Linear Algebra

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1 Objects

Definition 1.1 (Vector)**.**

$$\vec{x} = [x_1, \ldots, x_n] \in \mathbb{R}^n$$

Definition 1.2 (Matrix)**.**

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix} \in \mathbb{R}^{m \times n}$$

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2 Operations

Operation	Matrix/vector notation	Index notation
Scalar multiplication	$y\vec{x} = \vec{z}$	$yx_i = z_i$
Vector addition	$\vec{x} + \vec{y} = \vec{z}$	$x_i + y_i = z_i$
Vector dot product	$\vec{x} \cdot \vec{y} = z$	$\sum_{i=1}^{n} x_i y_i = z$
Vector norm	$\lVert\vec{x}\rVert = y$	$\sqrt{\sum_{i=1}^{n} x_i^2} = y$
Matrix transpose	$X^T = Y$	$x_{ij} = y_{ji}$
Matrix addition	$X + Y = Z$	$x_{ij} + y_{ij} = z_{ij}$
Matrix product	$XY = Z$	$\sum_{k=1}^{n} x_{ik} y_{kj} = z_{ij}$

3 Objects

Object	Matrix/vector notation	Index notation
Column-mean vector	$\vec{\mu} = \frac{1}{n} X^T \vec{1}$	$\mu_i = \frac{1}{n} \sum_{j=1}^{n} x_{ji}$
Zero-centred matrix	$X' = X - \vec{1}\vec{\mu}^T$	$x'_{ij} = x_{ij} - \mu_j$
Scatter matrix	$S = X'^T X'$	$s_{ij} = \sum_{k=1}^{n} x'_{ki} x'_{kj} = \sum_{k=1}^{n} (x_{ki} - \mu_i)(x_{kj} - \mu_j)$
Covariance matrix	$\Sigma = \frac{1}{n} S$	$\sigma_{ij} = \frac{1}{n} \sum_{k=1}^{n} x'_{ki} x'_{kj} = \frac{1}{n} \sum_{k=1}^{n} (x_{ki} - \mu_i)(x_{kj} - \mu_j)$

4 Models

It is assumed throughout that:

- $\vec{x} \in \mathbb{R}^n$ is a feature vector (instance)
- $X = \{\vec{x}_i \mid i \in 1..j\}$ is a set of j instances
- $\{X_i \mid i \in 1..k\}$ is a partition of X
- $\vec{w} \in \mathbb{R}^n$ is a weight vector
- y is a true label or value
- $\hat{y}$ is a predicted label or value

Definition 4.1 (Centroid, centre of mass)**.**

$$\vec{\mu}_i = \frac{1}{|X_i|} \sum_{\vec{x} \in X_i} \vec{x} \tag{1}$$

Definition 4.2 (Binary linear classifier)**.** *Let $\{X_+, X_-\}$ be a partition of X and $b \in \mathbb{R}$ be the decision boundary.*

$$\hat{y} = \operatorname{sign}\left(\vec{w} \cdot \vec{x} + b\right), \quad \vec{w} = \vec{\mu}_+ - \vec{\mu}_-, \quad b = \frac{1}{2}\left(\|\vec{\mu}_+\|^2 - \|\vec{\mu}_-\|^2\right) \tag{2}$$