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411 Homework 3

(1)

	a	b	c
a	0.2	0.2	0.6
b	0.8	0.2	0
c	0	0.7	0.3

$P(S_3 = a)$

b) $P(S_1 = a) = 1$ $\quad P(S_2 = a) = 0.2 \quad 0.2$

$P(S_2 = b) = 0.2 \quad 0.8$

$P(S_2 = c) = 0.6 \quad 0$

$(.2)(.2) + (0.2)(0.8)$

$= 0.20$

c) Stationary distribution when $\pi = \pi(P)$

$$\begin{cases} 0.2\pi(a) + 0.8\pi(b) + 0\pi(c) = \pi(a) \\ 0.2(a) + 0.2(b) + 0.7\pi(c) = \pi(b) \\ 0.6a + 0(b) \; 0.3\pi(c) = \pi(c) \end{cases}$$

$a + 4b = 5a \qquad 4b = 4a$

$2a + 2b + 7c = 10b \qquad 2a + 7c = 8b$

$6a + 3c = 10c \qquad 6a = 7c$

$a = b \qquad 7c = 6b \qquad 6a = 7c$

$c = \frac{6}{7}a$

$a + b + c = 1$

$2a + c = 1$

$\frac{20}{7}a = 1$

$a = \frac{7}{20}$

$b = 7/20$

$c = \frac{6}{20}$

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2. Blue = 90% of the cabs

Green = 10% of the cabs

① witness identified accident as Blue

↳ blue as blue 70%

↳ green as green 80%

What is probability the cab was Blue?

$P(A|B) = P(\text{cab was blue} \mid \text{witness said blue})$

$P(A) = \cancel{95}\% \ (0.90)$

$P(B) = P(B|A)P(A) + P(B|\neg A)P(\neg A)$

$= (0.70)(0.90) + (0.2)(0.10) = 0.65$

$P(B|A) = (0.70)$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

$= (0.7)(0.90) / (0.65) = 0.969$

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③ The product rule is $P(A,B) = P(A|B)\,P(B)$

Base Case: $P(X_1, X_2) = P(X_1|X_2)\,P(X_2)$ per product rule

Let us now consider $P(X_1 \ldots X_n) = P(X_1|X_2 \ldots X_n)\,P(X_2 \ldots X_n)$

We assume the following, which is the chain rule:

$$P(X_1 \ldots X_n) = P(X_1|X_2 \ldots X_n)\,P(X_2 \ldots X_n)$$

$$= \prod P(X_i|X_1, \ldots, X_{i-1}) = \prod_{k=1}^{n} P(X_k|\bigcap_{j=1}^{k-1} X_j)$$

Let us attempt to prove for X_{n+1}

$$P(X_1 \ldots X_{n+1}) = P(X_{n+1}|X_1 \ldots X_n)\,P(X_1 \ldots X_n) \quad \text{Product Rule}$$

$$\prod P(X_{n+1}|X_1 \ldots X_n) = P(X_{n+1}|X_1 \ldots X_n)\,P(X_1 \ldots X_n) \quad \text{assumption}$$

$$P(X_{n+1}|X_1 \ldots X_n)\prod P(X_n|X_1 \ldots X_{n-1}) = P(X_{n+1}|X_1 \ldots X_n)\,\underbrace{P(X_1 \ldots X_n)}_{\prod P(X_n|X_1 \ldots X_{n-1})} \quad \text{assumption}$$

Thus, by induction, we have shown the equality of left and right sides and prove the correctness.

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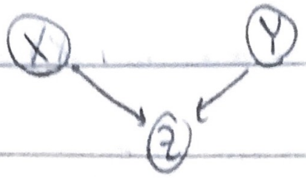
4. X, Y, Z are random variables

(a) X, Y are unconditionally independent

Per unconditional independence, we have:

$$P(X,Y) = P(X)P(Y)$$

We can consider the 2 following BNs:



This implies X, Y are conditionally dependent given Z,

however, we can also have:

The above implies no relation between X, Y, or Z. Thus, we cannot always assume new Z will make X, Y conditionally independent

(b) X, Y conditionally independent given Z

The definition is thus $P(X,Y|Z) = P(X|Z)P(Y|Z)$

In class, we were given 4 cases of 3 configurations that show conditional independence given Z. They are the following:

$A \rightarrow B \rightarrow C$ where B not observed

$A \leftarrow B \rightarrow C$, where B not observed

$A \rightarrow B \leftarrow C$ B or descendent observed

$\quad\;\; D$

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Now consider that we implement the "Common Effect" structure like so:

$$\begin{tikzcd} X \arrow[dr] & & Y \arrow[dl] \\ & Z & \end{tikzcd}$$

Then, per lecture, given that Z is observed, we find that these two events X and Y are not unconditionally independent of each other, at least, not always assumed. By counterexample, we dispel this notion.

5. a) $P(F, G) = P(F)P(G)$

False, FMH & MHG are active pathing, so not guaranteed

b) $P(A, T) = P(A)P(T)$

True, The two paths AMRT and AMHT are both inactive

c) $P(A, T \mid R, G) = P(A \mid R, G) P(T \mid R, G)$

False, G being observed makes AMHT active

d) $P(F, T \mid R) = P(F \mid R) P(T \mid R)$

False, the path FMH is active, so no independence

e) $P(A, M \mid G) = P(A \mid G) P(M \mid G)$

False, M directly after A, so is affected by A

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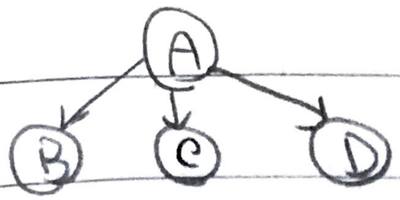

A.

B, C, D all depend on A

However, given A, all 3 are independent

- A only needs four parameters, technically 5 values

$A = 4$

$B, C, D = O(4 \cdot 3 \cdot 5) = 60$

$60 + 4 = 64$

B.

B, C, D all have 4 parameters, tech 5 values

A has 4 values, dependent on B, C, D

$B, C, D = 4 + 4 + 4$

$A = O(4 \cdot 1 \cdot 5^3) = 4 \cdot 125 = 500$

$12 + 500 = 512$