The Price of Human Nature

LILLIAN TSAI, MIT, USA
VIBHAALAKSHMI SIVARAMAN, MIT, USA
XIAOYUE GONG, MIT, USA

The impact of selfish actions on the latency incurred by a network users has historically been a problem of great interest to the algorithmic community. Our paper first presents a brief overview of the original selfish routing problem, the standard algorithms used to route in the selfish routing setting, and the limits on the optimality of these algorithms (termed as the "price of anarchy"), and then compare and contrasts other recent works that have formulated the problem in (perhaps more realistic) settings, i.e., by taking into account the possibility for altruistic, risk averse, and diverse-interest behaviors. This paper both presents and clarifies the findings of these papers in the context of the original selfish routing paper, and demonstrates how these papers' results can be synthesized into a more general framework addressing optimality of routing with various human behaviors or motivations.

Additional Key Words and Phrases: selfish routing, price of anarchy

1 INTRODUCTION

Finding the best strategy for network and traffic routing has historically been a problem of great importance combining the theoretical aspects of both game theory and computer science.

- Introduce noncooperative games / Nash equilibrium (in network setting)
- Briefly introduce selfish routing paper / price of anarchy
- Discuss how it is a limited view of human behavior
- Briefly discuss alternatives (papers including taxes, different objectives, atomic games, studies on network structure)
- Us: There are newer papers with more interesting versions of human behavior, and we present a survey of these newer papers to show how human behaviors affect price of anarchy

2 BACKGROUND

This section presents a brief history of traffic routing algorithms, defining the terminology and the traffic routing problem.

2.1 The Traffic Routing Problem

Traffic routing problems naturally arise in communication or transportation networks, where users are trying to minimize the latency that they or their data experiences. However, links in the network often becomes *congested* if too many users decide to route their data or cars through that link. Consequently, in these networks, the path each user chooses can affect the travel times of other users. Here, we describe Roughgarden and Tardos' formalization of the problem of minimizing latency as multicommodity flow networks [4, 5], and use Pigou's example network in Figure 1 as a running example. We use this to reason about the deviation from the opimum minimal latency when users make selfish routing decisions.

The input to a traffic routing problem consists of:

- A network $G = (V, E)$ of $|V|$ destinations (e.g., locations or servers) and $|E|$ links
- A set of k source-destination pairs $S = \{(s_1, t_1), \cdots (s_k, t_k)\}$ representing traffic demands
- A rate r_i of traffic for each $(s_i, t_i) \in S$ representing the demanded amount of traffic from s_i to t_i
- A latency cost function c that assigns a per-edge function c_e to each edge e describing how adding traffic (i.e., congestion) to e affects the time taken to travel across e. We can also think of c as assigning per-path

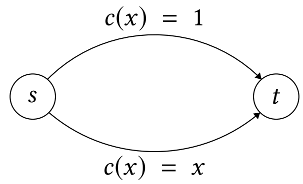


Fig. 1. Pigou's example traffic routing problem, with a demand of $r_{(s,t)} = 1$

costs: for any path p in the graph

$$c_p(f) = \sum_{e \in P} c_e(f_e)$$

We assume that c is continuous, nonnegative, and nondecreasing.

In Figure 1, we see a single source-destination input network with an example (linear) cost function with $r_{(s,t)} = 1$.

Solutions correspond to flow assignments to the set of simple paths P_i between s_i and t_i for all i. To describe a flow assignment f, we can consider f_p, the flow on a single path $p \in P_i$ (this adds an equal amount of flow f_p to all edges in p), as well as $f_e = \sum_p \sum_{e \in p} f_p$, the flow on edge e (the sum of flow on all paths that use e).

A *feasible* solution given such an input is an assignment of path flows such that the demand from s_i to t_i is met:

$$\forall 1 \leq i \leq k, \sum_{p \in P_i} f_p = r_i$$

An *optimal* (feasible) solution given such an input is the feasible flow assignment f that minimizes the **total weighted cost** $C(f)$, where

$$C(f) = \sum_i \sum_{p \in P_i} c_p(f) f_p = \sum_{e \in E} c_e(f_e) \cdot f_e$$

Intuitively, we are calculating the cost of each path of a given flow assignment, weighing each path's cost proportional to the amount of flow through the path. More concretely, if we were to let flow represent the routes chosen by (infinitely many) users, $C(f)$ calculates the average cost over all users. Thus, when minimizing $C(f)$, some users may incur more latency so that other users can go faster: the optimal flow is the *socially optimal* solution. Note that there exists an optimal flow f^* minimizing $C(f)$ because we assume c is continuous and the set of feasible flows is compact.

In our running example (Figure 1), a feasible flow is any flow that sends one unit from s to t (divided in any fashion between the top and bottom edges). The optimal flow is the flow that sends half the traffic through the lower edge and half through the upper edge: the users on the lower edge only experience a cost of $1/2$, while the users on the upper edge experience a cost of 1, making the total weighted cost $3/4$.

2.2 Coordination Models and the Price of Anarchy

Before we can create algorithms to solve the traffic routing problem, we must first assume a *coordination model* for our traffic network. There are two clear extremes: (1) centralized control, in which some entity (e.g., an air traffic controller) knows all traffic demands and routes accordingly, and (2) decentralization, i.e., a complete *lack* of coordination between entities in the network. In a centralized setting, there is a clear optimal solution, as shown in the previous section. However, in a decentralized and uncoordinated model, the lack of coordination and the exercise of freewill can result in inefficiencies.

Note that our solutions assume *nonatomic* entities: the flows we find may not be integral. Intuitively, this means that the demand from one s_i to t_i is generated by an infinite number of entities in the network, which allows us to reason about continuous, rather than discrete, functions.

The Price of Anarchy (PoA) allows us to measure the inefficiencies of a decentralized model given some notion of equilibrium (how the flow assignment is determined in the model), and was first introduced by Koutsoupias and Papadimitriou in 1999 [3]. The PoA is defined as the ratio between the optimal flow and the flow achieved at equilibrium. (This is similar to how we measured the distance from optimal of an approximation algorithm in a limited computational power model, and of online algorithms in an incomplete information model.)

2.3 The Selfish Routing Model

One example of an uncoordinated model is the *selfish routing* model, in which all entities in the network are selfish and choose a route minimizing their individual latency without caring (or knowing) about the effects on other users [5].

The selfish routing model corresponds to flows at a *Wardrop, or Nash equilibrium* [2, 6]. The set of flows in a Wardrop equilibrium are defined such that for all i source-destination pairs, all the paths from $s_i \longrightarrow t_i$ have the minimum-possible cost. In other words, the (nonzero) flow paths at Wardrop equilibrium have equal path costs:

$$\forall 0 \leq i \leq k, \ \forall p_1, p_2 \in P_i \ s.t. \ f_{p_1} > 0 \text{ and} f_{p_2} > 0, \ c_{p_1}(f) = c_{p_2}(f)$$

If we revisit our running example in Figure 1, we note that the flow at Wardrop equilibrium corresponds to a flow that sends the entire unit of traffic through the bottom edge (the 0 flow through the top path has a cost of 1, and the unit flow through the bottom path will have cost 1). Intuitively, each user routing from s to t will selfishly choose to take the bottom route because she will reason that the bottom route can have cost no worse than the top route. However, by doing so, the bottom route becomes more congested and leads to a total average cost $C(f) = 1$. Thus, in Figure 1, the price of anarchy ρ is $\frac{1}{3/4} = \frac{4}{3}$.

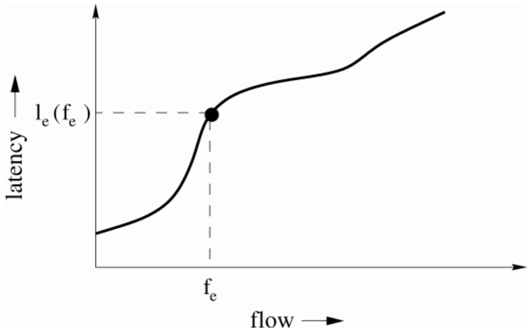
XXX TODO should put more results about flows at Wardrop equilibrium? Not sure if important... the altruism paper has a good short description about how to compute Nash equilibria in polynomial time to find the selfish routing solution

We next describe and present the main results regarding the PoA in this (decentralized) selfish routing model, which will act as a basis to which we will compare traffic routing results in more recently formulated models.

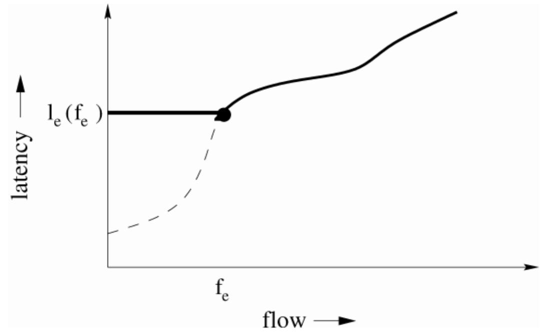
2.4 Main Results

Theorem 1. *If f is a flow at Nash Equlibrium for a given input set (G, r, c) and f^* is a feasible flow for $(G, 2r, c)$, then $C(f) \leq C(f^*)$*

Proof Sketch. In other words, this result suggests that the latency when all users of a network route with only their own interests in mind is utmost the optimal (minimum) latency for the same graph and latency functions with twice as much demand per path.



(a) Graph of latency function ℓ_e and its value at flow value f_e



(b) Graph of latency function $\overline{\ell}_e$

For any Nash Equilibrium solution f for (G, r, ℓ) and any feasible solution f^*, we can draw the figures so that: The area under the black line from 0 to $2f_e$ in subgraph (a) represents the cost of any feasible solution for $(G, 2r, \ell)$, and the area under the black line from 0 to $2f_e$ in subgraph (b) represents the cost of any feasible solution for $(G, 2r, \ell)$. The area under the black line on the left side of f_e in subgraph (a) represents the cost $C(f)$ of any Nash equilibrium solution for (G, r, ℓ). Note that since the area of t he area under the black line on the left side of f_e in subgraph (a) is at least half of the dashed line, we know that the cost of any feasible solution for $(G, 2r, \ell)$, and the area under the black line from 0 to $2f_e$ in subgraph (b) is at most $C(f)$ more than the cost of any feasible solution for $(G, 2r, \ell)$, and the area under the black line from 0 to $2f_e$ in subgraph (a). So the area of the generalized trapezoid under the black line from 0 to $2f_e$ is more than two times the area under the black line on the left side of f_e in subgraph (b).

The main idea of the proof for Theorem 1 lies in that the area under the black line between f_e and $2f_e$ is at least as big as the area under the black line on the left side of f_e in subgraph (b). This is always true because the latency $\bar{\ell}_e$ is a nondecreasing function and thus the graph of latency function gives a shape that looks like a generalized trapezoid (in particular, a right generalized trapezoid lying on the horizontal axis). Then we can obtain the result based on the property of a trapezoid shape. Therefore, the cost of any feasible solution for $(G, 2r, \ell)$ is at least twice the cost $C(f)$ of any Nash equilibrium solution for (G, r, ℓ). Then the area under the black line from 0 to $2f_e$ in subgraph (a) is at least $2C(f) - C(f)$ and thus larger than $C(f)$. Therefore we have the result that the cost of any Nash equilibrium flow for (G, r, ℓ) is no bigger than represents the cost of any feasible solution for $(G, 2r, \ell)$. □

Theorem 2. *If the edge latency functions are linear in that $c_e = a_e f_e + b_e$ for every edge $e \in E$, then the price of anarchy or $\rho(G, R, l) \leq 4/3$.*

Proof Sketch. - describe some intuition for the $r/2$ case leading to $1/4$ cost of nash equilibrium - describe the augmentation that adds the $1/2$ factor. □

XXX TODO put results here XXX Result about unbounded PoA (use Pigou's example, maybe add figure) XXX Result about 4/3 PoA if functions linear

3 ALTERNATIVE MODELS

In this section, we present and compare a subset of recent coordination models for the traffic routing problem against the selfish routing model. These models propose a more nuanced (and perhaps more accurate) description of human behavior. XXX TODO more summary once we have more insights?

3.1 Altruistic

The first alternative model we consider is that proposed by Chen and Kempe in 2008 [1], which assumes that users are "not entirely selfish."

3.1.1 Model.

3.1.2 Results.

3.2 Risk Adverse

3.2.1 Model.

3.2.2 Results.

3.3 Diverse in Interests

3.3.1 Model.

3.3.2 Results.

4 DISCUSSION

4.1 Importance of Human Understanding

4.2 Future Work

Clearly the models we present are a miniscule subset of all potential models for human behavior; furthermore, they are coarse-grained and oversimplistic in comparison with the complexity of the human brain. As understanding of the neurophysiological aspects of human behavior improves, we hope to see a matching evolution in the precision and accuracy of these models for traffic routing as well.

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