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Homework 4 - CS 411

① a) ~~$\theta =$ (Circle, Blue | 1) $= 1/5$ (Circle, Blue | 0) $= 1/5$~~

~~(Diamond, Red | 1) $= 1/5$ (Circle, Green | 0) $= 1/5$~~

~~(Square, Blue | 1) $= 1/5$ (Diamond, Blue | 0) $= 1/5$~~

~~(Square, Red | 1) $= 2/5$ (Diamond, Green | 0) $= 2/5$~~

(Circle | 1) $= 1/5$ (Blue | 1) $= 2/5$

(Circle | 0) $= 2/5$ (Blue | 0) $= 2/5$

(Diamond | 1) $= 1/5$ (Green | 1) $= 0/5$

(Diamond | 0) $= 3/5$ (Green | 0) $= 3/5$

(Sq | 1) $= 3/5$ (Red | 1) $= 3/5$

(Sq | 0) $= 0/5$ (Red | 0) $= 0/5$

$P(1) = 1/2$

b) (Shape | Color) $= \frac{0}{0} =$ (Circle, Red)...

c) (Circle | 1) $= 2/8$ (Blue | 1) $= 3/8$

(Circle | 0) $= 3/8$ (Blue | 0) $= 3/8$

(Diamond | 1) $= 2/8$ (Green | 1) $= 1/8$

(Dia | 0) $= 4/8$ (Green | 0) $= 4/8$

(Sq | 1) $= 4/8$ (Red | 1) $= 4/8$

(Sq | 0) $= 1/8$ (Red | 0) $= 1/8$

d) $1/8$

(2) $P_r[X = 2^k] = \frac{1}{2^k}$

a) $E[X] = \infty$

$E[x] = \sum_{i=1}^{k} x_i p_i$

$= (2^1)(\frac{1}{2}) + (2^2)(\frac{1}{2^2}) + \dots (2^k)(\frac{1}{2^k})$

$= 1 + 1 + \dots 1$

As k approaches ∞, then $\sum_{i=1}^{\infty} 1$

Thus $E[X] = \infty$

b) utility of x dollars is $\log_2 x$

Maximum to play the lottery?

$EMV(L_k) = (\frac{1}{2})(L_{k+1}) + (\frac{1}{2})(2^k)$

$U(L) = (\frac{1}{2})\log_2(L_{k+1}) + (\frac{1}{2})\log_2(2^k)$

$= \frac{1}{2}\log_2(L_{k+1}) + \frac{1}{2}(k)$

$EMV = \frac{1}{2}(2^1) + (\frac{1}{2})((\frac{1}{2})(2^2) + ((\frac{1}{2})(2^3 + \dots)))$

$\frac{1}{2}(2) + \frac{1}{4}(2^2) + \frac{1}{8}(2^3) + \dots$

$U(L) = (\frac{1}{2})(\log_2(2^1) + (\frac{1}{2})((\frac{1}{2})\log(2^2) + \dots$

$\frac{1}{2}(1) + (\frac{1}{4})(2) + (\frac{1}{8})(3)$

$\frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \dots = 1 +$ infinite dec series $\approx 1 + 1$

$\approx \$2$

Makes sense since our smallest payout is $2^1 = 2$, so we at minimum break even.

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P_2

③

P_1	4 L	M	2 R	
U	5,0	1,3	4,0	
~~C~~	~~2,4~~	~~1,4~~	~~2,5~~	1
~~D~~	~~0,1~~	~~1,0~~	~~2,0~~	3

1 - Center row is dominated by U for P_1

2 - Right col is dominated by M for P_2

3 - Down row is dominated by U for P_1

4 - Left col is dominated by M for P_2

Nash equilibrium is then (1, 3)

④ a) $\begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}$ probability

This is b/c this is intuitively best for both parties, but this is not possible via pure / mixed nash equilibria

b) $\forall_p \forall_{p'} \sum P(s_1, s_2) u_1(s_1, s_2) \geq \sum P(s_1, s_2) u_1(s_1', s_2)$

$\forall_s \; P(s_1, s_2) \geq 0$

$\forall_s \sum P(s_1, s_2) = 1$

* For all players p and p', they like s_1 and s_2 more than any other strategy

* Probability of all s_1, s_2 is at least 0

* Sum of probabilities of strats = 1 for p, p'

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5. Players 1 and 2 have value $v = [0,1]$

winner utility $= v - p$; loser utility $= \emptyset$

a) Assume player 2 price P_2 in range $[0, 0.5]$

Player 1 has value v_1

Player 2 is bidding at most max value $\frac{1}{2} V_2$

P_1 wins when $\frac{1}{2} V_2 \leq P_1$ $\quad V_2 \leq 2P_1$

$E[U_1] = P(\text{win})(v_1 - P_1) + P(\text{lose})(\emptyset)$

$= P(\text{win})(v_1 - P_1)$

$= \int_0^{2P_1} (v_1 - P_1) dv_2 = (v_1 - P_1)(v_2) \Big|_0^{2P_1}$

$= (2P_1)(v_1) - (2P_1)(P_1) + 0 + 0$

$= (2P_1)(v_1 - P_1)$ derive by P_1

$= 2P_1 v_1 - 2P_1^2$

$= 2v_1 - 4P_1 = 0$ $\quad \frac{1}{2} v_1 = P_1$

Therefore, P_1 should be $\frac{1}{2}$ value v

b) In this case, this is a Nash equilibrium.

Because we know $P_2 = [0, 0.5]$, then Player 1 never wants to bid more, since at most they want to bid 0.5 ($\frac{1}{2}$ max value). For Player 2, we do not want to deviate because if they assume the same strat is being played, if they increase their bid they will win but with a loss in utility.