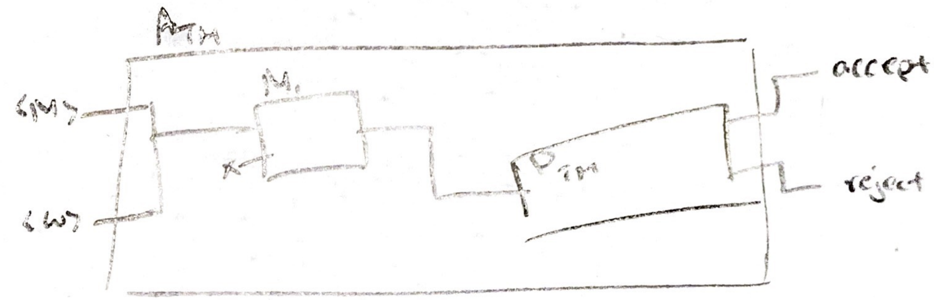
3. Palindrome_{TM} → accept at least one palindrome



we will prove by contradiction. Suppose P_{TM} is decidable. Then consider A_{TM} that takes in $\langle M, w \rangle$. Consider an M_1 that takes in M and input x. We say that M_1 will accept on the ~~emptystring~~ emptystring. That is, we simulate M on w. If N accepts w, M_1 will accept the empty string. If M rejects on w, then we reject on all string inputs x. This will then be sent to P_{TM}, which will reject accept, thus showing we can reduce A_{TM} to P_{TM}, so it is undecideable. We chose accept string as empty string, as it is also considered a palindrome, and we only need M_1 to accept a minimum 1 palindrome.

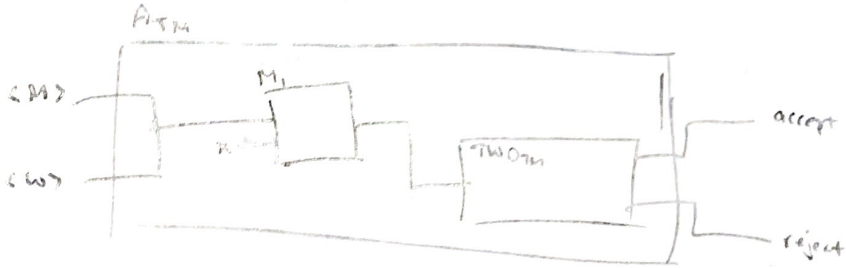
4. $HALTALL_{TM}$

We prove by contradiction. Let us assume HALTALL is decideable. Let us look at $HALT_{TM}$ as overarching, where we have M_1 as an intermediate TM, that takes in $\langle M, x \rangle$. Let us say that M_1 will take in its inputs, and remove x from the tape, consequently placing w onto the tape. This M_1 then runs M_1 (or actually M) on input x (which is now w). If M accepts w, then we say it accepts. Else, reject. This then gets pushed to $HALTALL_{TM}$. This works because on any input x, we are replacing it with w, so if M on w accepts, M_1 says it halts on any input w. With this reduction, we show HALT reduces to HALTALL and so both must be undecidable.

2. $TWO_{TM} = \{\langle M \rangle \mid M$ is a TM that accepts exactly 2 strings$\}$

We assume the above means that M will accept 2 distinct, particular strings, and not that it will take in 2 input strings.

Let us prove this is undecidable by contradiction.



First, we say that there is an A_{TM} that encompasses TWO_{TM}, that takes in $\langle M, w \rangle$. Let us construct M_1 TM such that it takes in $\langle M, x \rangle$. For M_1, we say it will accept if x is one of the two strings that TWO_{TM} will accept. We will also note that if x is not one of the 2 strings, then we reject. That is, how we have set it up, if A_{TM} accepts on $\langle M, w \rangle$, we say x will be an accepting string. If A_{TM} rejects on $\langle M, w \rangle$, we say x is any string but the accepting strings, so it will reject in TWO_{TM}. This reduces A_{TM} to TWO_{TM}, however, we know A_{TM} is undecidable, so TWO_{TM} must be as well.

Note, for the above M_1, let us set up the 2 accepted strings as '0' or '1'. Thus, upon acceptance of $\langle M, w \rangle$, x will be either or. Therefore, M_1 will be such that if x is 0 or 1, simulate M on w and accept if M accepts w. Else, reject. This is, if we reject $\langle M, w \rangle$, then M_1 will reject on our 2 strings, and then be rejected by TWO_{TM}.

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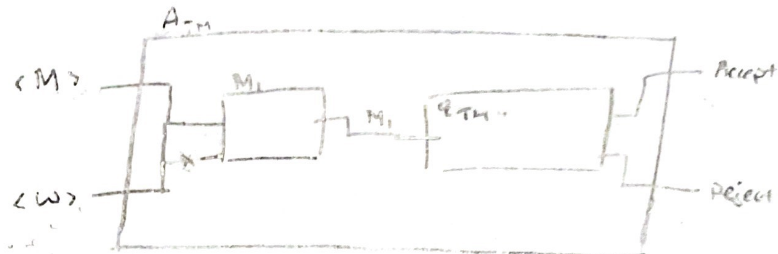
CS 301
HW 5

1.



We will prove E_{TM} is undecideable by contradiction. First, consider that E_{TM} can be decided. Then, we say that given a turing machine M_1, it will decide if it accepts or rejects on ε. Now, consider A_{TM} on input $\langle M,w\rangle$. Let us create a new TM M_1, such that it takes in the input $\langle M,x\rangle$ and then decides if it will accept ~~M~~ on x. That is, if M has $x \neq \varepsilon$, then we reject and generate M_1 to push into E_{TM}. If $x \neq \varepsilon$ is false, we simulate M on w and then if M accepts w, we generate an M_1 to push into E_{TM}. Here, we show a reduction from A_{TM} to E_{TM}, but we know A_{TM} cannot be decideable, so E_{TM} must be undecideable.

Note in the above that the way it is set up, we have it that M_1 will effectively accept ε only if M accepts on input w. We are constraining it such that By setting up M_1 such that it will accept on acceptance of $\langle M,w\rangle$, we are trying to push the A_{TM} to be isolated to the ε. On the case that M rejects w, then x must not be ε, giving us our possible above scenario.