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HW 2 - 411

① Cryptarithmetic

X_2 X_1

O N E

\+ O N E

T W O

Constraints

$E + E = O + X_1 \cdot 10$ $\quad O, T \neq \emptyset$

$N + N + X_1 = W + X_2 \cdot 10$

$O + O + X_2 = T$

$E \neq O \neq N \neq T \neq W$

Domains

$X_2, X_1 = \{0, 1\}$

$E, O, N, T, W = \{1, 9\}$

Variables

E, O, N, T, W, X_1, X_2

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1b

$E = 1$

↓

$O = 2 \quad X_1 = \emptyset$

$T = \{3,4,5,6,7,8,9\}$

$N = \{3,4,5,6,7,8,9\}$

$W = \{3,4,5,6,7,8,9\}$

$T = 2 + 2 + X_2$

$N + N = W + X_2 \cdot 10$

$E = 1$

↓

$O = 2, X_1 = \emptyset$

↓

$N = 3, W = 6, X_2 = \emptyset$

↓

$T = 4$

$E = 1$
$O = 2$
$N = 3$
$W = 6$
$T = 4$
$X_1 = \emptyset$
$X_2 = \emptyset$

$$\begin{array}{r} 2\ 3\ 1 \\ 2\ 3\ 1 \\ \hline 4\ 6\ 2 \end{array}$$

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(2) $X < Y + Z$

- Let us introduce a new variable V_1 which contains a domain of all X, Y, Z possible values
- Let the domain of X, Y, Z be any such values that the above formula holds true
- Then, we can say $V = (X, Y, Z)$

where V_1 denotes the X value,

V_2 denotes the y value

V_3 denotes the z value

We can then rewrite the above constraint into 3 binary constraints in the following way:

$X < V_2 + V_3$

$V_1 < V_2 + Z$

$V_1 < Y + V_3$

This way, we can reduce the constraints to binary via using a more holistic variable that can be drawn upon any X, Y, Z

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(3)(a) Let us consider that D_i and D_j are the domains of current X_i and X_j, and that arc (X_i, X_j) is not currently on the queue. Then, let us consider a case where on the queue, we pop off an arbitrary (X_j, X_{kl}) and check this arc. There are 2 ways this can go.

① The arc is consistent, we move on to the next item in the queue

② The arc was inconsistent, we make it consistent and remove from X_j, and then add any arcs where X_j is the head, This includes (X_i, X_j), then we move to the next in queue

Thus, we see that for any loop, we only introduce (X_i, X_j) to the queue when the D_j is insufficient for consistency. Also, D_i will stay consistent unless we have any changes to D_j, since any removal from D_i if D_i, D_j is already consistent cannot make it inconsistent. Thus, by induction, we can prove that so long as (X_i, X_j) is not on the queue, we cannot have arc inconsistency for X_i, X_j with D_i, D_j

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(3b) We have just checked (X_i, X_j) and have made it arc-consistent. That means for current D_i, D_j we have satisfied consistency (per part a). Then, any such constraint between X_i and X_j are now fulfilled or satisfiable.

Let us consider an issue is present in (X_j, X_i) and this has been added onto the queue. When this is checked, we find such that the arc is not consistent due to values in D_j. However, note we already just checked (X_i, X_j), meaning any values in D_i that would have made it inconsistent would have been removed. Then, (X_j, X_i) would not find any issue even if it came to the queue.

Thus, proof by contradiction shows us that detection in (X_i, X_j) enforces future consistency in

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(4) (a) With $k=1$, beam search will choose the most optimal of all the paths that it sees locally. And follow it, regardless of whether final soln is local/global best soln

(b) With infinite possible k, beam will look at every single enumerable possibility at every step, kind of like BFS

(c) $p(x) \propto e^{\frac{E(x)}{T}}$

If $T=0$ and we don't terminate on it, then we will only move when $\Delta E > 0$ (better state is available), but only locally

(d) If $T = \infty$, then we will approach areas where our ΔE shows promise in the next step. However, we ~~can't~~ will always also move even if it is not promising since $e^{1/\infty} = 1$ thus we will permanently be jumping around the successors.

$(F \vee D) \wedge (F \vee \neg D) \wedge (\cancel{F} \vee \neg(C \vee F) \wedge (\cancel{F} \vee C) \wedge (\neg F \vee \cancel{F})$

$D = T$ } contradiction, cannot be both T and F
$D = F$ }

so unsatisfiable

5. (a) we tackle this using the 2nd derivative test

observe $c_i x_i + d_i x_i^2$

where $d_i \geq 0$

Given the definition per class, a quadratic formula such that d_i, or the constant near x_i^2 is positive, is a convex function, we know that the summation function is convex. Also, given the constraint bounds, linear inequality constraints are representative of a convex feasible set. Therefore, this is a convex program.

(b) we know that f and g are convex. f being convex means the function is convex. Because h is affine, and per the lecture slides we know linear equality constraints are convex, h is convex. Then, because g and h are convex, there are convex constraints, so the feasible set is convex. Then, the whole is a convex program.

Thus, if unsatisfaibity probability < 1, then there must be some nonzero probability that it is satisfiable. Thus, there must be a satisfying assignment per the probabilistic method.

b) Consider where $k=1$, and $m=2$

$(A) \wedge (\neg A)$

This is unsatisfiable, where $m = 2^k$

(6a) MMS

$\forall i \in n, j \in M, \quad x_{ij} \in \{0,1\}$ # either own or don't own

$\forall j \in M, \sum_{i \in N} x_{ij} = 1$ # only one person owns an item

$\forall i \in N \sum_{j \in M} v_{ij} x_{ij} \geq MMS(i)$

6b

Objective: max (d)

S.t.

$\forall_{i \in M, T_j \in \Pi_i}$ $X_{i,T_j} = \{1,0\}$ # items are in/not in each subset T

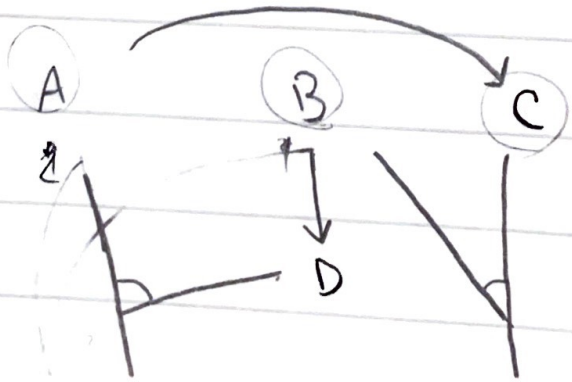
$\forall_{T_i \in \Pi, T_j \in \Pi, T_i \neq T_j}$ $V_i(T_j) \leq V_i(T_i)$ # $V_i(T_j)$ is the min of subsets

$d \geq V_i T_j$

$d \leq V_i T_j$

$A \wedge B \wedge (B \Rightarrow D) \wedge ((A \wedge D) \Rightarrow E) \wedge ((B \wedge C) \Rightarrow F) \wedge (A \Rightarrow C) \wedge (F \Rightarrow G) \models G$

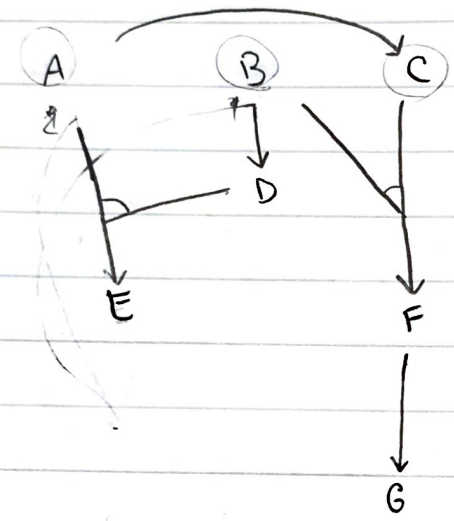
(a)



$KB \models \alpha$ because $KB \wedge \neg\alpha$ is unsatisfiable since $\neg G$ means $(F \Rightarrow G)$ is false, so

① $A \wedge B \wedge (B \Rightarrow D) \wedge ((A \wedge D) \Rightarrow E) \wedge ((B \wedge C) \Rightarrow F) \wedge (A \Rightarrow C) \wedge (F \Rightarrow G) \models G$

(a)



KB ⊨ α because KB ∧ ¬α is unsatisfiable since ¬G means (F ⇒ G) is false, so KB would have to be false

(b) A, B, C, D, F, E, G

(c) G, F, B, C, A

(d)

$A \wedge B \wedge (B \vee \neg D) \wedge ((A \wedge D) \vee \neg E) \wedge ((B \wedge C) \vee \neg F) \wedge (A \vee \neg C) \wedge (F \vee \neg G) \wedge \neg G$

$A \wedge B \wedge (\neg B \vee D) \wedge (\neg A \vee \neg D \vee E) \wedge (\neg B \vee \neg C \vee F) \wedge (\neg A \vee C) \wedge (\neg F \vee G) \wedge \neg G$ is unsatisfiable

$\neg G = T$, $G = F$

$\neg F = T$ $F = F$

$\neg B \vee \neg C = T$

$\neg B = F$

$\neg C = T$

$C = F$

$A = T$ $B = T$

$\neg B \vee D = T$

$D = T$

$\neg A \vee \neg D \vee E$

$F \vee F \vee E$

$E = T$

But, $(\neg A \vee C) \Rightarrow F \vee F$, which is false, so unsatisfiable

(e) $E = T$ (pure), $A = T$ (unit), $B = T$ (unit), $\neg G = T$, $G = F$

next: $D = T$, $C = T$

(e) $E = T$ (pure), $\neg G = T$ $(G = F)$

$A = B = T$ (unit)

$\neg(F \vee D) \wedge (F \vee \neg D) \wedge (F \vee \neg(C \vee F) \wedge (F \vee C) \wedge (\neg F \vee F)$

$D = T$ } contradiction, cannot be both T and F

$D = F$ } so unsatisfiable

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(8)

(a) Let us consider that for a clause, if we have specifically k literals, we can generate upto 2^k unique clauses - this is the enumeration of all T/F of k literals in a clause. Then, we can say the following -

if we have $(K_1 \vee K_2 \vee \dots K_k)$, we will find the eventual enumeration of $(\neg K_1 \vee K_2 \vee \dots \neg K_k)$ which is the opposite. This means at 2^k we may find ourselves unsatisfiable.

Also, for every clause, consider k elements with up to 2^k possible truth assignments (T/F). Every CNF clause is literals with or statements, so the only way to have the whole clause is if every literal is false, which is 1 of 2^k possible assignments. Then, the probability of being unsatisfactory is $1/2^k$ per clause. Figure max clauses have this, where $m < 2^k$. Then, we have a probability that is < 1 since $1/2^k \cdot m$ where $m < 2^k$. Per Bode's inequality, the probability that any one of

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these instances occuring is less than the sum total, which we just showed is < 1.

Thus, if unsatisfiability probability < 1, then there must be some nonzero probability that it is satisfiable. Thus, there must be a satisfying assignment per the probabilistic method.

b) Consider where $k=1$, and $m=2$

$(A) \wedge (\neg A)$

This is unsatisfiable, where $m = 2^k$

6a) MMS

$\forall i \in n, j \in M, \quad x_{ij} \in \{0,1\}$ # either own or don't own

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$\forall i \in N \quad \sum v_{ij} x_{ij} \geq MMS(i)$