Lesson 10:
Spatial Interaction Models

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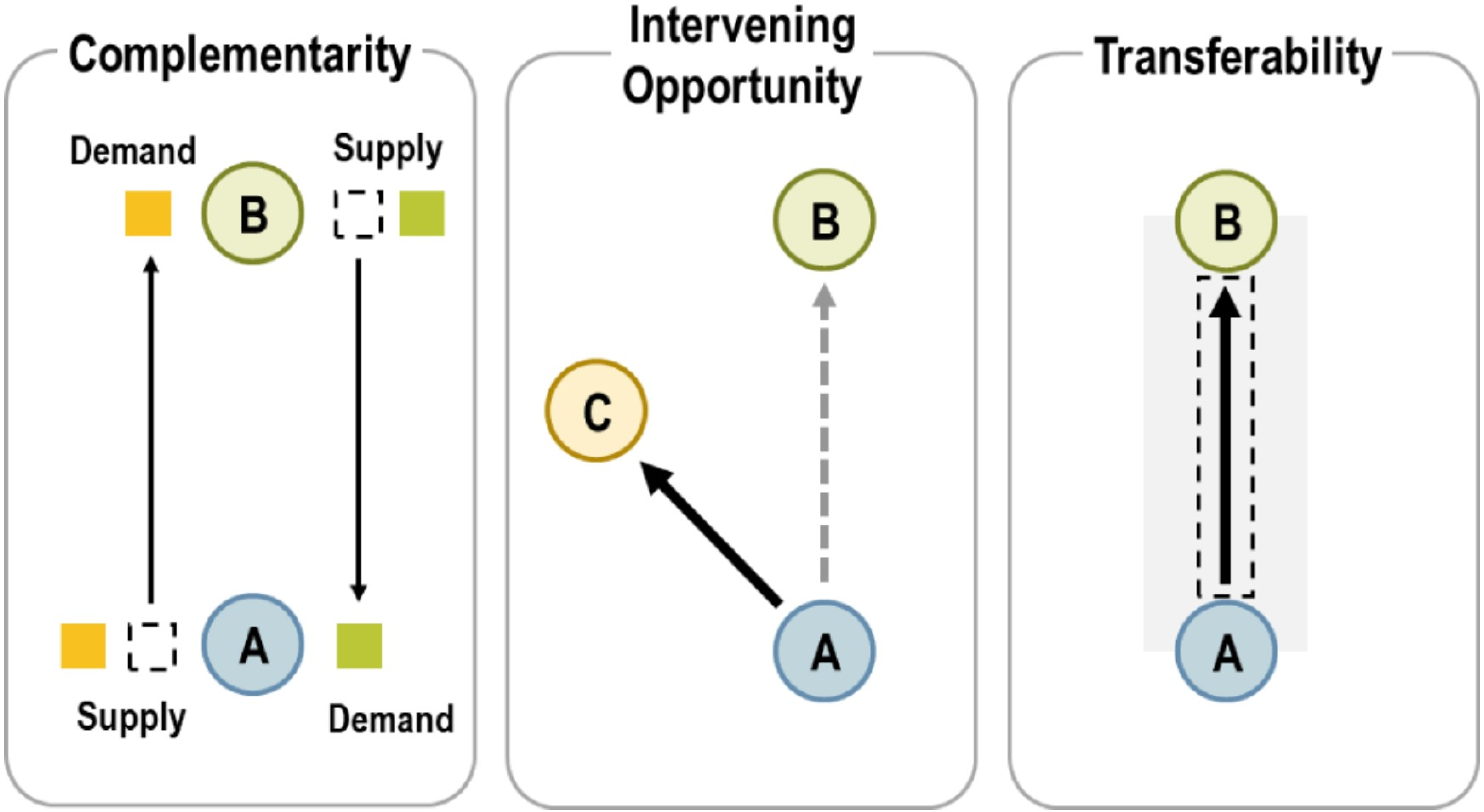
What Spatial Interaction Models are?

Spatial interaction or “gravity models” estimate the flow of people, material, or information between locations in geographical space.

> **Note**
>
> Spatial interaction models seek to explain existing spatial flows. As such it is possible to measure flows and predict the consequences of changes in the conditions generating them. When such attributes are known, it is possible to better allocate transport resources such as conveyances, infrastructure, and terminals.

Conditions for Spatial Flows

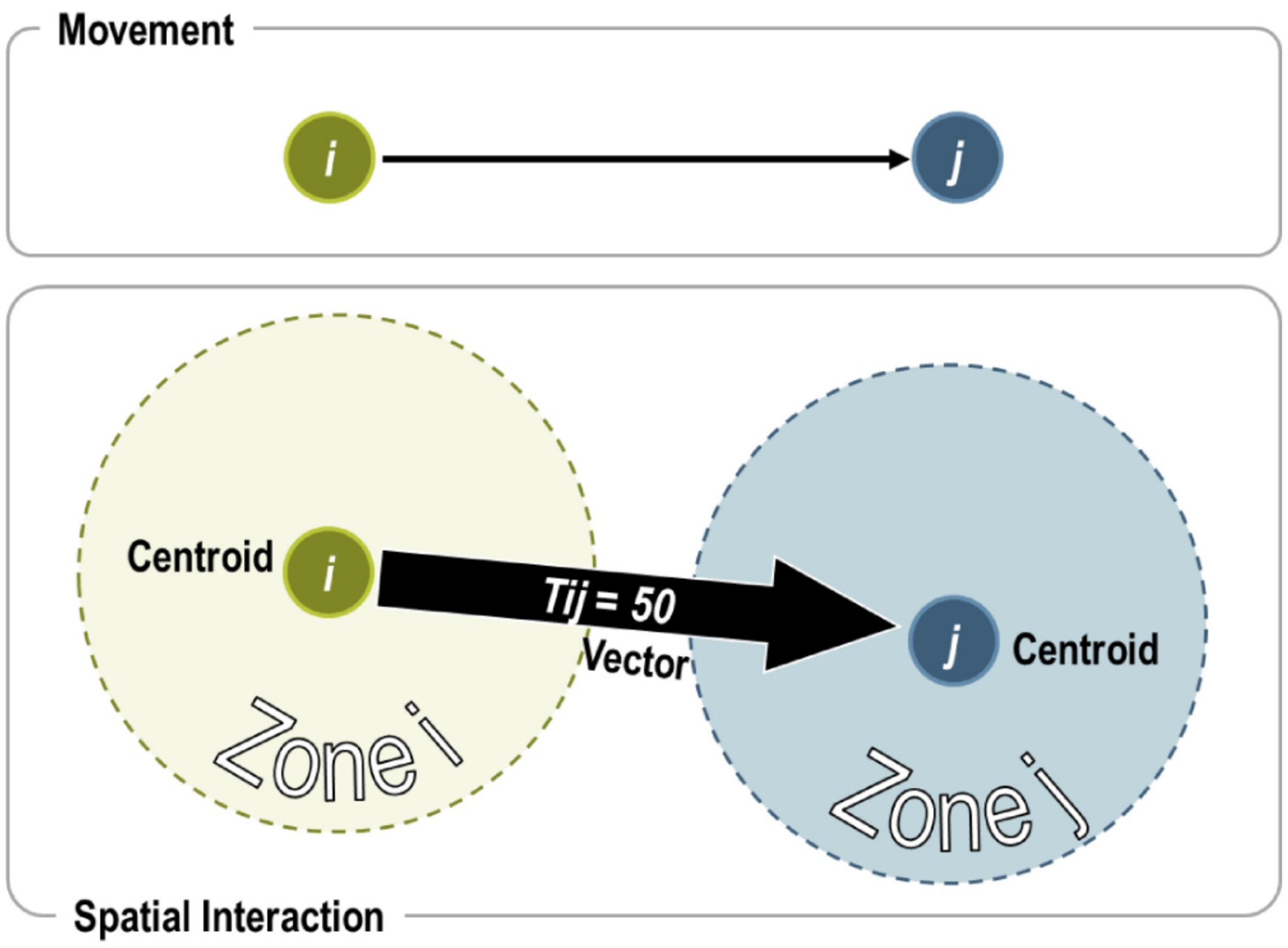
- Three interdependent conditions are necessary for a spatial interaction to occur:



Representation of a Movement as a Spatial Interaction

Representing mobility as a spatial interaction involves several considerations:

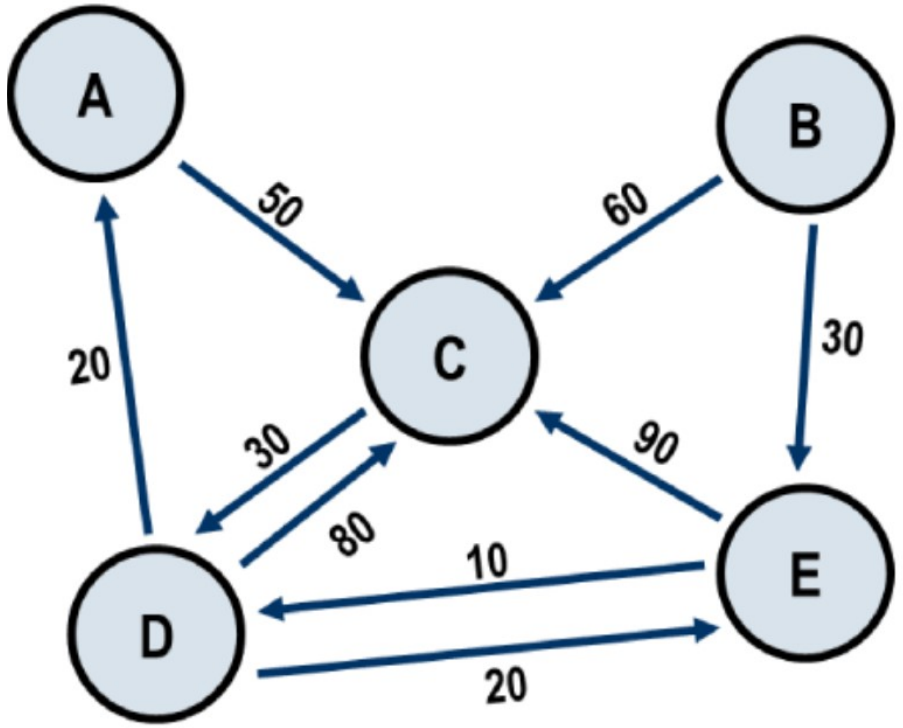
- Locations: A movement is occurring between a location of origin and a location of destination. i generally denotes an origin while j is a destination.
- Centroid: An abstraction of the attributes of a zone at a point.
- Flows: Flows are generally expressed by a valued vector Tij representing an interaction between locations i and j.
- Vectors: A vector Tij links two centroids and has a value assigned to it (50) which can represents movements.



Constructing an O/D Matrix

- The construction of an origin / destination matrix requires directional flow information between a series of locations.
- Figure below represents movements (O/D pairs) between five locations (A, B, C, D and E). From this graph, an O/D matrix can be built where each O/D pair becomes a cell. A value of 0 is assigned for each O/D pair that does not have an observed flow.

Spatial Interactions

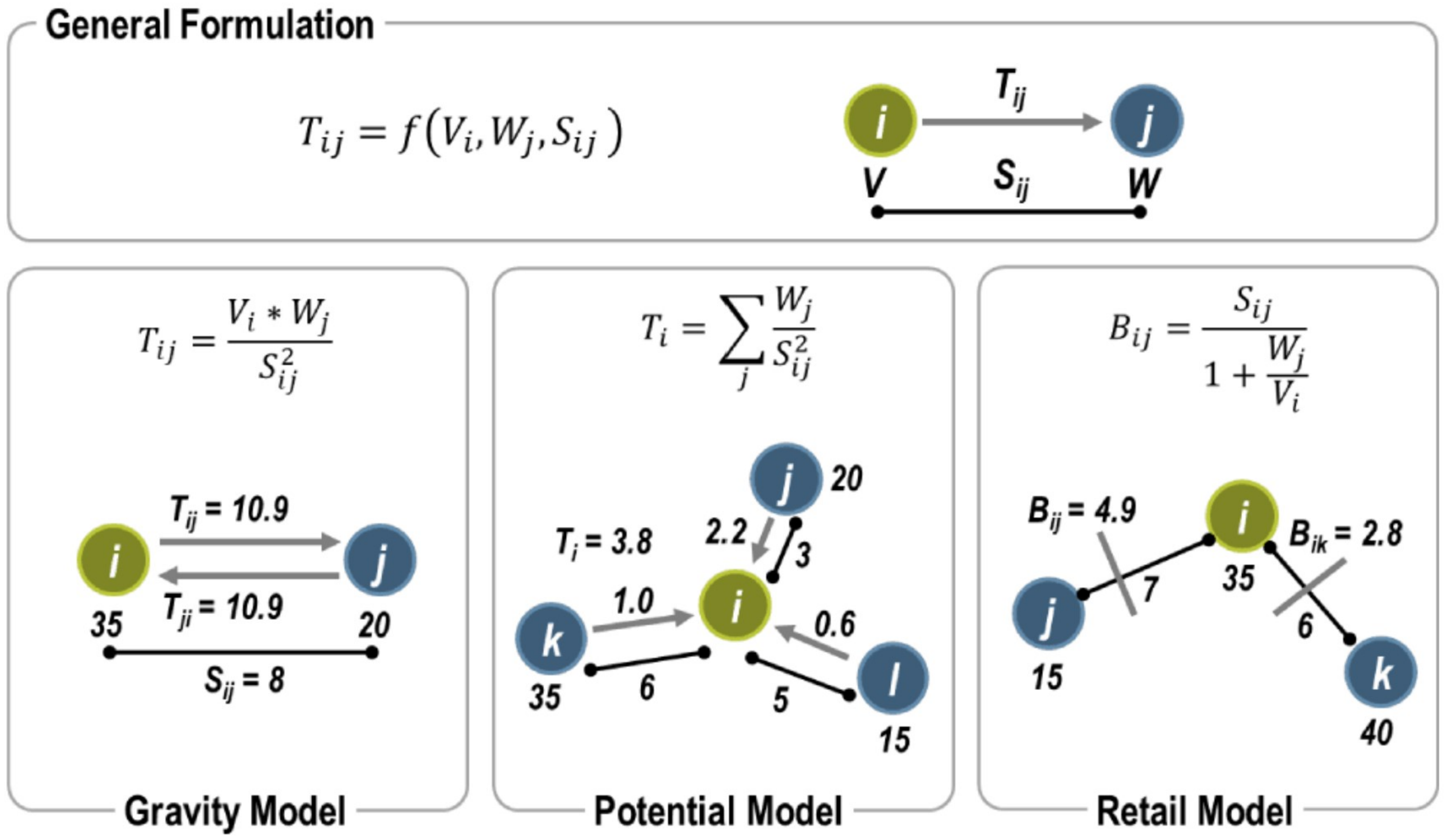


O/D Matrix

	A	B	C	D	E	Ti
A	0	0	50	0	0	50
B	0	0	60	0	30	90
C	0	0	0	30	0	30
D	20	0	80	0	20	120
E	0	0	90	10	0	100
Tj	20	0	280	40	50	**390**

Three Basic Types of Interaction Models

- The general formulation of the spatial interaction model is stated as **Tij**, which is the interaction between location i (origin) and location j (destination). **Vi** are the attributes of the location of origin i, **Wj** are the attributes of the location of destination j, and **Sij** are the attributes of separation between the location of origin i and the location of destination j.
- From this general formulation, three basic types of interaction models can be derived:



Gravity Models

The general formula (also known as unconstrained):

$$T_{ij} = k\frac{V_i^{\mu}W_j^{\alpha}}{d_{ij}^{\beta}}$$

- Tij is the transition/trip or flow, T, between origin i (always the rows in a matrix) and destination j (always the columns in a matrix). If you are not overly familiar with matrix notation, the i and j are just generic indexes to allow us to refer to any cell in the matrix.
- V is a vector (a 1 dimensional matrix – or, if you like, a single line of numbers) of origin attributes which relate to the emissivity of all origins in the dataset, i – this could be any of the origin-related variables.
- W is a vector of destination attributes relating to the attractiveness of all destinations in the dataset, j – similarly, this could be any of the destination related variables.
- d is a matrix of costs (frequently distances – hence, d) relating to the flows between i and j.
- k, μ, α and β are all model parameters to be estimated. β is assumed to be negative, as with an increase in cost/distance we would expect interaction to decrease.

Unconstrained (Totally constrained) case

The O-D Matrix

		Destinations (j)			Total Outflows
	from/to	1	2	3	($\sum_j T_{ij}$ or O_i)
	1	100	20	40	160
Origins (i)	2	60	300	90	450
	3	40	50	90	180
	Total Inflows ($\sum_i T_{ij}$ or D_j)	200	370	220	Grand Total 790 ($\sum_i \sum_j T_{ij}$ or T)

and distance matrix:

	1	2	3
1	2	15	5
2	15	2	10
3	5	10	2

Suppose $\lambda = 1$, $\alpha = 1$, and $\beta = -1$ for this system.

The estimated O-D matrix:

		Destinations (j)			$\sum_j \hat{T}_{ij}$
	from/to	1	2	3	
	1	79	19	35	133
Origins (i)	2	29	412	49	490
	3	36	33	98	167
$\sum_i \hat{T}_{ij}$		144	464	182	790 $\sum_i \sum_j \hat{T}_{ij}$

and the calculation T11

$$\frac{0.004944 \times 160 \times 200}{2} = 79$$

The Origin (Production) Constrained Model

In the Origin Constrained Model,

- Oi does not have a parameter as it is a known **constraint**.
- Ai is known as a **balancing factor** and is a vector of values which relate to each origin i which do the equivalent job as k in the unconstrained/total constrained model but ensure that flow estimates from each origin sum to the know totals Oi rather than just the overall total.

$$T_{ij} = A_i O_i W_j^{\alpha} d_{ij}^{-}\beta$$

where:

$$O_i = \sum_j T_{ij}$$

and:

$$A_i = \frac{1}{\sum_j W_j^{\alpha} d_{ij}^{-}\beta}$$

Oringin (Production) constrained case

The O-D Matrix

		Destinations (j)			Total Outlflows
	from/to	1	2	3	($\sum_j T_{ij}$ or O_i)
Origins (i)	1	100	20	40	160
	2	60	300	90	450
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	Total Inflows ($\sum_i T_{ij}$ or D_j)	200	370	220	Grand Total 790 ($\sum_i \sum_j T_{ij}$ or T)

and distance matrix:

	1	2	3
1	2	15	5
2	15	2	10
3	5	10	2

Suppose $\lambda = 1$, $\alpha = 1$, and $\beta = -1$ for this system.

The estimated O-D matrix:

		Destinations (j)			$\sum_j \hat{T}_{ij}$
	from/to	1	2	3	
Origins (i)	1	95	23	42	160
	2	27	378	45	450
	3	38	36	106	180
$\sum_i \hat{T}_{ij}$		160	437	193	790

A1 is calculated as shown below:

$$A_1 = \left[\frac{200}{2} + \frac{370}{15} + \frac{200}{5}\right]^{-1} = [168.67]^{-1} = 0.005929$$

Hence, T11 is calculated as shown below:

$$\hat{T}_{11} = \frac{0.005929 \times 160 \times 200}{2} = 95$$

The Destination (Attraction) Constrained Model

$$T_{ij} = D_j B_j V_i^{\mu} d_{ij}^{-}\beta$$

where:

$$D_j = \sum_i T_{ij}$$

and:

$$B_j = \frac{1}{\sum_i V_i^{\mu} d_{ij}^{-}\beta}$$

Destination (Attraction) constrained case

The O-D Matrix

		Destinations (j)			Total Outlflows
	from/to	1	2	3	($\sum_j T_{ij}$ or O_i)
	1	100	20	40	160
Origins (i)	2	60	300	90	450
	3	40	50	90	180
	Total Inflows ($\sum_i T_{ij}$ or D_j)	200	370	220	Grand Total 790 ($\sum_i \sum_j T_{ij}$ or T)

and distance matrix:

	1	2	3
1	2	15	5
2	15	2	10
3	5	10	2

Suppose $\lambda = 1$, $\alpha = 1$, and $\beta = -1$ for this system.

The estimated O-D matrix:

		Destinations (j)			$\sum_j \hat{T}_{ij}$
	from/to	1	2	3	
	1	110	16	42	168
Origins (i)	2	41	328	59	428
	3	49	26	119	194
$\sum_i \hat{T}_{ij}$		200	370	220	790

B1 is calculated as shown below:

$$B_1 = \left[\frac{160}{2} + \frac{450}{15} + \frac{180}{5}\right]^{-1} = [146]^{-1} = 0.006849$$

Hence, T11 is calculated as shown below:

$$\hat{T}_{11} = \frac{0.006849 \times 160 \times 200}{2} = 110$$

The Doubly Constrained Model

$$T_{ij} = A_i O_i B_j D_j d_{ij}^{-\beta}$$

where:

$$O_i = \sum_j T_{ij}$$

$$D_j = \sum_i T_{ij}$$

and:

$$A_i = \frac{1}{\sum_j B_j D_j d_{ij}^{-\beta}}$$

$$B_j = \frac{1}{\sum_i A_i O_i d_{ij}^{-\beta}}$$

Note

Note that the calculation of Ai relies on knowing Bj and the calculation of Bj relies on knowing Ai – something of a conundrum to which the solution is elegantly described by Senior (1979), who sketches out a very useful algorithm for iteratively arriving at values for Ai and Bj by setting each to equal 1 initially and then continuing to calculate each in turn until the difference between successive iterations of the Ai and Bj values is small enough not to matter.

Doubly constrained case

The O-D Matrix

	from/to	Destinations (j) 1	2	3	Total Outlflows ($\sum_j T_{ij}$ or O_i)
Origins (i)	1	100	20	40	160
	2	60	300	90	450
	3	40	50	90	180
	Total Inflows ($\sum_i T_{ij}$ or D_j)	200	370	220	Grand Total 790 ($\sum_i \sum_j T_{ij}$ or T)

and distance matrix:

	1	2	3
1	2	15	5
2	15	2	10
3	5	10	2

Suppose $\lambda = 1$, $\alpha = 1$, and $\beta = -1$ for this system.

The estimated O-D matrix:

	from/to	Destinations (j) 1	2	3	$\sum_j \hat{T}_{ij}$
Origins (i)	1	107	13	40	160
	2	47	334	69	450
	3	46	23	111	180
$\sum_i \hat{T}_{ij}$		200	370	220	790

Hence, T11 is calculated as shown below:

$$\hat{T}_{11} = \frac{0.0046 \times 160 \times 1.45 \times 200}{2} = 107$$

Notice that A1 and B1 are computed by using computer.