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СРАВНЕНИЕ МОДЕЛЕЙ СЛУЧАЙНЫХ ГРАФОВ ДЛЯ ЦЕЛЕЙ ОЦЕНКИ СИСТЕМНОГО РИСКА

RANDOM GRAPH MODELS COMPARISON FOR THE PURPOSE OF SYSTEMATIC RISK EVALUATION

Курсовая работа

по направлению подготовки 38.04.01 «Экономика»

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Introduction

Systemic risk measures total risk of the system. There is no one definition of systemic risk, but there are many of them.

Mishkin 1995 [1]: “the likelihood of a sudden, usually unexpected, event that disrupts information in financial markets, making them unable to effectively channel funds to those parties with the most productive investment opportunities.”

Kaufman 1995 [2] “The probability that cumulative losses will accrue from an event that sets in motion a series of successive losses along a chain of institutions or markets comprising a system. . . . That is, systemic risk is the risk of a chain reaction of falling interconnected dominos.”

Bank for International Settlements 1994 [3] “ the risk that the failure of a participant to meet its contractual obligations may in turn cause other participants to default with a chain reaction leading to broader financial difficulties.”

We need to measure systemic risk in order to be aware of risk of the whole system, not only some agents. We can evaluate systemic risk in many ways[4]. One of such ways is to use the concept of bank contagion. It uses a network of bank exposures to each other to simulate default cascades. Default of single bank can cause defaults of other banks; these banks can cause default of other banks and so on. Such defaults cascade forms a contagion. If we know the connections between banks, we can measure contagion and evaluate risk[5].

The main problem with this approach is that usually we do not know the real connections between banks and only know total debt for every bank from reports. To solve this problem one can use random graph models – a ways to generate random graph based on visible exposures[6].

The recent financial crisis has rekindled interest in the relationship between the structure of the financial network and systemic risk. Interesting question arises if we use random graph. How much does random graph model selection will affect the results? The main purpose of our work is to answer that question. This paper will describe the setup of the experiment to compare random graph models, provide and explain results.

The structure of the paper is the following. First, we make a review of main literature and empirical findings in this topic. Then, we introduce models for evaluating systemic risk and experiments’ setup. After conducting each experiment we provide results and our observation. In the conclusion, we finally summarize all finding and make some concluding remarks.

Literature review

In this part, a quick overview of previous works on financial contagion and evaluating systemic risk is presented. Many of these works explains different contagion propagation mechanisms and using random graphs to evaluate systemic risk.

Franklin Allen and Douglas Gale in their paper provide macroeconomic foundations for financial contagion[7]. They use only channel of contagion, overlapping claims, like we use in this paper. They work with perfect information and bank panics occur due to random liquidity shocks on the part of the depositors. There is another work by Amil Dasgupta[8], that features incomplete information. His model suggests that contagion occurs with positive probability even with complete interbank deposits and bank runs occur due to adverse information about asset returns.

Eisenberg and Noe completed one of the base works on systemic risk in financial networks[9]. In this paper they provide conditions for existing of clearing vector, that clears liabilities between financial institutions and this vector is unique. They explore clearing vectors and provide relation between clearing vectors and parameters of the underlying financial system.

Another important paper to mention is a work by Prasanna Gai and Sujit Kapadia about contagion in financial networks[10]. This paper provides analytical model of contagion in financial networks and explains concept of contagion in details. As like as Gai and Kapadia, James P. Gleeson, T.R. Hurd, Sergey Melnik, and AdamHackett completed a paper dedicated to calculating the contagion without using Monte-Carlo simulation[11].

Fabio Caccioli, Thomas A. Catanach, and J. Doyne Farmer made a research on heterogeneous degree distributions on contagion and tried to characterize some empirical features from real systems[12]. They showed, that such networks are more resilient to failure of random bank, but more fragile to failure of any highly-connected banks. And impact from defaulted highly-connected banks is even more, than from the biggest banks. Another interesting work by Daron Acemoglu, Asuman Ozdaglar, and Alireza Tahbaz-Salehi shows, that highly-connected networks are resilient to small shocks, but with large number of shocks such interconnectedness becomes a channel of contagion[13]. These works are very important because in real life we can often see such systems, when there are some highly-connected banks and small banks with fewer connections.

Tomas Hurd in his book “Contagion! Systemic Risk in Financial Networks”[14] explains the concept of contagion and provide instruments for evaluating systemic risk from contagion. He starts from the explaining the idea of systemic risks. It explains different sources of spreading contagions, such as default cascades, liquidity cascades and asset fire sales. In our work, we will focus on default cascades. Finally, random graph models are briefly explained in this book too. So this book can be used as a handbook on systemic risk and contagion in financial systems. Another review article was written by Fabio Caccioli, Paolo Barucca and Teruyoshi Kobayashi[5]. They described existing network models of systemic risk.

Stefano Pegoraro wrote a paper connected with the topic of our research. He explores how different network topologies affect systemic risk. He compares Erdős and Rényi random graphs, small world networks and scale free networks by probability of contagion[15]. In our work we focus mostly on Erdős and Rényi random graphs and explore how the parameter of the model affects systemic risk. Instead of using probability of contagion we use anther metrics for systemic risk which we will discuss later.

In 2012 Dimitrios Bisias, Mark Flood, Andrew W. Lo, and Stavros Valavanis completed a survey of systemic risk analytics[4]. It consists big number of different metrics to calculate systemic risk.

Last two papers are about applying theory and evaluating systemic risk for real bank system. Marco Espinosa-Vega and Juan Solé explore European Union national bank system[16] and Rama Cont, Amal Moussa, Edson B. Santos explore Brazil bank system[17].

Model setup and methods

Contagion model

Bank system can be represented as a directed graph, where nodes are banks, and edges are liabilities between banks[17]. Amount of liabilities is shown by edges' weights. If one bank defaults, his counterparties will lose assets from defaulted bank. In real life there is some recovery rate, because defaulted bank can at least partially satisfy his liabilities. In our model without loss of generality, we assume that recovery rate 0%, which means that bank lose 100% of defaulted asset.

For simplicity, each bank has assets, liabilities and capital buffer to accumulate losses. Bank is solvent if his capital buffer covers loses from defaulted assets. We recursively compute solvency condition for every non-defaulted banks until system stabilize, i.e. no new defaults. If we know exact graph we can run simulation and compute the number of induced defaults by any bank[18].

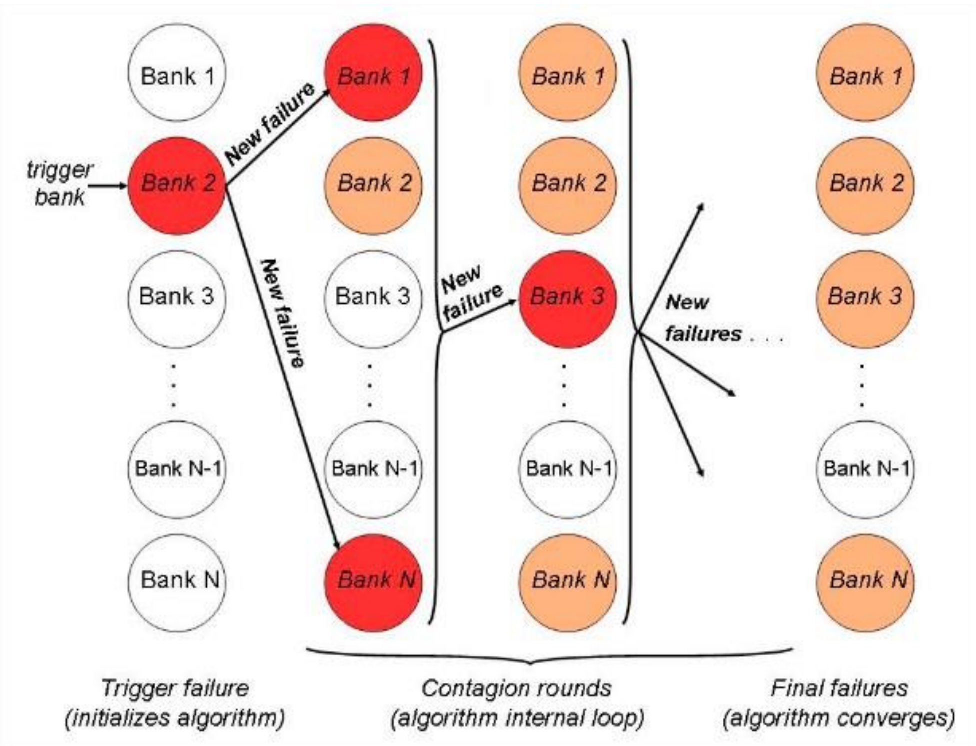


Figure 1.

Random Graph Models

To our regret, in most real cases, we do not have exact graph and we can only see visible exposures, such as total assets, total liabilities and capital buffer. We can generate infinite number of graphs consistent with visible exposures.

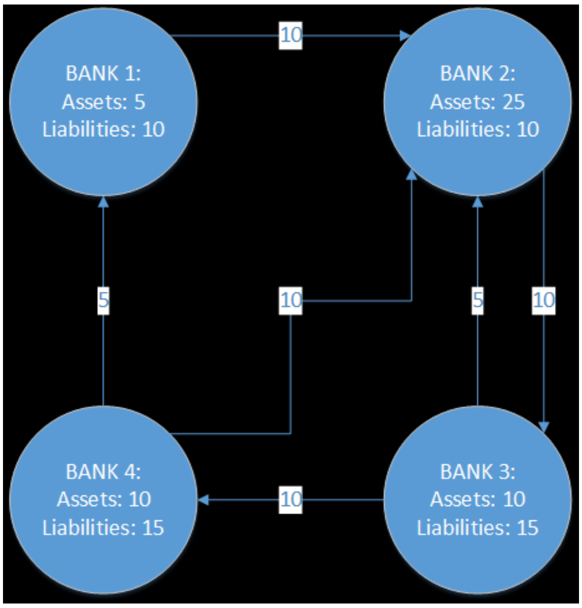


Figure 2.

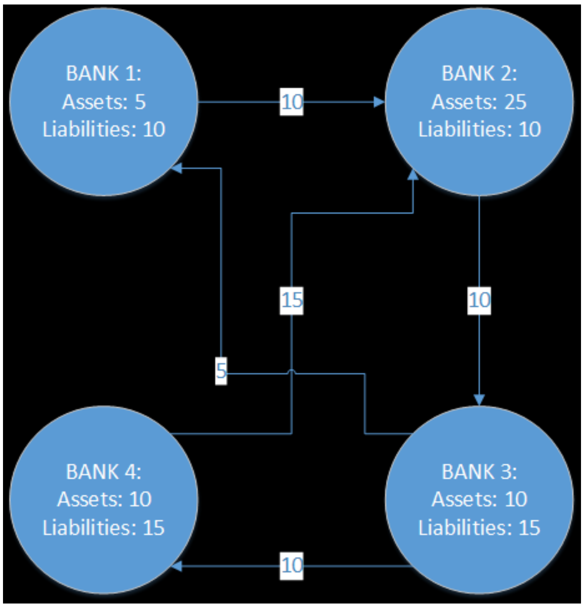


Figure 3.

Random graph models includes the distribution of edges and weights. From that model we can get the distribution of induced defaults. We can get exact solution, numerical solution or get distribution by Monte-Carlo simulation. To do initial comparing Monte-Carlo simulation gives enough precision, so we use it. In our work, we have used several models.

Simple model

In this model each banks has connection with each bank and weights are uniformly random. Algorithm selects random node and then randomly assign weights to liabilities of that node in order to satisfy total liabilities and total assets constraints.

Erdos–Rényi Model

Each edge has equal and independent probability to occur in graph[19], [20]. First, we generate skeleton of the graph, only edges without weights. Then we assume equal weights to each out-edge of every node.

We can extend this model by applying randomizing for weights. It means that distribution of liabilities will be not homogeneous.

It is important to notice, that in that class of models we ignore total assets constraints. We are not checking how that influence on results in this work, but it can be done in a future work.

Experiments

In most works on that topic probability of contagion is used as a metric to evaluate systemic risk. To have a look on the problem with the other side we have used two other metrics.

In order to compute these metrics we assume that banks have equal probability of default conditional on one bank defaulted. With that knowledge, we can compute expected value of induced defaults by one defaulted banks.

E (induced_defaults | one initially defaulted bank) = average of induced defaults by every bank.

Another important metric – 95 % quantile. From computed induced defaults we can build a distribution of defaults and compute 95% quantile of that distribution. This number means that with 95% probability number of induced defaults will be less or equal computed value.

Every experiment checks both metrics:

- Expected value of defaults
- 95% quantile value of defaults

For every metric experiment has following steps:

1. Generate 100 banks with total assets, total liabilities and capital buffer. This step is common for every metric, i.e. it is done only once at the beginning of experiment.
2. By Monte-Carlo simulation compute distributions of metrics for every random graph model, used in experiment
3. Compare this models visually if possible
4. Compute 95% VaR of distributions.
5. Compare numbers. Conduct statistical tests, if necessary.

In our research, we conducted three experiments to compare different models for generating random graphs.

Programming code for experiments can be found on github[21].

Results

Experiment 1

In this experiment, we compare Erdos–Rényi model with equally distributed liabilities and Erdos–Rényi with randomly distributed liabilities. We use three different random graph setups with 25%, 50%, 75% probability of edge in graph.

We conducted 20 experiments with different initial banks. They have more or less similar results, so we have looked at one of them to explain a tendency.

Let us setup a probability of edge P = 0.25 and look at the resulted probability density graphs.

Expected value of defaulted banks after default of any single banks:

Homogeneous

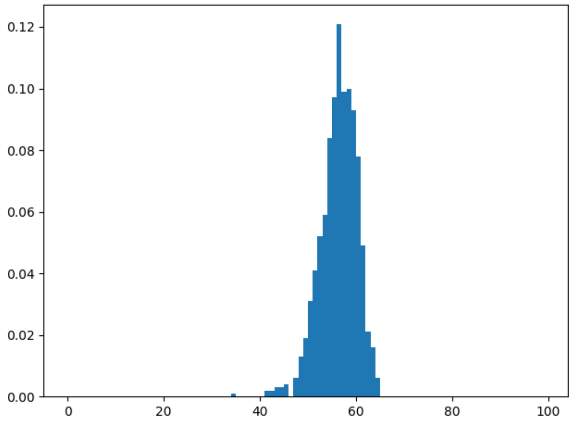


Figure 4.

Heterogeneous

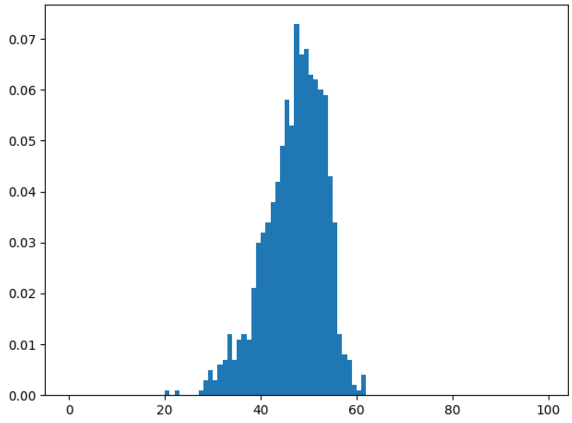


Figure 5.

Quantile 95% of defaulted banks after default of any single banks:

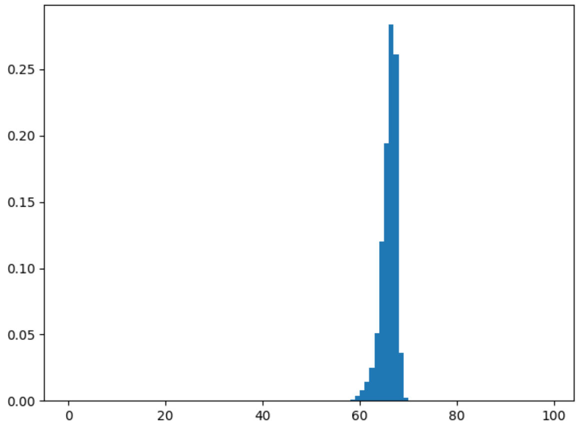


Figure 6.

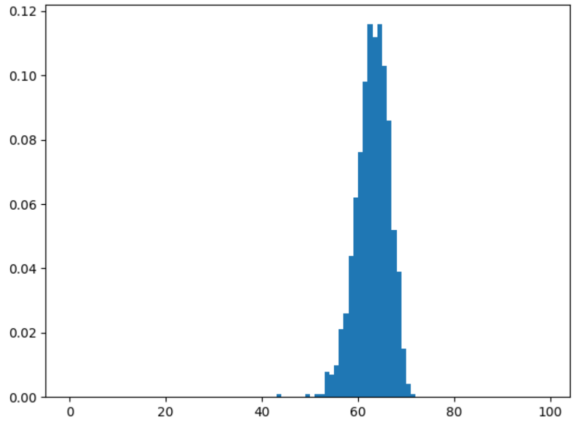


Figure 7.

As we can see both models in both metrics looks like normal distribution, but with slightly different expected values and dispersions. Because of these differences quantile 95% of distributions will be different as well.

On the following graphs, we compared 95% quantiles for both metrics: expected value of defaults and 95% quantile value of defaults.

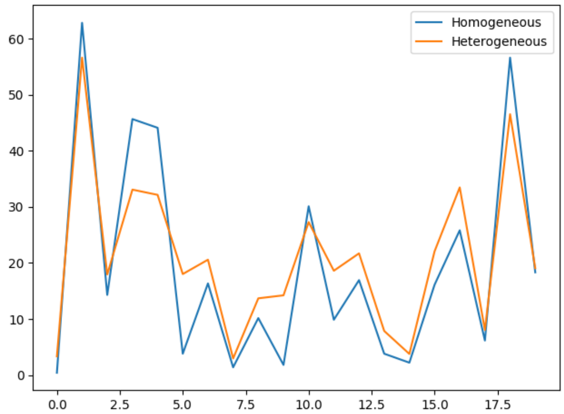


Figure 8. Expected value

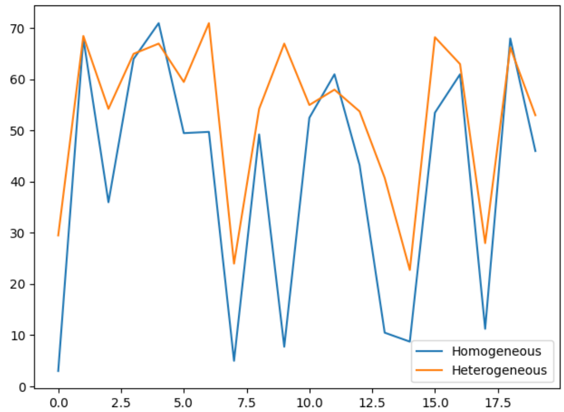


Figure 9. Quantile 95%

We can see that difference in metrics depends from generated banks. In some cases, heterogeneous distribution of liabilities does not affect metrics, but in some cases, it matters a lot. In some cases homogeneous and heterogeneous shows similar results in expected value, but quite different in quantile 95%. We can certainly say that for quantile 95% metric homogeneous and heterogeneous distributions are not interchangeable. However, for expected value sometimes we can use homogeneous instead of heterogeneous for simplicity.

If we set probability of edge P = 0.75, we can see this difference will increase. For 95 % quantile, it will rise dramatically, but for expected value, it will slightly rise. So conclusions, that we made about P = 0.25 can be still applied.

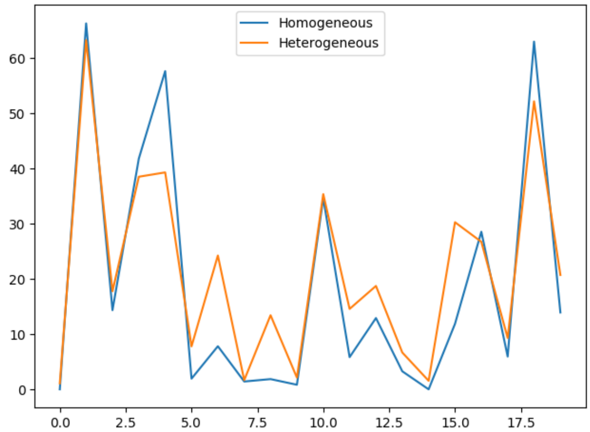


Figure 10. Expected value

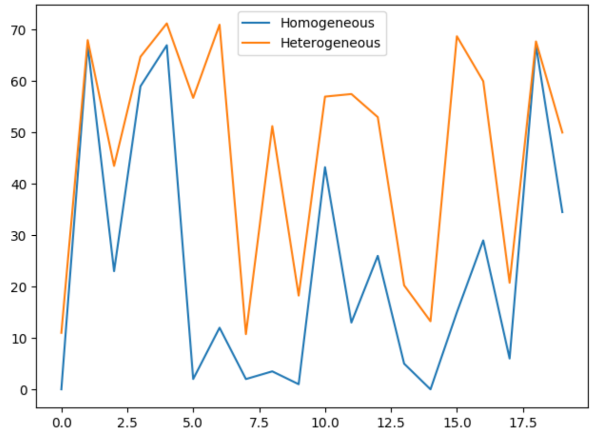


Figure 11. Quantile 95%

Experiment 2

This experiment has a goal to find dependence between probability of edge and distribution of defaults. This probability influence on connectivity of the graph, because more probability leads to more dense graphs [14].

For Erdos–Rényi graph with homogeneous weights, we run five experiments. Each experiment includes simulation for probabilities of edge P ∈ [0, 1] with step 0.01. Following graphs represent the result for expected value metric.

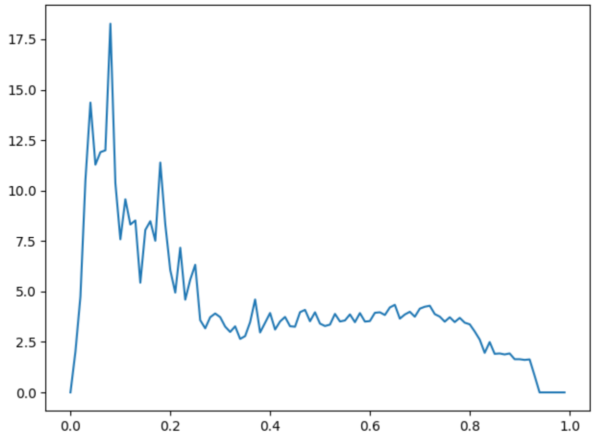


Figure 12.

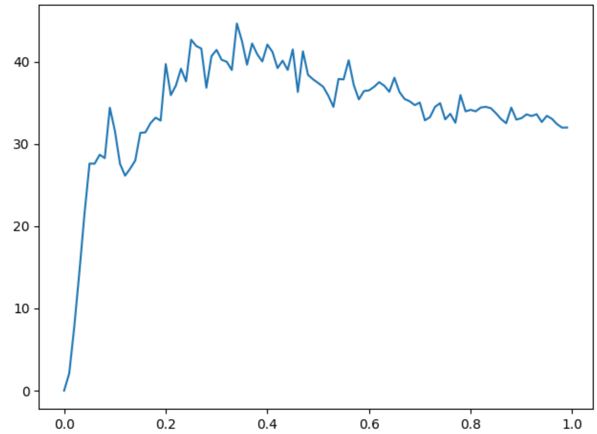


Figure 13.

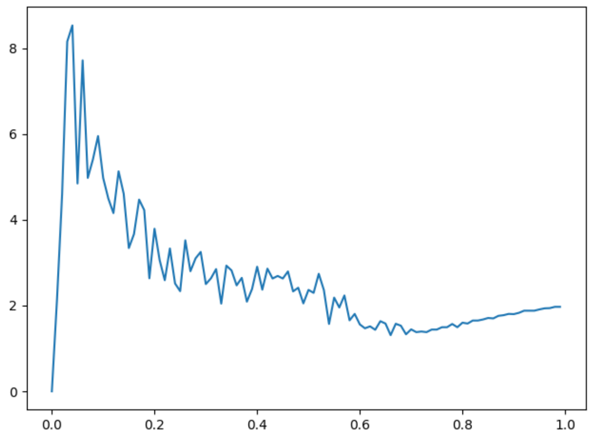


Figure 14.

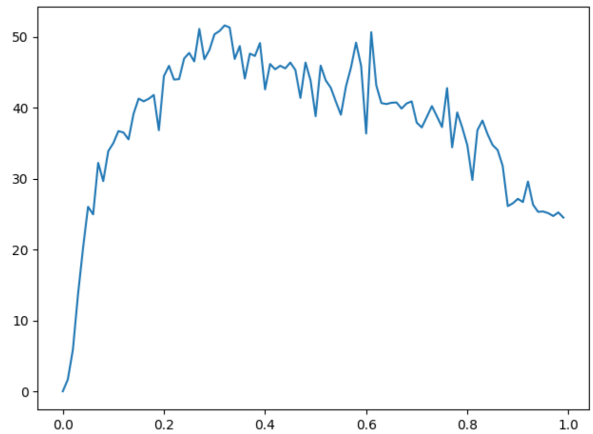


Figure 15.

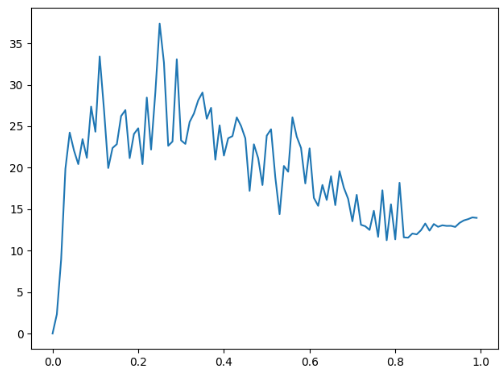
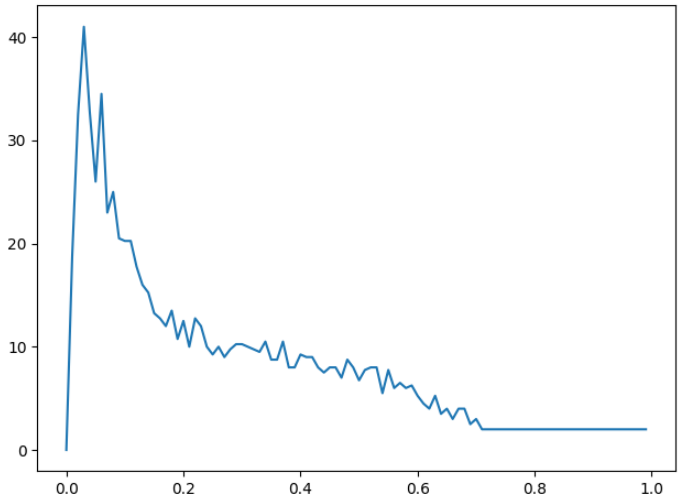


Figure 16

If we look at these graphs, we can see that they are quite different, so the first conclusion – bank parameters have a big influence. Despite differences, they have something in common. From zero to some point number of defaults increase dramatically. It happens because with low connectivity there is no possibilities for contagion spread. After number of defaults reaches the peak it goes down and reach a plateau with more or less stable values with small fluctuations.



For quantile 95% metric, we have similar situation. However, sometimes on the right side we can get stable value without any fluctuations at all.

Figure 17.

For Erdos–Rényi graph with heterogeneous weights, we run six experiments. Each experiment includes simulation for probabilities of edge P ∈ [0, 1] with step 0.01. In case of heterogeneity, we have quite different situations for expected value and for quantile 95% metrics. Following graphs are about expected value metric.

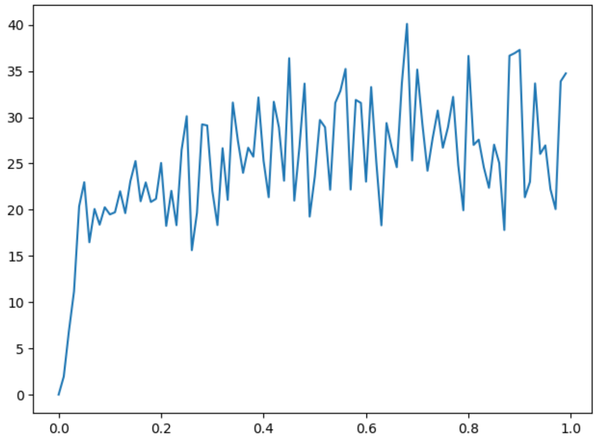


Figure 18.

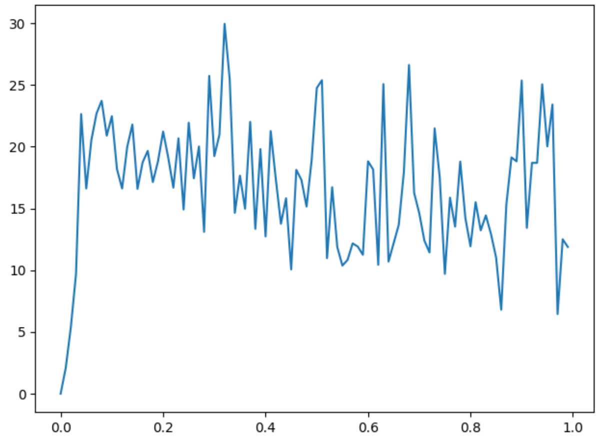


Figure 19.

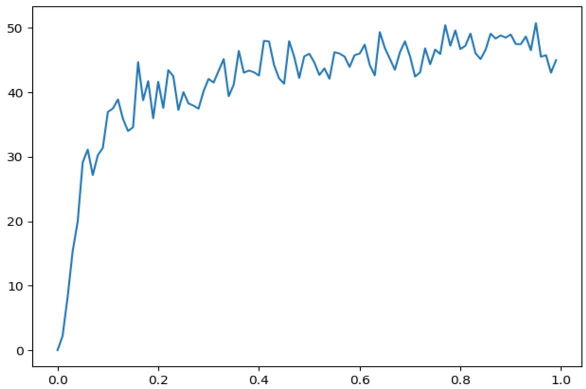


Figure 20.

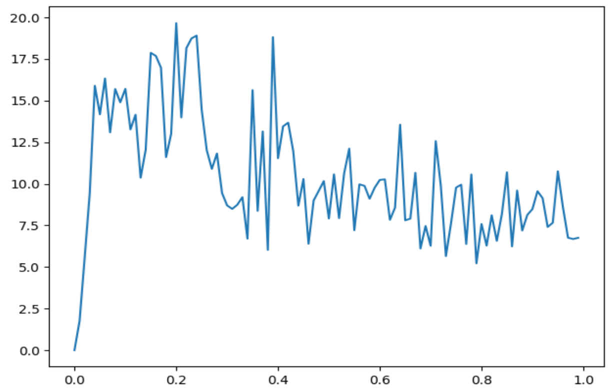


Figure 21.

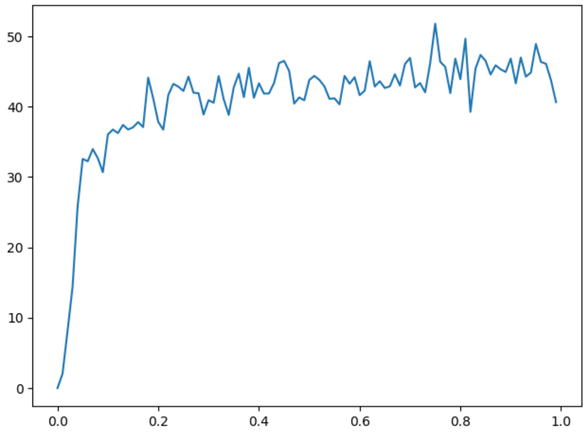


Figure 22.

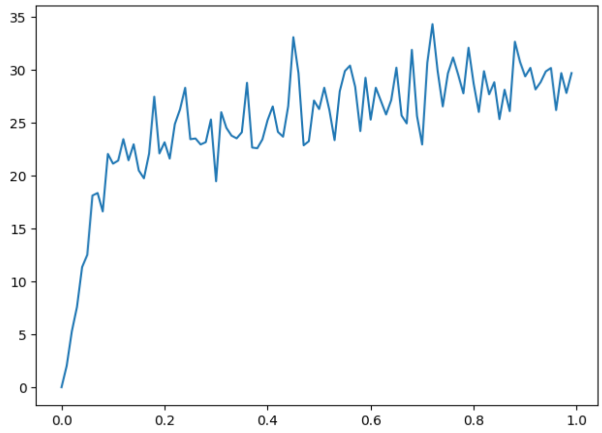


Figure 23.

Comparing with homogeneous case we can see, that there is no visible peak like in homogeneous. We can also notice that heterogeneity brings more fluctuations.

Now let us turn to the quantile 95% case.

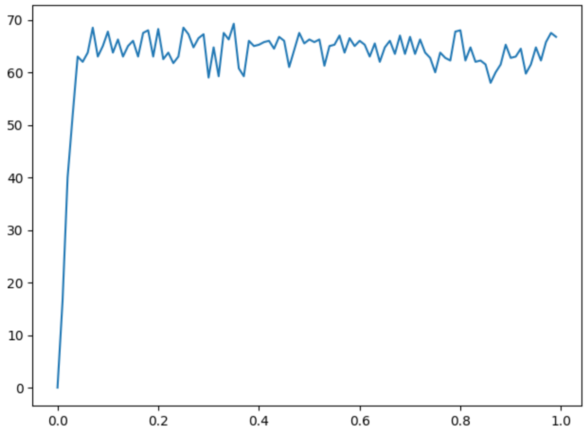


Figure 24.

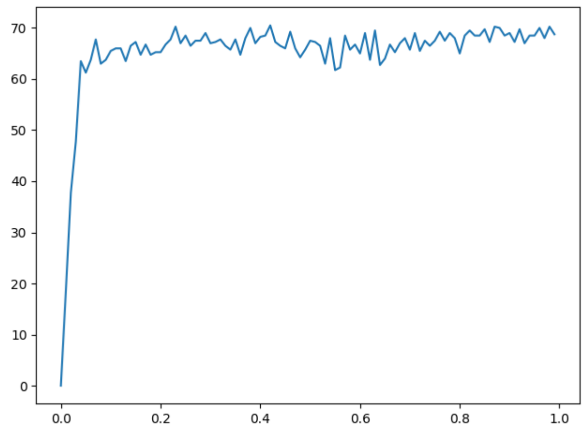


Figure 25.

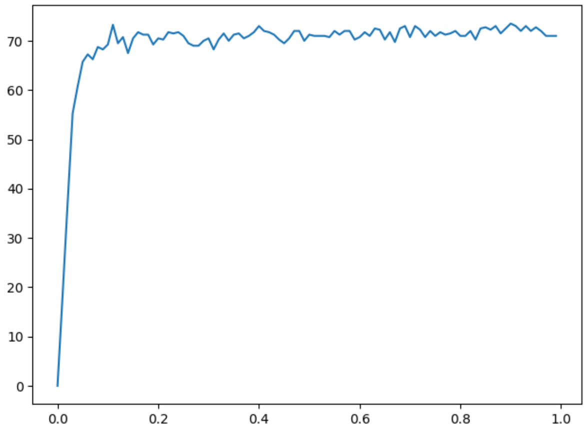


Figure 26.

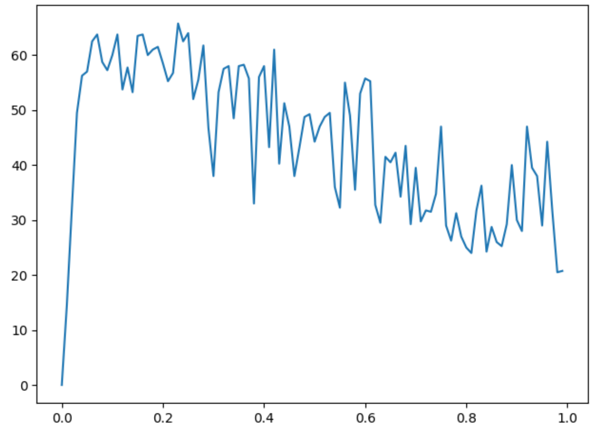


Figure 27.

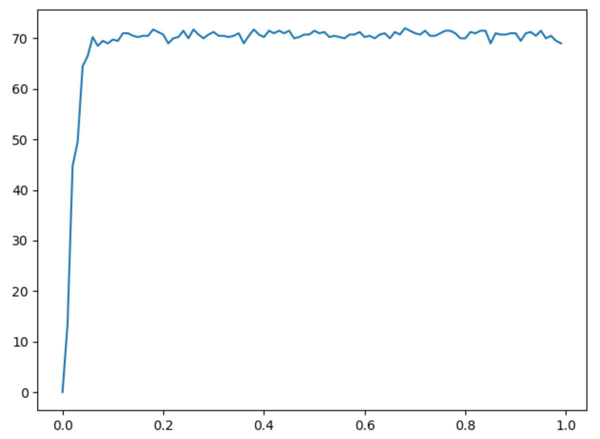


Figure 28.

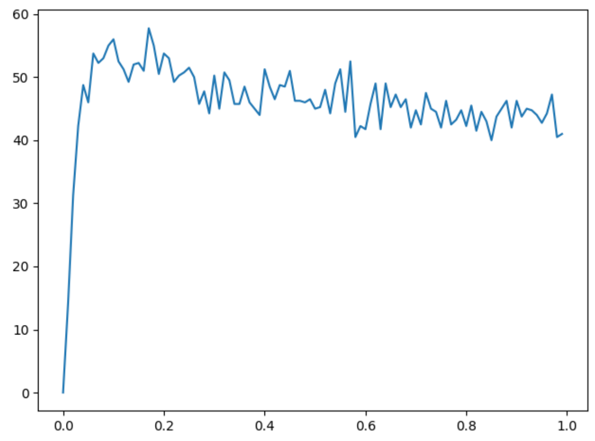


Figure 29.

We can see a very interesting situation. There are also no peaks like in expected value metric. However, in most cases fluctuations are minimal after some threshold point. That leads us to the conclusion that for some bank systems we do not have to know the probability of establishing connection to calculate quantile 95% of defaults, if we are sure that this probability is more than threshold.

Experiment 3

Last experiment compares Erdos–Rényi model and completely random model. We compare both homogeneous and heterogeneous models. Probability of edge P = 0.25.

Our completely random graph model is similar to heterogeneous Erdos–Rényi with P=1, but there is one important difference. When we distribute weights we check not only total liabilities constraint, but total assets too.

In this experiment we conducted simulations with 20 different banks systems. First, we compare homogeneous weights model and random graph.

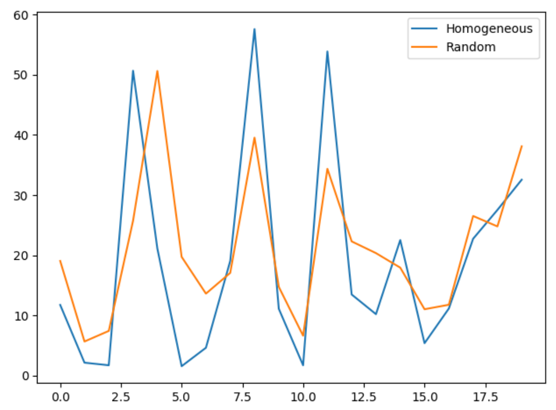


Figure 30. Expected value

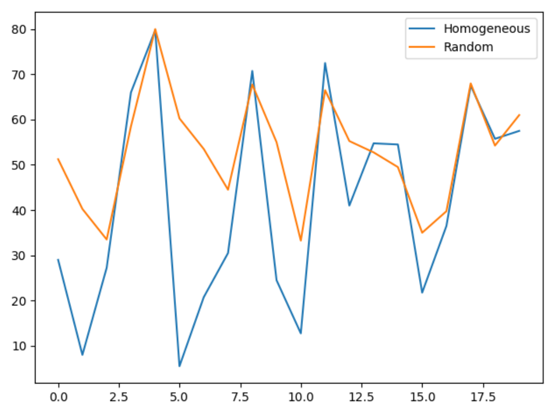


Figure 31. Quantile 95%

For both metrics, we can see relatively big differences between models. We can compute absolute differences between models and ratios.

Table 1.

Metric	Homogeneous -Random			Homogeneous/Random		
	Average	Minimum	Maximum	Average	Minimum	Maximum
Expected	-2.23	-29.52	24.94	0.79	0.08	1.97
Quantile 95%	-11.16	-54.75	7.5	0.76	0.09	1.13

From this table we can see that differences between models can be quite big. That means, that there is no possibility to use one model instead another model.

Now let us compare Erdos–Rényi graph with heterogeneous weights and Random graph.

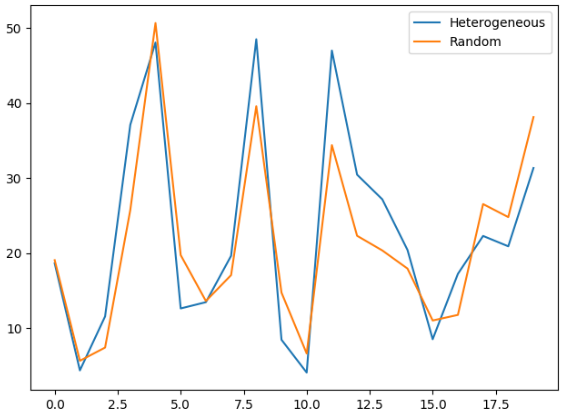


Figure 32. Expected value

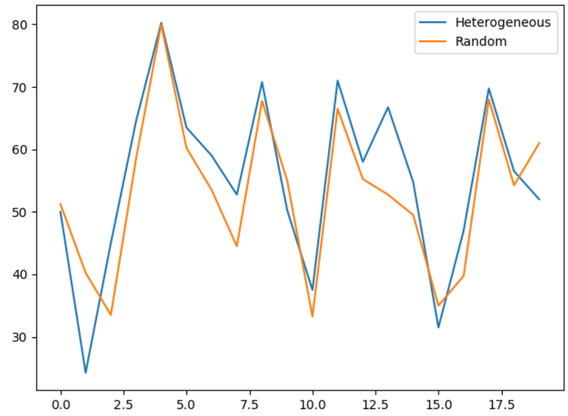


Figure 33. Quantile 95%

We can see a better picture. For both metrics graphs looks very similar. Now let us check differences.

Table 2.

Metric	Heterogeneous-Random			Heterogeneous/Random		
	Average	Minimum	Maximum	Average	Minimum	Maximum
Expected	1.24	-7.11	12.59	1.04	0.57	1.56
Quantile 95%	2.26	-16	14	1.05	0.6	1.34

Average ratio for both metrics is close to one and distance between minimum and maximum is low, comparing to homogeneous case. That means, that Erdos–Rényi with heterogeneous weights and P=0.25 without restrictions on total assets have very similar results to completely random graph, but with restriction on total assets.

Conclusion

In this paper, we compared three types of models:

1. Erdos–Rényi random graph model with homogeneous weights
2. Erdos–Rényi random graph model with heterogeneous weights
3. Random graph model with all banks connected and restrictions on total assets

At the beginning of our work, we asked a question: how much does random graph model selection will affect the results? Therefore, the general answer is it affects it a lot. However, there are some cases, when we can change model parameters and get similar results.

If we evaluate expected number of defaults, we can use homogeneous model instead of heterogeneous to increase performance of the simulations, however trade-off is losing some precise.

Another interesting observation is that in heterogeneous model 95% quantile metric is almost stable after some threshold, which depend on bank system, but it is about 10% usually. Therefore, it means if we know that each bank has connection with at least 10% banks, we can use any P more than 10% to evaluate 95% quantile metric.

As for complete random graph model. It is close to Erdos–Rényi random graph model with heterogeneous weights and P=0.25, so we can use them to evaluate desired metrics. However, we have to be aware of differences in results, as they can be as much as 50% more or 50% less.

Generally, there is no universal random graph that can represent any desired bank system. Nevertheless, maybe there are some patterns that give stable results with any random graph, which includes those patterns. We can consider looking for such patterns as a possible development of this work.

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