Lesson 1: Spatial Weights and Applications

Dr. Kam Tin Seong
Assoc. Professor of Information Systems

School of Computing and Information Systems,
Singapore Management University

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Content

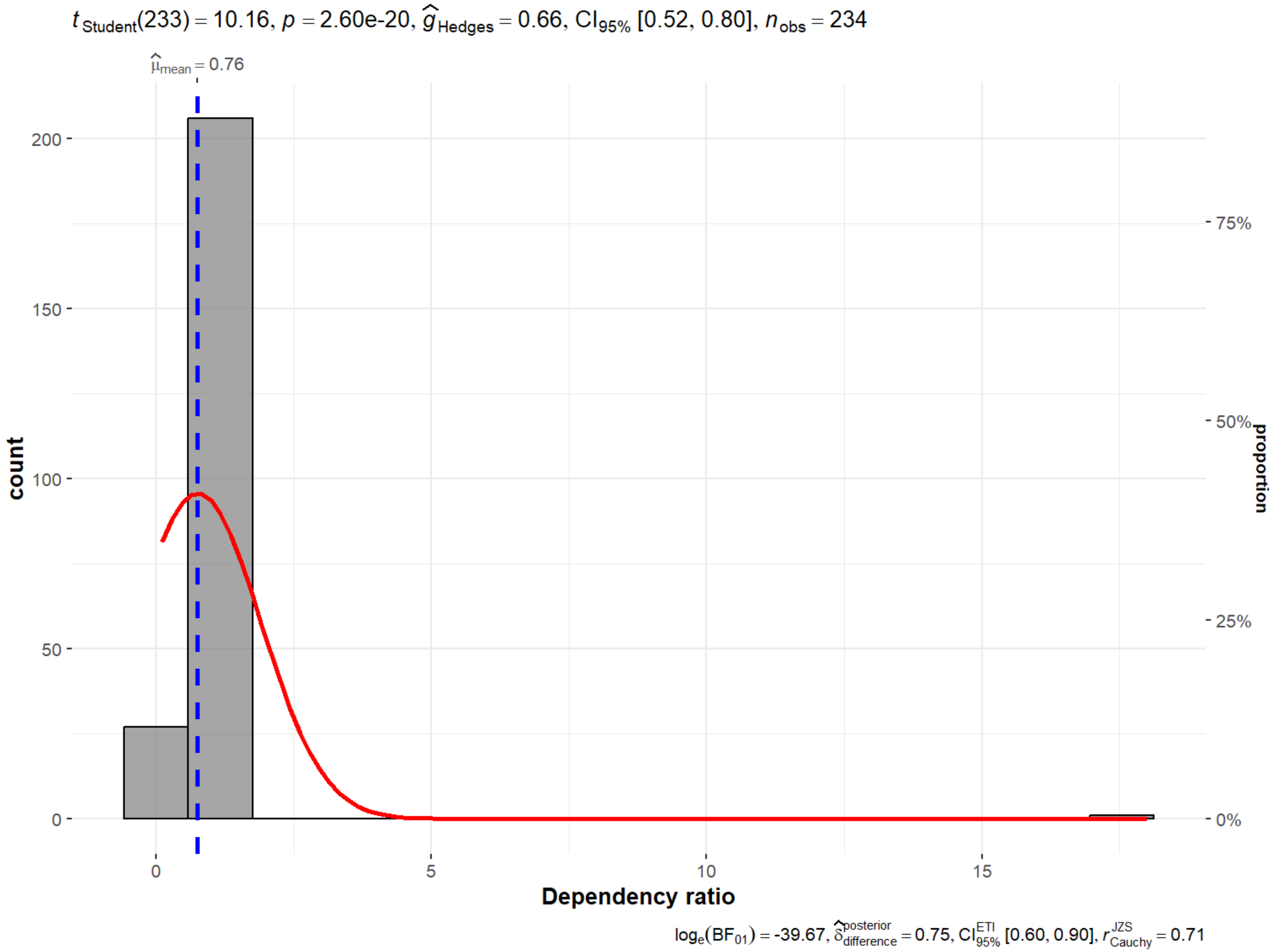
- Introduction to Spatial Weights
- Contiguity-Based Spatial Weights
    - Rook’s
    - Queen’s
- Distance-Band Spatial Weights
- Applications of Spatial Weights

What is geographically referenced attribute?

SUBZONE_N	TOTAL	DEPENDENCY	geometry
LOYANG WEST	190	18.0000000	*MULTIPOLYGON (((43756.39 39...*
NATIONAL UNIVERSITY OF S'PORE	340	1.2666667	*MULTIPOLYGON (((22653.1 306...*
SUNGEI ROAD	2010	0.9514563	*MULTIPOLYGON (((30371.78 31...*
PEARL'S HILL	6630	0.9385965	*MULTIPOLYGON (((29092.28 30...*
MALCOLM	2990	0.9290323	*MULTIPOLYGON (((28959.61 34...*
CORONATION ROAD	6550	0.8985507	*MULTIPOLYGON (((25771.96 33...*
DUNEARN	3940	0.8761905	*MULTIPOLYGON (((27284.61 34...*
HILLCREST	9240	0.8704453	*MULTIPOLYGON (((25392.54 35...*
SEMBAWANG HILLS	6770	0.8650138	*MULTIPOLYGON (((27744.03 38...*
BUKIT MERAH	1100	0.8644068	*MULTIPOLYGON (((26750.09 29...*
HOLLAND ROAD	10710	0.8561525	*MULTIPOLYGON (((23191.9 337...*
SIGLAP	6500	0.8518519	*MULTIPOLYGON (((39348.48 32...*

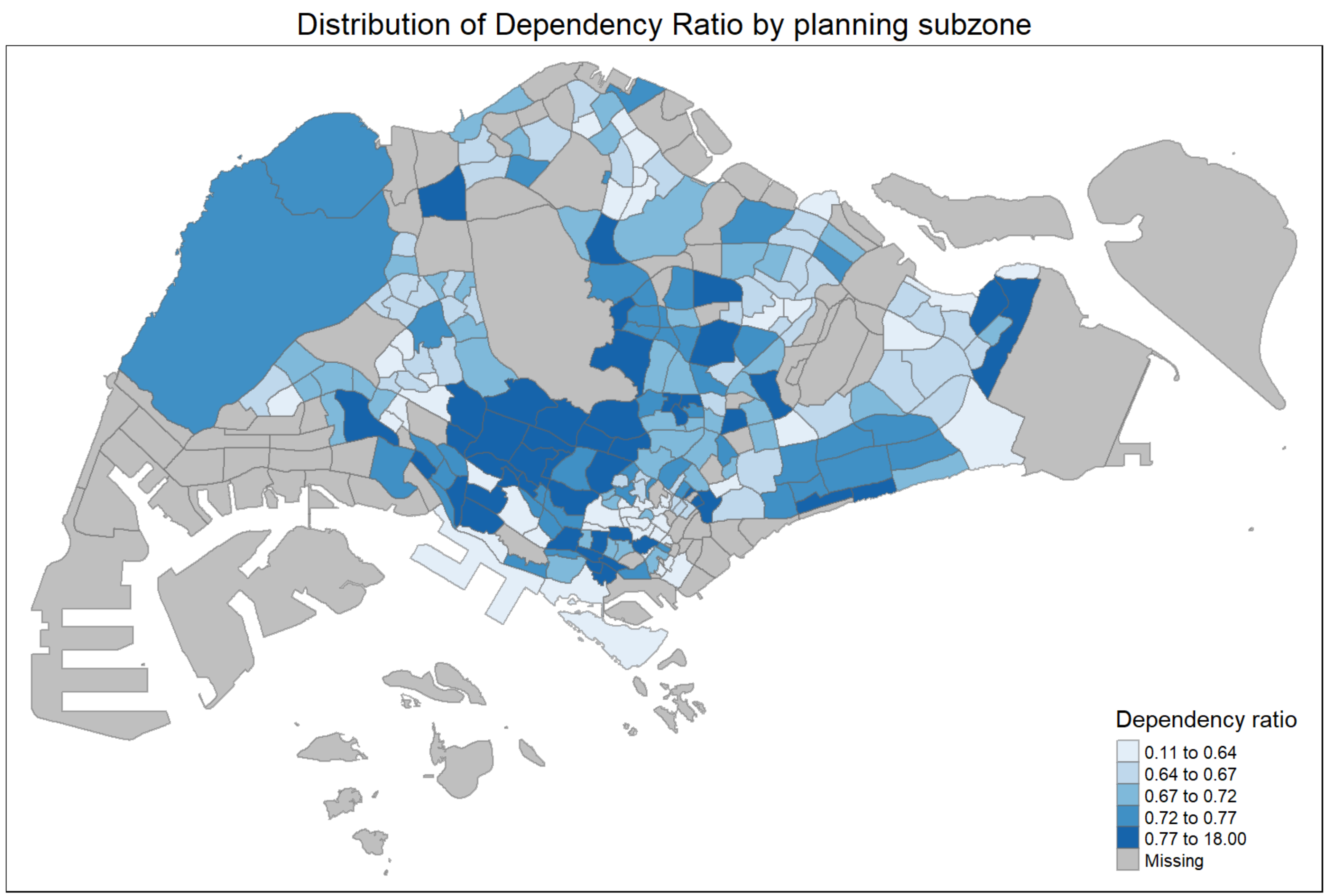
Describing attribute distribution

Are the values of the dependency ratio values by planning subzone normally distributed?



Geographical distribution question

Are the planning subzones with high proportion of dependency ratio randomly distributed over space?



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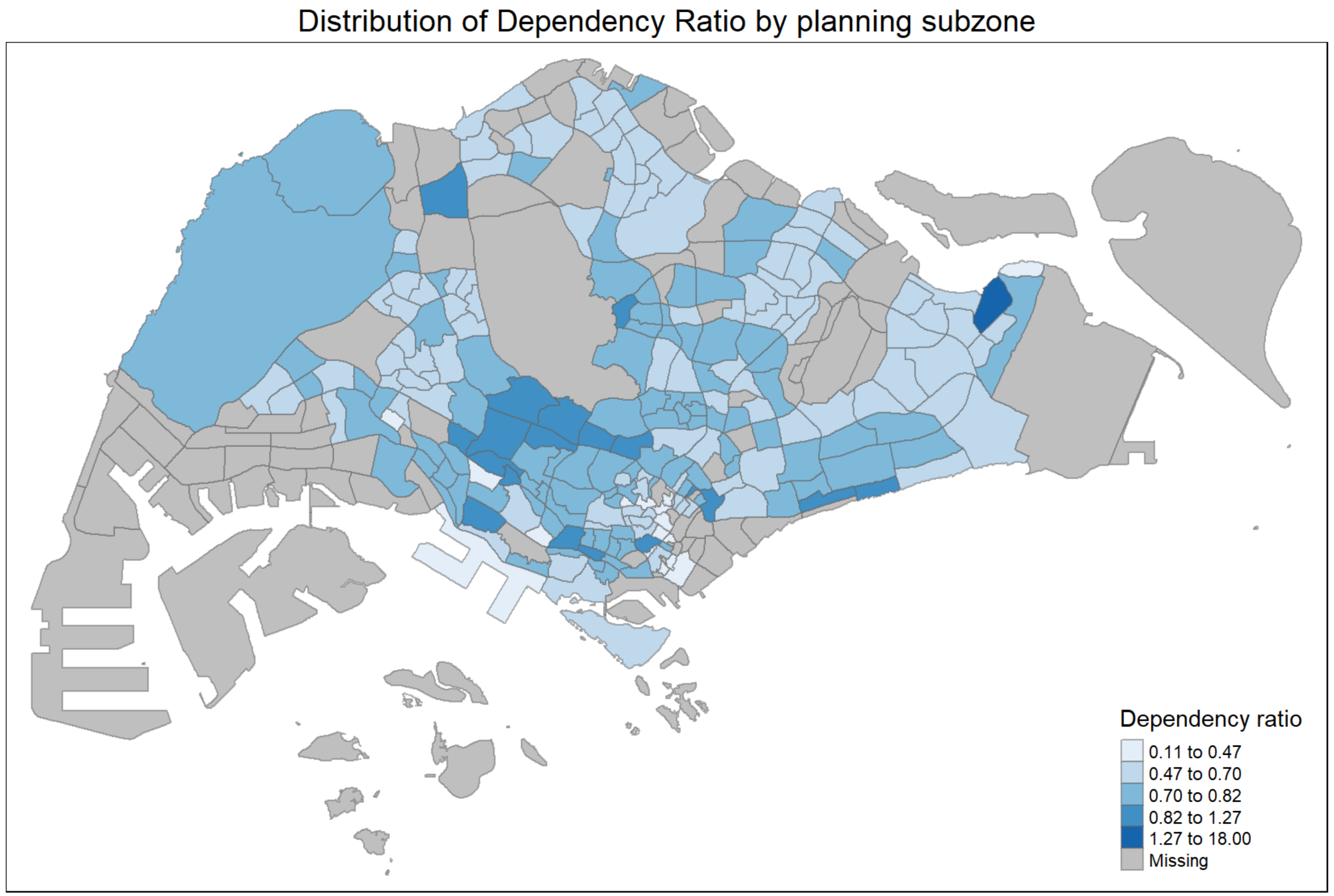
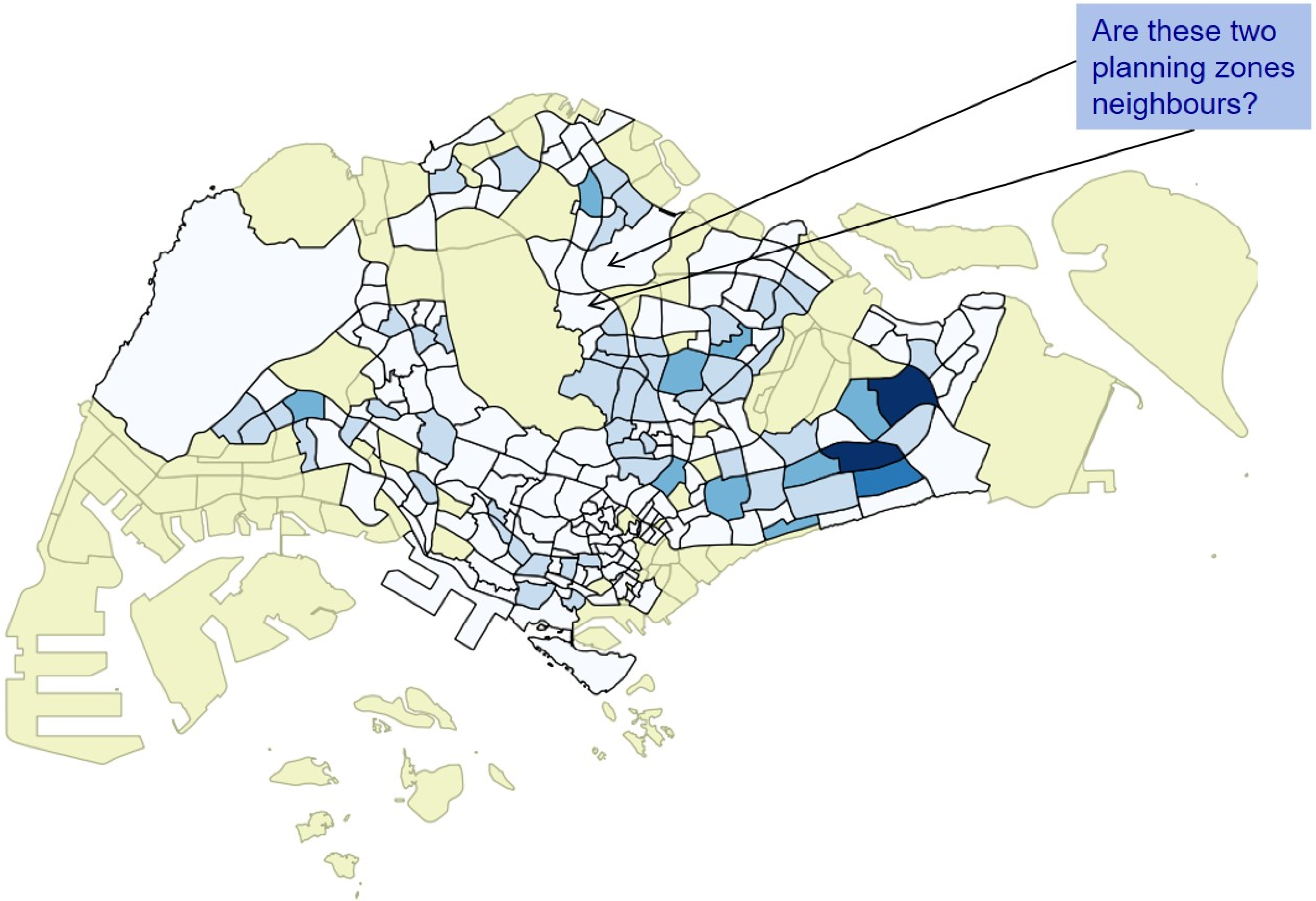


What are Spatial Weights *(wij)*

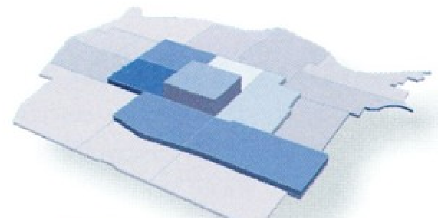
- A way to define spatial neighbourhood.



Defining Spatial Weights

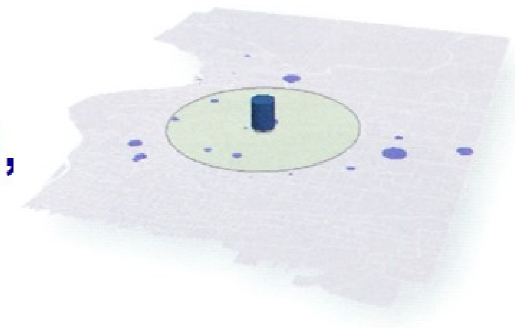
***Adjacency criterion*:**

$$w_{ij} = \begin{cases} 1 & \text{if location j is adjacent to i,} \\ 0 & \text{if location j is not adjacent to i.} \end{cases}$$


Adjacency

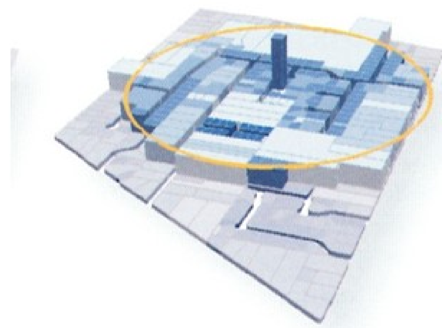
***Distance criterion*:**

$$w_{ij}(d) = \begin{cases} 1 & \text{if location j is within distance d from i,} \\ 0 & \text{otherwise.} \end{cases}$$


Distance: discrete features

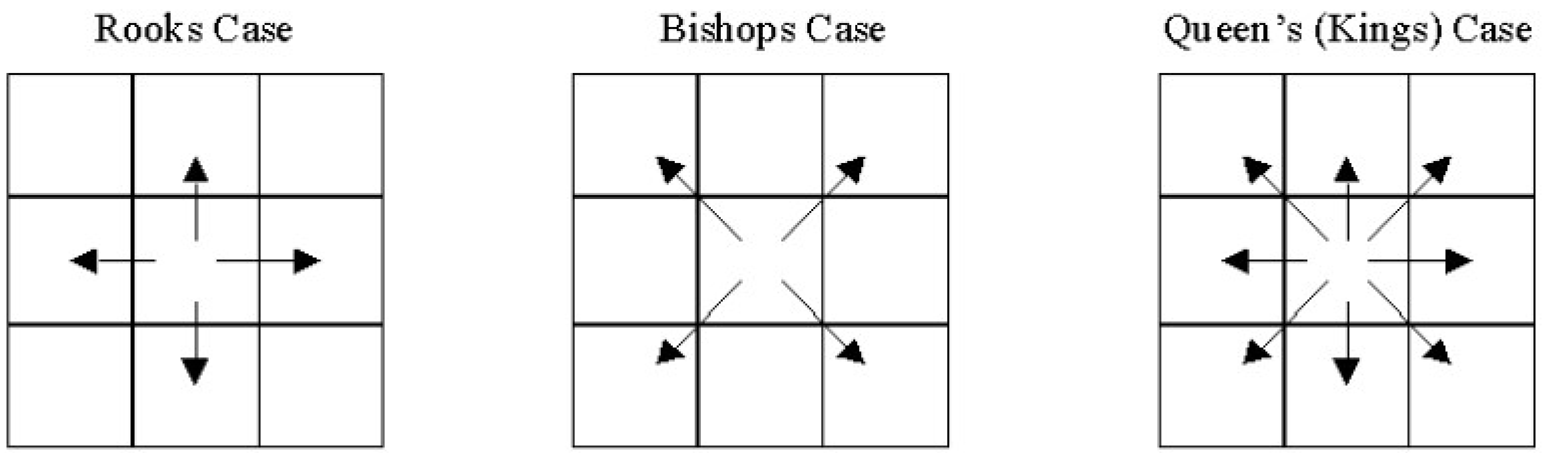
A general spatial distance weight matrices:

$$w_{ij}(\mathrm{d}) = d_{ij}^{-a} \cdot \beta^{b}$$


Distance: contiguous features

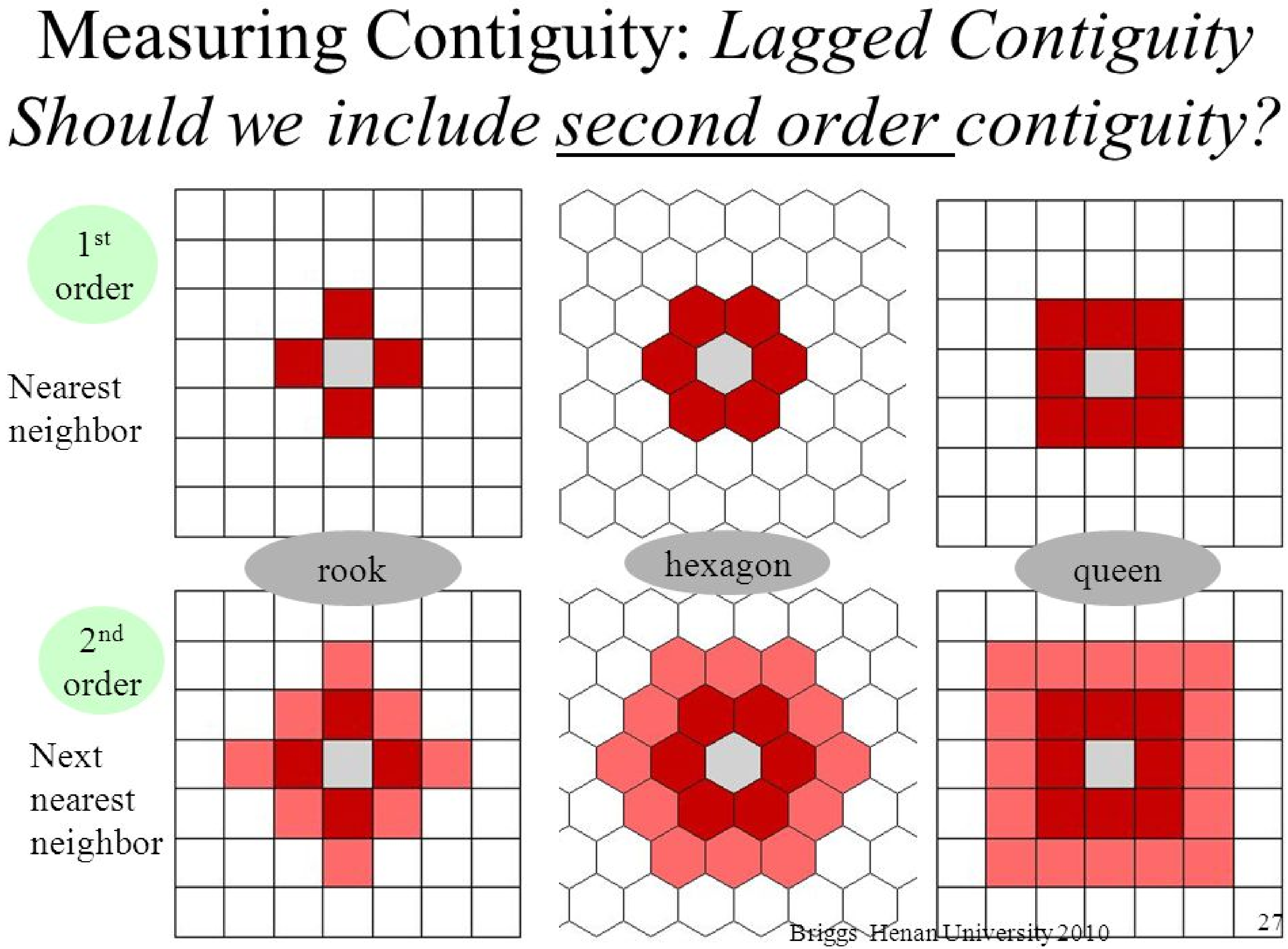
Contiguity Neighbours

- Contiguity (common boundary)
- What is a “shared” boundary?



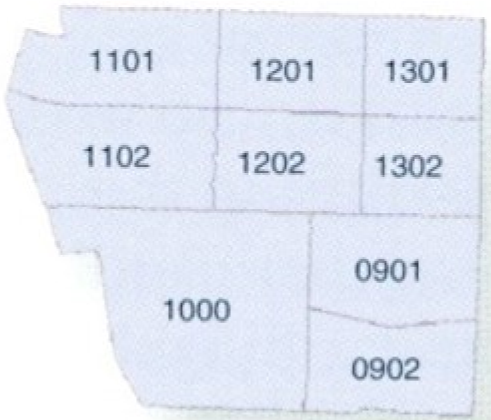
Beyond the basic contiguity neighbours

There are also second-order, third-order, forth-order, etc contiguity.



Weights matrix: Adjacency-based neighbours

Quiz: With reference to the figure below, list down the neighbour(s) of area 1202 using Rook case.

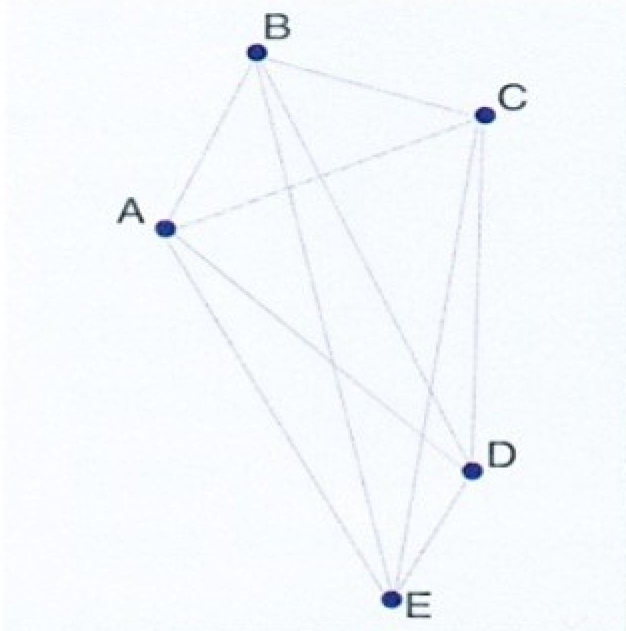


Weights matrix for an adjacency-based neighborhood

	1101	1201	1301	1102	1202	1302	1000	0901	0902
1101	0	1	0	1	1	0	0	0	0
1201	1	0	1	1	1	1	0	0	0
1301	0	1	0	0	1	1	0	0	0
1102	1	1	0	0	1	0	1	0	0
1202	1	1	1	1	0	1	1	1	0
1302	0	1	1	0	1	0	0	1	0
1000	0	0	0	1	1	0	0	1	1
0901	0	0	0	0	1	1	1	0	1
0902	0	0	0	0	0	0	1	1	0

Weights matrix: Distance-based neighbours

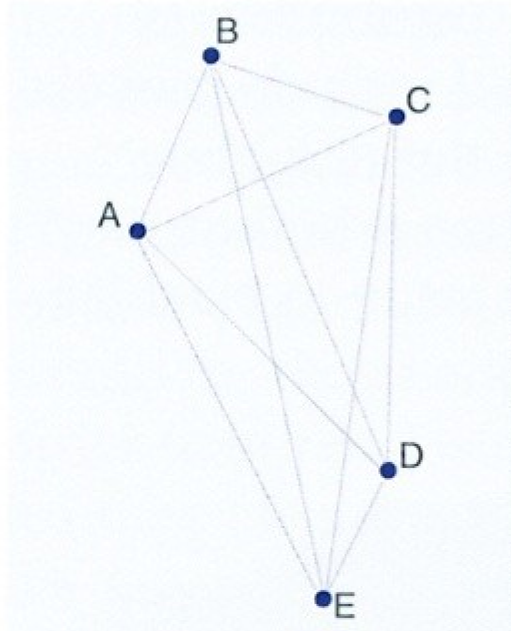
Quiz: With reference to the figure below, create a weights matrix for d = 650.



Weights matrix for a distance-based neighborhood

	A	B	C	D	E
A	0	353	516	641	757
B	353	0	357	837	1025
C	516	357	0	659	901
D	641	837	659	0	263
E	757	1025	901	263	0

Weights matrix: Measured distances



	A	B	C	D	E
A	0	353	516	641	757
B	353	0	357	837	1025
C	516	357	0	659	901
D	641				
E	757				

Weights matrix with measured distances

$$w_{ij} = \frac{1}{d_{ij}}$$

	A	B	C	D	E
A	0	0.00283	0.00194	0.00156	0.00132
B	0.00283	0	0.00280	0.00119	0.00098
C	0.00194	0.00280	0	0.00152	0.00111
D	0.00156	0.00119	0.00152	0	0.00380
E	0.00132	0.00098	0.00111	0.00380	0

Weights matrix with inverse distances

Row standardisation

In practice, row-standardised weights instead of spatial weights will be used.

Binary W matrix:

$$\tilde{W} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Row standardized W matrix:

$$W = \begin{bmatrix} 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \end{bmatrix}$$

Applications of Spatial Weights

Formally, for observation i, the spatial lag of yi, referred to as [Wy]i (the variable Wy observed for location i) is:

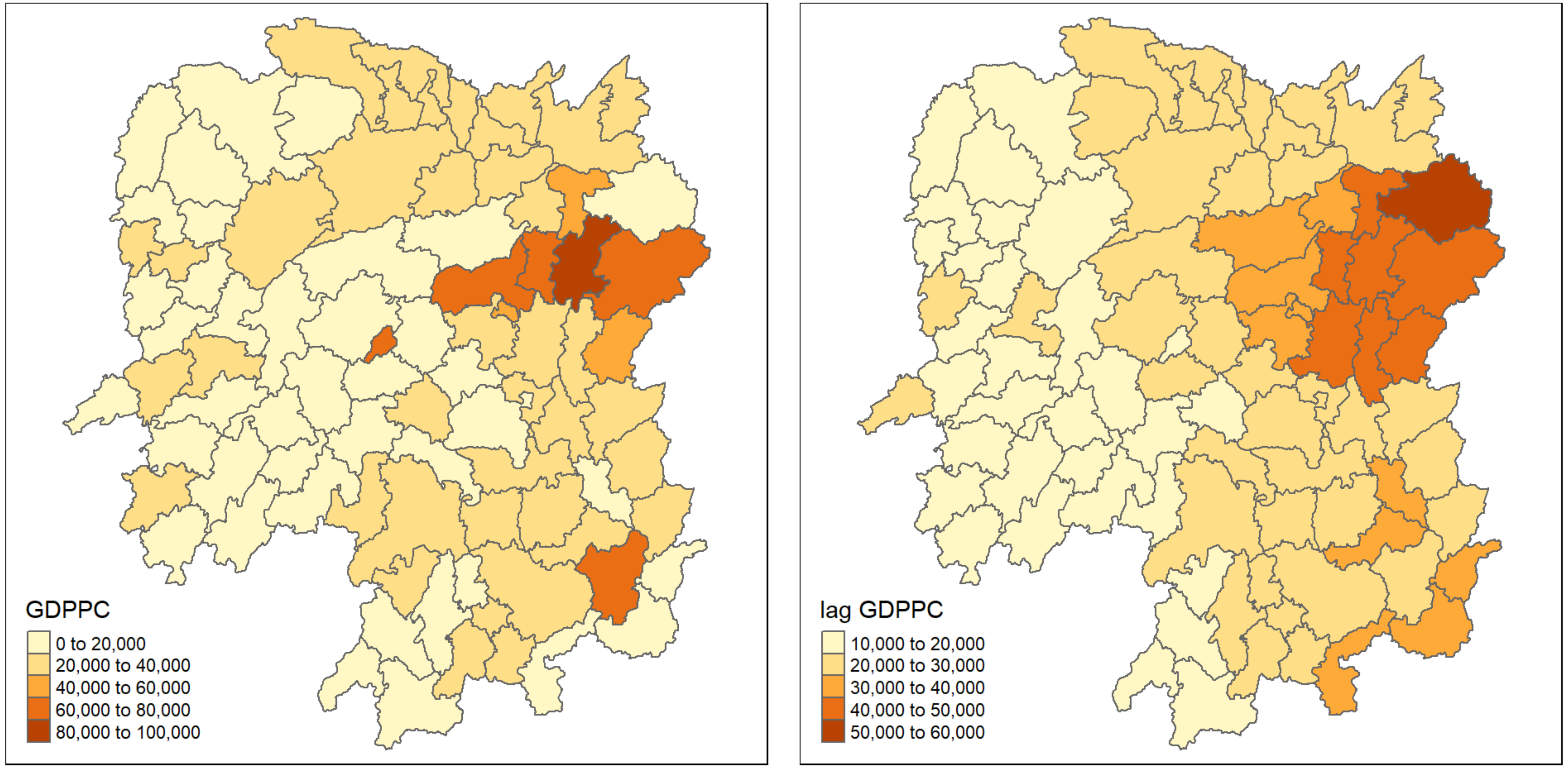
$$[Wy]_i = w_{i1}y_1 + w_{i2}y_2 + \cdots + w_{in}y_n,$$

$$[Wy]_i = \sum_{j=1}^{n} w_{ij}y_j,$$

where the weights wij consist of the elements of the i-th row of the matrix W, matched up with the corresponding elements of the vector y.

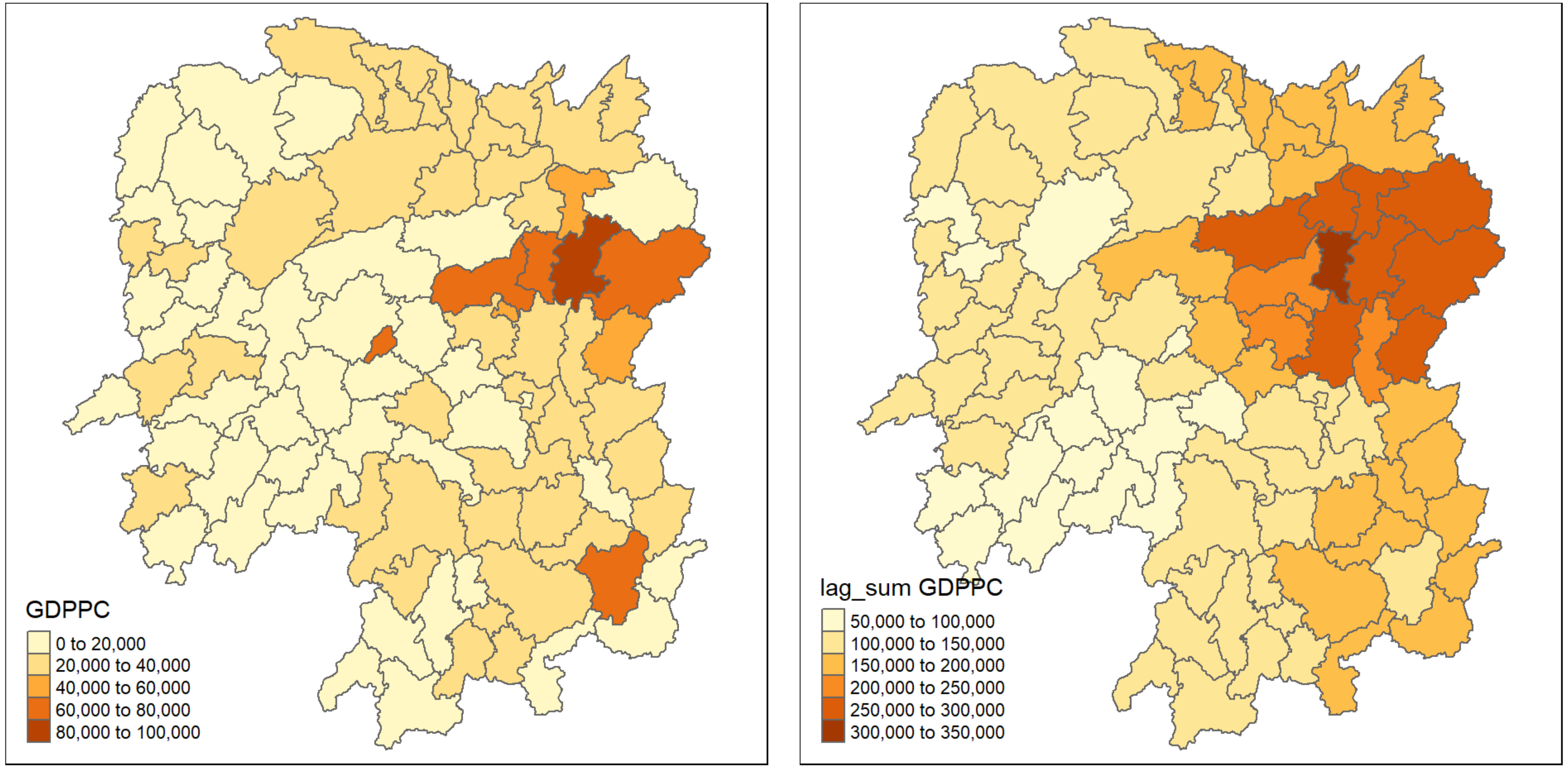
Spatially Lagged Variables

Spatial lag with row-standardized weights



Spatial window sum

The spatial window sum uses and includes the diagonal element.



References

- Chapter 2. Codifying the neighbourhood structure of Handbook of Spatial Analysis: Theory and Application with R.
- François Bavaud (2010) "Models for Spatial Weights: A Systematic Look" *Geographical Analysis*, Vol. 30, No.2, pp 153-171.
- Tony H. Grubesic and Andrea L. Rosso (2014) "The Use of Spatially Lagged Explanatory Variables for Modeling Neighborhood Amenities and Mobility in Older Adults", *Cityscape*, Vol. 16, No. 2, pp. 205-214.