# Econometrics I: Econometric Methods

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# Welcome

Welcome to your first course in econometrics!

Q: Wait! What is econometrics?

## Definition

**Econometrics** is the science of using economic theory and statistical techniques to analyze economic data.

Econometric methods are used in many branches of economics and business, including finance, labor economics, development economics, behavioral economics, macroeconomics, microeconomics, marketing, economic policy

It is also used in other social sciences such as political science and sociology

Econometrics is a nice combination of economics and statistics

Econometrics gives you skills that are rewarded in the workplace (private banks, central banks, consulting firms, insurance companies, government agencies all have big teams of econometricians trying to make sense of a broad array of data)

Econometrics can be quite mathematical, but this semester I will focus on the big ideas and the important concepts and intuition

But before we get started with econometrics, let's first briefly discuss …

# Logistics

You can seek help on matters *academic* from

- your friendly lecturer (me): Juergen Meinecke
- your friendly head tutor: Luis Uzeda Garcia
- your friendly tutor
  - Zain Virani
  - Daniel Payten

Feel free to e-mail us anytime, stop by our offices, randomly stop us on campus or call us on a Sunday afternoon (or not)

You can seek help on matters *administrative* from

- Course administrator: Karissa Carkeet
- School administrator: Finola Wijnberg

Karissa and Finola are very friendly, they are happy to help and you can find them in the first floor of the Arndt building

Indicative work load

- two hours of lecture per week
- one hour of problem solving tutorial per week
- one hour of computer tutorial per week
- 6 hours of private study per week

These are guidelines

If you miss a lecture or tute you should make up for it as soon as possible!

Now let's take a look at the course website

`http://EMET2007.Readthedocs.org`

(That's right, I'm not using Wattle)

(One exception however: audio and video recordings will go up on Wattle automatically after each session.)

# Roadmap

Introduction

Review of Univariate Statistics

- Random Variables, Probability Distributions
- Expected Value
- Standard Deviation and Variance
- Population versus Sample
- Sample Average

# This Week's Textbook References

Every week, in addition to the lecture, you should read along in the Stock and Watson textbook (updated third edition)

I design the lecture in such a way that reading the textbook will actually help you considerably in deepening your understanding of the course material

The content of this week's lecture is contained within the following sections of the textbook:

- 2.1, 2.2, 2.5

### Definition

The mutually exclusive potential results of a random process are called **outcomes**.

### Definition

The set of all possible outcomes is called **sample space**.

### Definition

An **event** is a subset of the sample space.

Example:

random process ‘rolling a die’

- outcomes: e.g., rolling ‘five dots’
- sample space: {one dot, two dots, . . . , six dots}
- event: e.g., {three dots, five dots}
  (there are many more, how many?)

Example:

random process *number of dead kangaroos b/w Canberra and Sydney* (along the freeway)

- outcomes: e.g., five dead kangaroos
- sample space:
  {one dead kangaroo, two dead kangaroos, . . . ,
  fifty dead kangaroos}
  (this one's tricky, surely (hopefully) there wouldn't be more than fifty of them?)
- example of an event: more than ten dead kangaroos

## Definition

A **random variable** $Y$ is the numerical representation of an outcome in a random process.

Die example

- the outcome 'one dot' is represented by the number 1
- the outcome 'two dots' is represented by the number 2 and so forth

Note: outcomes can be represented by any number

For instance, the outcome 'one dot' could also be represented by the number 247

I picked the obvious and sensible candidates

Random variables save us a lot of notation

Consider the event

*not less than ten but fewer than thirty dead kangaroos*

(sounds clumsy, doesn't it?)

Using random variables, this can be concisely summarized mathematically as

$$10 \leq Y < 30$$

## Definition

The **probability distribution** of a random variable $Y$ is the full characterization of probabilities for all values the random variable can take on.

(this applies to *discrete* random variables; the definition for *continuous* random variables would be slightly different)

Example

- age of EMET2007 students
- suppose ages vary between 18 and 26
  (just to keep things simple; sorry if you are older!)

Example: probability distribution of age

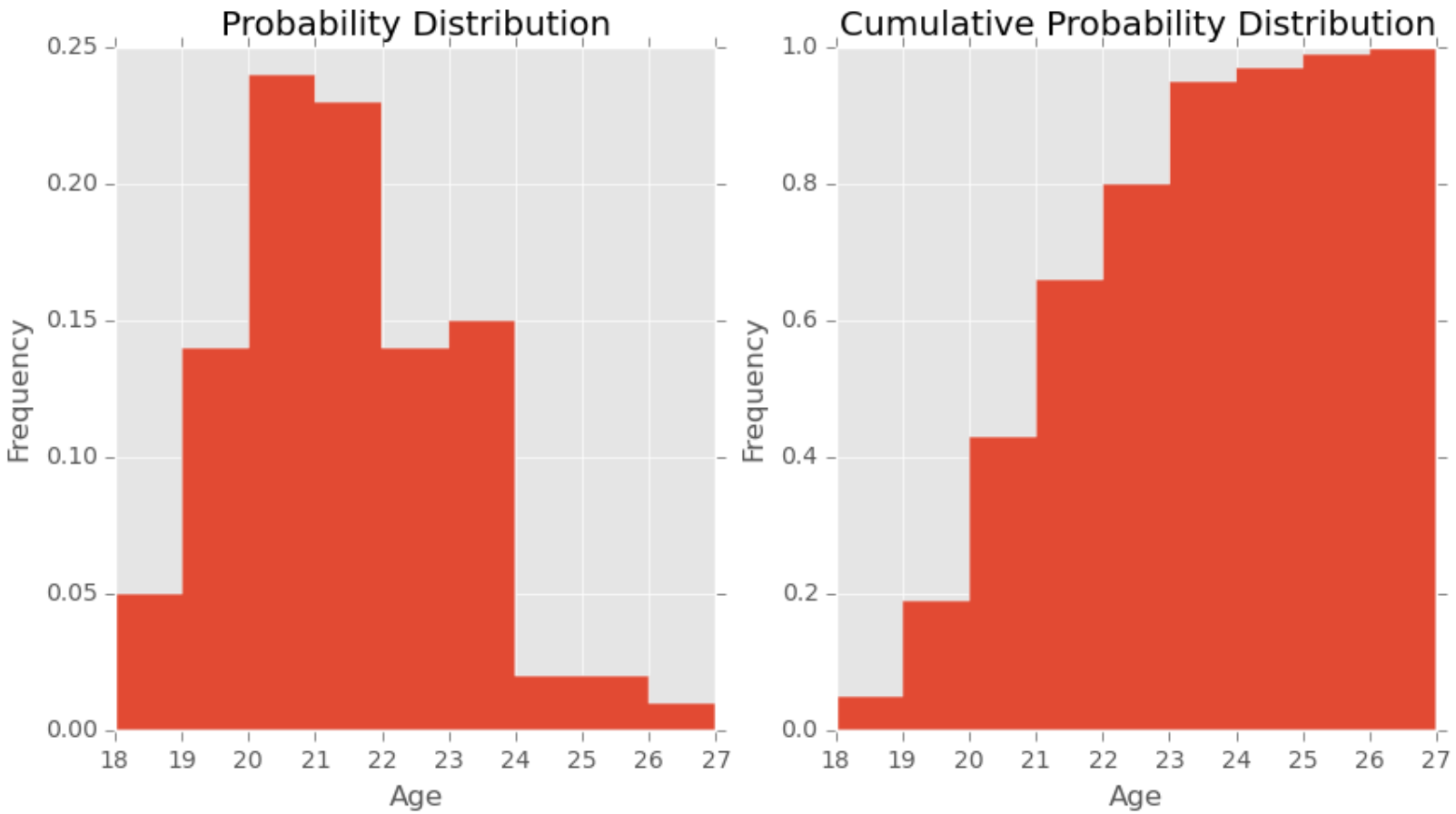
$$\Pr(Y = y) = \begin{cases} 0.05 & \text{if } y = 18 \\ 0.14 & \text{if } y = 19 \\ 0.24 & \text{if } y = 20 \\ 0.23 & \text{if } y = 21 \\ 0.14 & \text{if } y = 22 \\ 0.15 & \text{if } y = 23 \\ 0.02 & \text{if } y = 24 \\ 0.02 & \text{if } y = 25 \\ 0.01 & \text{if } y = 26 \end{cases}$$

Note: little $y$ is called the *realization* of the random variable, it's merely a placeholder for a number between 18 and 26

Example: **cumulative** probability distribution of heights

$$\Pr(Y \leq y) = \begin{cases} 0.05 & \text{if } y = 18 \\ 0.19 & \text{if } y = 19 \\ 0.43 & \text{if } y = 20 \\ 0.66 & \text{if } y = 21 \\ 0.80 & \text{if } y = 22 \\ 0.95 & \text{if } y = 23 \\ 0.97 & \text{if } y = 24 \\ 0.99 & \text{if } y = 25 \\ 1.00 & \text{if } y = 26 \end{cases}$$

# Frequency plot (histogram)



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## Definition

Suppose the random variable $Y$ takes on $k$ possible values $y_1, \ldots, y_k$. The **expected value** is given by

$$E[Y] := \sum_{j=1}^{k} y_j \cdot \Pr(Y = y_j) \tag{1}$$

Occasionally we also call this the **population mean** or simply the **mean** or the **expectation**.

Often times, the expected value is also denoted $\mu_Y$.

Example: age distribution

We have $y_1 = 18, y_2 = 19, \ldots, y_9 = 26$

Doing the math

$$\begin{aligned} E[Y] &= \sum_{j=1}^{9} y_j \cdot \Pr(Y = y_j) \\ &= 18 \cdot 0.05 + 19 \cdot 0.14 + \cdots + 26 \cdot 0.01 \\ &= 20.96 \end{aligned}$$

## Properties of the expected value

- Let $c$ be a constant, then $E[c] = c$
- Let $c$ be a constant and $Y$ be a random variable, then

$$\mathrm{E}[c + Y] = c + \mathrm{E}[Y]$$
$$\mathrm{E}[c \cdot Y] = c \cdot \mathrm{E}[Y]$$

  It follows that for two constants $c$ and $d$,
  $E[c + d \cdot Y] = c + d \cdot \mathrm{E}[Y]$
- Let $X$ and $Y$ be random variables, then
  $\mathrm{E}[X + Y] = \mathrm{E}[X] + \mathrm{E}[Y]$

(Can you prove all of these?)

## Definition

The $r^{th}$ **moment** of a random variable $Y$ is given by

$$m_r(Y) := E[Y^r], \qquad \text{for } r = 1, 2, 3, \ldots$$

It is obivous that the first moment and the expected value are the same

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## Definition

The **population variance** is defined by

$$\mathrm{Var}[Y] := \sum_{j=1}^{k} (y_j - \mu_y)^2 \cdot \Pr(Y = y_j)$$

Often times, the variance is denoted by $\sigma_Y^2$.

## Definition

The **population standard deviation** is defined by

$$\mathrm{StD}[Y] := \sqrt{\mathrm{Var}[Y]}$$

It follows immediately that the standard deviation is simply $\sigma_Y$.

Example: age distribution

We have $y_1 = 18, y_2 = 19, \ldots, y_9 = 26$

Doing the math

$$\begin{aligned}
\mathrm{Var}[Y] &= \sum_{j=1}^{9} (y_j - \mu_y)^2 \cdot \Pr(Y = y_j) \\
&= (18 - 20.96)^2 \cdot 0.05 + (19 - 20.96)^2 \cdot 0.14 + \cdots \\
&\quad (26 - 20.96)^2 \cdot 0.01 \\
&= 2.74
\end{aligned}$$

Therefore

$$\mathrm{StD}[Y] = 1.66$$

Properties of the variance

- Let $c$ be a constant, then $\text{Var}[c] = 0$
- Let $c$ be a constant and $Y$ be a random variable, then

$$\text{Var}[c + Y] = \text{Var}[Y]$$
$$\text{Var}[c \cdot Y] = c^2 \cdot \text{Var}[Y]$$

- Let $X$ and $Y$ be random variables, then
  $\text{Var}[X + Y] = \text{Var}[X] + \text{Var}[Y] + 2 \cdot \text{Cov}(X, Y)$

(Can you prove all of these?)

We haven't yet defined what we mean by '$\text{Cov}(X, Y)$', we'll do this later when we discuss bivariate analysis

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## Definition

A **population** is a well defined group of subjects.

The population contains all the information on the underlying probability distribution

Subjects don't need to be people only

Examples

- Australian citizens
- kangaroos in Tidbinbilla
- leukocytes in the bloodstream
- protons in an atom
- lactobacilli in yogurt

## Definition

The **population size** $N$ is the number of subjects in the population.

We typically think that $N$ is 'very large'

In fact, it is so large that observing the entire population becomes impossible

Mathematically, we think that $N = \infty$, even though in many applications this is clearly not the case

Setting $N = \infty$ merely symbolizes that we are not able to observe the entire population

Example: population of Australian citizens

Clearly, $N = 23,667,800$

For all practical purposes it is so large that it might as well have been $N = \infty$

Example: kangaroos in Tidbinbilla

I have no idea how many kangaroos live in Tidbinbilla (therefore, I do not know the actual population size)

I could ask the park ranger, but suppose she also doesn't know

We treat the population size as unimaginable: $N = \infty$

The point is:

for some reason we are not able to observe the entire population

(too difficult, too big, too costly)

Instead, we only have a random sample of the population

## Definition

In a **random sample**, $n$ subjects are selected (without replacement) at random from the population.

Each subject of the population is equally likely to be included in the random sample.

Typically, $n$ is much smaller than $N$

Most importantly, $n < N \leq \infty$

The random variable for the $i$-th randomly drawn subject is denoted $Y_i$

## Definition

Because each subject is equally likely to be drawn and the distribution is the same for all $i$, the random variables $Y_1, \ldots, Y_n$ are **independently and identically distributed (i.i.d.)** with mean $\mu_Y$ and variance $\sigma_Y^2$.

We write $Y_i \sim \text{i.i.d.}(\mu_Y, \sigma_Y^2)$.

Given a random sample, we observe the $n$ realizations $y_1, \ldots, y_n$ of the i.i.d. random variables $Y_1, \ldots, Y_n$

What do we do with a random sample of i.i.d. data?

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In analogy to the mean of a population,
we define the mean of a subset of the population:

## Definition

The **sample average** is the average outcome in the sample:

$$\bar{Y} := \frac{1}{n} \sum_{i=1}^{n} y_i$$

Sometimes we call the sample average also the sample mean.

It should be obvious that this is a sensible definition

Let's say we are interested in learning about the weights of kangaroos in Tidbinbilla

We drive to Tidbinbilla and somehow randomly collect 30 roos and measure their weights

This will give us a random sample of size 30 of kangaroo weights

It's easy to calculate the average weight of these 30 roos

Suppose we obtain a sample average of 70kg

There is a huge difference between the population mean and the sample mean

There is only one population, therefore there is only one population mean

But there are many different random subsets (samples) of the population, each of which results in a (potentially) different sample average

Let's say we drive to Tidbinbilla for a second time, again randomly collect 30 roos and measure their weights

Should we expect to obtain a sample average of 70kg?

It is unlikely that the second time around we collect exactly the same 30 roos (while it is possible, it is not probable)

If we collect a different subset of 30 kangaroos, chances are that we come up with a different sample average

Suppose we obtain a sample average of 66kg

And now we collect a third random sample …

…and obtain a sample average of 75kg

And so forth …

This illustrates that the sample average itself is a random variable!

Random variables have statistical distributions

What distribution does the sample average have?

- what is its expected value?
- what is its variance?
- what is its standard deviation?
- what is its shape?

Let $Y_i \sim \text{i.i.d.}(\mu_Y, \sigma_Y^2)$ for all $i$

We don't know exactly which distribution generates the $Y_i$, but at least we know its expected value and its variance (turns out this is all we need to know!)

Each random variable $Y_i$ has

- population mean $\mu_Y$
- variance $\sigma_Y^2$

## Expected value

$$
\begin{aligned}
\mathrm{E}[\bar{Y}] &= \mathrm{E}\left[\frac{1}{n}\sum_{i=1}^{n} Y_i\right] \\
&= \frac{1}{n}\mathrm{E}\left[\sum_{i=1}^{n} Y_i\right] \\
&= \frac{1}{n}\sum_{i=1}^{n} \mathrm{E}[Y_i] \\
&= \frac{1}{n}\sum_{i=1}^{n} \mu_Y \\
&= \frac{1}{n} n \mu_Y \\
&= \mu_Y
\end{aligned}
$$

(all of this follows by the properties of expected values)

## Variance

$$
\begin{aligned}
\mathrm{Var}[\bar{Y}] &= \mathrm{Var}\left[\frac{1}{n}\sum_{i=1}^{n} Y_i\right] \\
&= \frac{1}{n^2}\mathrm{Var}\left[\sum_{i=1}^{n} Y_i\right] \\
&= \frac{1}{n^2}\sum_{i=1}^{n}\mathrm{Var}[Y_i] \\
&= \frac{1}{n^2}\sum_{i=1}^{n}\sigma_Y^2 \\
&= \frac{1}{n^2} n\sigma_Y^2 \\
&= \sigma_Y^2/n
\end{aligned}
$$

(all of this follows by the properties of variances, and realizing that $\mathrm{Cov}(Y_i, Y_j) = 0$ for $i \neq j$ (why?))

Standard deviation

$$\mathrm{StD}(\bar{Y}) = \sigma_Y / \sqrt{n}$$

(that's an easy one, given that we know the variance)

In summary, we have figured out these three *parameters* for the sample average:

- expected value is $\mu_Y$
- variance is $\sigma^2/n$
- standard deviation is $\sigma/\sqrt{n}$

Also, we understand that the sample average itself is a random variable

It therefore must have a statistical distribution, we write

$$\bar{Y} \sim \mathrm{P}(\mu_Y, \sigma_Y^2/n)$$

where P abbreviates some unknown statistical distribution

But what is the actual *distribution* P?

Is it binomial, normal, logistic, exponential, gamma, or what? (you do not need to know exactly what these are, just accept that they are different shapes of probability distributions)

Perhaps not too surprisingly, the *exact* distribution of $\bar{Y}$ depends on the distribution of the underlying components of $\bar{Y}$, i.e., the distribution of $Y_1, \ldots, Y_n$

In our fantasy, we'd like to be able to say something like this:

- if the underlying distribution of $Y_1, \ldots, Y_n$ is binomial, the resulting distribution of $\bar{Y}$ is also binomial

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- if the underlying distribution of $Y_1, \ldots, Y_n$ is binomial, the resulting distribution of $\bar{Y}$ is also binomial
- if the underlying distribution of $Y_1, \ldots, Y_n$ is normal, the resulting distribution of $\bar{Y}$ is also normal
- if the underlying distribution of $Y_1, \ldots, Y_n$ is logistic, the resulting distribution of $\bar{Y}$ is also logistic

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- if the underlying distribution of $Y_1, \ldots, Y_n$ is binomial, the resulting distribution of $\bar{Y}$ is also binomial
- if the underlying distribution of $Y_1, \ldots, Y_n$ is normal, the resulting distribution of $\bar{Y}$ is also normal
- if the underlying distribution of $Y_1, \ldots, Y_n$ is logistic, the resulting distribution of $\bar{Y}$ is also logistic
- if the underlying distribution of $Y_1, \ldots, Y_n$ is exponential, the resulting distribution of $\bar{Y}$ is also exponential

In our fantasy, we'd like to be able to say something like this:

- if the underlying distribution of $Y_1, \ldots, Y_n$ is binomial, the resulting distribution of $\bar{Y}$ is also binomial
- if the underlying distribution of $Y_1, \ldots, Y_n$ is normal, the resulting distribution of $\bar{Y}$ is also normal
- if the underlying distribution of $Y_1, \ldots, Y_n$ is logistic, the resulting distribution of $\bar{Y}$ is also logistic
- if the underlying distribution of $Y_1, \ldots, Y_n$ is exponential, the resulting distribution of $\bar{Y}$ is also exponential
- if the underlying distribution of $Y_1, \ldots, Y_n$ is gamma, the resulting distribution of $\bar{Y}$ is also gamma

- if the underlying distribution of $Y_1, \ldots, Y_n$ is normal, the resulting distribution of $\bar{Y}$ is also normal

Unfortunately, only this statement here is true