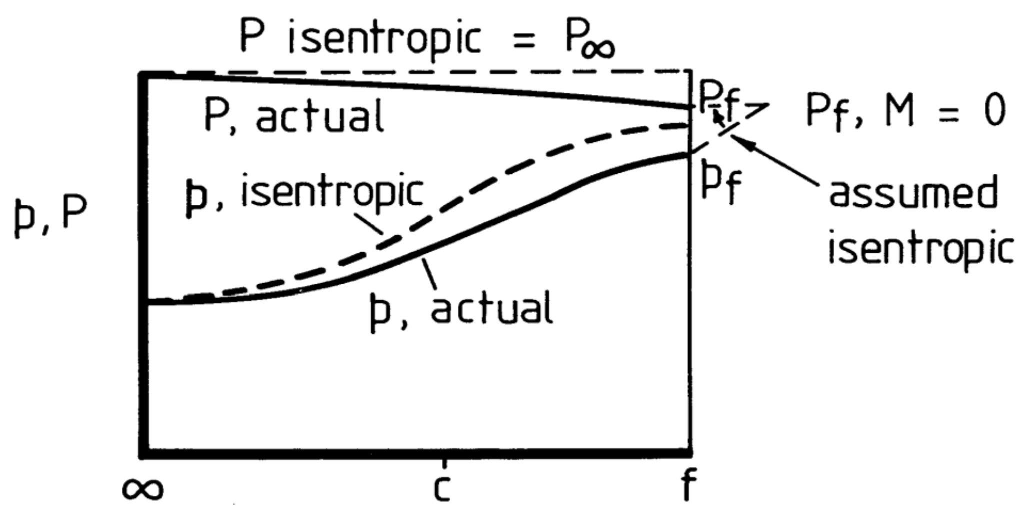

Figure 1: Inlet Pressure Recovery

From the graph, we can see that **static pressure** p_∞ rises to p_f and the **stagnation pressure** P_∞ drops to P_f. If we assume the flow at duct entry is uniform, then we have the relation:

$$\frac{P_f}{p_f} = (1 + \frac{\gamma - 1}{2} M_f^2)^{\frac{\gamma}{\gamma-1}} \tag{1}$$

2 Pressure Recovery Definition

The efficiency for this compression process is:

$$\eta_\sigma = \frac{\text{Work done in compression}}{\text{Kinetic energy available}} \tag{2}$$

After some magic math, we can get the expression:

$$\eta_\sigma = \frac{(\frac{p_f}{p_\infty})^{(\gamma-1)/\gamma} - 1}{\frac{\gamma-1}{2}(M_\infty^2 - M_f^2)} \tag{3}$$

If we want to avoid the sensitivity to particular value of M_f, so we assume that the state reached at station f **the compression is continued isentropically to zero velocity**, which means $M_f = 0$. This way, the static pressure becomes stagnation pressure, and we can get the **Whilst Equation**:

$$\eta_\sigma = \frac{(\frac{P_f}{p_\infty})^{(\gamma-1)/\gamma} - 1}{\frac{\gamma-1}{2}M_\infty^2} \tag{4}$$

For incompressible flow (only), also from some magic math, we have:

$$\eta_{\sigma i} = (P_f - p_\infty)/q_\infty \tag{5}$$

For general high speed flow (compressible), the equation is simplified to:

$$\eta_P = P_f/P_\infty \tag{6}$$

The correlation between the definitions of two efficiencies is given by:

$$(1 + \frac{\gamma-1}{2}M_\infty^2)\eta_P^{(\gamma-1)/\gamma} = 1 + \frac{\gamma-1}{2}M_\infty^2\eta_\sigma \tag{7}$$

The relation expressed by graph is shown below:

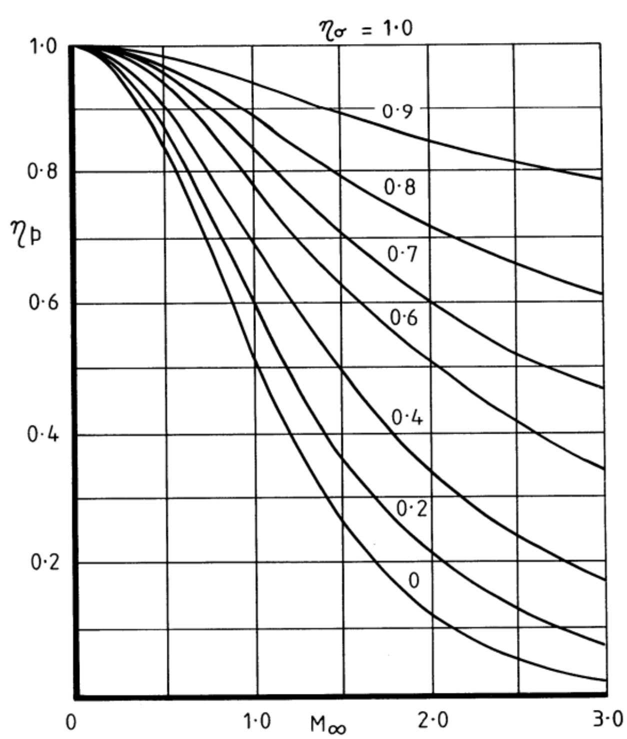

Figure 2: Relations between η_P and η_σ

In summary, η_σ is the formally correct measure of efficiency, but in practice if the flow is incompressible, using $\eta_{\sigma i}$ is more convenient. For all other times, η_P is more convenient. All of them are the **pressure recovery** of the inlet.

Using the pressure loss, we can have:

$$\boxed{\eta_{\sigma i} = 1 - \Delta P / q_\infty} \tag{8}$$

$$\boxed{\eta_P = 1 - \Delta P / P_\infty} \tag{9}$$

Loss of total pressure can occur in three ways:

1. By **frictions** on the walls of the duct and on any external surface which is wetted by flow going into the duct.
2. From **turbulent mixing**, associate with flow separation. The boundary layers in the duct and on forward surfaces are subjected to **adverse pressure gradient**, which may cause flow separation and turbulent mixing.
3. Shock waves. The interaction of a boundary layer and a shock wave is critical.

3 Pressure Recovery Analysis

3.1 Podded vs Integrated Installation

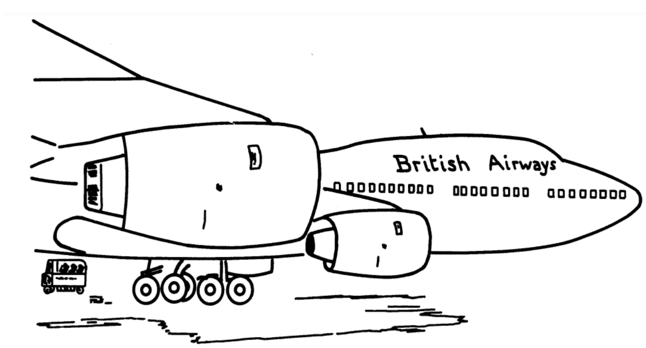

Figure 3: Podded Installation

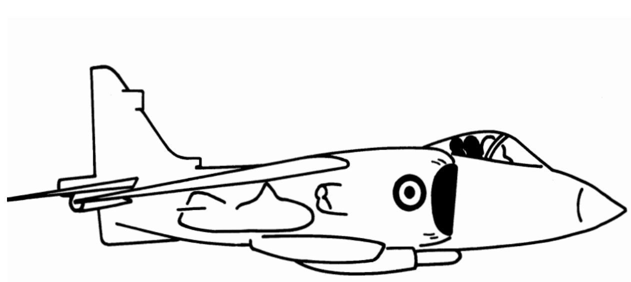

Figure 4: Integrated Installation

The ways to analyze the pressure recovery of podded and integrated installations are different.

1. **For podded installation:** the internal has shortest and most direct route possible to the engine, **PR is nearly** 100%. The significant problem of this type of installation relate to the **external flow**, for example external cowl shaping and the merging to wing and fuselage.

2. **For integrated installation:** the **internal flow** is the most dominant concern. The duct being longer, usually containing bends and shape changes. The presence of aircraft surface ahead of the intake, wetted by the internal flow.

3.2 Wetted Area

Before actually analyzing the PR, we need to know what is the **wetted area (S)**:

1. **For inlet on the side of fuselage:** S is taken to be the surface area between **generators from the ends of the entry** where it meets the surface to the **foremost point of the fuselage nose.**

2. **For inlet on the wing:** S is taken to be the surface area between chordwise lines from ends of the entry to the leading edge.

3.3 Friction Loss Approximation

In this section, we focus on the loss of pressure only caused by friction on the walls of the duct and on the approach, ignore the flow separations. First, we need to relate the friction force at any position to a change in total pressure.

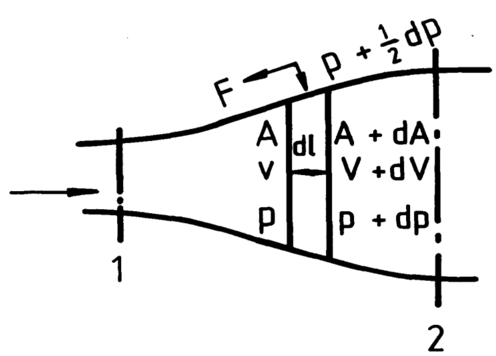

Figure 5: Friction Loss Setup

Parameters:

1. g: local perimeter length along which the friction force is applied

2. dl: length increment

3. F: friction force on an element of the boundary. that is on an area gdl.

4. q: local dynamic pressure, $\frac{1}{2}\rho U^2$

5. C_f: local friction coefficient

Therefore, we have:

$$F = qC_f g dl \tag{10}$$

Now apply the momentum theorem to the streamtube element:

$$\rho AU(U + dU - U) = pA + (p + \frac{1}{2}dp)dA - (p + dp)(A + dA) - F \tag{11}$$

Simplify:

$$\rho AU dU = -Adp - F \tag{12}$$

$$d(p + \frac{1}{2}\rho U^2) = dP = -F/A \tag{13}$$

After integration (and reckoned positively):

$$\Delta P = \int_1^2 F/A = \int_1^2 qC_f \frac{g}{A} dl \tag{14}$$

Recall the continuity equation:

$$AU = A_1 U_1 \tag{15}$$

Therefore we have:

$$\boxed{\frac{\Delta P}{q_1} = \int_1^2 \frac{q}{q_1} C_f \frac{g}{A} dl = \int_1^2 (\frac{A_1}{A})^2 C_f \frac{g}{A} dl} \tag{16}$$

3.4 Approach and Duct Loss

Now we divide the geometry into 2 sections: approach and duct:

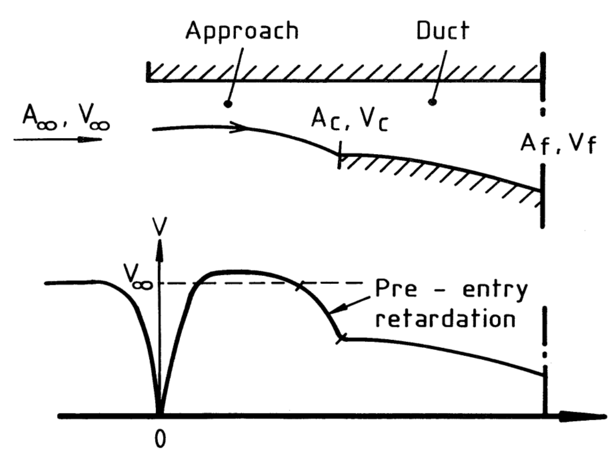

Figure 6: Approach Duct

Then, we can express the pressure loss in terms of two parts.
Approach loss:

$$\frac{\Delta P_a}{q_c} = \int_0^{l_c} (\frac{A_c}{A})^2 C_f \frac{g}{A} dl \tag{17}$$

Duct loss:

$$\frac{\Delta P_d}{q_c} = \int_{l_c}^{l_f} (\frac{A_c}{A})^2 C_f \frac{g}{A} dl \tag{18}$$

From experimental evidence, it follows that the situation on the approach may be approximated by the assumption of a **constant velocity** close to U_∞.

$$\frac{\Delta P_a}{q_c} = \int_0^{l_c} kC_f (\frac{A_c}{A_\infty})^3 \frac{g}{A_c} dl = C_{Fa} (\frac{A_c}{A_\infty})^3 k \frac{S}{A_c} \tag{19}$$

The parameters include:

1. C_{Fa}: overall friction coefficient of the approach
2. A_c/A_∞: inverse of the intake flow ratio
3. S: equal to $\int gdl$ is the wetted surface area
4. k: an empirical factor with a value close to 1.0, related to the difference from U_∞ of the assumed constant approach velocity. Usually, $k = 0.8$.
5. J: **corrected position ratio**, which is equal to $\dfrac{kS}{A_c}$

Therefore, we can also write the approach loss as:

$$\boxed{\frac{\Delta P_a}{q_c} = JC_{Fa} (\frac{A_c}{A_\infty})^3} \tag{20}$$

For the duct loss, based on Squire's result, the Reynolds number for the **equivalent flat plate** to be defined in terms of the **hydraulic diameter at entry**:

$$R_{eff} = \frac{4A_c}{g_c} \frac{U_c}{\nu} \tag{21}$$

Then we have the duct loss:

$$\frac{\Delta P_d}{q_c} = C_{Fd} \int_{l_c}^{l_f} (\frac{A_c}{A})^2 \frac{g}{A} dl = IC_{Fd} \tag{22}$$

Where:

$$I = \int_{l_c}^{l_f} (\frac{A_c}{A})^2 \frac{g}{A} dl \tag{23}$$

Which is purely geometrical function, defined as **duct integral**.
Here we define the inverse flow ratio as:

$$\mu_A = \frac{A_c}{A_\infty} \tag{24}$$

Then we have the total loss for a fully ducted intake:

$$\boxed{\frac{\Delta P}{q_c} = IC_{Fd} + JC_{Fa}\mu_A^3} \tag{25}$$

4 Pressure Recovery Characteristics

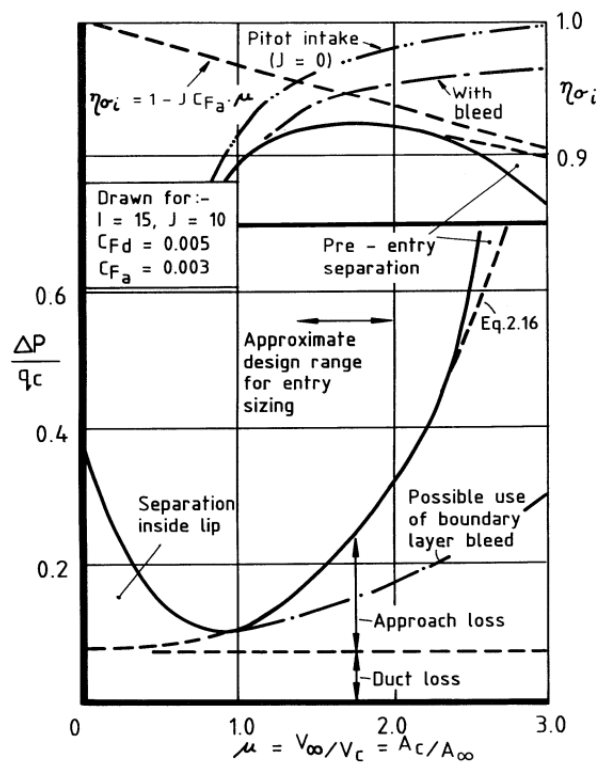

Figure 7: Pressure Recovery Characteristics

1. Using the inverse flow ratio μ_A enables the ground running characteristic ($\mu = 0$) to be examined on an equivalent basis with flight performance.

2. Use of the **entry dynamic head** q_c for non-dimensionalising the total-pressure loss is analytically convenient, because it covers both approach loss and duct loss. However, sometimes the entry area is an **exploitable variable**, so we also can express the loss in terms of **free-stream dynamic head** q_∞:

$$\frac{\Delta P}{q_\infty} = IC_{Fd}(\frac{A_\infty}{A_c})^2 + JC_{Fa}(\frac{A_c}{A_\infty}) = \frac{IC_{Fd}}{\mu_A^2} + JC_{Fa}\mu_A \tag{26}$$

In terms of the efficiency:

$$\eta_{\sigma i} = 1 - \frac{\Delta P}{q_\infty} = 1 - \frac{IC_{Fd}}{\mu_A^2} - JC_{Fa}\mu_A \tag{27}$$

3. The approach loss, is the result of natural boundary layer development on the approach surface. This could be reduced by the flow active control. For example, **a lip standing the entry off from the approach surface** or the use of **suction slot or boundary layer bleed.**

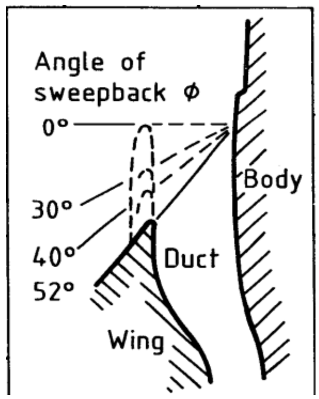

Figure 8: Lip Geometry