**Research Question regarding Conditions that Guarantee no FPSBS**

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**Research Question**

Fourier Peak Shortening, Broadening, and Shifting (FPSBS) complicates determining prominent frequencies in a periodic time series. For example, FPSBS interferes with determining the frequency of A440 in a recorded audio signal. In the case where feature frequencies are sufficiently similar and FPSBS is sufficiently significant, only one feature is recognized. In the case where Real Fourier Spectra are aggregated into a boosted signal, FPSBS introduces error into the signal.

What conditions guarantee no Fourier Peak Shortening, Broadening, and Shifting?

**Frequency of Audio Signal Recorded in a Column Vector**

Suppose a continuous audio signal with a signal frequency of (i.e., the pitch A440) is sampled at a sampling frequency of and sampling period for time , generating a time series of data. The time series is recorded in a column vector

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The Discrete Fourier Transform Matrix for performing a Fourier Transform of the vector is given by

Question: Is the DFTM always square, regardless of length of the times series?

The Fourier Transform of the vector involves generating a column vector Complex Fourier Spectrum of

From the Complex Fourier Spectrum may be generated a Real Two-Sided Spectrum of of Half Fourier Powers.

From the Real Two-Sided Spectrum may be generated a Real Single-Sided Spectrum of composed of Full Fourier Powers.

Question: This definition of RSSS is derived the MATLAB FFT usage documentation at <https://www.mathworks.com/help/matlab/ref/fft.html>. Does R**T**SS, calculated using the DFTM, truly represent first the offset Half Power, then the Half Fourier Powers ordered from lowest-magnitude positive frequency to the highest-magnitude positive frequency, then the Half Fourier Powers ordered from the lowest-magnitude negative frequency to the highest-magnitude negative frequency?

A column vector of Positive Frequencies is given by

The frequency of the original audio signal may be extracted by finding the index of the maximum value in the Real Single-Sided Spectrum of and finding the value at that index in the Positive Frequencies.

A graph of Full Fourier Power versus Positive Frequency corresponding to is presented in Figure 1. The frequency corresponding to the maximum power of is .

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| **Figure 1:**  **Graph of Fourier Power vs. Frequency**  **corresponding to with**  **No Fourier Peak Shortening, Broading, or Shifting** |

**Fourier Peak Shortening, Broadening, and Shifting**

Suppose we wanted to find the pitch of one note with a very short duration in our audio signal. Suppose specifically that we wanted to find the frequency associated with a vector representing the first data points in our times series . Our graph of Full Fourier Power versus Frequency corresponding to is presented in Figure 2. The frequency corresponding to the maximum Full Frequency Power of is .

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| **Figure 2:**  **Graph Fourier Power vs. Frequency**  **corresponding to with**  **Fourier Peak Shortening, Broading, and Shifting** |

**Conditions Guaranteeing No Fourier Peak Shortening, Broadening, or Shifting**

What conditions guarantee no Fourier Peak Shortening, Broadening, and Shifting?