Gibbs Sampling

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Gibbs Sampling

- Gibbs sampling is a commonly used and highly flexible MCMC technique
- Widely used for sampling from multidimensional distributions
- Suppose the target is $p(\mathbf{x}) = p(x_1, \ldots, x_M)$, and we can also use it as a proposal distribution
- Initialize $\mathbf{x}^{(\tau)}$
- Each step replaces one of the variables, say x_i, with a draw from the distribution with that variable removed $p(x_i|x_1, \ldots, x_{i-1}, x_{i_+1}, \ldots, x_M)$
- Cycle through the variables or randomly select the update variable

Gibbs Sampling Algorithm

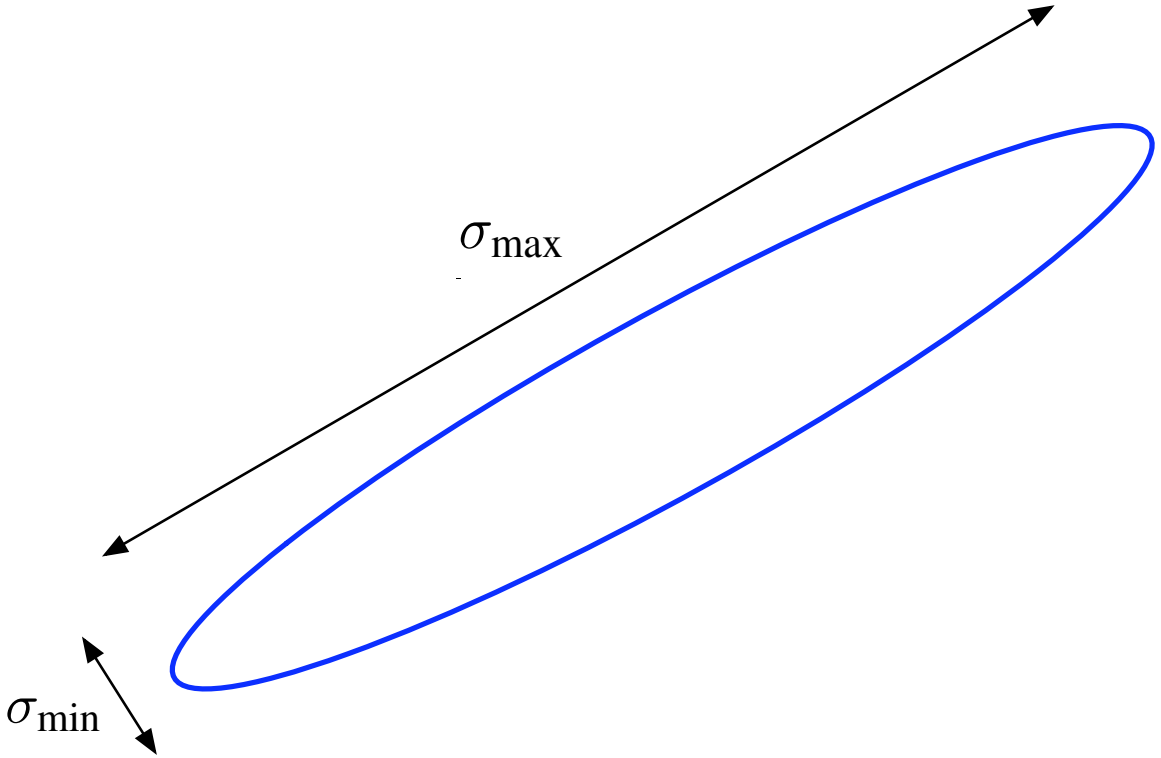
1. Initialize $\mathbf{x} = (x_1, \ldots, x_M)$
2. For $t = 1, \ldots, T$
   - For $m = 1, \ldots, M$
     - Sample
       $x_m^{(t)} \sim p(x_m | \{x_i^{(t)}, 1 \leq i \leq m-1\}, \{x_i^{(t-1)}, m+1 \leq i \leq M\})$

Markov Chain

- When we sample from $p(x_i|x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_M)$, the distribution for $p(x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_M)$ is invariant

  since $(x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_M)$ does not change
- This means the distribution $p(\mathbf{x})$ is invariant for each of the Gibbs sampling steps
- Also each step samples from the conditional, $p(x_i|x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_M)$ and that combined with the marginal, $p(x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_M)$, gives us the correct joint distribution $p(\mathbf{x})$
- Gibbs sampling is a special case of the Metropolis-Hastings algorithm where acceptance is assured

Example



Gibbs Sampling Comments

- Used for many multidimensional Bayesian models
- *Blocking Gibbs sampling*: Rather than one variable at a time, draw samples of sets of variables
- Usefulness of Gibbs sampling depends on the ease of sampling from $p(x_i|x_1,\ldots,x_{i-1},x_{i+1},\ldots,x_M)$