3D Matching

Prove that 3D Matching is a NP-Problem

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NP問題

- 此類決策問題(decision problem)可用非確定性演算法(nondeterministic algorithm)解決。
- 若非確定性演算法的猜測與檢查階段都是多項式時間複雜度，則此演算法稱為NP演算法。

What is 3D Matching

Let $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{Z}$ be finite sets, and let $\mathbf{P}$ be a subset of $\mathbf{X} \times \mathbf{Y} \times \mathbf{Z}$. That is, $\mathbf{P}$ consists of triples $(\mathbf{x}, \mathbf{y}, \mathbf{z})$ such that $\mathbf{x} \in \mathbf{X}$, $\mathbf{y} \in \mathbf{Y}$, and $\mathbf{z} \in \mathbf{Z}$. Now $\mathbf{M} \subseteq \mathbf{P}$ is a 3-dimensional matching if the following holds: for any two distinct triples $(\mathbf{x}_1, \mathbf{y}_1, \mathbf{z}_1) \in \mathbf{M}$ and $(\mathbf{x}_2, \mathbf{y}_2, \mathbf{z}_2) \in \mathbf{M}$, we have $\mathbf{x}_1 \neq \mathbf{x}_2$, $\mathbf{y}_1 \neq \mathbf{y}_2$, and $\mathbf{z}_1 \neq \mathbf{z}_2$.

NDA of 3D Matching

Input: three set **S**[1], **S**[2], **S**[3] with same amount of values, respectively

Output: **Success** if there exist subset **M** satisfy 3D Matching Problem; **Failure** otherwise.

Process:

```
for i <- 1 to 3              // guessing
    P[i] = choice(S[i])
    Q[i] = choice(S[i])
if(P[1] ≠ Q[1] and P[2] ≠ Q[2] and P[3] ≠ Q[3]) return Success
else return Failure
```

Analysis of NDA of 3D Matching

Process:

1. for i <- 1 to 3 // guessing
2. **P**[i] = choice(**S**[i])
3. **Q**[i] = choice(**S**[i])
4. if(**P**[1] ≠ **Q**[1] and **P**[2] ≠ **Q**[2] and **P**[3] ≠ **Q**[3]) return **Success**
5. else return **Failure**

Analysis:

在猜測階段，無論元素多少我們皆只會猜測三次，故時間複雜度是O(1)

而在檢查階段，我們也只會檢查三次，故時間複雜度也是O(1)

因為都是多項式時間複雜度，故3D Matching是NP問題

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