Lesson 6: Spatial Weights and Applications

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12 Feb 2023

Content

- Introduction to Spatial Weights
- Contiguity-Based Spatial Weights
  - Rook's
  - Queen's
- Distance-Band Spatial Weights
- Applications of Spatial Weights

What is geographically referenced attribute?

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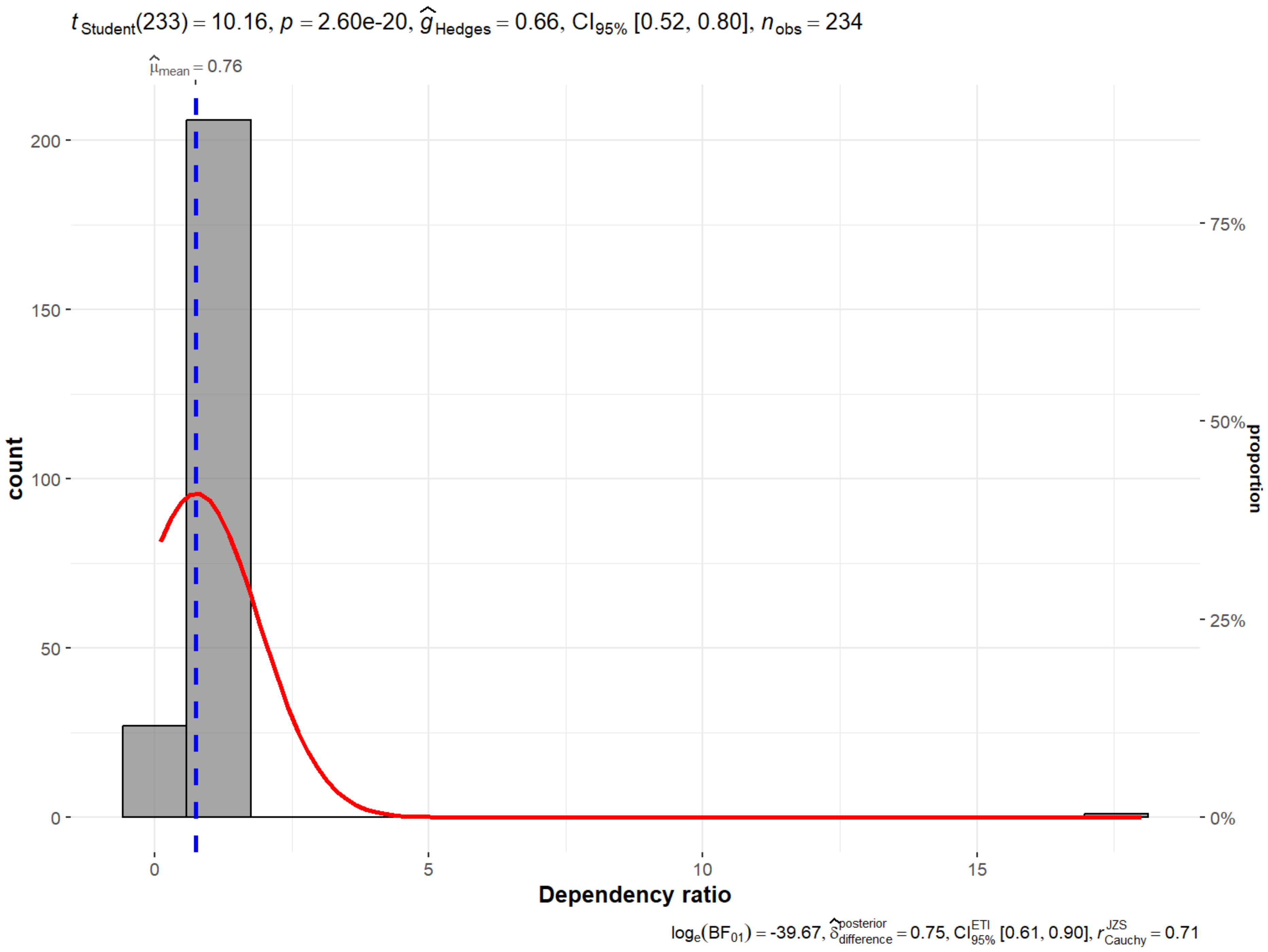
Rows: 323
Columns: 12
$ SUBZONE_N        <chr> "MARINA SOUTH", "PEARL'S HILL", "BOAT QUAY", "HENDERS…
$ SUBZONE_C        <fct> MSSZ01, OTSZ01, SRSZ03, BMSZ08, BMSZ03, BMSZ07, BMSZ0…
$ PLN_AREA_N       <fct> MARINA SOUTH, OUTRAM, SINGAPORE RIVER, BUKIT MERAH, B…
$ PLN_AREA_C       <fct> MS, OT, SR, BM, BM, BM, BM, SR, QT, QT, QT, BM, ME, R…
$ REGION_N         <fct> CENTRAL REGION, CENTRAL REGION, CENTRAL REGION, CENTR…
$ REGION_C         <fct> CR, CR, CR, CR, CR, CR, CR, CR, CR, CR, CR, CR, CR, C…
$ YOUNG            <dbl> NA, 1100, 0, 2620, 2840, 2910, 2850, 0, 1120, 30, NA,…
$ `ECONOMY ACTIVE` <dbl> NA, 3420, 50, 7500, 6260, 7560, 8340, 50, 2750, 210, …
$ AGED             <dbl> NA, 2110, 20, 3260, 1630, 3310, 3590, 10, 560, 50, NA…
$ TOTAL            <dbl> NA, 6630, 70, 13380, 10730, 13780, 14780, 60, 4430, 2…
$ DEPENDENCY       <dbl> NA, 0.9385965, 0.4000000, 0.7840000, 0.7140575, 0.822…
$ geometry         <MULTIPOLYGON [m]> MULTIPOLYGON (((31495.56 30..., MULTIPOL…
```

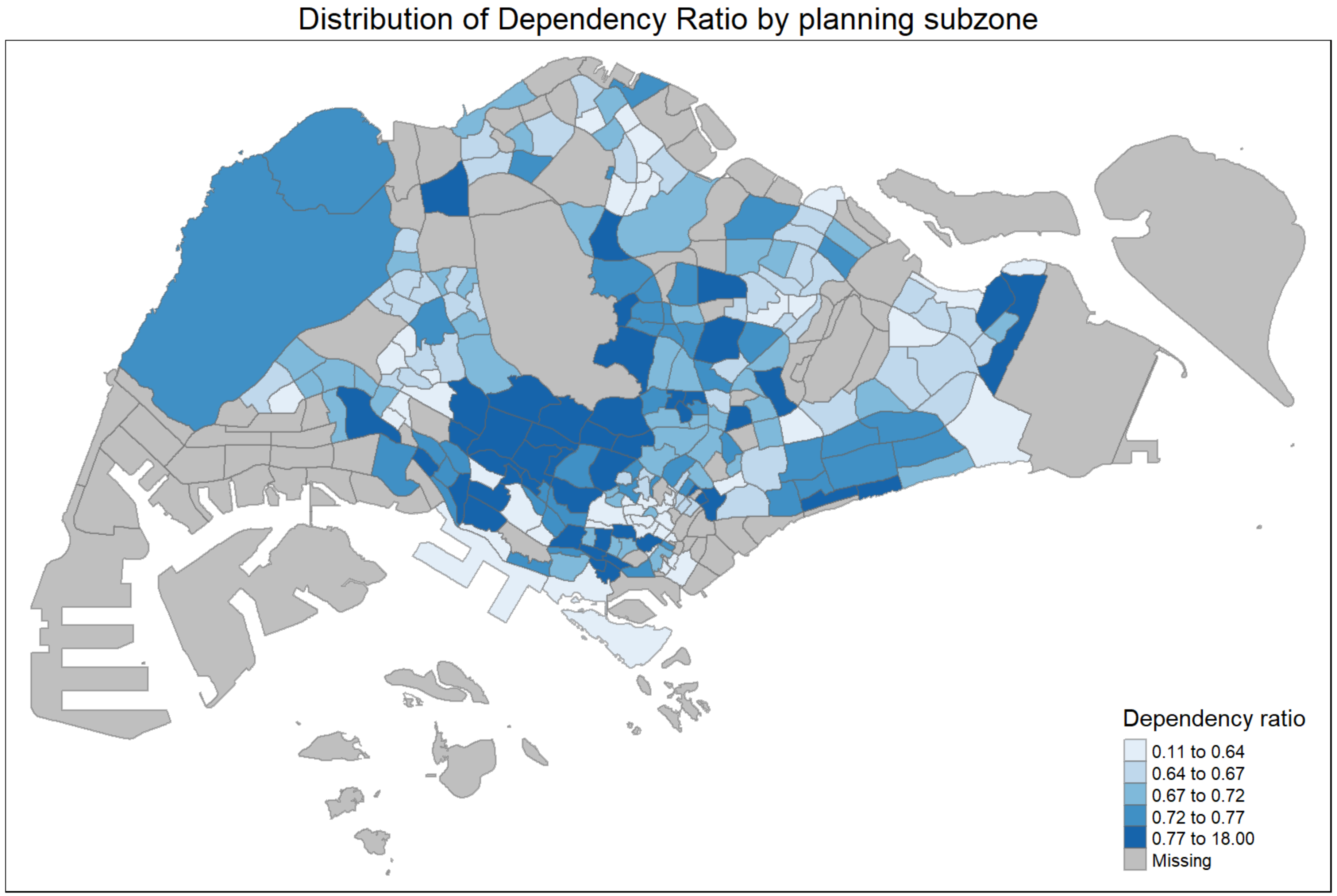
Describing attribute distribution

The dependency ratio values by planning subzone are normally distributed.



Geographical distribution question

Are the planning subzones with high proportion of dependency ratio randomly distributed over space?



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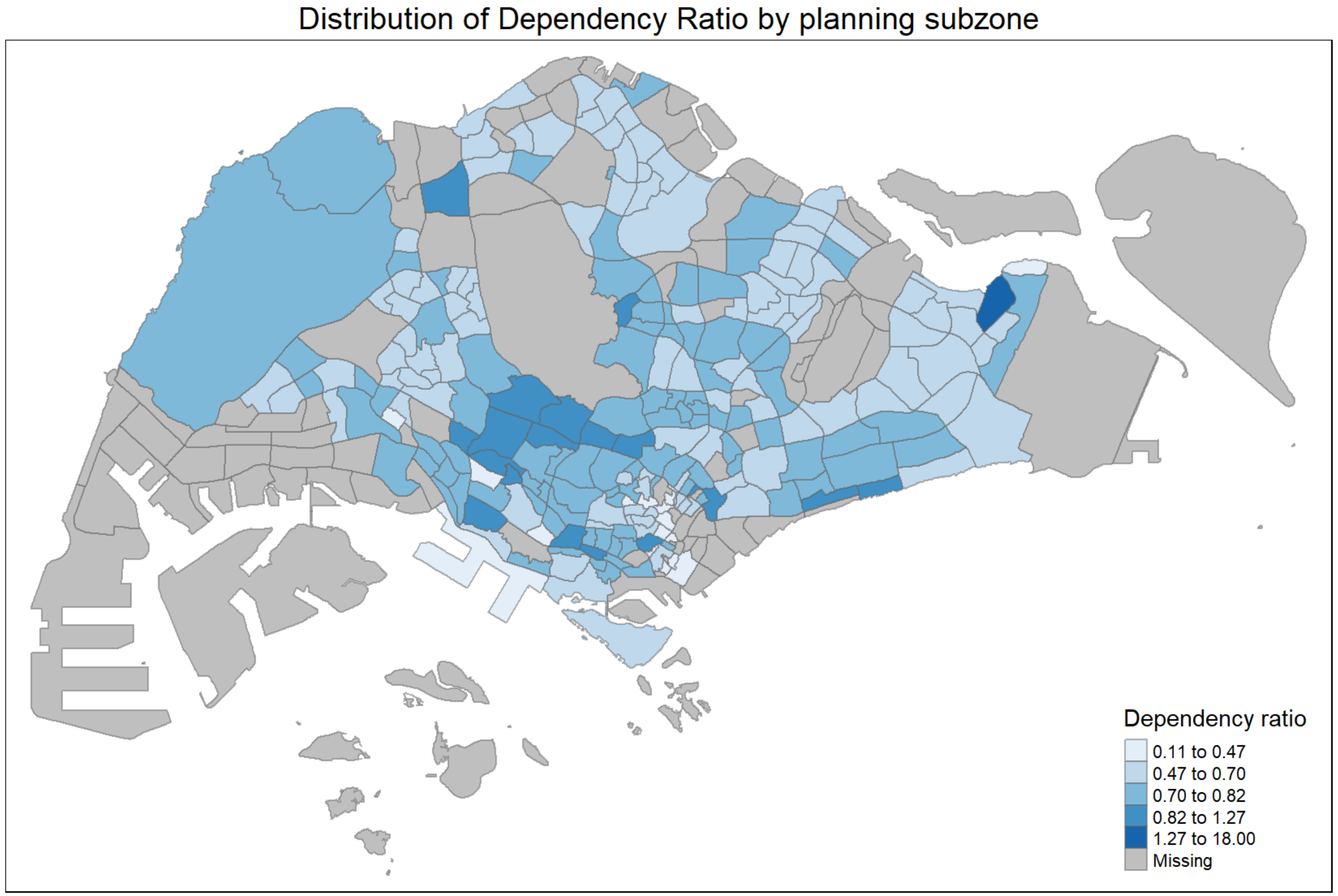

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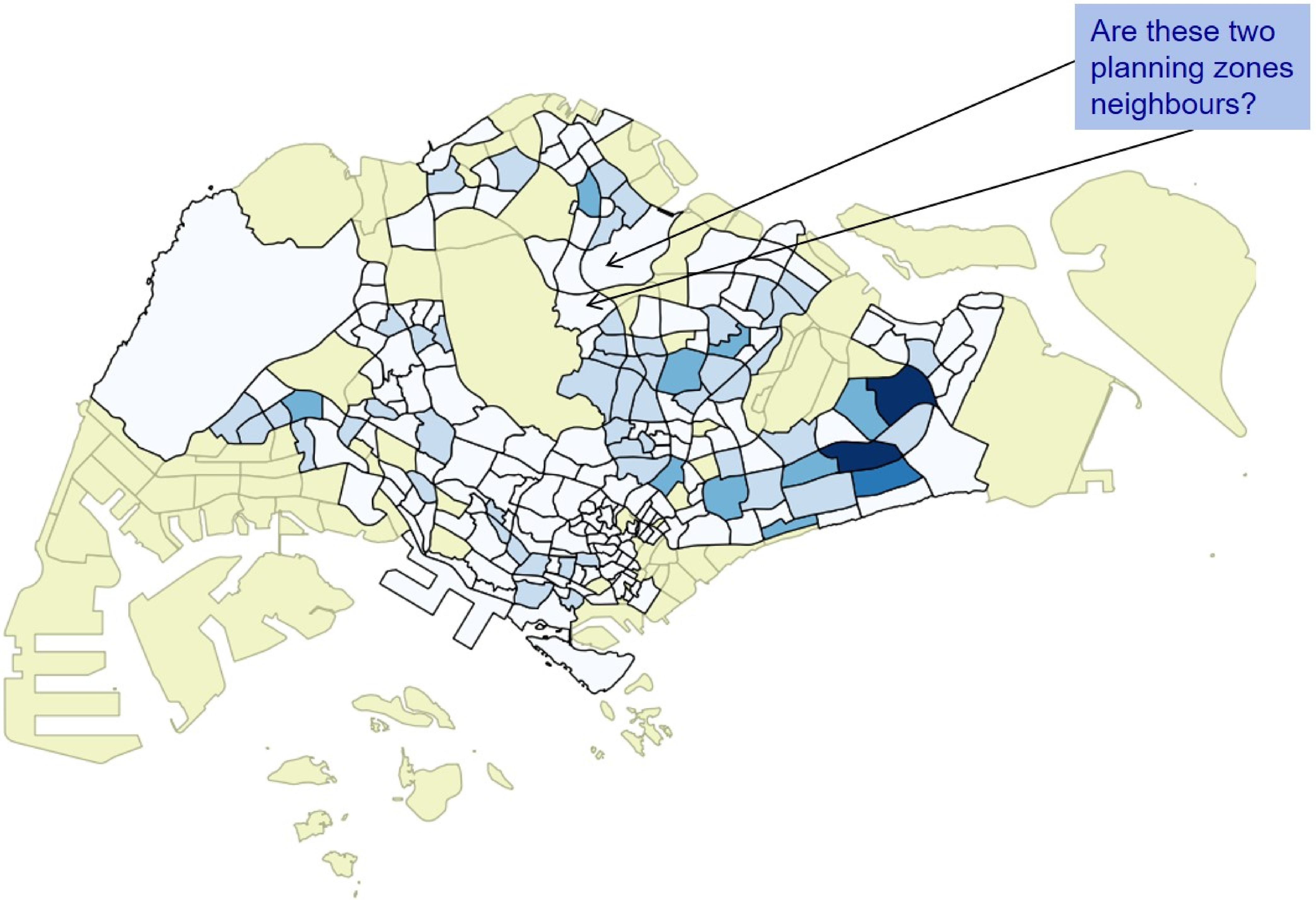
What are Spatial Weights *(wij)*

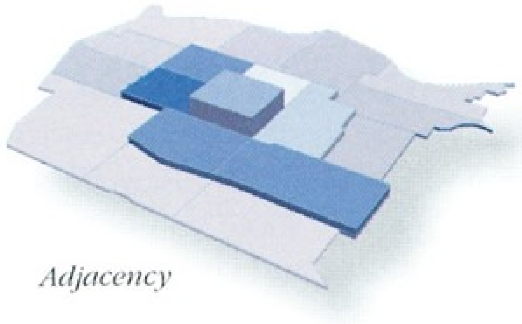
- A way to define spatial neighbourhood.



Defining Spatial Weights

Adjacency criterion:

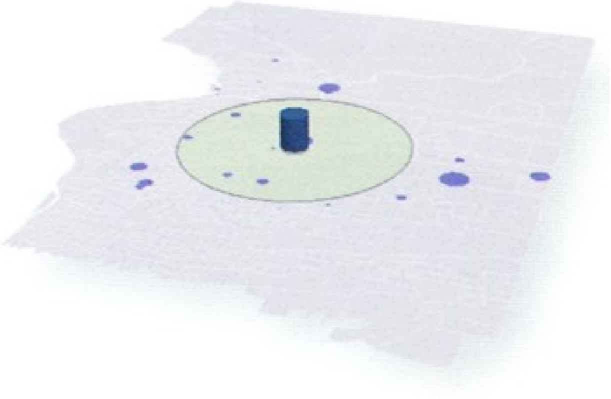
$$w_{ij} = \begin{cases} 1 & \text{if location } j \text{ is adjacent to } i, \\ 0 & \text{if location } j \text{ is not adjacent to } i. \end{cases}$$



Adjacency

Distance criterion:

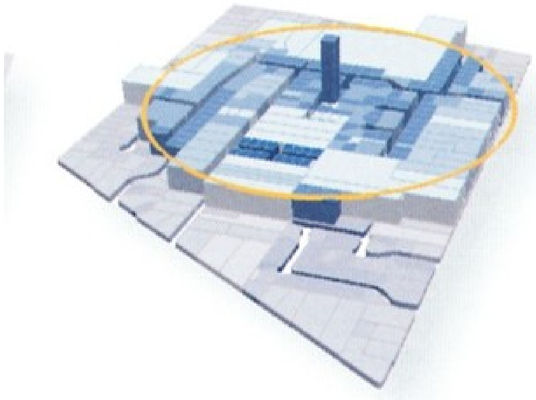
$$w_{ij}(d) = \begin{cases} 1 & \text{if location } j \text{ is within distance } d \text{ from } i, \\ 0 & \text{otherwise.} \end{cases}$$



Distance: discrete features

A general spatial distance weight matrices:

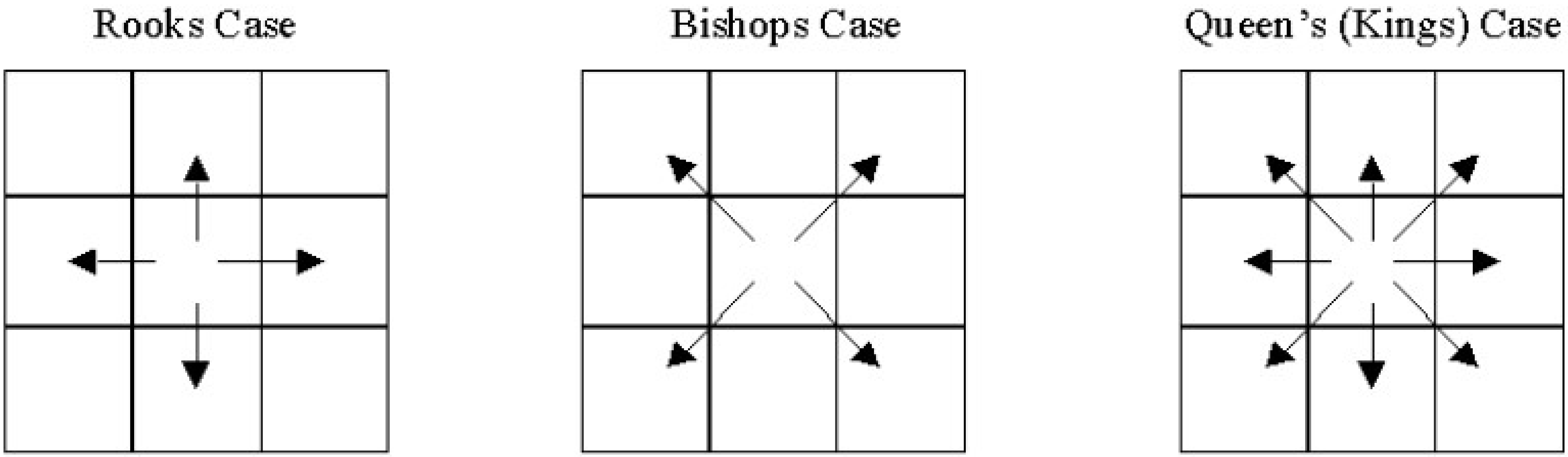
$$w_{ij}(d) = d_{ij}^{-a} \cdot \beta^{b}$$



Distance: contiguous features

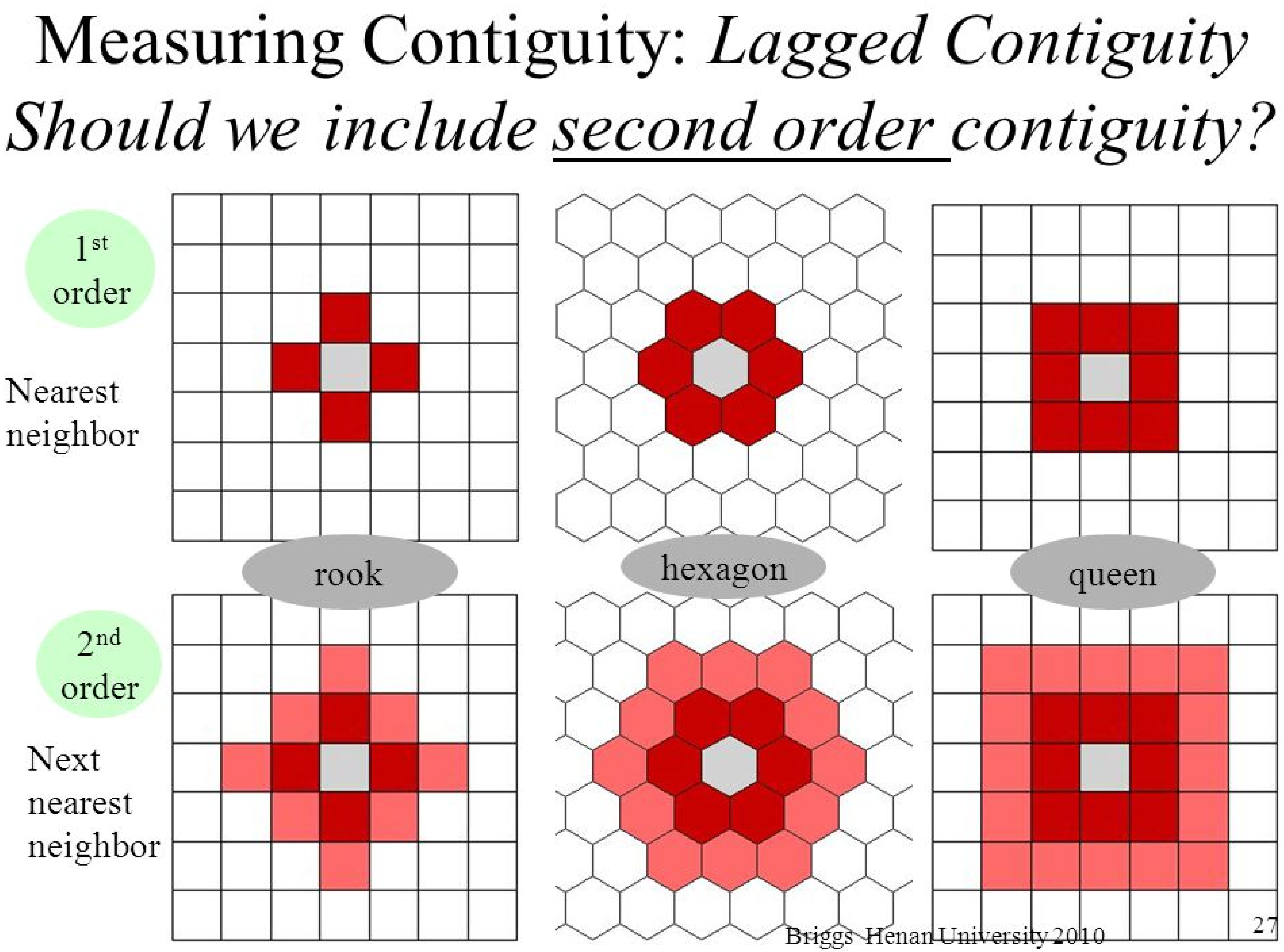
Contiguity Neighbours

- Contiguity (common boundary)
- What is a “shared” boundary?



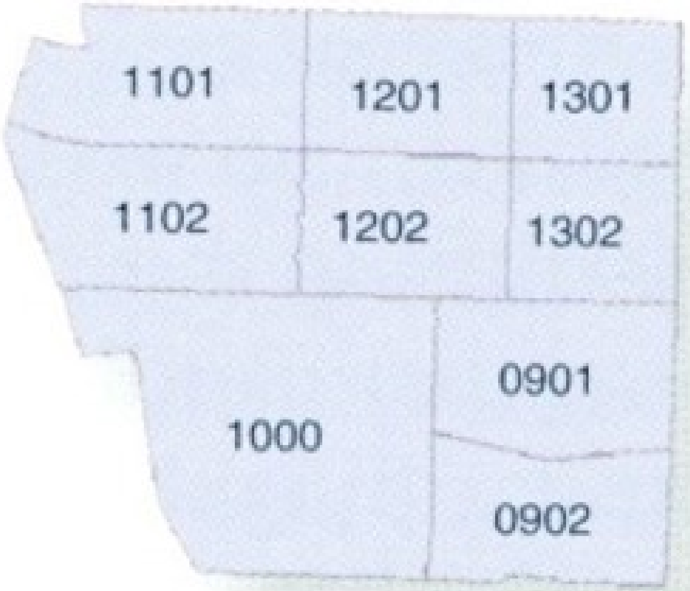
Beyond the basic contiguity neighbours

There are also second-order, third-order, forth-order, etc contiguity



Weights matrix: Adjacency-based neighbours

Quiz: With reference to the figure below, list down the neighbour(s) of area 1202 using Rook case.

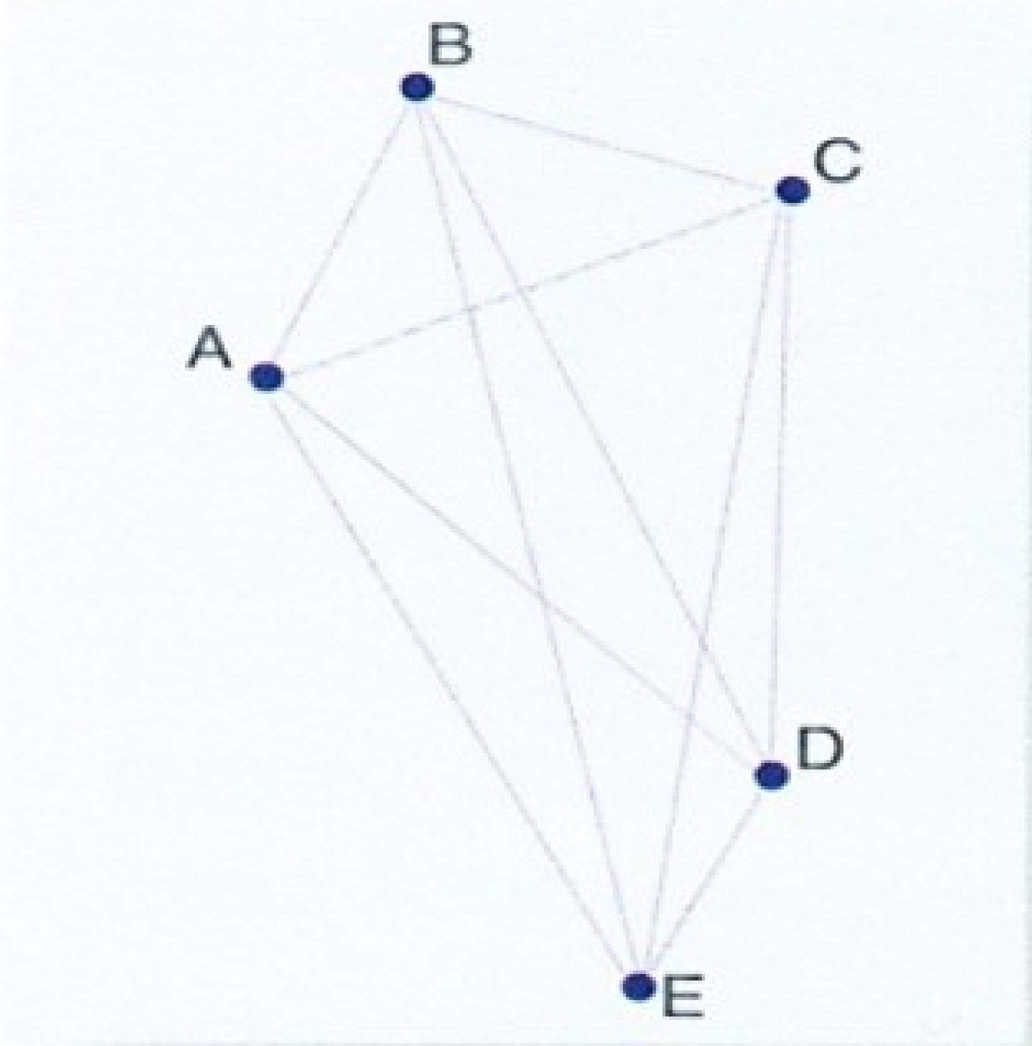


Weights matrix for an adjacency-based neighborhood

	1101	1201	1301	1102	1202	1302	1000	0901	0902
1101	0	1	0	1	1	0	0	0	0
1201	1	0	1	1	1	1	0	0	0
1301	0	1	0	0	1	1	0	0	0
1102	1	1	0	0	1	0	1	0	0
1202	1	1	1	1	0	1	1	1	0
1302	0	1	1	0	1	0	0	1	0
1000	0	0	0	1	1	0	0	1	1
0901	0	0	0	0	1	1	1	0	1
0902	0	0	0	0	0	0	1	1	0

Weights Matrix: Distance-based neighbours

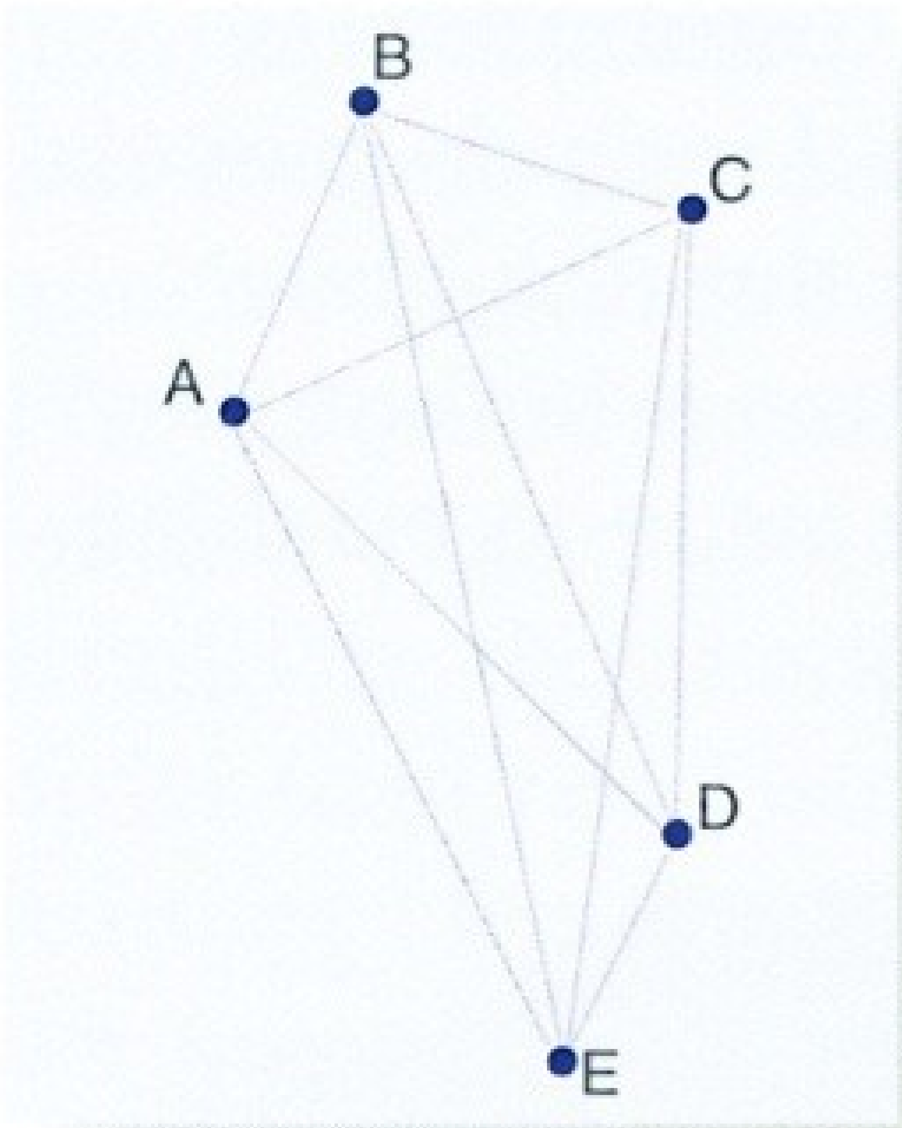
Quiz: With reference to the figure below, create a weights matrix for d = 650.



Weights matrix for a distance-based neighborhood

	A	B	C	D	E
A	0	353	516	641	757
B	353	0	357	837	1025
C	516	357	0	659	901
D	641	837	659	0	263
E	757	1025	901	263	0

Weights matrix: Measured distances



	A	B	C	D	E
A	0	353	516	641	757
B	353	0	357	837	1025
C	516	357	0	659	901
D	641				
E	757				

Weights matrix with measured distances

$$w_{ij} = \frac{1}{d_{ij}}$$

	A	B	C	D	E
A	0	0.00283	0.00194	0.00156	0.00132
B	0.00283	0	0.00280	0.00119	0.00098
C	0.00194	0.00280	0	0.00152	0.00111
D	0.00156	0.00119	0.00152	0	0.00380
E	0.00132	0.00098	0.00111	0.00380	0

Weights matrix with inverse distances

Row standardisation

In practice, row-standardised weights instead of spatial weights will be used.

Binary W matrix:

$$\tilde{W} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Row standardized W matrix:

$$W = \begin{bmatrix} 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \end{bmatrix}$$

Applications of Spatial Weights

Formally, for observation i, the spatial lag of yi, referred to as [Wy]i (the variable Wy observed for location i) is:

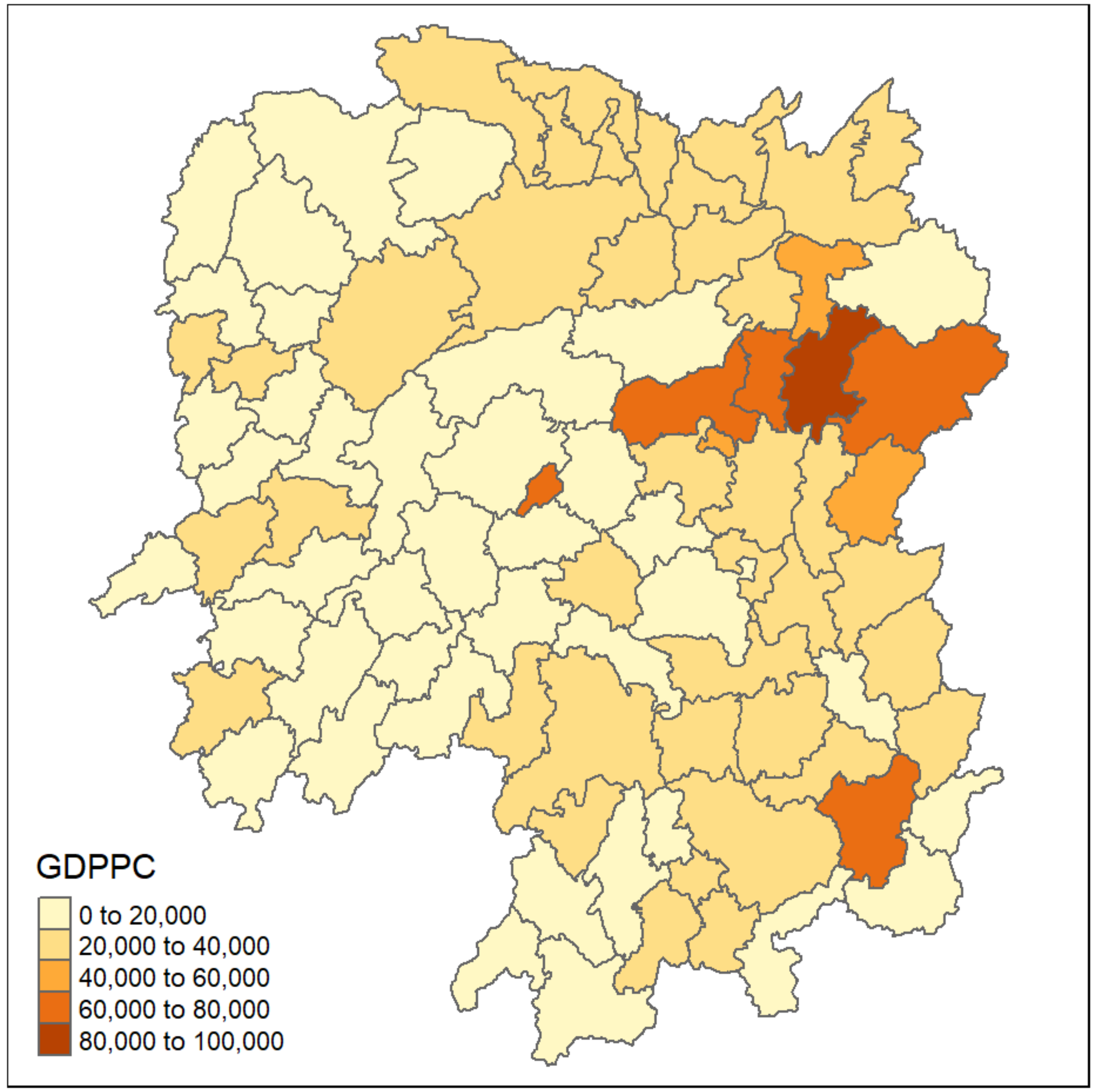
$$[Wy]_i = w_{i1} y_1 + w_{i2} y_2 + \cdots + w_{in} y_n,$$

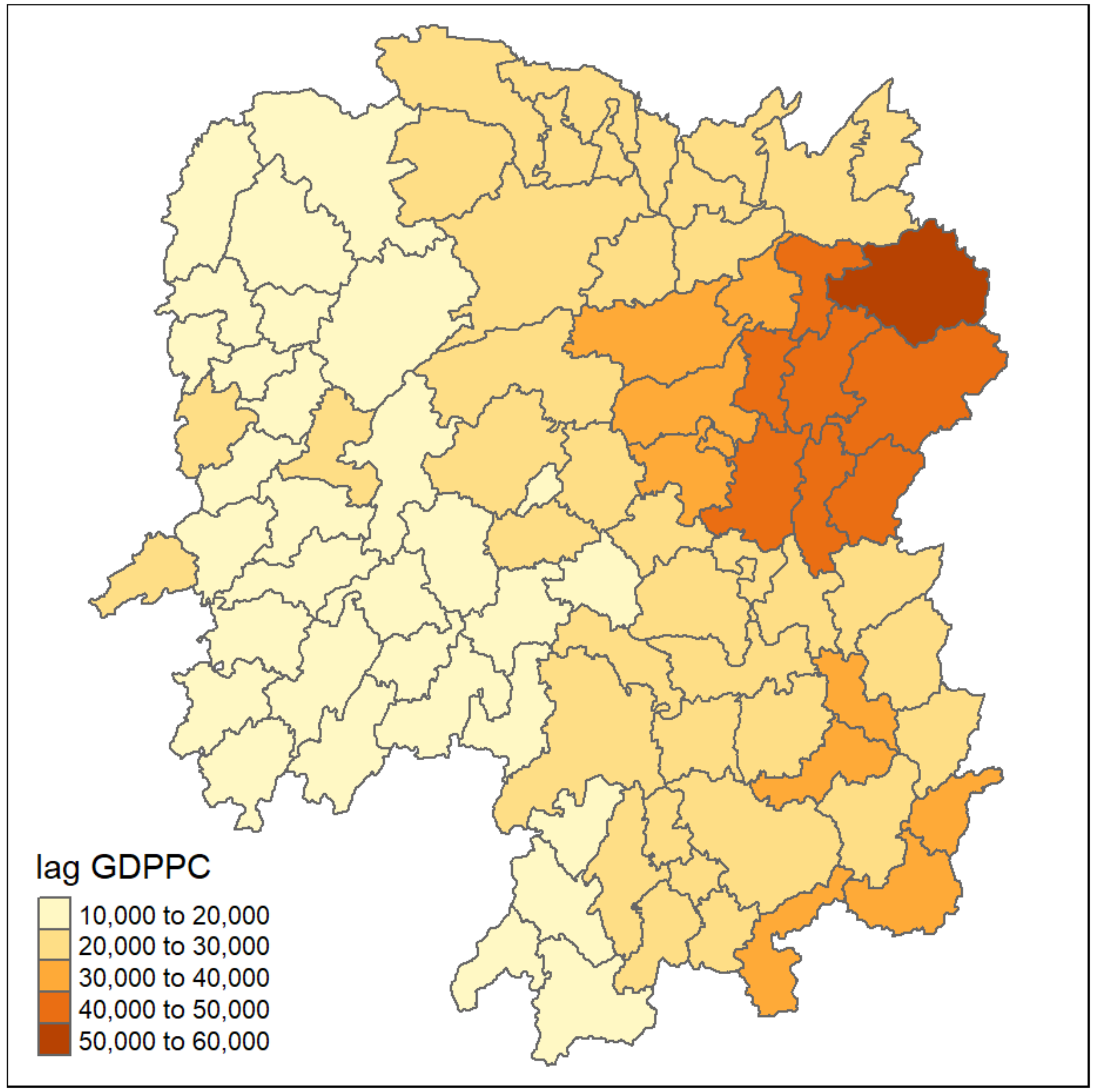
$$[Wy]_i = \sum_{j=1}^{n} w_{ij} y_j,$$

where the weights wij consist of the elements of the i-th row of the matrix W, matched up with the corresponding elements of the vector y.

Spatially Lagged Variables

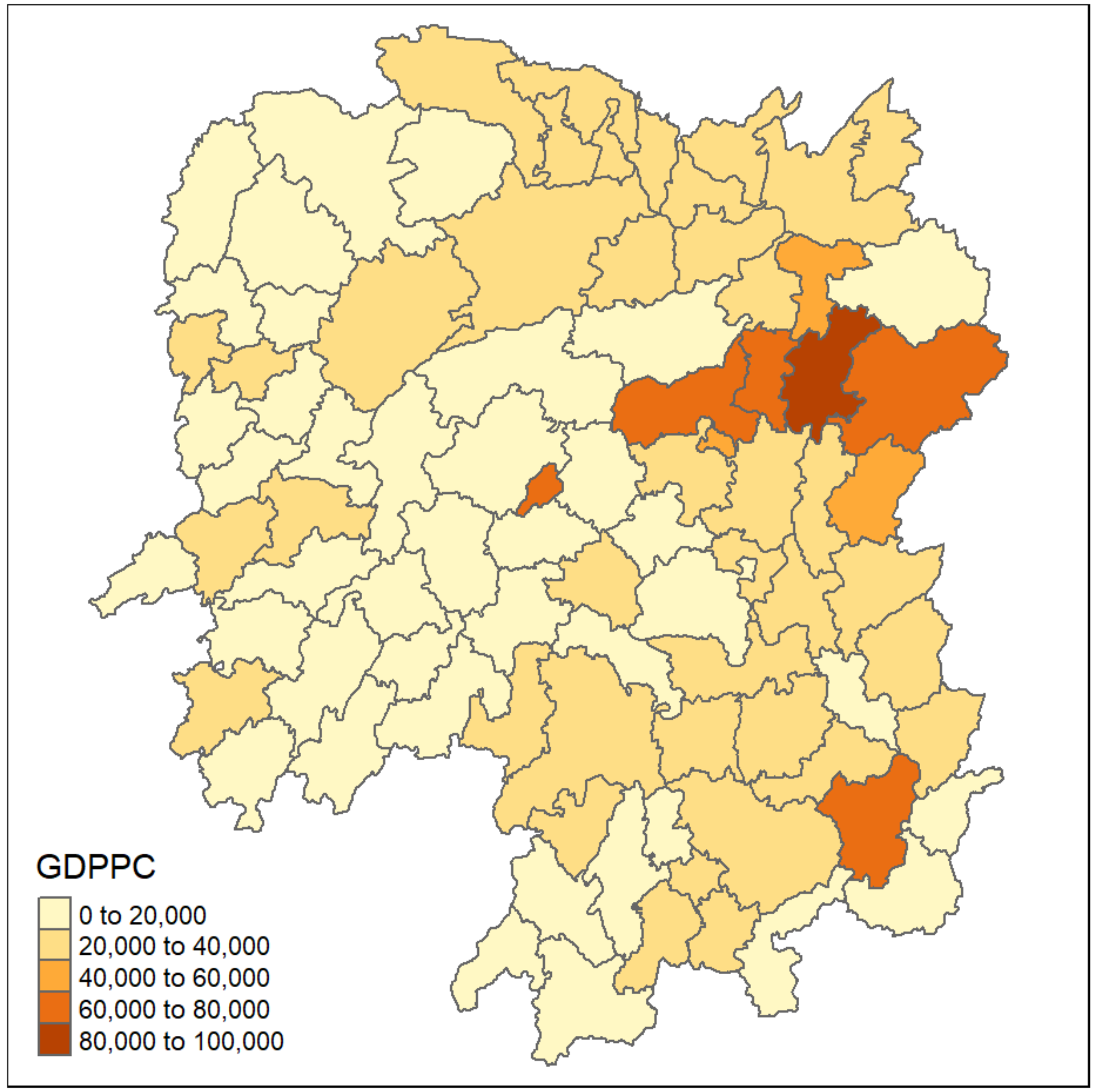
Spatial lag with row-standardized weights.

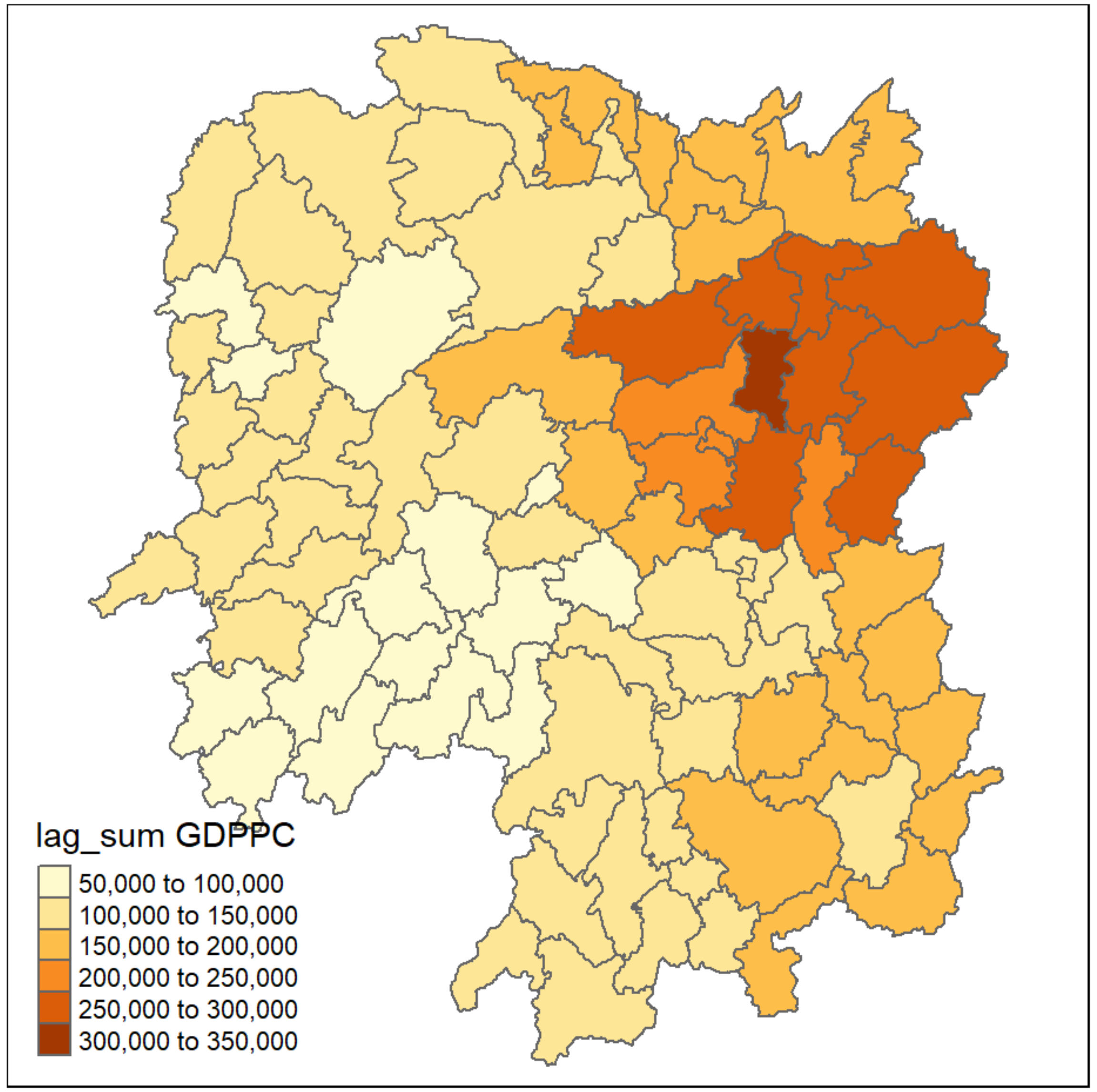




Spatial window sum

The spatial window sum uses and includes the diagonal element.





References

- Chapter 2. Codifying the neighbourhood structure of Handbook of Spatial Analysis: Theory and Application with R.
- François Bavaud (2010) “Models for Spatial Weights: A Systematic Look” *Geographical Analysis*, Vol. 30, No.2, pp 153-171.
- Tony H. Grubesic and Andrea L. Rosso (2014) “The Use of Spatially Lagged Explanatory Variables for Modeling Neighborhood Amenities and Mobility in Older Adults”, *Cityscape*, Vol. 16, No. 2, pp. 205-214.