Migration, pop size $M^{(k)}, G^{(k)}, N^{(k)}$

on $t_{k-1} \le t < t_k$.

$t_0 = 0$
let $S_k = t_k - t_{k-1}$.

1) if $t < t_1$, then

$$r_{xy}(t) = \sum_z \left(e^{tG^{(1)}}\right)_{xy,zz} \frac{1}{N_z^{(1)}}$$

2) if $t_1 \le t < t_2$.

$$r_{xy}(t) = \sum_{uv} P^{(1)}_{xy,uv} \left(e^{(t-t_1)G^{(2)}}\right)_{uv,zz} \frac{1}{N_z^{(2)}}$$

$$P^{(1)}_{xy,uv} = \left(e^{S_1 G^{(1)}}\right)_{xy,uv}$$

3) if $t_{k-1} \le t < t_k$

$$r_{xy}(t) = \sum_{uv} P^{(k)}_{xy,uv} \left(e^{(t-t_{k-1})G^{(k)}}\right)_{uv,zz} \frac{1}{N_z^{(k)}}$$

$$P^{(k)}_{xy,uv} = \left(e^{S_1 G^{(1)}} e^{S_2 G^{(2)}} \cdots e^{S_{k-1} G^{(k-1)}}\right)_{xy,uv}$$

Let $q_{xy}(t) = \sum_{uv} \left(P^{(k)} e^{(t-t_{k-1})G^{(k)}}\right)_{xy,uv}$

$t_{k-1} \le t < t_k$

$$r_{xy}(t) = \sum_z \left(P^{(k)} e^{(t-t_{k-1})G^{(k)}}\right)_{xy,zz} \frac{1}{N_z^{(k)}}$$

For $k_x$ samples in loc $x$, ...

the coal rate at time $t$ is

$$r(t) = \frac{\sum_{x=1}^{n} \sum_{y=1}^{n} k_x (k_y - \delta_{xy}) r_{xy}(t)}{\sum_{x=1}^{n} \sum_{y=1}^{n} k_x (k_y - \delta_{xy}) q_{xy}(t)}$$

$$\delta_{xy} = \begin{cases} 1 & \text{if } x = y \\ 0 & \text{o.w.} \end{cases}$$

$$\begin{array}{cccc} 1 & 2 & 3 & 4 \\ \hline 1 & & & \\ & 6 & & \\ & & 11 & \\ & & & 16 \end{array}$$

$N = (1, 5, 10)$

$$C = \begin{bmatrix} \frac{1}{2} & \cdots & & \\ \vdots & \ddots & & \\ & & \frac{1}{10} & \\ & & & \ddots \\ & & & \frac{1}{20} \end{bmatrix}$$

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 2 & 3 \\ 2 & 4 & 5 & 6 \\ 3 & 7 & 8 & 9 \end{array}$$