



Mixture Models

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Overview

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- Mixture models provide a general framework for data analysis applicable to a range of problems
- Expectation maximization is an algorithm that makes mixture modeling computationally feasible



Density Estimation

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- A mixture model density combines a set of component models:

$$p(\mathbf{x}|\boldsymbol{\theta}) = \sum_{k=1}^K p(\mathbf{x}|\boldsymbol{\theta}_k, k)p(k)$$

- The data in the variables in \mathbf{x} are ‘visible’ or ‘observable’
- $\boldsymbol{\theta}$ is the vector of parameters, not visible or latent; in general, we can estimate with Bayes rule, but now they are a function of k
- $k = 1, \dots, K$ indexes each component model $p(\mathbf{x}|\boldsymbol{\theta}, k)$, along with its weight $p(k)$ and $\sum_{k=1}^K p(k) = 1$ and; these are also latent
- Mixture models have a natural interpretation in terms of clustering with each k , corresponding to a cluster model $p(\mathbf{x}|\boldsymbol{\theta}_k, k)$.



Univariate Mixture

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- Suppose a population consists of observations from two Gaussians, $N(\mu_i, \sigma_i)$, $i = 1, 2$.
- Let

$$\phi(\mu_1, \sigma_1) = \frac{1}{\sigma_1 \sqrt{2\pi}} \exp \left[-\frac{(x - \mu_1)^2}{2\sigma_1^2} \right]$$

$$\phi(\mu_2, \sigma_2) = \frac{1}{\sigma_2 \sqrt{2\pi}} \exp \left[-\frac{(x - \mu_2)^2}{2\sigma_2^2} \right]$$

- Let π_1 be the probability that an observation comes from distribution 1.
- The mixture density is

$$f(x) = \pi_1 \phi(\mu_1, \sigma_1) + (1 - \pi_1) \phi(\mu_2, \sigma_2)$$



Univariate Mixture Model Fitting

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- The parameters are $\theta = \{\pi_1, \mu_1, \mu_2, \sigma_1, \sigma_2\}$
- Log-likelihood is

$$\ell(\theta|x) = \sum_{i=1}^N \log[\pi_1 \phi(\mu_1, \sigma_1) + (1 - \pi_1) \phi(\mu_2, \sigma_2)]$$

- Cannot easily maximize this
- So we iteratively maximize it
 - Start with guessed values for the parameters and calculate the probability of membership in the Gaussians
 - Then calculate the new estimates for the parameters and repeat until the change in values is small

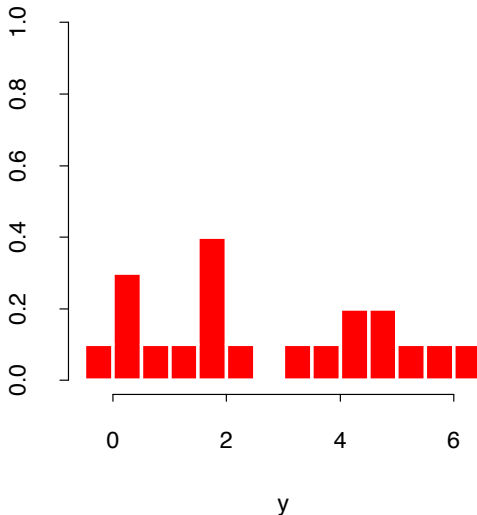


Mixture: Univariate Example

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Mixture Example: Results

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