hw05

September 21, 2022

1 hw05

1.1 Metadata

```
Name:    hw05
URL:     https://github.com/tslever/DS5100-2022-08-tsl2b/blob/main/lessons/M05/hw05.ipynb
Course:  DS 5100
Term:    Fall 2022 Online
Module:  M05: numpy
Topic:   Capital Asset Pricing Model (CAPM)
Author:  Tom Lever
Net ID:  tsl2b
Created: 20 September 2022
Updated: 21 September 2022
```

1.2 Overview

In finance, a capital asset pricing model (CAPM) is a single-factor regression model used to explain and predict excess returns for a stock i.

There are better, more accurate models, but a CAPM has its uses.

For example, the market beta β_i is a parameter of a CAPM.

Here is the formula for calculating the expected excess return for stock i.

$$E\left[R_i\right] - R_f = \beta_i \left(E\left[R_m\right] - R_f\right)$$

where

- $E\left[R_i\right]$ is the expected excess return of stock i
- R_f is the risk-free treasury rate
- β_i is the market beta of stock i
- $E\left[R_m\right] - R_f$ is the market-risk premium

1.3 Setting Up

Import numpy.

[1]:
```
import numpy as np
```

Define Risk-free treasury rate.

[1]:
```python
R_f: float = 0.0175 / 252.0
```

1.4 Preparing the Data

We import CAPM market data and convert the data into usable numpy arrays.

1.4.1 Read in the Market Data

The values are closing prices, adjusted for splits and dividends.

The prefixes of the second two columns are based on the following codes:

- SPY is an Exchange-Traded Fund (ETF) for the S&P 500, a stock market index tracking the stock performance of 500 large companies listed on exchanges in the United States.
- AAPL stands for Apple.

[3]:
```python
import os
path: str = None
if os.name == 'posix':
    path = "~/Documents/DS5100-2022-08-O/lessons/M05_NumPy/HW/capm_market_data.
  ↪csv"
elif os.name == 'nt':
    path = os.environ['USERPROFILE'] + "/Documents/DS5100-2022-08-O/lessons/
  ↪M05_NumPy/HW/capm_market_data.csv"
list_of_file_lines: list[str] = None
with open(path, 'r') as file:
    list_of_file_lines = file.readlines()
print(type(list_of_file_lines))
```

```
<class 'list'>
```

1.4.2 Create numpy Array of Column Names

[4]:
```python
!pip install nptyping
```

```
Requirement already satisfied: nptyping in c:\users\tom\anaconda3\lib\site-
packages (2.3.1)
Requirement already satisfied: typing-extensions<5.0.0,>=4.0.0 in
c:\users\tom\anaconda3\lib\site-packages (from nptyping) (4.1.1)
Requirement already satisfied: numpy<2.0.0,>=1.20.0 in
c:\users\tom\appdata\roaming\python\python39\site-packages (from nptyping)
(1.21.4)
```

[5]:
```python
from nptyping import Float64, NDArray, Shape, String
list_of_column_names: list[str] = list_of_file_lines[0].strip().split(',')
numpy_array_of_column_names: NDArray[Shape['3'], String] = np.
  ↪array(list_of_column_names)
print(numpy_array_of_column_names)
```

```
['date' 'spy_adj_close' 'aapl_adj_close']
```

1.4.3 Create numpy Arrays of Dates and Returns

[6]:
```python
list_of_data_lines: list[str] = [line.strip().split(',') for line in␣
↪list_of_file_lines[1:]]
print(len(list_of_data_lines))
numpy_array_of_data: NDArray[Shape['135, 3'], String] = np.
↪array(list_of_data_lines)
numpy_array_of_dates: NDArray[Shape['135'], String] = numpy_array_of_data[:, 0]
numpy_array_of_returns: NDArray[Shape['135, 2'], Float64] =␣
↪numpy_array_of_data[:, 1:3].astype('float')
print(numpy_array_of_dates[0:3])
print(numpy_array_of_returns[0:3, :])
```

```
135
['2020-01-02' '2020-01-03' '2020-01-06']
[[321.55578613 298.82995605]
 [319.12091064 295.92471313]
 [320.33837891 298.28271484]]
```

1.5 Tasks

1.5.1 Task 1

(1 point)

Print the first 5 rows of the numpy array of returns.

[7]:
```python
print(numpy_array_of_returns[0:5, :])
```

```
[[321.55578613 298.82995605]
 [319.12091064 295.92471313]
 [320.33837891 298.28271484]
 [319.43765259 296.87988281]
 [321.1401062  301.6555481 ]]
```

1.5.2 Task 2

(1 point)

Print the first five values from the SPY column in the numpy array of returns.

Then do the same for the AAPL column.

Use one cell for each operation.

[8]:
```python
numpy_array_of_stock_names: NDArray[Shape['2'], String] =␣
↪numpy_array_of_column_names[np.where(numpy_array_of_column_names != 'date')]
index_of_SPY_column: int = numpy_array_of_stock_names.tolist().
↪index('spy_adj_close')
```

```
SPY_returns: NDArray[Shape['135'], Float64] = numpy_array_of_returns[:,␣
↪index_of_SPY_column]
_ = [print(return_) for return_ in SPY_returns[0:5].tolist()]
```

```
321.555786132812
319.120910644531
320.33837890625
319.437652587891
321.140106201172
```

[9]:
```
index_of_AAPL_column: int = numpy_array_of_stock_names.tolist().
↪index('aapl_adj_close')
AAPL_returns: NDArray[Shape['135'], Float64] = numpy_array_of_returns[:,␣
↪index_of_AAPL_column]
_ = [print(return_) for return_ in AAPL_returns[0:5].tolist()]
```

```
298.829956054687
295.924713134766
298.28271484375
296.8798828125
301.655548095703
```

1.5.3 Task 3

(1 point)

Compute the excess returns by subtracting the constant R_f from the numpy array of returns.

Save the results as a two-dimensional numpy array named EXCESS.

Print the last five rows from EXCESS.

[10]:
```
EXCESS = numpy_array_of_returns - R_f
print(EXCESS[-5:, :])
```

```
[[314.37993544 383.00994032]
 [317.58992689 383.67992323]
 [314.83992689 381.90993422]
 [318.91994398 388.22994154]
 [321.84993666 390.89992445]]
```

1.5.4 Task 4

(1 point)

Make a simple scatterplot using Matplotlib with SPY excess returns on the x-axis, and AAPL excess returns on the y-axis.

Hint: Use the following code:

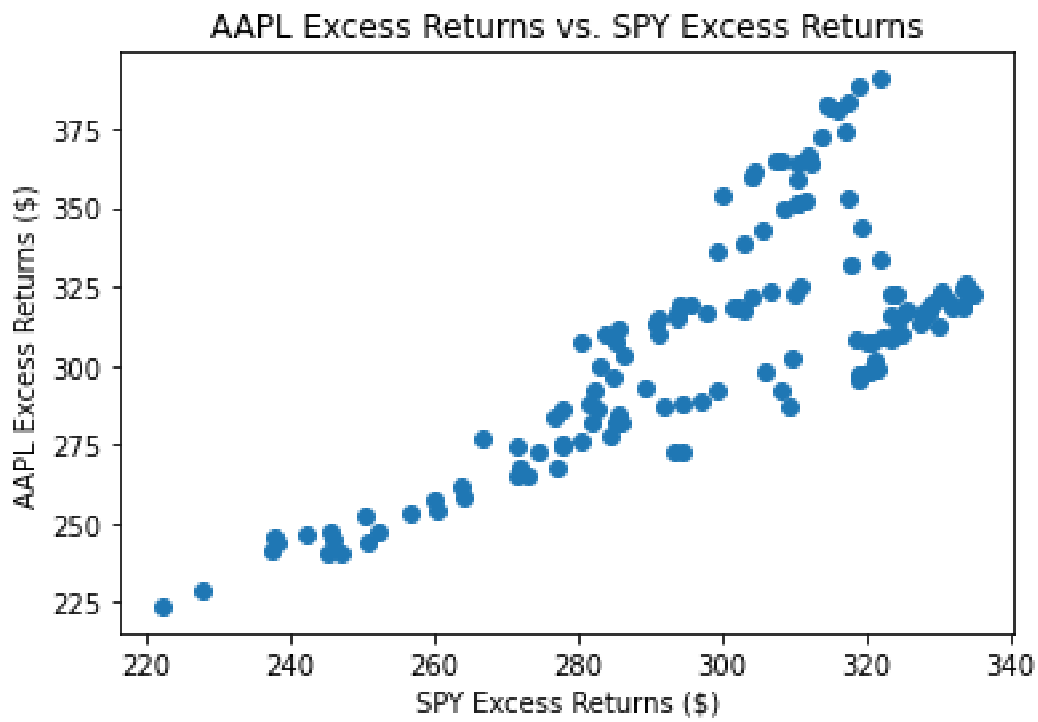
```
from matplotlib.pyplot import scatter
```

```
scatter(<x>, <y>)
```

Replace `<x>` and `<y>` with the appropriate vectors.

You may want to save the vectors for the SPY and AAPL columns as `x` and `y` respectively. This will make it visually easier to perform Task 6.

[11]:
```python
SPY_excess_returns: NDArray[Shape['135'], Float64] = EXCESS[:, index_of_SPY_column]
AAPL_excess_returns: NDArray[Shape['135'], Float64] = EXCESS[:, index_of_AAPL_column]
from matplotlib import pyplot as plt
plt.scatter(SPY_excess_returns, AAPL_excess_returns)
plt.title("AAPL Excess Returns vs. SPY Excess Returns")
plt.xlabel("SPY Excess Returns ($)")
plt.ylabel("AAPL Excess Returns ($)")
plt.show()
```



1.5.5 Taks 5

(3 points)

Use the **normal equation**, listed below, to compute the Regression Coefficient Estimate of the data plotted above, $\hat{\beta}_i$.

Note that x^T denotes the transpose of x.

$\hat{\beta}_i = (x^T x)^{-1} x^T y$

Use the Numpy functions for matrix to do this — multiplication, transpose, and inverse.

Note, however, that since x in this case a single column matrix, i.e. a vector, the result of $x'x$ will be a scalar, which is not invertible. So you can just invert the result by division, i.e.

$\hat{\beta}_i = \frac{1}{x^T x}(x^T y)$

Be sure to review what these operations do, and how they work, if you're a bit rusty.

[12]:
```python
def calculate_regression_coefficient_estimate(x: NDArray[Shape['135'],␣
↪Float64], y: NDArray[Shape['135'], Float64]) -> float:
    regression_coefficient_estimate = 1 / np.matmul(np.transpose(x), x) * np.
↪matmul(np.transpose(x), y)
    return regression_coefficient_estimate

calculate_regression_coefficient_estimate(SPY_excess_returns,␣
↪AAPL_excess_returns)
```

[12]: 1.029980294240815

1.5.6 Task 6

(3 points)

Measuring Beta Sensitivity to Dropping Observations (Jackknifing)

Let's understand how sensitive the market beta is to each data point.
We want to drop each data point (one at a time), compute $\hat{\beta}_i$ using our formula from above, and save each measurement.

Write a function called `beta_sensitivity()` with these specs:

- Take numpy arrays x and y as inputs.
- For each observation i, compute the beta without the current observation. You can use a `lambda` function for this.
- Return a list of tuples each containing the observation row dropped and the beta estimate, i.e. something like `(i, beta_est)`, depending how you've named your variables.

Hint: `np.delete(x, i)` will delete observation i from array x.

Call `beta_sensitivity()` and print the first five tuples of output.

[13]:
```python
def beta_sensitivity(x: NDArray[Shape['135'], Float64], y:␣
↪NDArray[Shape['135'], Float64]) -> list[tuple[int, float]]:
    indices_of_removed_observation_and_redacted_market_beta_estimates:␣
↪list[tuple[int, float]] = []
    for index_of_removed_observation in range(0, len(x)):
        redacted_x: NDArray[Shape['134'], Float64] = np.delete(x,␣
↪index_of_removed_observation)
        redacted_y: NDArray[Shape['134'], Float64] = np.delete(y,␣
↪index_of_removed_observation)
```

```
        redacted_market_beta_estimate: float =␣
↪calculate_regression_coefficient_estimate(redacted_x, redacted_y)
        index_of_removed_observation_and_redacted_market_beta_estimate:␣
↪tuple[int, float] = (index_of_removed_observation,␣
↪redacted_market_beta_estimate)
        indices_of_removed_observation_and_redacted_market_beta_estimates.
↪append(index_of_removed_observation_and_redacted_market_beta_estimate)
    return indices_of_removed_observation_and_redacted_market_beta_estimates
```

[14]:

```
indices_of_removed_observations_and_market_beta_estimates: 'np.
↪ndarray[tuple[int], np.dtype[float]]' = beta_sensitivity(SPY_excess_returns,␣
↪AAPL_excess_returns)
print(indices_of_removed_observations_and_market_beta_estimates[0:5])
```

```
[(0, 1.030847730172396), (1, 1.0308516176393125), (2, 1.0308255236222597), (3,
1.0308357542837525), (4, 1.030759501843587)]
```

[ ]:

```
```