

# Linear Algebra

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## 1 Objects

**Definition 1.1** (Vector).

$$\vec{x} = [x_1, \dots, x_n] \in \mathbb{R}^n$$

**Definition 1.2** (Matrix).

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix} \in \mathbb{R}^{m \times n}$$

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## 2 Operations

Operation	Matrix/vector notation	Index notation
Scalar multiplication	$y\vec{x} = \vec{z}$	$yx_i = z_i$
Vector addition	$\vec{x} + \vec{y} = \vec{z}$	$x_i + y_i = z_i$
Vector dot product	$\vec{x} \cdot \vec{y} = z$	$\sum_{i=1}^n x_i y_i = z$
Vector norm	$\ \vec{x}\  = y$	$\sqrt{\sum_{i=1}^n x_i^2} = y$
Matrix transpose	$X^T = Y$	$x_{ij} = y_{ji}$
Matrix addition	$X + Y = Z$	$x_{ij} + y_{ij} = z_{ij}$
Matrix product	$XY = Z$	$\sum_{k=1}^n x_{ik} y_{kj} = z_{ij}$

### 3 Objects

Object	Matrix/vector notation	Index notation
Column-mean vector	$\vec{\mu} = \frac{1}{n} X^T \vec{1}$	$\mu_i = \frac{1}{n} \sum_{j=1}^n x_{ji}$
Zero-centred matrix	$X' = X - \vec{1} \vec{\mu}^T$	$x'_{ij} = x_{ij} - \mu_j$
Scatter matrix	$S = X'^T X'$	$s_{ij} = \sum_{k=1}^n x'_{ki} x'_{kj} = \sum_{k=1}^n (x_{ki} - \mu_i)(x_{kj} - \mu_j)$
Covariance matrix	$\Sigma = \frac{1}{n} S$	$\sigma_{ij} = \frac{1}{n} \sum_{k=1}^n x'_{ki} x'_{kj} = \frac{1}{n} \sum_{k=1}^n (x_{ki} - \mu_i)(x_{kj} - \mu_j)$

## 4 Models

It is assumed throughout that:

- $\vec{x} \in \mathbb{R}^n$  is a feature vector (instance)
- $X = \{\vec{x}_i \mid i \in 1..j\}$  is a set of  $j$  instances
- $\{X_i \mid i \in 1..k\}$  is a partition of  $X$
- $\vec{w} \in \mathbb{R}^n$  is a weight vector
- $y$  is a true label or value
- $\hat{y}$  is a predicted label or value

**Definition 4.1** (Centroid, centre of mass).

$$\vec{\mu}_i = \frac{1}{|X_i|} \sum_{\vec{x} \in X_i} \vec{x} \quad (4.1)$$

**Definition 4.2** (Binary linear classifier). *Let  $\{X_+, X_-\}$  be a partition of  $X$  and  $b \in \mathbb{R}$  be the decision boundary.*

$$\hat{y} = \text{sign}(\vec{w} \cdot \vec{x} + b), \quad \vec{w} = \vec{\mu}_+ - \vec{\mu}_-, \quad b = \frac{1}{2} (\|\vec{\mu}_+\|^2 - \|\vec{\mu}_-\|^2) \quad (4.2)$$