

Lesson 10:

Spatial Interaction Models

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- Characteristics of Spatial Interaction Data
- Spatial Interaction Models
 - Unconstrained
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What Spatial Interaction Models are?

Spatial interaction or “gravity models” estimate the flow of people, material, or information between locations in geographical space.

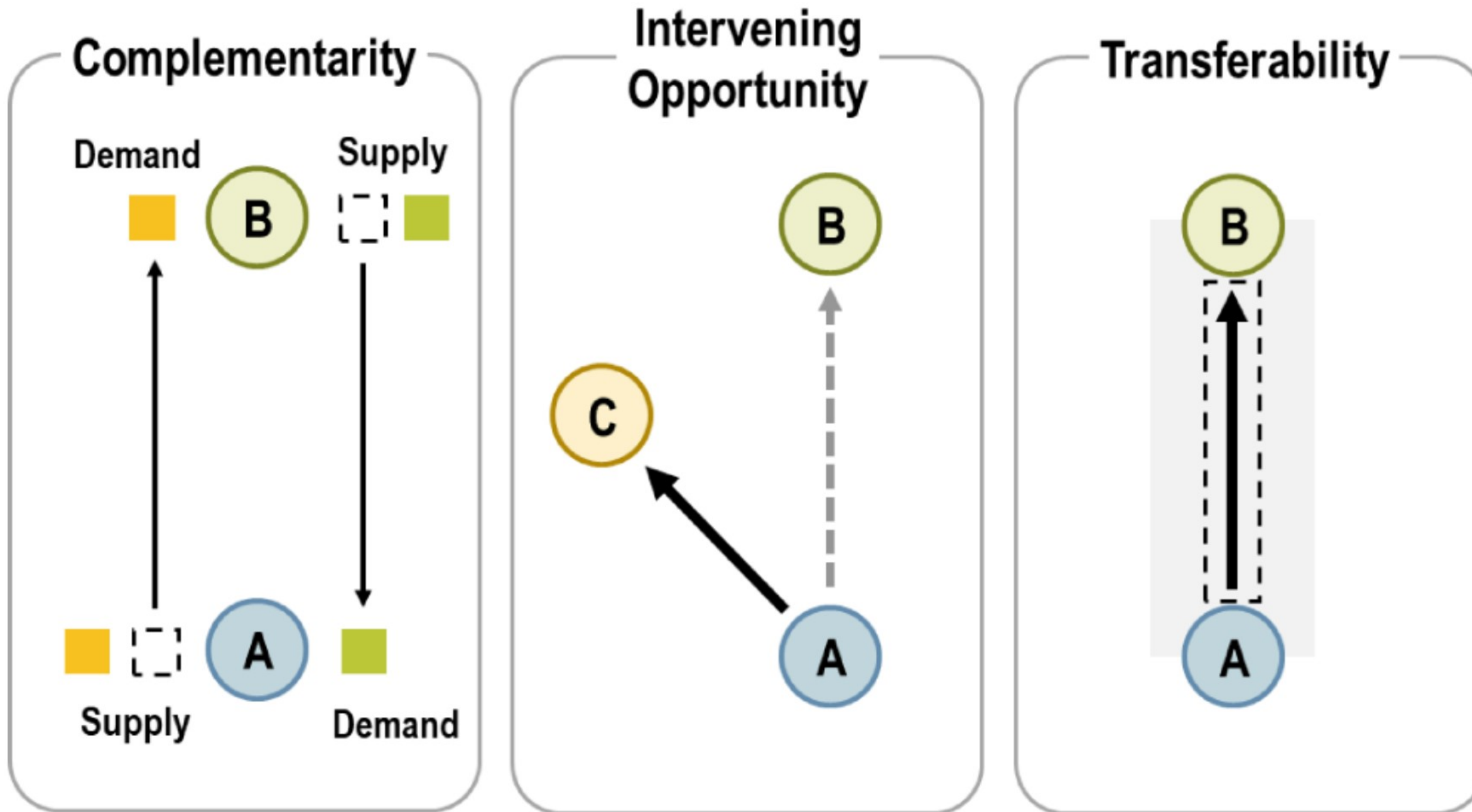
Note

Spatial interaction models seek to explain existing spatial flows. As such it is possible to measure flows and predict the consequences of changes in the conditions generating them. When such attributes are known, it is possible to better allocate transport resources such as conveyances, infrastructure, and terminals.



Conditions for Spatial Flows

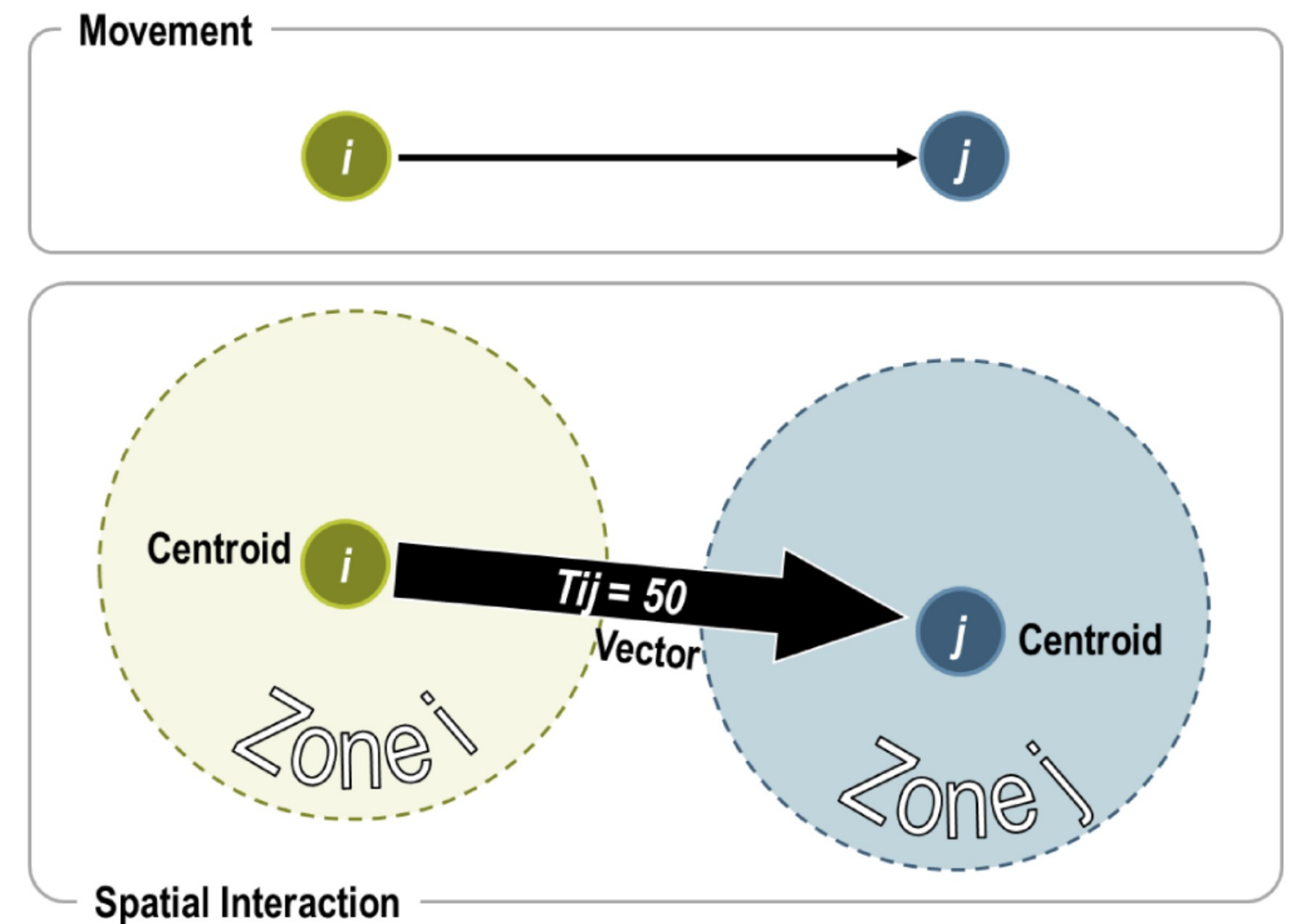
- Three interdependent conditions are necessary for a spatial interaction to occur:



Representation of a Movement as a Spatial Interaction

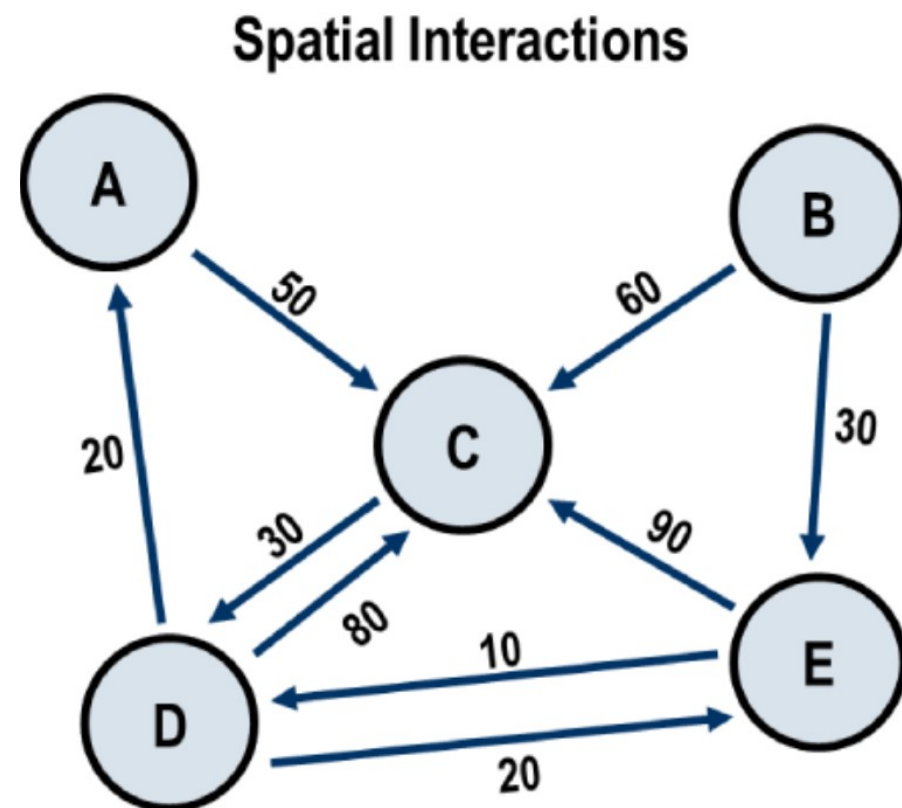
Representing mobility as a spatial interaction involves several considerations:

- **Locations:** A movement is occurring between a location of origin and a location of destination. i generally denotes an origin while j is a destination.
- **Centroid:** An abstraction of the attributes of a zone at a point.
- **Flows:** Flows are generally expressed by a valued vector T_{ij} representing an interaction between locations i and j .
- **Vectors:** A vector T_{ij} links two centroids and has a value assigned to it (50) which can represents movements.



Constructing an O/D Matrix

- The construction of an origin / destination matrix requires directional flow information between a series of locations.
- Figure below represents movements (O/D pairs) between five locations (A, B, C, D and E). From this graph, an O/D matrix can be built where each O/D pair becomes a cell. A value of 0 is assigned for each O/D pair that does not have an observed flow.



O/D Matrix

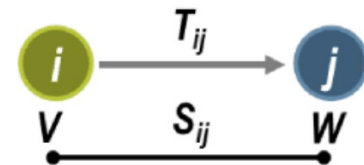
	A	B	C	D	E	Ti
A	0	0	50	0	0	50
B	0	0	60	0	30	90
C	0	0	0	30	0	30
D	20	0	80	0	20	120
E	0	0	90	10	0	100
Tj	20	0	280	40	50	390

Three Basic Types of Interaction Models

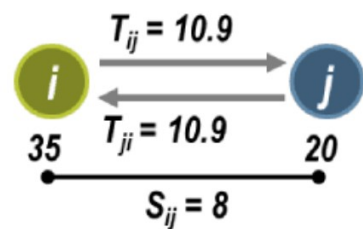
- The general formulation of the spatial interaction model is stated as T_{ij} , which is the interaction between location i (origin) and location j (destination). V_i are the attributes of the location of origin i , W_j are the attributes of the location of destination j , and S_{ij} are the attributes of separation between the location of origin i and the location of destination j .
- From this general formulation, three basic types of interaction models can be derived:

General Formulation

$$T_{ij} = f(V_i, W_j, S_{ij})$$

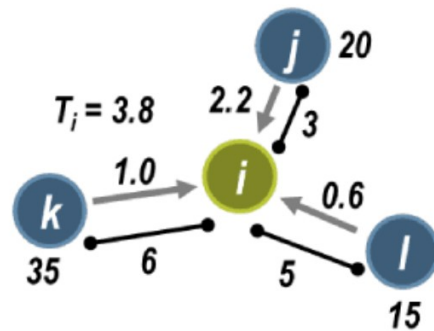


$$T_{ij} = \frac{V_i * W_j}{S_{ij}^2}$$



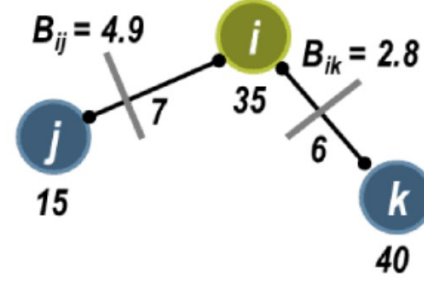
Gravity Model

$$T_i = \sum_j \frac{W_j}{S_{ij}^2}$$



Potential Model

$$B_{ij} = \frac{S_{ij}}{1 + \frac{W_j}{V_i}}$$



Retail Model

Gravity Models

The general formula (also known as unconstrained):

$$T_{ij} = k \frac{V_i^\mu W_j^\alpha}{d_{ij}^\beta}$$

- T_{ij} is the transition/trip or flow, T , between origin i (always the rows in a matrix) and destination j (always the columns in a matrix). If you are not overly familiar with matrix notation, the i and j are just generic indexes to allow us to refer to any cell in the matrix.
- V is a vector (a 1 dimensional matrix – or, if you like, a single line of numbers) of origin attributes which relate to the emissivity of all origins in the dataset, i – this could be any of the origin-related variables.
- W is a vector of destination attributes relating to the attractiveness of all destinations in the dataset, j – similarly, this could be any of the destination related variables.
- d is a matrix of costs (frequently distances – hence, d) relating to the flows between i and j .
- k , μ , α and β are all model parameters to be estimated. β is assumed to be negative, as with an increase in cost/distance we would expect interaction to decrease.

Unconstrained (Totally constrained) case

The O-D Matrix

		Destinations (j)			Total Outflows	
		from/to	1	2	3	$(\sum_j T_{ij} \text{ or } O_i)$
Origins (i)	1		100	20	40	160
	2		60	300	90	450
	3		40	50	90	180
		Total Inflows				Grand Total
		$(\sum_i T_{ij} \text{ or } D_j)$	200	370	220	790
						$(\sum_i \sum_j T_{ij} \text{ or } T)$

The estimated O-D matrix:

		Destinations (j)			$\sum_j \hat{T}_{ij}$	
		from/to	1	2	3	
Origins (i)	1	79	19	35		133
	2	29	412	49		490
	3	36	33	98		167
$\sum_i \hat{T}_{ij}$			144	464	182	790
					$\sum_i \sum_j \hat{T}_{ij}$	

and distance matrix:

	1	2	3
1	2	15	5
2	15	2	10
3	5	10	2

and the calculation T11

$$\frac{0.004944 \times 160 \times 200}{2} = 79$$

Suppose $\lambda = 1$, $\alpha = 1$, and $\beta = -1$ for this system.

The Origin (Production) Constrained Model

In the Origin Constrained Model,

- O_i does not have a parameter as it is a known **constraint**.
- A_i is known as a **balancing factor** and is a vector of values which relate to each origin i which do the equivalent job as k in the unconstrained/total constrained model but ensure that flow estimates from each origin sum to the known totals O_i rather than just the overall total.

$$T_{ij} = A_i O_i W_j^\alpha d_{ij}^{-\beta}$$

where:

$$O_i = \sum_j T_{ij}$$

and:

$$A_i = \frac{1}{\sum_j W_j^\alpha d_{ij}^{-\beta}}$$

Oringin (Production) constrained case

The O-D Matrix

		Destinations (j)			Total Outflows	
		from/to	1	2	3	$(\sum_j T_{ij} \text{ or } O_i)$
Origins (i)	1		100	20	40	160
	2		60	300	90	450
	3		40	50	90	180
		Total Inflows				Grand Total
		$(\sum_i T_{ij} \text{ or } D_j)$	200	370	220	790
						$(\sum_i \sum_j T_{ij} \text{ or } T)$

and distance matrix:

	1	2	3
1	2	15	5
2	15	2	10
3	5	10	2

Suppose $\lambda = 1$, $\alpha = 1$, and $\beta = -1$ for this system.

The estimated O-D matrix:

		Destinations (j)			$\sum_j \hat{T}_{ij}$	
		from/to	1	2	3	
Origins (i)	1	95	23	42		160
	2	27	378	45		450
	3	38	36	106		180
$\sum_i \hat{T}_{ij}$		160	437	193		790

A1 is calculated as shown below:

$$A_1 = \left[\frac{200}{2} + \frac{370}{15} + \frac{200}{5} \right]^{-1} = [168.67]^{-1} = 0.005929$$

Hence, T11 is calculated as shown below:

$$\hat{T}_{11} = \frac{0.005929 \times 160 \times 200}{2} = 95$$

The Destination (Attraction) Constrained Model

$$T_{ij} = D_j B_j V_i^\mu d_{ij}^- \beta$$

where:

$$D_j = \sum_i T_{ij}$$

and:

$$B_j = \frac{1}{\sum_i V_i^\mu d_{ij}^- \beta}$$

Destination (Attraction) constrained case

The O-D Matrix

		Destinations (j)			Total Outflows	
		from/to	1	2	3	($\sum_j T_{ij}$ or O_i)
Origins (i)	1		100	20	40	160
	2		60	300	90	450
	3		40	50	90	180
Total Inflows ($\sum_i T_{ij}$ or D_j)						Grand Total
						790
						($\sum_i \sum_j T_{ij}$ or T)

and distance matrix:

	1	2	3
1	2	15	5
2	15	2	10
3	5	10	2

Suppose $\lambda = 1$, $\alpha = 1$, and $\beta = -1$ for this system.

The estimated O-D matrix:

		Destinations (j)			$\sum_j \hat{T}_{ij}$	
		from/to	1	2	3	
Origins (i)	1	110	16	42		168
	2	41	328	59		428
	3	49	26	119		194
$\sum_i \hat{T}_{ij}$		200	370	220		790

B1 is calculated as shown below:

$$B_1 = \left[\frac{160}{2} + \frac{450}{15} + \frac{180}{5} \right]^{-1} = [146]^{-1} = 0.006849$$

Hence, T11 is calculated as shown below:

$$\hat{T}_{11} = \frac{0.006849 \times 160 \times 200}{2} = 110$$

The Doubly Constrained Model

$$T_{ij} = A_i O_i B_j D_j d_{ij}^- \beta$$

where:

$$O_i = \sum_j T_{ij}$$

$$D_j = \sum_i T_{ij}$$

and:

$$A_i = \frac{1}{\sum_j B_j D_j d_{ij}^- \beta}$$

$$B_j = \frac{1}{\sum_i A_i O_i d_{ij}^- \beta}$$

Note

Note that the calculation of A_i relies on knowing B_j and the calculation of B_j relies on knowing A_i – something of a conundrum to which the solution is elegantly described by Senior (1979), who sketches out a very useful algorithm for iteratively arriving at values for A_i and B_j by setting each to equal 1 initially and then continuing to calculate each in turn until the difference between successive iterations of the A_i and B_j values is small enough not to matter.

Doubly constrained case

The O-D Matrix

		Destinations (j)			Total Outflows	
		from/to	1	2	3	$(\sum_j T_{ij} \text{ or } O_i)$
Origins (i)	1		100	20	40	160
	2		60	300	90	450
	3		40	50	90	180
Total Inflows						Grand Total
		$(\sum_i T_{ij} \text{ or } D_j)$	200	370	220	790
						$(\sum_i \sum_j T_{ij} \text{ or } T)$

and distance matrix:

	1	2	3
1	2	15	5
2	15	2	10
3	5	10	2

The estimated O-D matrix:

		Destinations (j)			$\sum_j \hat{T}_{ij}$	
		from/to	1	2	3	
Origins (i)	1	107	13	40		160
	2	47	334	69		450
	3	46	23	111		180
$\sum_i \hat{T}_{ij}$		200	370	220		790

Hence, T11 is calculated as shown below:

$$\hat{T}_{11} = \frac{0.0046 \times 160 \times 1.45 \times 200}{2} = 107$$

Notice that A1 and B1 are computed by using computer.

Suppose $\lambda = 1$, $\alpha = 1$, and $\beta = -1$ for this system.

