Hands-on Exercise 9: Calibrating Hedonic Pricing Model for Private Highrise Property with GWR Method

In this hands-on exercise, you will learn how to calibrate geographically weighted regression models by using GWmodel package of R.

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PUBLISHED

Oct. 12, 2021

Contents

Overview

Geographically weighted regression (GWR) is a spatial statistical technique that takes non-stationary variables into consideration (e.g., climate; demographic factors; physical environment characteristics) and models the local relationships between these independent variables and an outcome of interest (also known as dependent variable). In this hands-on exercise, you will learn how to build hedonic pricing models by using GWR methods. The dependent variable is the resale prices of condominium in 2015. The independent variables are divided into either structural and locational.

The data

Two data sets will be used in this model building exercise, they are:

- URA Master Plan subzone boundary in shapefile format (i.e. MP14_SUBZONE_WEB_PL)
- condo_resale_2015 in csv format (i.e. condo_resale_2015.csv)

Getting Started

Before we get started, it is important for us to install the necessary R packages into R and launch these R packages into R environment.

The R packages needed for this exercise are as follows:

- Geospatial statistical modelling package
    - **GWmodel**
- Spatial data handling
    - **sf**
- Attribute data handling
    - **tidyverse**, especially **readr**, **ggplot2** and **dplyr**
- Choropleth mapping
    - **tmap**

The code chunks below installs and launches these R packages into R environment.

```
packages = c('olsrr', 'corrplot', 'ggpubr', 'sf', 'spdep',
'GWmodel', 'tmap', 'tidyverse')
for (p in packages) {
  if (!require(p, character.only = T
)) {
    install.packages(p)
  }
  library(p,character.only = T)
}
```

A shirt note about GWmodel

GWmodel package provides a collection of localised spatial statistical methods, namely: GW summary statistics, GW principal components analysis, GW discriminant analysis and various forms of GW regression; some of which are provided in basic and robust (outlier resistant) forms. Commonly, outputs or parameters of the GWmodel are mapped to provide a useful exploratory tool, which can often precede (and direct) a more traditional or sophisticated statistical analysis.

Geospatial Data Wrangling

Importing geospatial data

The geospatial data used in this hands-on exercise is called MP14_SUBZONE_WEB_PL. It is in ESRI shapefile format. The shapefile consists of URA Master Plan 2014's planning subzone boundaries. Polygon features are used to represent these geographic boundaries. The GIS data is in svy21 projected coordinates systems.

The code chunk below is used to import *MP_SUBZONE_WEB_PL* shapefile by using *st_read()* of **sf** packages.

```
mpsz = st_read(dsn = "data/geospatial", layer =
```

```
mpsz = st_read(dsn = "data/geospatial", layer =
"MP14_SUBZONE_WEB_PL")
```

```
Reading layer `MP14_SUBZONE_WEB_PL' from data source
  `D:\tskam\IS415\Hands-on_Ex\Hands-on_Ex09\data\geospatial'
  using driver `ESRI Shapefile'
Simple feature collection with 323 features and 15 fields
Geometry type: MULTIPOLYGON
Dimension:     XY
Bounding box:  xmin: 2667.538 ymin: 15748.72 xmax: 56396.44 ymax: 50256.33
Projected CRS: SVY21
```

The report above shows that the R object used to contain the imported MP14_SUBZONE_WEB_PL shapefile is called *mpsz* and it is a simple feature object. The geometry type is *multipolygon*. it is also important to note that mpsz simple feature object does not have EPSG information.

Updating CRS information

The code chunk below updates the newly imported *mpsz* with the correct ESPG code (i.e. 3414)

```
mpsz_svy21 <- st_transform(mpsz, 3414)
```

After transforming the projection metadata, you can varify the projection of the newly transformed *mpsz_svy21* by using *st_crs()* of sf package.

The code chunk below will be used to varify the newly transformed *mpsz_svy21*.

```
st_crs(mpsz_svy21)
```

```
Coordinate Reference System:
  User input: EPSG:3414
  wkt:
PROJCRS["SVY21 / Singapore TM",
    BASEGEOGCRS["SVY21",
        DATUM["SVY21",
            ELLIPSOID["WGS 84",6378137,298.257223563,
                LENGTHUNIT["metre",1]]],
        PRIMEM["Greenwich",0,
            ANGLEUNIT["degree",0.0174532925199433]],
        ID["EPSG",4757]],
    CONVERSION["Singapore Transverse Mercator",
        METHOD["Transverse Mercator",
            ID["EPSG",9807]],
        PARAMETER["Latitude of natural origin",1.36666666666667,
            ANGLEUNIT["degree",0.0174532925199433],
            ID["EPSG",8801]],
        PARAMETER["Longitude of natural origin",103.833333333333,
            ANGLEUNIT["degree",0.0174532925199433],
            ID["EPSG",8802]],
```

```
        PARAMETER["Scale factor at natural origin",1,
            SCALEUNIT["unity",1],
            ID["EPSG",8805]],
        PARAMETER["False easting",28001.642,
            LENGTHUNIT["metre",1],
            ID["EPSG",8806]],
        PARAMETER["False northing",38744.572,
            LENGTHUNIT["metre",1],
            ID["EPSG",8807]]],
    CS[Cartesian,2],
        AXIS["northing (N)",north,
            ORDER[1],
            LENGTHUNIT["metre",1]],
        AXIS["easting (E)",east,
            ORDER[2],
            LENGTHUNIT["metre",1]],
    USAGE[
        SCOPE["Cadastre, engineering survey, topographic mapping."],
        AREA["Singapore - onshore and offshore."],
        BBOX[1.13,103.59,1.47,104.07]],
    ID["EPSG",3414]]
```

Notice that the EPSG: is indicated as *3414* now.

Next, you will reveal the extent of *mpsz_svy21* by using *st_bbox()* of sf package.

```
st_bbox(mpsz_svy21) #view extent
```

```
     xmin      ymin      xmax      ymax 
 2667.538 15748.721 56396.440 50256.334 
```

Aspatial Data Wrangling

Importing the aspatial data

The *condo_resale_2015* is in csv file format. The codes chunk below uses *read_csv()* function of **readr** package to import *condo_resale_2015* into R as a tibble data frame called *condo_resale*.

```
condo_resale = read_csv("data/aspatial/Condo_resale_2015.csv")
```

After importing the data file into R, it is important for us to examine if the data file has been imported correctly.

The codes chunks below uses *glimpse()* to display the data structure of will do the job.

```
glimpse(condo_resale)
```

```
Rows: 1,436
Columns: 23
$ LATITUDE             <dbl> 1.287145, 1.328698, 1.313727, 1.308563,~
$ LONGITUDE            <dbl> 103.7802, 103.8123, 103.7971, 103.8247,~
$ POSTCODE             <dbl> 118635, 288420, 267833, 258380, 467169,~
$ SELLING_PRICE        <dbl> 3000000, 3880000, 3325000, 4250000, 140~
$ AREA_SQM             <dbl> 309, 290, 248, 127, 145, 139, 218, 141,~
$ AGE                  <dbl> 30, 32, 33, 7, 28, 22, 24, 24, 27, 31, ~
$ PROX_CBD             <dbl> 7.941259, 6.609797, 6.898000, 4.038861,~
$ PROX_CHILDCARE       <dbl> 0.16597932, 0.28027246, 0.42922669, 0.3~
$ PROX_ELDERLYCARE     <dbl> 2.5198118, 1.9333338, 0.5021395, 1.9910~
$ PROX_URA_GROWTH_AREA <dbl> 6.618741, 7.505109, 6.463887, 4.906512,~
$ PROX_HAWKER_MARKET   <dbl> 1.76542207, 0.54507614, 0.37789301, 1.6~
$ PROX_KINDERGARTEN    <dbl> 0.05835552, 0.61592412, 0.14120309, 0.3~
$ PROX_MRT             <dbl> 0.5607188, 0.6584461, 0.3053433, 0.6910~
$ PROX_PARK            <dbl> 1.1710446, 0.1992269, 0.2779886, 0.9832~
$ PROX_PRIMARY_SCH     <dbl> 1.6340256, 0.9747834, 1.4715016, 1.4546~
$ PROX_TOP_PRIMARY_SCH <dbl> 3.3273195, 0.9747834, 1.4715016, 2.3006~
$ PROX_SHOPPING_MALL   <dbl> 2.2102717, 2.9374279, 1.2256850, 0.3525~
$ PROX_SUPERMARKET     <dbl> 0.9103958, 0.5900617, 0.4135583, 0.4162~
$ PROX_BUS_STOP        <dbl> 0.10336166, 0.28673408, 0.28504777, 0.2~
$ NO_Of_UNITS          <dbl> 18, 20, 27, 30, 30, 31, 32, 32, 32, 32,~
$ FAMILY_FRIENDLY      <dbl> 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0, 0, ~
$ FREEHOLD             <dbl> 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 1, ~
$ LEASEHOLD_99YR       <dbl> 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, ~
```

```
head(condo_resale$LONGITUDE) #see the data in XCOORD column
```

```
[1] 103.7802 103.8123 103.7971 103.8247 103.9505 103.9386
```

```
head(condo_resale$LATITUDE) #see the data in YCOORD column
```

```
[1] 1.287145 1.328698 1.313727 1.308563 1.321437 1.314198
```

```
summary(condo_resale)
```

```
    LATITUDE       LONGITUDE        POSTCODE      SELLING_PRICE
 Min.   :1.240   Min.   :103.7   Min.   : 18965   Min.   :  540000
 1st Qu.:1.309   1st Qu.:103.8   1st Qu.:259849   1st Qu.: 1100000
 Median :1.328   Median :103.8   Median :469298   Median : 1383222
 Mean   :1.334   Mean   :103.8   Mean   :440439   Mean   : 1751211
 3rd Qu.:1.357   3rd Qu.:103.9   3rd Qu.:589486   3rd Qu.: 1950000
 Max.   :1.454   Max.   :104.0   Max.   :828833   Max.   :18000000
    AREA_SQM          AGE           PROX_CBD       PROX_CHILDCARE
 Min.   : 34.0   Min.   : 0.00   Min.   : 0.3869   Min.   :0.004927
 1st Qu.:103.0   1st Qu.: 5.00   1st Qu.: 5.5574   1st Qu.:0.174481
 Median :121.0   Median :11.00   Median : 9.3567   Median :0.258135
 Mean   :136.5   Mean   :12.14   Mean   : 9.3254   Mean   :0.326313
 3rd Qu.:156.0   3rd Qu.:18.00   3rd Qu.:12.6661   3rd Qu.:0.368293
 Max.   :619.0   Max.   :37.00   Max.   :19.1804   Max.   :3.465726
```

```
 PROX_ELDERLYCARE  PROX_URA_GROWTH_AREA PROX_HAWKER_MARKET
 Min.   :0.05451   Min.   :0.2145       Min.   :0.05182
 1st Qu.:0.61254   1st Qu.:3.1643       1st Qu.:0.55245
 Median :0.94179   Median :4.6186       Median :0.90842
 Mean   :1.05351   Mean   :4.5981       Mean   :1.27987
 3rd Qu.:1.35122   3rd Qu.:5.7550       3rd Qu.:1.68578
 Max.   :3.94916   Max.   :9.1554       Max.   :5.37435
 PROX_KINDERGARTEN     PROX_MRT          PROX_PARK
 Min.   :0.004927   Min.   :0.05278   Min.   :0.02906
 1st Qu.:0.276345   1st Qu.:0.34646   1st Qu.:0.26211
 Median :0.413385   Median :0.57430   Median :0.39926
 Mean   :0.458903   Mean   :0.67316   Mean   :0.49802
 3rd Qu.:0.578474   3rd Qu.:0.84844   3rd Qu.:0.65592
 Max.   :2.229045   Max.   :3.48037   Max.   :2.16105
 PROX_PRIMARY_SCH  PROX_TOP_PRIMARY_SCH PROX_SHOPPING_MALL
 Min.   :0.07711   Min.   :0.07711      Min.   :0.0000
 1st Qu.:0.44024   1st Qu.:1.34451      1st Qu.:0.5258
 Median :0.63505   Median :1.88213      Median :0.9357
 Mean   :0.75471   Mean   :2.27347      Mean   :1.0455
 3rd Qu.:0.95104   3rd Qu.:2.90954      3rd Qu.:1.3994
 Max.   :3.92899   Max.   :6.74819      Max.   :3.4774
 PROX_SUPERMARKET PROX_BUS_STOP       NO_Of_UNITS
 Min.   :0.0000   Min.   :0.001595   Min.   :  18.0
 1st Qu.:0.3695   1st Qu.:0.098356   1st Qu.: 188.8
 Median :0.5687   Median :0.151710   Median : 360.0
 Mean   :0.6141   Mean   :0.193974   Mean   : 409.2
 3rd Qu.:0.7862   3rd Qu.:0.220466   3rd Qu.: 590.0
 Max.   :2.2441   Max.   :2.476639   Max.   :1703.0
 FAMILY_FRIENDLY     FREEHOLD       LEASEHOLD_99YR
 Min.   :0.0000   Min.   :0.0000   Min.   :0.0000
 1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000
 Median :0.0000   Median :0.0000   Median :0.0000
 Mean   :0.4868   Mean   :0.4227   Mean   :0.4882
 3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.0000
 Max.   :1.0000   Max.   :1.0000   Max.   :1.0000
```

Converting aspatial data frame into a sf object

Currently, the condo_resale data frame is aspatial. We will convert it to a *sf* object. The code chunk below converts condo_resale data frame into a simple feature data frame by using *st_as_sf()* of **sf** packages.

Notice that *st_transform()* of **sf** package is used to convert the coordinates from wgs84 (i.e. crs:4326) to svy21 (i.e. crs=3414).

```
condo_resale.sf <- st_as_sf(condo_resale,
                            coords = c("LONGITUDE", "LATITUDE"),
                            crs=4326) %>%
  st_transform(crs=3414)
head(condo_resale.sf)
```

```
head(condo_resale.sf)
```

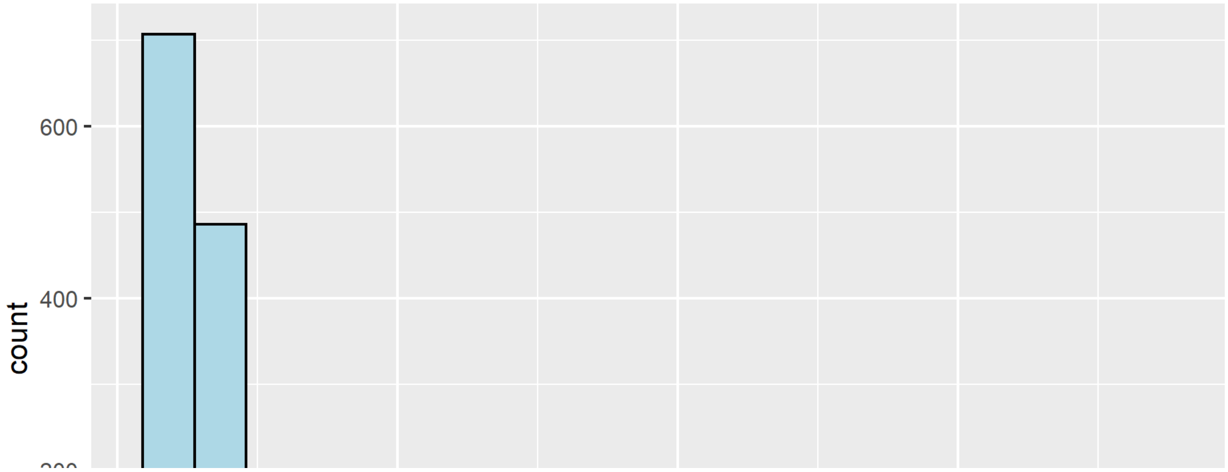
```
Simple feature collection with 6 features and 21 fields
Geometry type: POINT
Dimension:     XY
Bounding box:  xmin: 22085.12 ymin: 29951.54 xmax: 41042.56 ymax: 34546.2
Projected CRS: SVY21 / Singapore TM
# A tibble: 6 x 22
  POSTCODE SELLING_PRICE AREA_SQM   AGE PROX_CBD PROX_CHILDCARE
     <dbl>         <dbl>    <dbl> <dbl>    <dbl>          <dbl>
1   118635       3000000      309    30     7.94          0.166
2   288420       3880000      290    32     6.61          0.280
3   267833       3325000      248    33     6.90          0.429
4   258380       4250000      127     7     4.04          0.395
5   467169       1400000      145    28    11.8           0.119
6   466472       1320000      139    22    10.3           0.125
# ... with 16 more variables: PROX_ELDERLYCARE <dbl>,
#   PROX_URA_GROWTH_AREA <dbl>, PROX_HAWKER_MARKET <dbl>,
#   PROX_KINDERGARTEN <dbl>, PROX_MRT <dbl>, PROX_PARK <dbl>,
#   PROX_PRIMARY_SCH <dbl>, PROX_TOP_PRIMARY_SCH <dbl>,
#   PROX_SHOPPING_MALL <dbl>, PROX_SUPERMARKET <dbl>,
#   PROX_BUS_STOP <dbl>, NO_Of_UNITS <dbl>, FAMILY_FRIENDLY <dbl>,
#   FREEHOLD <dbl>, LEASEHOLD_99YR <dbl>, geometry <POINT [m]>
```

Exploratory Data Analysis

EDA using statistical graphics

We can plot the distribution of ***SELLING_PRICE*** by using appropriate Exploratory Data Analysis (EDA) as shown in the code chunk below.

```
ggplot(data=condo_resale.sf, aes(x=`SELLING_PRICE`)) +
  geom_histogram(bins=20, color="black", fill="light blue")
```



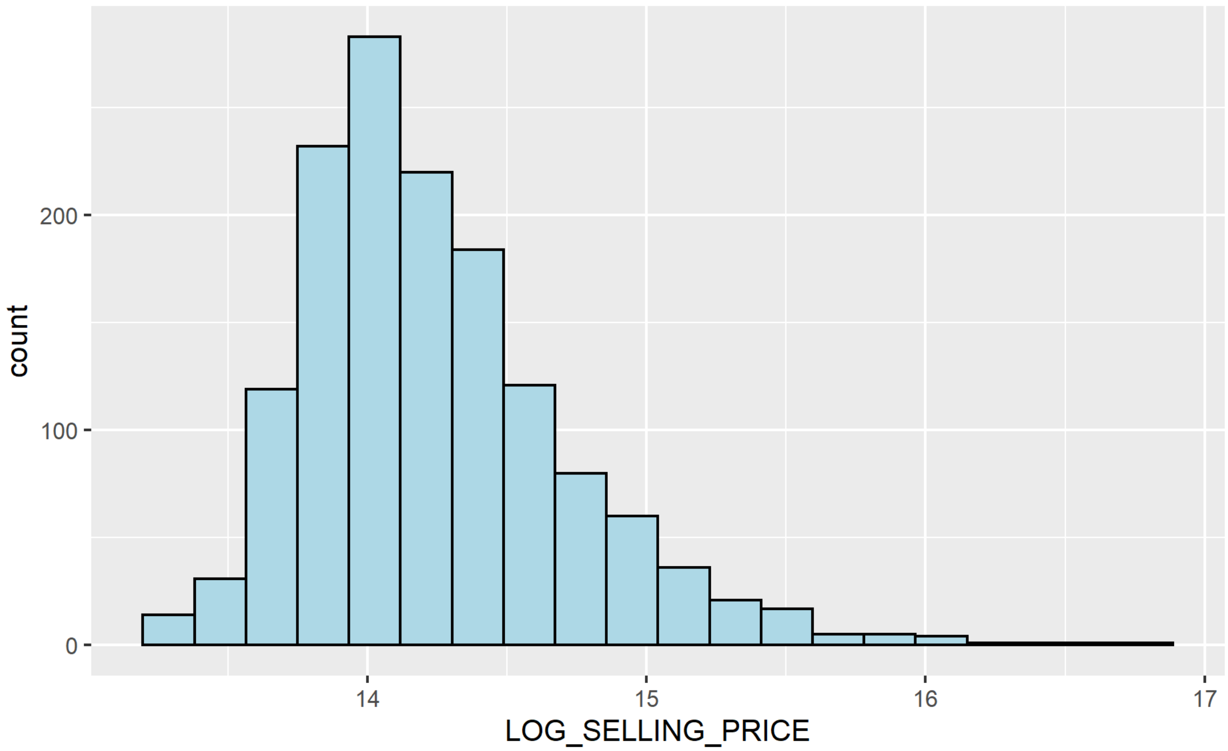
The figure above reveals a right skewed distribution. This means that more condominium units were transacted at relative lower prices.

Statistically, the skewed dsitribution can be normalised by using log transformation. The code chunk below is used to derive a new variable called ***LOG_SELLING_PRICE*** by using a log transformation on the variable ***SELLING_PRICE***. It is performed using *mutate()* of **dplyr** package.

```
condo_resale.sf <- condo_resale.sf %>%
  mutate(`LOG_SELLING_PRICE` = log(SELLING_PRICE))
```

Now, you can plot the ***LOG_SELLING_PRICE*** using the code chunk below.

```
ggplot(data=condo_resale.sf, aes(x=`LOG_SELLING_PRICE`)) +
  geom_histogram(bins=20, color="black", fill="light blue")
```



Notice that the distribution is relatively less skewed after the transformation.

Multiple Histogram Plots distribution of variables

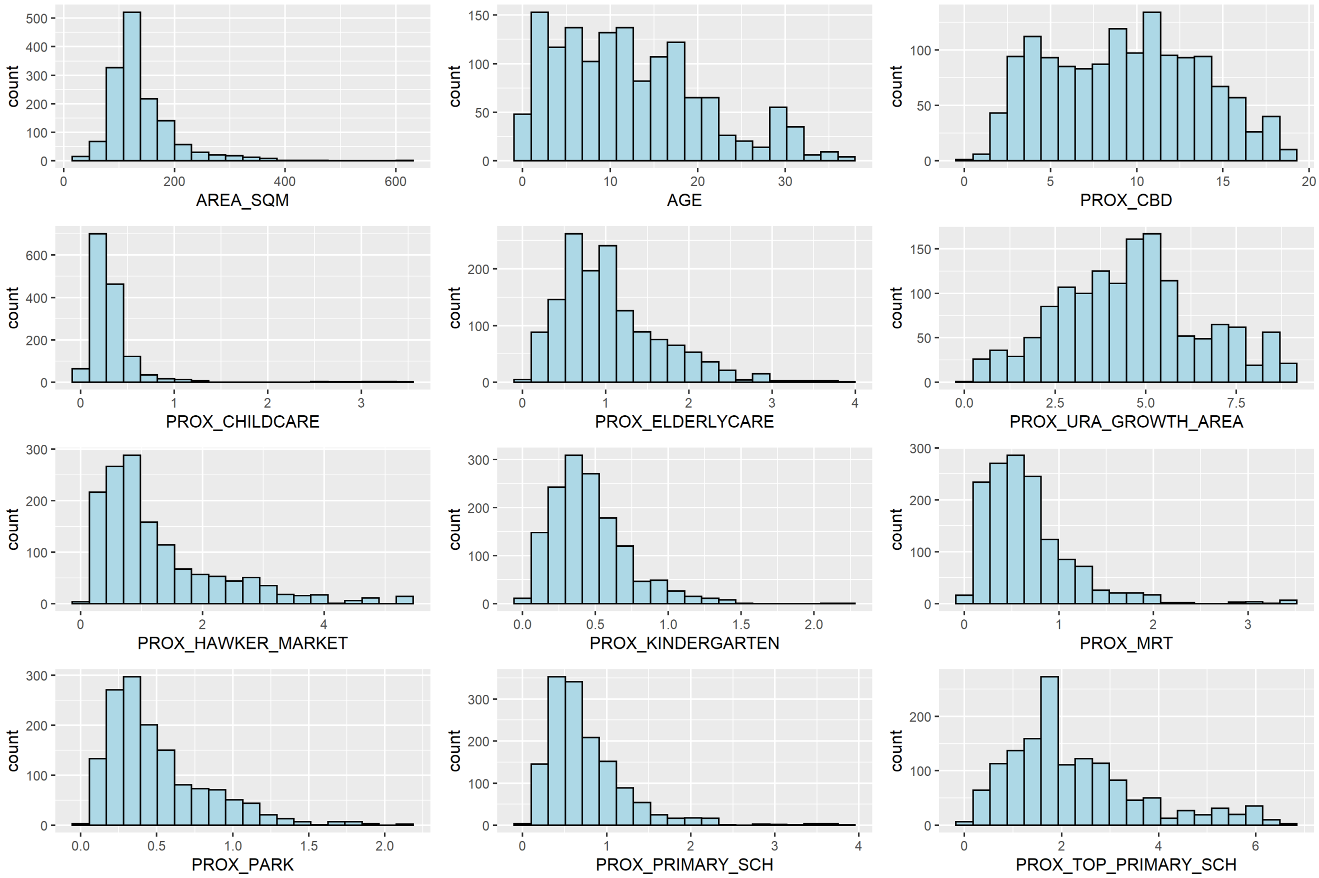
In this section, you will learn how to draw a small mutliple histograms (also known as treltis plot) by using *ggarrange()* of **ggpubr** package.

The code chunk below is used to create 12 histograms. Then, ggarrnage() is used to organised these histogram into a 3 columns by 4 rows small multiple plot.

```
AREA_SQM <- ggplot(data=condo_resale.sf, aes(x=
`AREA_SQM`)) +
  geom_histogram(bins=20, color="black", fill=
"light blue")
AGE <- ggplot(data=condo_resale.sf, aes(x=
`AGE`)) +
  geom_histogram(bins=20, color="black", fill=
"light blue")
PROX_CBD <- ggplot(data=condo_resale.sf, aes(x=
`PROX_CBD`)) +
  geom_histogram(bins=20, color="black", fill=
"light blue")
PROX_CHILDCARE <- ggplot(data=condo_resale.sf, aes(x
= `PROX_CHILDCARE`)) +
  geom_histogram(bins=20, color="black", fill=
"light blue")
PROX_ELDERLYCARE <- ggplot(data=condo_resale.sf, aes(x
= `PROX_ELDERLYCARE`)) +
  geom_histogram(bins=20, color="black", fill=
"light blue")
PROX_URA_GROWTH_AREA <- ggplot(data=condo_resale.sf, aes(
x= `PROX_URA_GROWTH_AREA`)) +
  geom_histogram(bins=20, color="black", fill=
"light blue")
PROX_HAWKER_MARKET <- ggplot(data=condo_resale.sf, aes(x
= `PROX_HAWKER_MARKET`)) +
  geom_histogram(bins=20, color="black", fill=
"light blue")
PROX_KINDERGARTEN <- ggplot(data=condo_resale.sf, aes(x
= `PROX_KINDERGARTEN`)) +
  geom_histogram(bins=20, color="black", fill=
"light blue")
PROX_MRT <- ggplot(data=condo_resale.sf, aes(x=
`PROX_MRT`)) +
  geom_histogram(bins=20, color="black", fill=
"light blue")
PROX_PARK <- ggplot(data=condo_resale.sf, aes(x=
`PROX_PARK`)) +
  geom_histogram(bins=20, color="black", fill=
"light blue")
PROX_PRIMARY_SCH <- ggplot(data=condo_resale.sf, aes(x
```

```
= `PROX_PRIMARY_SCH`)) +
  geom_histogram(bins=20, color="black", fill="light blue")
PROX_TOP_PRIMARY_SCH <- ggplot(data=condo_resale.sf, aes(
x= `PROX_TOP_PRIMARY_SCH`)) +
  geom_histogram(bins=20, color="black", fill=
"light blue")


ggarrange(AREA_SQM, AGE, PROX_CBD, PROX_CHILDCARE, PROX_ELDERLYCARE,
PROX_URA_GROWTH_AREA, PROX_HAWKER_MARKET, PROX_KINDERGARTEN, PROX_MRT, PROX_PARK,
PROX_PRIMARY_SCH, PROX_TOP_PRIMARY_SCH, ncol = 3, nrow = 4
)
```



Drawing Statistical Point Map

Lastly, we want to reveal the geospatial distribution condominium resale prices in Singapore. The map will be prepared by using **tmap** package.

First, we will turn on the interactive mode of tmap by using the code chunk below.

```
tmap_mode("plot")
```

Next, the code chunks below is used to create an interactive point symbol map.

```
tm_shape(mpsz_svy21) +
  tm_polygons() +
tm_shape(condo_resale.sf) +
  tm_dots(col = "SELLING_PRICE",
          alpha = 0.6,
          style="quantile") +
  tm_view(set.zoom.limits = c(11,14))
```

Notice that *tm_dots()* is used instead of *tm_bubbles()*.

set.zoom.limits argument of *tm_view()* sets the minimum and maximum zoom level to 11 and 14 respectively.

Before moving on to the next section, the code below will be used to turn R display into **plot** mode.

```
tmap_mode("plot")
```

Hedonic Pricing Modelling in R

In this section, you will learn how to building hedonic pricing models for condominium resale units using *lm()* of R base.

Simple Linear Regression Method

First, we will build a simple linear regression model by using *SELLING_PRICE* as the dependent variable and *AREA_SQM* as the independent variable.

```
condo.slr <- lm(formula=SELLING_PRICE ~ AREA_SQM, data = condo_resale.sf)
```

lm() returns an object of class "lm" or for multiple responses of class c("mlm", "lm").

The functions *summary()* and *anova()* can be used to obtain and print a summary and analysis of variance table of the results. The generic accessor functions coefficients, effects, fitted.values and residuals extract various useful features of the value returned by **lm**.

```
summary(condo.slr)
```

```
Call:
lm(formula = SELLING_PRICE ~ AREA_SQM, data = condo_resale.sf)

Residuals:
     Min       1Q   Median       3Q      Max 
-3695815  -391764   -87517   258900 13503875 
```

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -258121.1    63517.2  -4.064 5.09e-05 ***
AREA_SQM      14719.0      428.1  34.381  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 942700 on 1434 degrees of freedom
Multiple R-squared:  0.4518,    Adjusted R-squared:  0.4515
F-statistic:  1182 on 1 and 1434 DF,  p-value: < 2.2e-16
```

The output report reveals that the SELLING_PRICE can be explained by using the formula:

```
*y = -258121.1 + 14719x1*
```

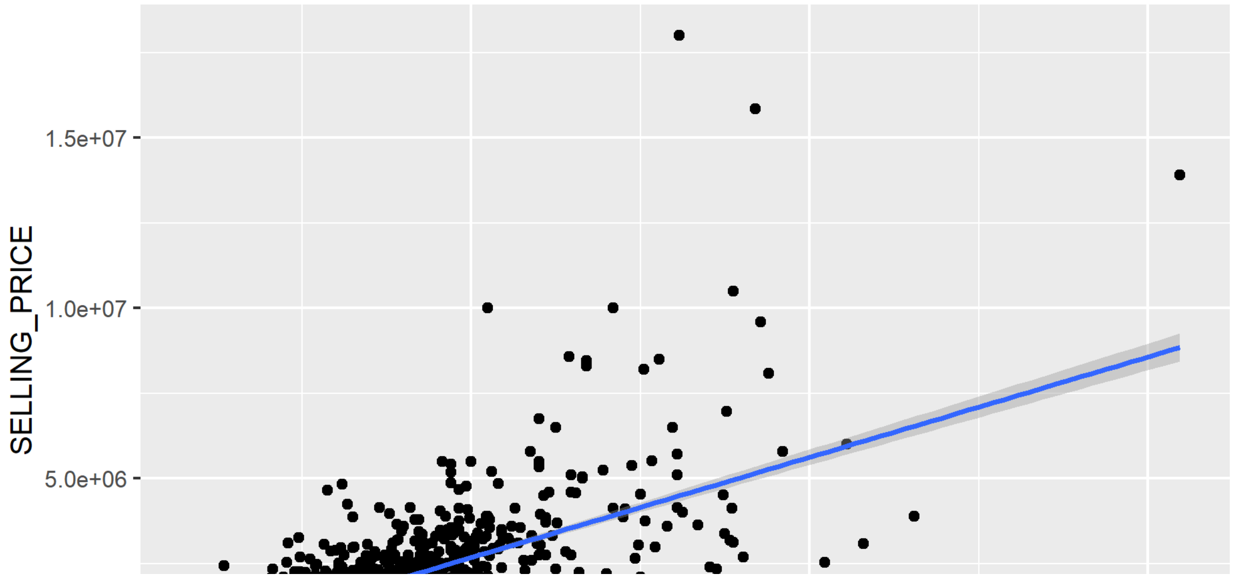
The R-squared of 0.4518 reveals that the simple regression model built is able to explain about 45% of the resale prices.

Since p-value is much smaller than 0.0001, we will reject the null hypothesis that mean is a good estimator of SELLING_PRICE. This will allow us to infer that simple linear regression model above is a good estimator of SELLING_PRICE.

The **Coefficients:** section of the report reveals that the p-values of both the estimates of the Intercept and ARA_SQM are smaller than 0.001. In view of this, the null hypothesis of the B0 and B1 are equal to 0 will be rejected. As a results, we will be able to infer that the B0 and B1 are good parameter estimates.

To visualise the best fit curve on a scatterplot, we can incorporate *lm()* as a method function in ggplot's geometry as shown in the code chunk below.

```
ggplot(data=condo_resale.sf,
       aes(x=`AREA_SQM`, y=`SELLING_PRICE`)) +
  geom_point() +
  geom_smooth(method = lm)
```



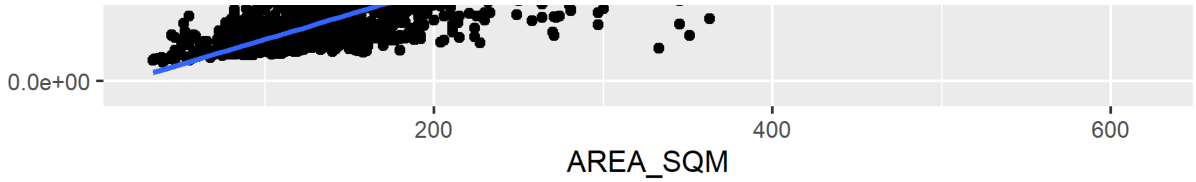


Figure above reveals that there are a few statistical outliers with relatively high selling prices.

Multiple Linear Regression Method

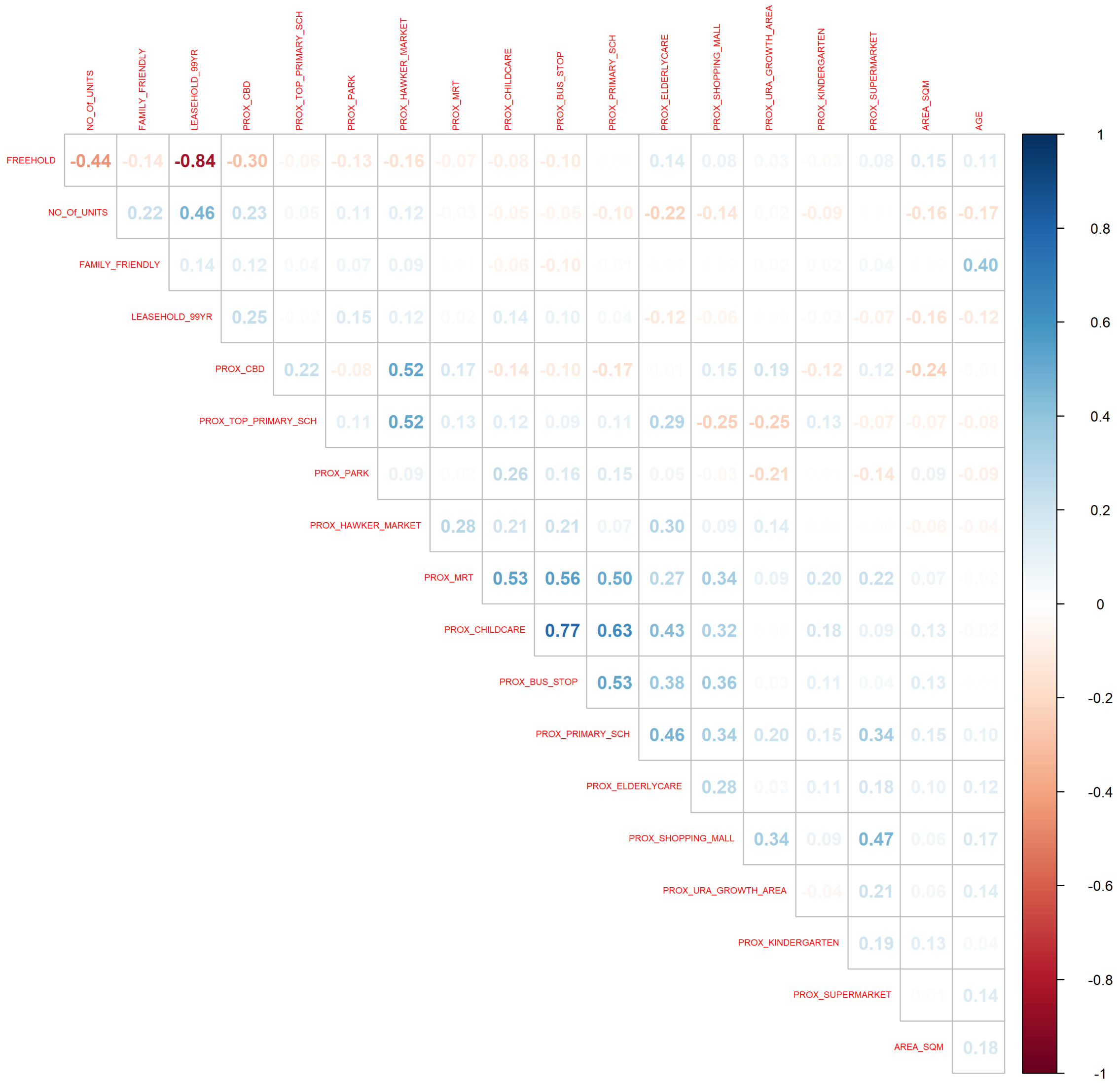
Visualising the relationships of the independent variables

Before building a multiple regression model, it is important to ensure that the indepdent variables used are not highly correlated to each other. If these highly correlated independent variables are used in building a regression model by mistake, the quality of the model will be compromised. This phenominan is known as ***multicollinearity*** in statistics.

Correlation matrix is commonly used to visualise the relationships between the independent variables. Beside the *pairs()* of R, there are many packages support the display of a correlation matrix. In this section, the *corrplot* package will be used.

The code chunk below is used to plot a scatterplot matrix of the relationship between the independent variables in ***condo_resale*** data.frame.

```
corrplot(cor(condo_resale[, 5:23]), diag = FALSE, order = "AOE",
         tl.pos = "td", tl.cex = 0.5, method = "number", type = "upper")
```



Matrix reorder is very important for mining the hiden structure and patter in the matrix. There are four methods in corrplot (parameter order), named “AOE”, “FPC”, “hclust”, “alphabet”. In the code chunk above, alphabet order is used. It orders the variables alphabetically.

From the scatterplot matrix, it is clear that ***Freehold*** is highly correlated to ***LEASE_99YEAR***. In view of this, it is wiser to only include either one of them in the subsequent model building. As a result, ***LEASE_99YEAR*** is excluded in the subsequent model building.

Building a hedonic pricing model using multiple linear regression method

The code chunk below using **lm()** to calibrate the multiple linear regression model.

```r
condo.mlr <- lm ( formula = SELLING_PRICE ~ AREA_SQM
+ AGE + PROX_CBD + PROX_CHILDCARE + PROX_ELDERLYCARE
+ PROX_URA_GROWTH_AREA + PROX_HAWKER_MARKET + PROX_KINDERGARTEN
+ PROX_MRT + PROX_PARK + PROX_PRIMARY_SCH +
PROX_TOP_PRIMARY_SCH + PROX_SHOPPING_MALL + PROX_SUPERMARKET +
PROX_BUS_STOP + NO_Of_UNITS + FAMILY_FRIENDLY + FREEHOLD , data=
condo_resale.sf)
summary ( condo.mlr)
```

```
Call:
lm(formula = SELLING_PRICE ~ AREA_SQM + AGE + PROX_CBD + PROX_CHILDCARE +
    PROX_ELDERLYCARE + PROX_URA_GROWTH_AREA + PROX_HAWKER_MARKET +
    PROX_KINDERGARTEN + PROX_MRT + PROX_PARK + PROX_PRIMARY_SCH +
    PROX_TOP_PRIMARY_SCH + PROX_SHOPPING_MALL + PROX_SUPERMARKET +
    PROX_BUS_STOP + NO_Of_UNITS + FAMILY_FRIENDLY + FREEHOLD,
    data = condo_resale.sf)

Residuals:
     Min       1Q   Median       3Q      Max
-3475964  -293923   -23069   241043 12260381

Coefficients:
                       Estimate Std. Error t value Pr(>|t|)
(Intercept)           481728.40  121441.01   3.967 7.65e-05 ***
AREA_SQM               12708.32     369.59  34.385  < 2e-16 ***
AGE                   -24440.82    2763.16  -8.845  < 2e-16 ***
PROX_CBD              -78669.78    6768.97 -11.622  < 2e-16 ***
PROX_CHILDCARE       -351617.91  109467.25  -3.212  0.00135 **
PROX_ELDERLYCARE      171029.42   42110.51   4.061 5.14e-05 ***
PROX_URA_GROWTH_AREA   38474.53   12523.57   3.072  0.00217 **
PROX_HAWKER_MARKET     23746.10   29299.76   0.810  0.41782
PROX_KINDERGARTEN     147468.99   82668.87   1.784  0.07466 .
PROX_MRT             -314599.68   57947.44  -5.429 6.66e-08 ***
PROX_PARK             563280.50   66551.68   8.464  < 2e-16 ***
PROX_PRIMARY_SCH      180186.08   65237.95   2.762  0.00582 **
PROX_TOP_PRIMARY_SCH    2280.04   20410.43   0.112  0.91107
PROX_SHOPPING_MALL   -206604.06   42840.60  -4.823 1.57e-06 ***
PROX_SUPERMARKET      -44991.80   77082.64  -0.584  0.55953
PROX_BUS_STOP         683121.35  138353.28   4.938 8.85e-07 ***
NO_Of_UNITS             -231.18      89.03  -2.597  0.00951 **
FAMILY_FRIENDLY       140340.77   47020.55   2.985  0.00289 **
FREEHOLD              359913.01   49220.22   7.312 4.38e-13 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 755800 on 1417 degrees of freedom
Multiple R-squared:  0.6518,    Adjusted R-squared:  0.6474
F-statistic: 147.4 on 18 and 1417 DF,  p-value: < 2.2e-16
```

With reference to the report above, it is clear that not all the indepent variables are statistically significant. We will revised the model by removing those variables which are not statistically significant.

Now, we are ready to calibrate the revised model by using the code chunk below.

```
condo.mlr1 <-         lm        (         formula =         SELLING_PRICE ~         AREA_SQM
+         AGE         +         PROX_CBD  +         PROX_CHILDCARE +         PROX_ELDERLYCARE
+         PROX_URA_GROWTH_AREA +         PROX_MRT    +         PROX_PARK  +
PROX_PRIMARY_SCH +         PROX_SHOPPING_MALL  +         PROX_BUS_STOP  +         NO_Of_UNITS
+         FAMILY_FRIENDLY +         FREEHOLD , data=         condo_resale.sf)
ols_regress(         condo.mlr1)
```

```
                             Model Summary
-------------------------------------------------------------------------
R                       0.807       RMSE                     755957.289
R-Squared               0.651       Coef. Var                    43.168
Adj. R-Squared          0.647       MSE                571471422208.591
Pred R-Squared          0.638       MAE                      414819.628
-------------------------------------------------------------------------
 RMSE: Root Mean Square Error
 MSE: Mean Square Error
 MAE: Mean Absolute Error

                                     ANOVA
---------------------------------------------------------------------------------
                    Sum of
                   Squares          DF         Mean Square       F          Sig.
---------------------------------------------------------------------------------
Regression    1.512586e+15          14        1.080418e+14    189.059     0.0000
Residual      8.120609e+14        1421    571471422208.591
Total         2.324647e+15        1435
---------------------------------------------------------------------------------


                                          Parameter Estimates
-----------------------------------------------------------------------------------------------------
               model           Beta    Std. Error    Std. Beta        t        Sig           lower
-----------------------------------------------------------------------------------------------------
         (Intercept)     527633.222    108183.223                   4.877     0.000      315417.244
            AREA_SQM      12777.523       367.479        0.584     34.771     0.000       12056.663
                 AGE     -24687.739      2754.845       -0.167     -8.962     0.000      -30091.739
            PROX_CBD     -77131.323      5763.125       -0.263    -13.384     0.000      -88436.469
      PROX_CHILDCARE    -318472.751    107959.512       -0.084     -2.950     0.003     -530249.889
    PROX_ELDERLYCARE     185575.623     39901.864        0.090      4.651     0.000      107302.737
PROX_URA_GROWTH_AREA      39163.254     11754.829        0.060      3.332     0.001       16104.571
            PROX_MRT    -294745.107     56916.367       -0.112     -5.179     0.000     -406394.234
           PROX_PARK     570504.807     65507.029        0.150      8.709     0.000      442003.938
    PROX_PRIMARY_SCH     159856.136     60234.599        0.062      2.654     0.008       41697.849
  PROX_SHOPPING_MALL    -220947.251     36561.832       -0.115     -6.043     0.000     -292668.213
       PROX_BUS_STOP     682482.221    134513.243        0.134      5.074     0.000      418616.359
         NO_Of_UNITS       -245.480        87.947       -0.053     -2.791     0.005        -418.000
     FAMILY_FRIENDLY     146307.576     46893.021        0.057      3.120     0.002       54320.593
            FREEHOLD     350599.812     48506.485        0.136      7.228     0.000      255447.802
-----------------------------------------------------------------------------------------------------
```

Checking for multicolinearity

In this section, we would like to introduce you a fantastic R package specially programmed for performing OLS regression. It is called **olsrr**. It provides a collection of very useful methods for building better multiple linear regression models:

- comprehensive regression output
- residual diagnostics
- measures of influence
- heteroskedasticity tests
- collinearity diagnostics
- model fit assessment
- variable contribution assessment
- variable selection procedures

In the code chunk below, the *ols_vif_tol()* of **olsrr** package is used to test if there are sign of multicollinearity.

```
ols_vif_tol(        condo.mlr1)
```

```
              Variables Tolerance      VIF
1              AREA_SQM 0.8728554 1.145665
2                   AGE 0.7071275 1.414172
3              PROX_CBD 0.6356147 1.573280
4        PROX_CHILDCARE 0.3066019 3.261559
5      PROX_ELDERLYCARE 0.6598479 1.515501
6   PROX_URA_GROWTH_AREA 0.7510311 1.331503
7              PROX_MRT 0.5236090 1.909822
8             PROX_PARK 0.8279261 1.207837
9      PROX_PRIMARY_SCH 0.4524628 2.210126
10    PROX_SHOPPING_MALL 0.6738795 1.483945
11         PROX_BUS_STOP 0.3514118 2.845664
12           NO_Of_UNITS 0.6901036 1.449058
13       FAMILY_FRIENDLY 0.7244157 1.380423
14              FREEHOLD 0.6931163 1.442759
```

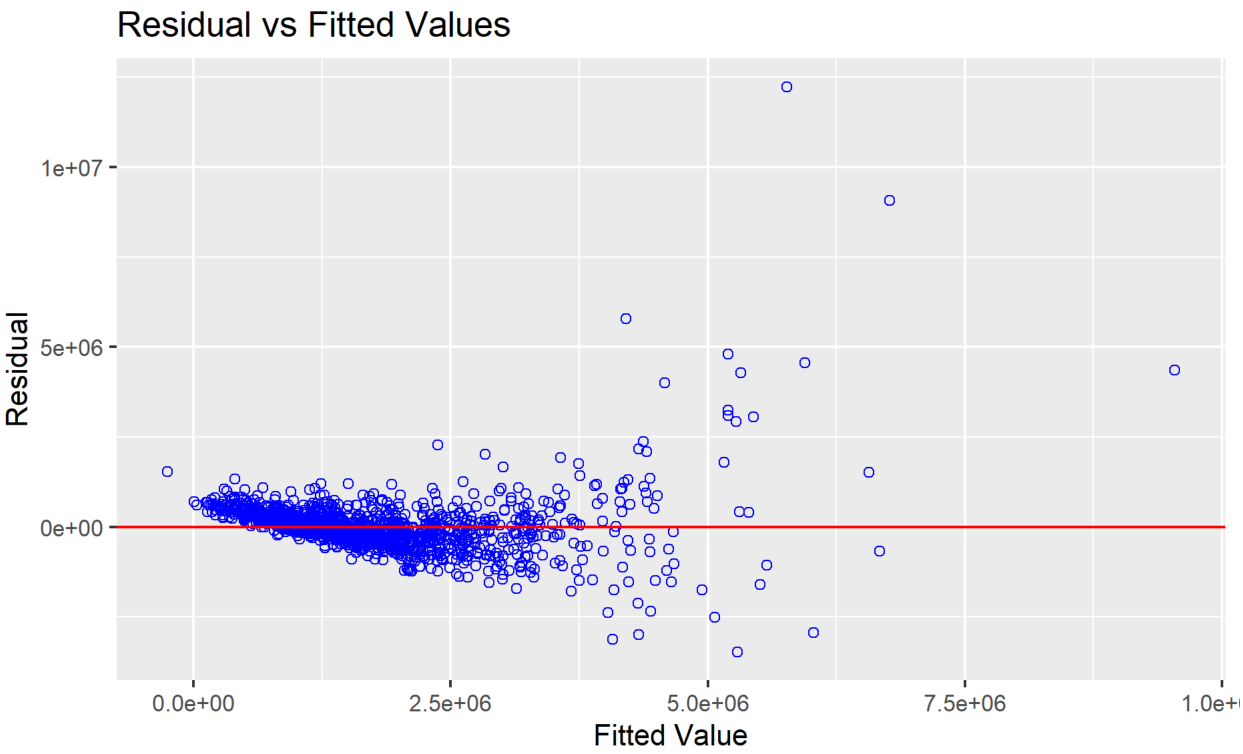
Since the VIF of the independent variables are less than 10. We can safely conclude that there are no sign of multicollinearity among the independent variables.

Test for Non-Linearity

In multiple linear regression, it is important for us to test the assumption that linearity and additivity of the relationship between dependent and independent variables.

In the code chunk below, the *ols_plot_resid_fit()* of **olsrr** package is used to perform linearity assumption test.

```
ols_plot_resid_fit(condo.mlr1)
```

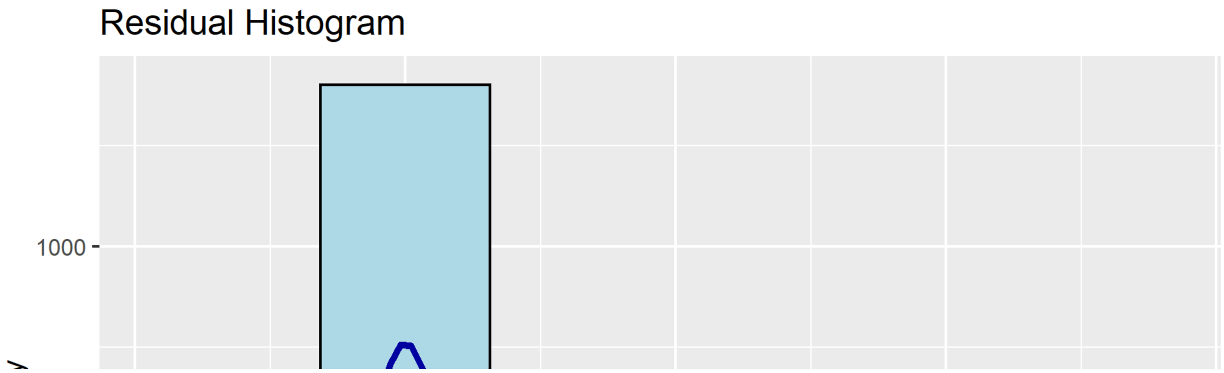


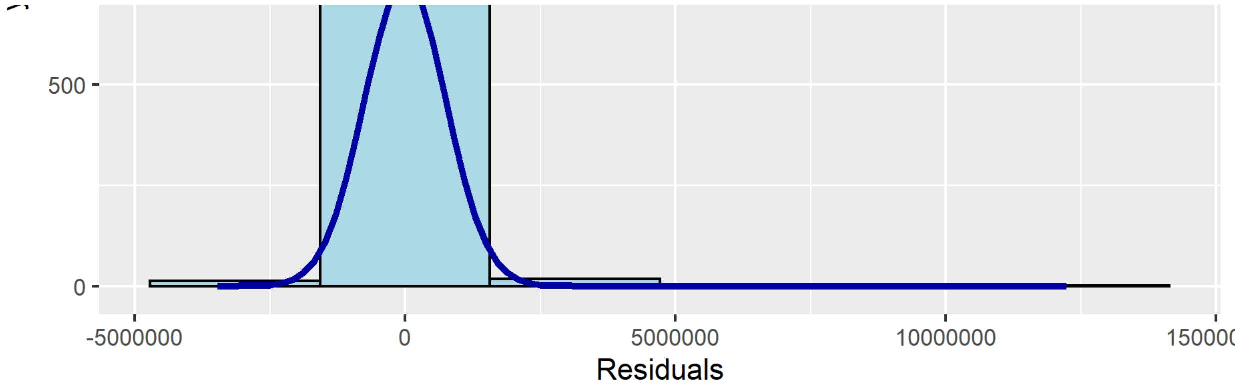
The figure above reveals that most of the data poitns are scattered around the 0 line, hence we can safely conclude that the relationships between the dependent variable and independent variables are linear.

Test for Normality Assumption

Lastly, the code chunk below uses *ols_plot_resid_hist()* of *olsrr* package to perform normality assumption test.

```
ols_plot_resid_hist(condo.mlr1)
```





The figure reveals that the residual of the multiple linear regression model (i.e. condo.mlr1) is resemble normal distribution.

If you prefer formal statistical test methods, the *ols_test_normality()* of **olsrr** package can be used as shown in the code chun below.

```
ols_test_normality(condo.mlr1)
```

```
-----------------------------------------------
       Test             Statistic       pvalue
-----------------------------------------------
Shapiro-Wilk              0.6856         0.0000
Kolmogorov-Smirnov        0.1366         0.0000
Cramer-von Mises         121.0768        0.0000
Anderson-Darling         67.9551         0.0000
-----------------------------------------------
```

The summary table above reveals that the p-values of the four tests are way smaller than the alpha value of 0.05. Hence we will reject the null hypothesis that the residual is NOT resemble normal distribution.

Testing for Spatial Autocorrelation

The hedonic model we try to build are using geographically referenced attributes, hence it is also important for us to visual the residual of the hedonic pricing model.

In order to perform spatial autocorrelation test, we need to convert ***condo_resale.sf*** simple into a SpatialPointsDataFrame.

First, we will export the residual of the hedonic pricing model and save it as a data frame.

```
mlr.output <- as.data.frame(condo.mlr1$residuals)
```

Next, we will join the newly created data frame with ***condo_resale.sf*** object.

```
condo_resale.res.sf <- cbind(condo_resale.sf,
                        condo.mlr1$residuals) %>%
rename(`MLR_RES` = `condo.mlr1.residuals`)
```

Next, we will convert ***condo_resale.res.sf*** simple feature object into a SpatialPointsDataFrame because spdep package can only process sp conformed spatial data objects.

The code chunk below will be used to perform the data conversion process.

```
condo_resale.sp <- as_Spatial(condo_resale.res.sf)
condo_resale.sp
```

```
class       : SpatialPointsDataFrame
features    : 1436
extent      : 14940.85, 43352.45, 24765.67, 48382.81  (xmin, xmax, ymin, ymax)
crs         : +proj=tmerc +lat_0=1.36666666666667 +lon_0=103.833333333333 +k=1 +x_0=28001.642 +y_0=38
variables   : 23
names       : POSTCODE, SELLING_PRICE, AREA_SQM, AGE,    PROX_CBD, PROX_CHILDCARE, PROX_ELDERLYCARE,
min values  :    18965,        540000,       34,   0, 0.386916393,    0.004927023,      0.054508623,
max values  :   828833,       1.8e+07,      619,  37, 19.18042832,     3.46572633,      3.949157205,
```

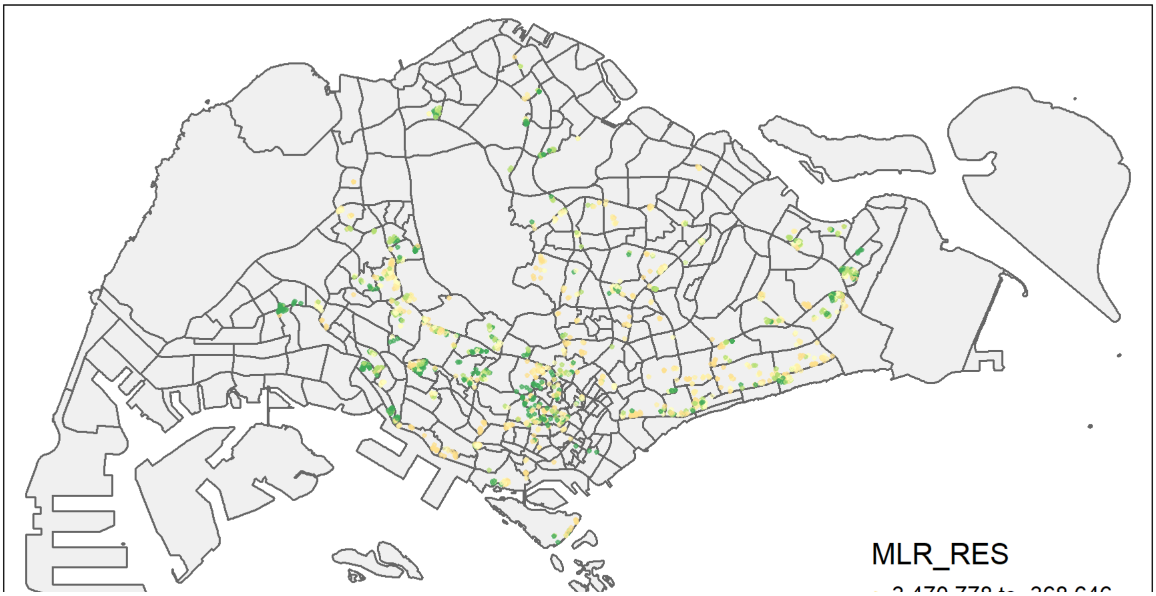
Next, we will use tmap package to display the distribution of the residuals on an interactive map.

The code churn below will turn on the interactive mode of tmap.

```
tmap_mode("plot")
```

The code chunks below is used to create an interactive point symbol map.

```
tm_shape(mpsz_svy21) +
  tm_polygons(alpha = 0.4) +
tm_shape(condo_resale.res.sf) +
  tm_dots(col = "MLR_RES",
          alpha = 0.6,
          style="quantile") +
  tm_view(set.zoom.limits = c(11,14))
)
```



Remember to switch back to “plot” mode before continue.

```
tmap_mode("plot")
```

The figure above reveal that there is sign of spatial autocorrelation.

To proof that our observation is indeed true, the Moran’s I test will be performed

First, we will compute the distance-based weight matrix by using *dnearneigh()* function of **spdep**.

```
nb <- dnearneigh(coordinates(condo_resale.sp), 0, 1500, longlat = FALSE)
summary(nb)
```

```
Neighbour list object:
Number of regions: 1436
Number of nonzero links: 66266
Percentage nonzero weights: 3.213526
Average number of links: 46.14624
Link number distribution:

  1   3   5   7   9  10  11  12  13  14  15  16  17  18  19  20  21
  3   3   9   4   3  15  10  19  17  45  19   5  14  29  19   6  35
 22  23  24  25  26  27  28  29  30  31  32  33  34  35  36  37  38
 45  18  47  16  43  22  26  21  11   9  23  22  13  16  25  21  37
 39  40  41  42  43  44  45  46  47  48  49  50  51  52  53  54  55
 16  18   8  21   4  12   8  36  18  14  14  43  11  12   8  13  12
 56  57  58  59  60  61  62  63  64  65  66  67  68  69  70  71  72
 13   4   5   6  12  11  20  29  33  15  20  10  14  15  15  11  16
 73  74  75  76  77  78  79  80  81  82  83  84  85  86  87  88  89
 12  10   8  19  12  14   9   8   4  13  11   6   4   9   4   4   4
 90  91  92  93  94  95  96  97  98  99 100 101 102 103 104 105 106
  6   2  16   9   4   5   9   3   9   4   2   1   2   1   1   1   5
107 108 109 110 112 116 125
  9   2   1   3   1   1   1
3 least connected regions:
193 194 277 with 1 link
1 most connected region:
285 with 125 links
```

Next, *nb2listw()* of **spdep** packge will be used to convert the output neighbours lists (i.e. nb) into a spatial weights.

```
nb_lw <- nb2listw(nb, style = 'W')
summary(nb_lw)
```

```
Characteristics of weights list object:
Neighbour list object:
Number of regions: 1436
Number of nonzero links: 66266
Percentage nonzero weights: 3.213526
Average number of links: 46.14624
Link number distribution:

  1   3   5   7   9  10  11  12  13  14  15  16  17  18  19  20  21
  3   3   9   4   3  15  10  19  17  45  19   5  14  29  19   6  35
 22  23  24  25  26  27  28  29  30  31  32  33  34  35  36  37  38
 45  18  47  16  43  22  26  21  11   9  23  22  13  16  25  21  37
 39  40  41  42  43  44  45  46  47  48  49  50  51  52  53  54  55
 16  18   8  21   4  12   8  36  18  14  14  43  11  12   8  13  12
 56  57  58  59  60  61  62  63  64  65  66  67  68  69  70  71  72
 13   4   5   6  12  11  20  29  33  15  20  10  14  15  15  11  16
 73  74  75  76  77  78  79  80  81  82  83  84  85  86  87  88  89
 12  10   8  19  12  14   9   8   4  13  11   6   4   9   4   4   4
 90  91  92  93  94  95  96  97  98  99 100 101 102 103 104 105 106
  6   2  16   9   4   5   9   3   9   4   2   1   2   1   1   1   5
107 108 109 110 112 116 125
  9   2   1   3   1   1   1
3 least connected regions:
193 194 277 with 1 link
1 most connected region:
285 with 125 links

Weights style: W
Weights constants summary:
     n      nn   S0       S1       S2
W 1436 2062096 1436 94.81916 5798.341
```

Next, *lm.morantest()* of **spdep** package will be used to perform Moran's I test for residual spatial autocorrelation

```
lm.morantest(        condo.mlr1, nb_lw    )
```

```

    Global Moran I for regression residuals

data:
model: lm(formula = SELLING_PRICE ~ AREA_SQM + AGE + PROX_CBD
+ PROX_CHILDCARE + PROX_ELDERLYCARE + PROX_URA_GROWTH_AREA +
PROX_MRT + PROX_PARK + PROX_PRIMARY_SCH + PROX_SHOPPING_MALL +
PROX_BUS_STOP + NO_Of_UNITS + FAMILY_FRIENDLY + FREEHOLD, data
= condo_resale.sf)
weights: nb_lw

Moran I statistic standard deviate = 24.366, p-value < 2.2e-16
alternative hypothesis: greater
```

```
sample estimates:
Observed Moran I      Expectation         Variance
    1.438876e-01    -5.487594e-03     3.758259e-05
```

The Global Moran's I test for residual spatial autocorrelation shows that it's p-value is less than 0.00000000000000022 which is less than the alpha value of 0.05. Hence, we will reject the null hypothesis that the residuals are randomly distributed.

Since the Observed Global Moran I = 0.1424418 which is greater than 0, we can infer than the residuals resemble cluster distribution.

Building Hedonic Pricing Models using GWmodel

In this section, you are going to learn how to modelling hedonic pricing using both the fixed and adaptive bandwidth schemes

Building Fixed Bandwidth GWR Model

Computing fixed bandwith

In the code chunk below *bw.gwr()* of GWModel package is used to determine the optimal fixed bandwidth to use in the model. Notice that the argument ***adaptive*** is set to **FALSE** indicates that we are interested to compute the fixed bandwidth.

There are two possible approaches can be uused to determine the stopping rule, they are: CV cross-validation approach and AIC corrected (AICc) approach. We define the stopping rule using ***approach*** argement.

```
bw.fixed  <-        bw.gwr   (         formula =         SELLING_PRICE ~         AREA_SQM
+         AGE          +          PROX_CBD  +          PROX_CHILDCARE +         PROX_ELDERLYCARE
+         PROX_URA_GROWTH_AREA +          PROX_MRT    +          PROX_PARK  +
PROX_PRIMARY_SCH +          PROX_SHOPPING_MALL  +          PROX_BUS_STOP  +          NO_Of_UNITS
+         FAMILY_FRIENDLY +          FREEHOLD , data=        condo_resale.sp, approach=
"CV"      , kernel=        "gaussian", adaptive=        FALSE     , longlat=        FALSE
)
```

```
Fixed bandwidth: 17660.96 CV score: 8.259118e+14
Fixed bandwidth: 10917.26 CV score: 7.970454e+14
Fixed bandwidth: 6749.419 CV score: 7.273273e+14
Fixed bandwidth: 4173.553 CV score: 6.300006e+14
Fixed bandwidth: 2581.58 CV score: 5.404958e+14
Fixed bandwidth: 1597.687 CV score: 4.857515e+14
Fixed bandwidth: 989.6077 CV score: 4.722431e+14
Fixed bandwidth: 613.7939 CV score: 1.378294e+16
Fixed bandwidth: 1221.873 CV score: 4.778717e+14
Fixed bandwidth: 846.0596 CV score: 4.791629e+14
```

```
Fixed bandwidth: 1078.325 CV score: 4.751406e+14
Fixed bandwidth: 934.7772 CV score: 4.72518e+14
Fixed bandwidth: 1023.495 CV score: 4.730305e+14
Fixed bandwidth: 968.6643 CV score: 4.721317e+14
Fixed bandwidth: 955.7206 CV score: 4.722072e+14
Fixed bandwidth: 976.6639 CV score: 4.721387e+14
Fixed bandwidth: 963.7202 CV score: 4.721484e+14
Fixed bandwidth: 971.7199 CV score: 4.721293e+14
Fixed bandwidth: 973.6083 CV score: 4.721309e+14
Fixed bandwidth: 970.5527 CV score: 4.721295e+14
Fixed bandwidth: 972.4412 CV score: 4.721296e+14
Fixed bandwidth: 971.2741 CV score: 4.721292e+14
Fixed bandwidth: 970.9985 CV score: 4.721293e+14
Fixed bandwidth: 971.4443 CV score: 4.721292e+14
Fixed bandwidth: 971.5496 CV score: 4.721293e+14
Fixed bandwidth: 971.3793 CV score: 4.721292e+14
Fixed bandwidth: 971.3391 CV score: 4.721292e+14
Fixed bandwidth: 971.3143 CV score: 4.721292e+14
Fixed bandwidth: 971.3545 CV score: 4.721292e+14
Fixed bandwidth: 971.3296 CV score: 4.721292e+14
Fixed bandwidth: 971.345 CV score: 4.721292e+14
Fixed bandwidth: 971.3355 CV score: 4.721292e+14
Fixed bandwidth: 971.3413 CV score: 4.721292e+14
Fixed bandwidth: 971.3377 CV score: 4.721292e+14
Fixed bandwidth: 971.34 CV score: 4.721292e+14
Fixed bandwidth: 971.3405 CV score: 4.721292e+14
Fixed bandwidth: 971.3408 CV score: 4.721292e+14
Fixed bandwidth: 971.3403 CV score: 4.721292e+14
Fixed bandwidth: 971.3406 CV score: 4.721292e+14
Fixed bandwidth: 971.3404 CV score: 4.721292e+14
Fixed bandwidth: 971.3405 CV score: 4.721292e+14
Fixed bandwidth: 971.3405 CV score: 4.721292e+14
```

The result shows that the recommended bandwidth is 971.3398 metres. (Quiz: Do you know why it is in metre?)

GWModel method - fixed bandwith

Now we can use the code chunk below to calibrate the gwr model using fixed bandwidth and gaussian kernel.

```
gwr.fixed <- gwr.basic(formula = SELLING_PRICE ~ AREA_SQM
+ AGE + PROX_CBD + PROX_CHILDCARE + PROX_ELDERLYCARE
+ PROX_URA_GROWTH_AREA + PROX_MRT + PROX_PARK +
PROX_PRIMARY_SCH + PROX_SHOPPING_MALL + PROX_BUS_STOP + NO_Of_UNITS
+ FAMILY_FRIENDLY + FREEHOLD , data= condo_resale.sp, bw= bw.fixed
, kernel = 'gaussian', longlat = FALSE )
```

The output is saved in a list of class “gwrm”. The code below can be used to display the model output.

```
gwr.fixed
```

```
   ***********************************************************************
   *                       Package   GWmodel                             *
   ***********************************************************************
   Program starts at: 2021-10-12 15:07:43
   Call:
   gwr.basic(formula = SELLING_PRICE ~ AREA_SQM + AGE + PROX_CBD +
    PROX_CHILDCARE + PROX_ELDERLYCARE + PROX_URA_GROWTH_AREA +
    PROX_MRT + PROX_PARK + PROX_PRIMARY_SCH + PROX_SHOPPING_MALL +
    PROX_BUS_STOP + NO_Of_UNITS + FAMILY_FRIENDLY + FREEHOLD,
    data = condo_resale.sp, bw = bw.fixed, kernel = "gaussian",
    longlat = FALSE)

   Dependent (y) variable:  SELLING_PRICE
   Independent variables:  AREA_SQM AGE PROX_CBD PROX_CHILDCARE PROX_ELDERLYCARE PROX_URA_GROWTH_ARE/
   Number of data points: 1436
   ***********************************************************************
   *                    Results of Global Regression                     *
   ***********************************************************************

   Call:
    lm(formula = formula, data = data)

   Residuals:
     Min       1Q   Median       3Q      Max
-3470778  -298119   -23481   248917 12234210

   Coefficients:
                          Estimate Std. Error t value Pr(>|t|)
   (Intercept)           527633.22  108183.22   4.877 1.20e-06 ***
   AREA_SQM               12777.52     367.48  34.771  < 2e-16 ***
   AGE                   -24687.74    2754.84  -8.962  < 2e-16 ***
   PROX_CBD              -77131.32    5763.12 -13.384  < 2e-16 ***
   PROX_CHILDCARE       -318472.75  107959.51  -2.950 0.003231 **
   PROX_ELDERLYCARE      185575.62   39901.86   4.651 3.61e-06 ***
   PROX_URA_GROWTH_AREA   39163.25   11754.83   3.332 0.000885 ***
   PROX_MRT             -294745.11   56916.37  -5.179 2.56e-07 ***
   PROX_PARK             570504.81   65507.03   8.709  < 2e-16 ***
   PROX_PRIMARY_SCH      159856.14   60234.60   2.654 0.008046 **
   PROX_SHOPPING_MALL   -220947.25   36561.83  -6.043 1.93e-09 ***
   PROX_BUS_STOP         682482.22  134513.24   5.074 4.42e-07 ***
   NO_Of_UNITS             -245.48      87.95  -2.791 0.005321 **
   FAMILY_FRIENDLY       146307.58   46893.02   3.120 0.001845 **
   FREEHOLD              350599.81   48506.48   7.228 7.98e-13 ***

   ---Significance stars
   Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
   Residual standard error: 756000 on 1421 degrees of freedom
   Multiple R-squared: 0.6507
   Adjusted R-squared: 0.6472
```

```
F-statistic: 189.1 on 14 and 1421 DF,  p-value: < 2.2e-16
***Extra Diagnostic information
Residual sum of squares: 8.120609e+14
Sigma(hat): 752522.9
AIC:  42966.76
AICc:  42967.14
BIC:  41731.39
***********************************************************************
*          Results of Geographically Weighted Regression              *
***********************************************************************

*********************Model calibration information*********************
Kernel function: gaussian
Fixed bandwidth: 971.3405
Regression points: the same locations as observations are used.
Distance metric: Euclidean distance metric is used.

****************Summary of GWR coefficient estimates:******************
                            Min.     1st Qu.      Median
Intercept            -3.5988e+07 -5.1998e+05  7.6780e+05
AREA_SQM              1.0003e+03  5.2758e+03  7.4740e+03
AGE                  -1.3475e+05 -2.0813e+04 -8.6260e+03
PROX_CBD             -7.7047e+07 -2.3608e+05 -8.3600e+04
PROX_CHILDCARE       -6.0097e+06 -3.3667e+05 -9.7425e+04
PROX_ELDERLYCARE     -3.5000e+06 -1.5970e+05  3.1971e+04
PROX_URA_GROWTH_AREA -3.0170e+06 -8.2013e+04  7.0749e+04
PROX_MRT             -3.5282e+06 -6.5836e+05 -1.8833e+05

PROX_PARK            -1.2062e+06 -2.1732e+05  3.5383e+04
PROX_PRIMARY_SCH     -2.2695e+07 -1.7066e+05  4.8472e+04
PROX_SHOPPING_MALL   -7.2585e+06 -1.6684e+05 -1.0517e+04
PROX_BUS_STOP        -1.4676e+06 -4.5207e+04  3.7601e+05
NO_Of_UNITS          -1.3170e+03 -2.4822e+02 -3.0846e+01
FAMILY_FRIENDLY      -2.2749e+06 -1.1140e+05  7.6214e+03
FREEHOLD             -9.2067e+06  3.8073e+04  1.5169e+05
                         3rd Qu.      Max.
Intercept             1.7412e+06 112793548
AREA_SQM              1.2301e+04     21575
AGE                  -3.7784e+03    434201
PROX_CBD              3.4646e+04   2704596
PROX_CHILDCARE        2.9007e+05   1654087
PROX_ELDERLYCARE      1.9577e+05  38867814
PROX_URA_GROWTH_AREA  2.2612e+05  78515730
PROX_MRT              3.6922e+04   3124316
PROX_PARK             4.1335e+05  18122425
PROX_PRIMARY_SCH      5.1555e+05   4637503
PROX_SHOPPING_MALL    1.5923e+05   1529952
PROX_BUS_STOP         1.1664e+06  11342182
NO_Of_UNITS           2.5496e+02     12907
FAMILY_FRIENDLY       1.6107e+05   1720744
FREEHOLD              3.7528e+05   6073636
************************Diagnostic information*************************
```

```
Diagnostic information
Number of data points: 1436
Effective number of parameters (2trace(S) - trace(S'S)): 438.3804
Effective degrees of freedom (n-2trace(S) + trace(S'S)): 997.6196
AICc (GWR book, Fotheringham, et al. 2002, p. 61, eq 2.33): 42263.61
AIC (GWR book, Fotheringham, et al. 2002,GWR p. 96, eq. 4.22): 41632.36
BIC (GWR book, Fotheringham, et al. 2002,GWR p. 61, eq. 2.34): 42515.71
Residual sum of squares: 2.53407e+14
R-square value:  0.8909912
Adjusted R-square value:  0.8430417

***********************************************************************
Program stops at: 2021-10-12 15:07:45
```

The report shows that the adjusted r-square of the gwr is 0.8430418 which is significantly better than the globel multiple linear regression model of 0.6472.

Building Adaptive Bandwidth GWR Model

In this section, we will calibrate the gwr-absed hedonic pricing model by using adaptive bandwidth approach.

Computing the adaptive bandwidth

Similar to the earlier section, we will first use *bw.ger()* to determine the recommended data point to use.

The code chunk used look very similar to the one used to compute the fixed bandwidth except the ***adaptive*** argument has changed to **TRUE**.

```
bw.adaptive <- bw.gwr(formula = SELLING_PRICE ~ AREA_SQM + AGE + PROX_CBD + PROX_CHILDCARE + PROX_ELDERLYCARE + PROX_URA_GROWTH_AREA + PROX_MRT + PROX_PARK + PROX_PRIMARY_SCH + PROX_SHOPPING_MALL + PROX_BUS_STOP + NO_Of_UNITS + FAMILY_FRIENDLY + FREEHOLD, data=condo_resale.sp, approach="CV", kernel="gaussian",
adaptive=TRUE, longlat=FALSE)
```

```
Adaptive bandwidth: 895 CV score: 7.952401e+14
Adaptive bandwidth: 561 CV score: 7.667364e+14
Adaptive bandwidth: 354 CV score: 6.953454e+14
Adaptive bandwidth: 226 CV score: 6.15223e+14
Adaptive bandwidth: 147 CV score: 5.674373e+14
Adaptive bandwidth: 98 CV score: 5.426745e+14
Adaptive bandwidth: 68 CV score: 5.168117e+14
Adaptive bandwidth: 49 CV score: 4.859631e+14
Adaptive bandwidth: 37 CV score: 4.646518e+14
Adaptive bandwidth: 30 CV score: 4.422088e+14
Adaptive bandwidth: 25 CV score: 4.430816e+14
```

```
Adaptive bandwidth: 32 CV score: 4.505602e+14
Adaptive bandwidth: 27 CV score: 4.462172e+14
Adaptive bandwidth: 30 CV score: 4.422088e+14
```

The result shows that the 30 is the recommended data points to be used.

Constructing the adaptive bandwidth gwr model

Now, we can go ahead to calibrate the gwr-based hedonic pricing model by using adaptive bandwidth and gaussian kernel as shown in the code chunk below.

```
gwr.adaptive <-        gwr.basic(        formula =        SELLING_PRICE ~        AREA_SQM
+        AGE        +        PROX_CBD  +        PROX_CHILDCARE +        PROX_ELDERLYCARE
+        PROX_URA_GROWTH_AREA +        PROX_MRT   +        PROX_PARK  +
PROX_PRIMARY_SCH +        PROX_SHOPPING_MALL  +        PROX_BUS_STOP  +        NO_Of_UNITS
+        FAMILY_FRIENDLY +        FREEHOLD , data=        condo_resale.sp, bw=
bw.adaptive, kernel =        'gaussian', adaptive=        TRUE      , longlat =        FALSE
)
```

The code below can be used to display the model output.

```
gwr.adaptive
```

```
   ***********************************************************************
   *                       Package   GWmodel                             *
   ***********************************************************************
   Program starts at: 2021-10-12 15:07:51
   Call:
   gwr.basic(formula = SELLING_PRICE ~ AREA_SQM + AGE + PROX_CBD +
    PROX_CHILDCARE + PROX_ELDERLYCARE + PROX_URA_GROWTH_AREA +
    PROX_MRT + PROX_PARK + PROX_PRIMARY_SCH + PROX_SHOPPING_MALL +
    PROX_BUS_STOP + NO_Of_UNITS + FAMILY_FRIENDLY + FREEHOLD,
    data = condo_resale.sp, bw = bw.adaptive, kernel = "gaussian",
    adaptive = TRUE, longlat = FALSE)

   Dependent (y) variable:  SELLING_PRICE
   Independent variables:  AREA_SQM AGE PROX_CBD PROX_CHILDCARE PROX_ELDERLYCARE PROX_URA_GROWTH_AREA
   Number of data points: 1436
   ***********************************************************************
   *                    Results of Global Regression                     *
   ***********************************************************************

   Call:
    lm(formula = formula, data = data)

   Residuals:
       Min       1Q    Median       3Q      Max
-3470778  -298119   -23481   248917 12234210

   Coefficients:
```

```
Coefficients:
                      Estimate Std. Error t value Pr(>|t|)
(Intercept)          527633.22  108183.22   4.877 1.20e-06 ***
AREA_SQM              12777.52     367.48  34.771  < 2e-16 ***
AGE                  -24687.74    2754.84  -8.962  < 2e-16 ***
PROX_CBD             -77131.32    5763.12 -13.384  < 2e-16 ***
PROX_CHILDCARE      -318472.75  107959.51  -2.950 0.003231 **
PROX_ELDERLYCARE     185575.62   39901.86   4.651 3.61e-06 ***
PROX_URA_GROWTH_AREA  39163.25   11754.83   3.332 0.000885 ***
PROX_MRT            -294745.11   56916.37  -5.179 2.56e-07 ***
PROX_PARK            570504.81   65507.03   8.709  < 2e-16 ***
PROX_PRIMARY_SCH     159856.14   60234.60   2.654 0.008046 **
PROX_SHOPPING_MALL  -220947.25   36561.83  -6.043 1.93e-09 ***
PROX_BUS_STOP        682482.22  134513.24   5.074 4.42e-07 ***
NO_Of_UNITS            -245.48      87.95  -2.791 0.005321 **
FAMILY_FRIENDLY      146307.58   46893.02   3.120 0.001845 **
FREEHOLD             350599.81   48506.48   7.228 7.98e-13 ***

---Significance stars
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 756000 on 1421 degrees of freedom
Multiple R-squared: 0.6507
Adjusted R-squared: 0.6472
F-statistic: 189.1 on 14 and 1421 DF,  p-value: < 2.2e-16
***Extra Diagnostic information
Residual sum of squares: 8.120609e+14
Sigma(hat): 752522.9
AIC:  42966.76
AICc:  42967.14
BIC:  41731.39
***********************************************************************
*          Results of Geographically Weighted Regression              *
***********************************************************************

*********************Model calibration information*********************
Kernel function: gaussian
Adaptive bandwidth: 30 (number of nearest neighbours)
Regression points: the same locations as observations are used.
Distance metric: Euclidean distance metric is used.

****************Summary of GWR coefficient estimates:******************
                           Min.     1st Qu.      Median
Intercept            -1.3487e+08 -2.4669e+05  7.7928e+05
AREA_SQM              3.3188e+03  5.6285e+03  7.7825e+03
AGE                  -9.6746e+04 -2.9288e+04 -1.4043e+04
PROX_CBD             -2.5330e+06 -1.6256e+05 -7.7242e+04
PROX_CHILDCARE       -1.2790e+06 -2.0175e+05  8.7158e+03
PROX_ELDERLYCARE     -1.6212e+06 -9.2050e+04  6.1029e+04
PROX_URA_GROWTH_AREA -7.2686e+06 -3.0350e+04  4.5869e+04
PROX_MRT             -4.3781e+07 -6.7282e+05 -2.2115e+05

PROX_PARK            -2.9020e+06 -1.6782e+05  1.1601e+05
PROX_PRIMARY_SCH     -8.6418e+05 -1.6627e+05  7.7853e+03
```

```
PROX_PRIMARY_SCH     -8.6418e+05 -1.6627e+05 -7.7853e+03
PROX_SHOPPING_MALL   -1.8272e+06 -1.3175e+05 -1.4049e+04
PROX_BUS_STOP        -2.0579e+06 -7.1461e+04  4.1104e+05
NO_Of_UNITS          -2.1993e+03 -2.3685e+02 -3.4699e+01
FAMILY_FRIENDLY      -5.9879e+05 -5.0927e+04  2.6173e+04
FREEHOLD             -1.6340e+05  4.0765e+04  1.9023e+05
                         3rd Qu.     Max.
Intercept             1.6194e+06 18758355
AREA_SQM              1.2738e+04    23064
AGE                  -5.6119e+03    13303
PROX_CBD              2.6624e+03 11346650
PROX_CHILDCARE        3.7778e+05  2892127
PROX_ELDERLYCARE      2.8184e+05  2465671
PROX_URA_GROWTH_AREA  2.4613e+05  7384059
PROX_MRT             -7.4593e+04  1186242
PROX_PARK             4.6572e+05  2588497
PROX_PRIMARY_SCH      4.3222e+05  3381462
PROX_SHOPPING_MALL    1.3799e+05 38038564
PROX_BUS_STOP         1.2071e+06 12081592
NO_Of_UNITS           1.1657e+02     1010
FAMILY_FRIENDLY       2.2481e+05  2072414
FREEHOLD              3.7960e+05  1813995
************************Diagnostic information*************************
Number of data points: 1436
Effective number of parameters (2trace(S) - trace(S'S)): 350.3088
Effective degrees of freedom (n-2trace(S) + trace(S'S)): 1085.691
AICc (GWR book, Fotheringham, et al. 2002, p. 61, eq 2.33): 41982.22
AIC (GWR book, Fotheringham, et al. 2002,GWR p. 96, eq. 4.22): 41546.74
BIC (GWR book, Fotheringham, et al. 2002,GWR p. 61, eq. 2.34): 41914.08
Residual sum of squares: 2.528227e+14
R-square value:  0.8912425
Adjusted R-square value:  0.8561185

***********************************************************************
Program stops at: 2021-10-12 15:07:53
```

The report shows that the adjusted r-square of the gwr is 0.8561185 which is significantly better than the globel multiple linear regression model of 0.6472.

Visualising GWR Output

In addition to regression residuals, the output feature class table includes fields for observed and predicted y values, condition number (cond), Local R2, residuals, and explanatory variable coefficients and standard errors:

- Condition Number: this diagnostic evaluates local collinearity. In the presence of strong local collinearity, results become unstable. Results associated with condition numbers larger than 30, may be unreliable.

- Local R2: these values range between 0.0 and 1.0 and indicate how well the local regression model fits observed y values. Very low values indicate the local model is performing poorly. Mapping the Local R2 values to see where GWR predicts well and where it predicts poorly may provide clues about important variables that may be missing from the regression model.

- Predicted: these are the estimated (or fitted) y values 3. computed by GWR.
- Residuals: to obtain the residual values, the fitted y values are subtracted from the observed y values. Standardized residuals have a mean of zero and a standard deviation of 1. A cold-to-hot rendered map of standardized residuals can be produce by using these values.
- Coefficient Standard Error: these values measure the reliability of each coefficient estimate. Confidence in those estimates are higher when standard errors are small in relation to the actual coefficient values. Large standard errors may indicate problems with local collinearity.

They are all stored in a SpatialPointsDataFrame or SpatialPolygonsDataFrame object integrated with fit.points, GWR coefficient estimates, y value, predicted values, coefficient standard errors and t-values in its "data" slot in an object called **SDF** of the output list.

Converting SDF into sf data.frame

To visualise the fields in ***SDF***, we need to first covert it into **sf** data.frame by using the code chunk below.

```
condo_resale.sf.adaptive <-         st_as_sf (         gwr.adaptive$         SDF       )
%>%
  st_transform(         crs=         3414      )
```

```
condo_resale.sf.adaptive.svy21 <-         st_transform(         condo_resale.sf.adaptive, 3414
)
condo_resale.sf.adaptive.svy21
```

```
gwr.adaptive.output <-         as.data.frame(         gwr.adaptive$         SDF       )
condo_resale.sf.adaptive <-         cbind     (         condo_resale.res.sf, as.matrix(
gwr.adaptive.output)         )
```

```
glimpse   (         condo_resale.sf.adaptive)
```

```
Rows: 1,436
Columns: 52
$ Intercept               <dbl> 2050011.67, 1633128.24, 3433608.17, ~
$ AREA_SQM                <dbl> 9561.892, 16576.853, 13091.861, 2073~
$ AGE                     <dbl> -9514.634, -58185.479, -26707.386, -~
$ PROX_CBD                <dbl> -120681.94, -149434.22, -259397.77, ~
$ PROX_CHILDCARE          <dbl> 319266.925, 441102.177, -120116.816,~
$ PROX_ELDERLYCARE        <dbl> -393417.79, 325188.74, 535855.81, 31~
$ PROX_URA_GROWTH_AREA    <dbl> -159980.203, -142290.389, -253621.20~
$ PROX_MRT                <dbl> -299742.96, -2510522.23, -936853.28,~
```

```
$ PROX_PARK               <dbl> -172104.47, 523379.72, 209099.85, -7~
$ PROX_PRIMARY_SCH        <dbl> 242668.03, 1106830.66, 571462.33, 31~
$ PROX_SHOPPING_MALL      <dbl> 300881.390, -87693.378, -126732.712,~
$ PROX_BUS_STOP           <dbl> 1210615.44, 1843587.22, 1411924.90, ~
$ NO_Of_UNITS             <dbl> 104.8290640, -288.3441183, -9.553294~
$ FAMILY_FRIENDLY         <dbl> -9075.370, 310074.664, 5949.746, 155~
$ FREEHOLD                <dbl> 303955.61, 396221.27, 168821.75, 121~
$ y                       <dbl> 3000000, 3880000, 3325000, 4250000, ~
$ yhat                    <dbl> 2886531.8, 3466801.5, 3616527.2, 543~
$ residual                <dbl> 113468.16, 413198.52, -291527.20, -1~
$ CV_Score                <dbl> 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, ~
$ Stud_residual           <dbl> 0.38207013, 1.01433140, -0.83780678,~
$ Intercept_SE            <dbl> 516105.5, 488083.5, 963711.4, 444185~
$ AREA_SQM_SE             <dbl> 823.2860, 825.2380, 988.2240, 617.40~
$ AGE_SE                  <dbl> 5889.782, 6226.916, 6510.236, 6010.5~
$ PROX_CBD_SE             <dbl> 37411.22, 23615.06, 56103.77, 469337~
$ PROX_CHILDCARE_SE       <dbl> 319111.1, 299705.3, 349128.5, 304965~
$ PROX_ELDERLYCARE_SE     <dbl> 120633.34, 84546.69, 129687.07, 1271~
$ PROX_URA_GROWTH_AREA_SE <dbl> 56207.39, 76956.50, 95774.60, 470762~
$ PROX_MRT_SE             <dbl> 185181.3, 281133.9, 275483.7, 279877~
$ PROX_PARK_SE            <dbl> 205499.6, 229358.7, 314124.3, 227249~
$ PROX_PRIMARY_SCH_SE     <dbl> 152400.7, 165150.7, 196662.6, 240878~
$ PROX_SHOPPING_MALL_SE   <dbl> 109268.8, 98906.8, 119913.3, 177104.~
$ PROX_BUS_STOP_SE        <dbl> 600668.6, 410222.1, 464156.7, 562810~
$ NO_Of_UNITS_SE          <dbl> 218.1258, 208.9410, 210.9828, 361.77~
$ FAMILY_FRIENDLY_SE      <dbl> 131474.7, 114989.1, 146607.2, 108726~
$ FREEHOLD_SE             <dbl> 115954.0, 130110.0, 141031.5, 138239~
$ Intercept_TV            <dbl> 3.9720784, 3.3460017, 3.5629010, 0.5~
$ AREA_SQM_TV             <dbl> 11.614302, 20.087361, 13.247868, 33.~
$ AGE_TV                  <dbl> -1.6154474, -9.3441881, -4.1023685, ~
$ PROX_CBD_TV             <dbl> -3.22582173, -6.32792021, -4.6235352~
$ PROX_CHILDCARE_TV       <dbl> 1.000488185, 1.471786337, -0.3440475~
$ PROX_ELDERLYCARE_TV     <dbl> -3.2612693, 3.8462625, 4.1319138, 2.~
$ PROX_URA_GROWTH_AREA_TV <dbl> -2.846248368, -1.848971738, -2.64810~
$ PROX_MRT_TV             <dbl> -1.61864578, -8.92998600, -3.4007572~
$ PROX_PARK_TV            <dbl> -0.83749312, 2.28192684, 0.66565951,~
$ PROX_PRIMARY_SCH_TV     <dbl> 1.59230221, 6.70194543, 2.90580089, ~
$ PROX_SHOPPING_MALL_TV   <dbl> 2.75358842, -0.88662640, -1.05686949~
$ PROX_BUS_STOP_TV        <dbl> 2.0154464, 4.4941192, 3.0419145, 12.~
$ NO_Of_UNITS_TV          <dbl> 0.480589953, -1.380026395, -0.045279~
$ FAMILY_FRIENDLY_TV      <dbl> -0.06902748, 2.69655779, 0.04058290,~
$ FREEHOLD_TV             <dbl> 2.6213469, 3.0452799, 1.1970499, 8.7~
$ Local_R2                <dbl> 0.8846744, 0.8899773, 0.8947007, 0.9~
$ geometry                <POINT [m]> POINT (22085.12 29951.54), POI~
```

```
summary(gwr.adaptive$SDF$yhat)
```

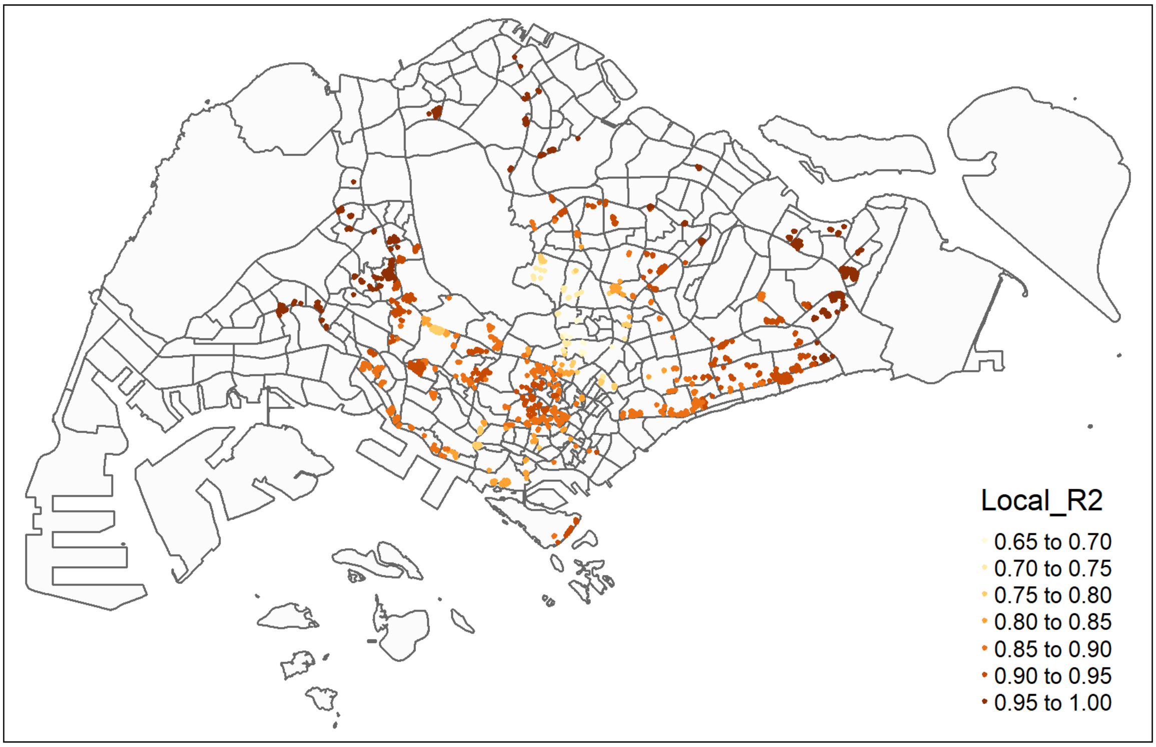
```
   Min.  1st Qu.   Median     Mean  3rd Qu.     Max. 
 171347  1102001  1385528  1751842  1982307 13887901 
```

Visualising local R2

Visualising local R2

The code chunks below is used to create an interactive point symbol map.

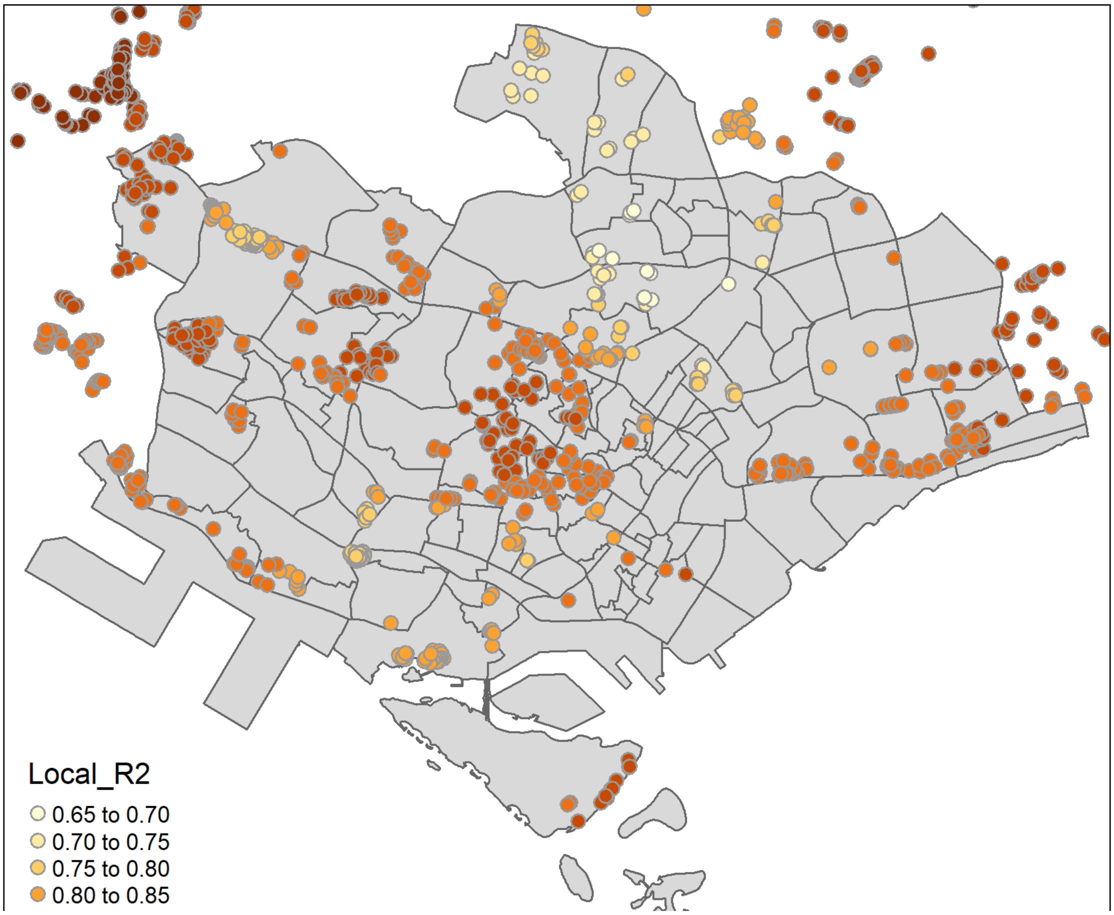
```
tmap_mode("plot")
tm_shape(mpsz_svy21) +
  tm_polygons(alpha = 0.1) +
tm_shape(condo_resale.sf.adaptive) +
  tm_dots(col = "Local_R2",
          border.col = "gray60",
          border.lwd = 1) +
  tm_view(set.zoom.limits = c(11,14))
```

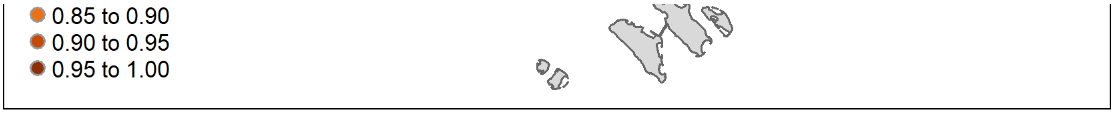



```
tmap_mode("plot")
```

By URA Plannign Region

```
tm_shape(mpsz_svy21[mpsz_svy21$REGION_N=="CENTRAL REGION", ]) +
  tm_polygons() +
tm_shape(condo_resale.sf.adaptive) +
  tm_bubbles(col = "Local_R2",
             size = 0.15,
             border.col = "gray60",
             border.lwd = 1)
```


Local_R2
0.65 to 0.70
0.70 to 0.75
0.75 to 0.80
0.80 to 0.85



Reference

Gollini I, Lu B, Charlton M, Brunsdon C, Harris P (2015) “GWmodel: an R Package for exploring Spatial Heterogeneity using Geographically Weighted Models”. *Journal of Statistical Software*, 63(17):1-50, http://www.jstatsoft.org/v63/i17/

Lu B, Harris P, Charlton M, Brunsdon C (2014) “The GWmodel R Package: further topics for exploring Spatial Heterogeneity using GeographicallyWeighted Models”. *Geo-spatial Information Science* 17(2): 85-101, http://www.tandfonline.com/doi/abs/10.1080/1009502.2014.917453