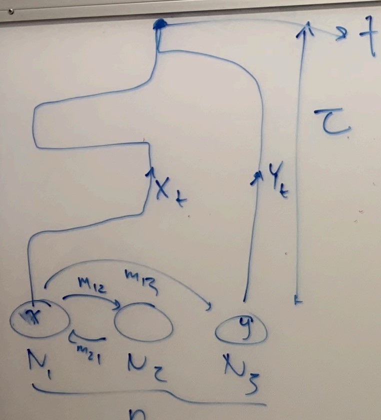

goal: coal rate for lineages started at (x,y) at time t ago

$$r_{xy}(t) = \sum_{\substack{z \\ \text{locations}}} \underbrace{\mathbb{P}\{\text{both } X_t = Y_t = z\}} \frac{1}{2N_z}$$

maybe

diploid

Let $G = M \otimes I + I \otimes M - C\left(\frac{1}{2N}\right)$

$\otimes$: Kronecker

the matrix

$C_{zz,zz} = \frac{1}{2N_z}$

$= 0$ otherwise

$n \times n \begin{cases} M_{xy} = \text{rate of lineage migr from } x \text{ to } y, \ x \neq y \\ M_{xx} = -\sum_{y \neq x} M_{xy} \end{cases}$

$$\underbrace{G_{xy,uv}}_{n^2 \times n^2} = \begin{cases} M_{xu} & \text{if } y = v \\ M_{yv} & \text{if } x = u \\ \underbrace{-\sum_z M_{xz} - \sum_z M_{yz}} & \text{if } x = u,\ y = v,\ x \neq y \\ (\text{same}) - \frac{1}{2N_z} & \text{if } x = u = y = v \\ 0 & \text{otherwise} \end{cases}$$

$$A \otimes B = \begin{bmatrix} A_{11}B & A_{12}B & A_{13}B \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & A_{33}B \end{bmatrix}$$

$$e^{tG} = \sum_{n \geq 0} \frac{t^n G^n}{n!}$$

$$r_{xy}(t) = \sum_z \left(e^{tG}\right)_{xy,zz} \frac{1}{2N_z}$$

$$\mathbb{P}\{X_t = u, Y_t = v \mid X_0 = x, Y_0 = y\} = \left(e^{tG}\right)_{xy,uv}$$

Let $f_{xy}(t) = \mathbb{P}\{\text{haven't yet coalesced} \mid X_0 = x, Y_0 = y\}$

$$= \sum_{uv} \left(e^{tG}\right)_{xy,uv}$$