

Part 1:

Exercise 1:

CPU time: 0.011s

SAT -1 -2 3 -4 -5 -6 -7 -8 -9 10 11 -12 -13 -14 -15 -16 -17 -18 19 -20 -21 22 -23 -24 -
25 -26 -27 -28 -29 30 31 -32 -33 -34 -35 -36 -37 -38 39 -40 -41 42 -43 -44 -45 -46 -
47 48 -49 -50 51 -52 -53 -54 -55 -56 57 -58 -59 -60 -61 -62 -63 64 -65 -66 -67 68 -69
-70 -71 -72 -73 -74 75 76 -77 -78 -79 -80 -81 82 -83 -84 -85 -86 -87 88 -89 -90 -91 -
92 -93 94 -95 -96 -97 -98 -99 100 -101 -102 -103 -104 105 106 -107 -108 -109 -110
-111 -112 -113 114 -115 -116 -117 118 -119 -120 -121 122 -123 -124 -125 -126 -
127 128 -129 -130 -131 -132 -133 -134 -135 -136 137 -138 139 -140 -141 142 -143
-144 -145 -146 -147 148 149 -150 151 -152 -153 154 -155

A. Japanese owns the zebra (31)

B. Japanese drinks coffee (106). Japanese lives in green house (57)

Exercise 2:

A. Create a clause that basically states (And.. not this satisfiability solution). This basically means that I am going to create an inverse of the satisfiability to state that it is NOT the existing satisfiability solution.

B. 1 2 -3 4 5 6 7 8 9 -10 -11 12 13 14 15 16 17 18 -19 20 21 -22 23 24 25 26 27 28
29 -30 -31 32 33 34 35 36 37 38 -39 40 41 -42 43 44 45 46 47 -48 49 50 -51 52 53
54 55 56 -57 58 59 60 61 62 63 -64 65 66 67 -68 69 70 71 72 73 74 -75 -76 77 78 79
80 81 -82 83 84 85 86 87 -88 89 90 91 92 93 -94 95 96 97 98 99 -100 101 102 103
104 -105 -106 107 108 109 110 111 112 113 -114 115 116 117 -118 119 120 121 -
122 123 124 125 126 127 -128 129 130 131 132 133 134 135 136 -137 138 -139
140 141 -142 143 144 145 146 147 -148 -149 150 -151 152 153 -154 155 0

C. CPU time: 0.009s UNSAT

Exercise 3:

A. Under the rule that says "82 0" I modified to say "82 81 0" to account for 82 or 81, which is the Japanese will smoke Parliament or Old Gold

B.CPU time: 0.012s

SAT -1 -2 -3 -4 5 -6 -7 8 -9 -10 11 -12 -13 -14 -15 -16 -17 -18 19 -20 -21 22 -23 -24 -
25 26 -27 -28 -29 -30 -31 -32 -33 -34 35 -36 -37 -38 39 -40 -41 42 -43 -44 -45 -46 -
47 48 -49 -50 51 -52 -53 -54 -55 -56 -57 58 -59 -60 -61 -62 -63 64 -65 -66 67 -68 -69 -
-70 -71 -72 -73 -74 75 -76 -77 -78 79 -80 81 -82 -83 -84 -85 -86 -87 88 -89 -90 -91
92 -93 -94 -95 -96 -97 -98 -99 100 -101 -102 103 -104 -105 -106 -107 -108 -109
110 -111 -112 -113 114 -115 116 -117 -118 -119 -120 -121 122 -123 -124 -125 -
126 -127 -128 -129 -130 -131 132 -133 -134 -135 -136 137 138 -139 -140 -141 -
142 143 -144 -145 -146 -147 148 -149 -150 151 152 -153 154 -155

This is a different truth assignment due to clause 3 (Englishman in house 2). This was originally true but under the modified solution, is now false.

Exercise 4:

A. My step in figuring out the remaining non-unique solutions is to include the new satisfiability rules as a "And ... not this" rule. For instance, under exercise 3, the englishman did not live in house 0. I would then include a rule that says "And the englishman did live in house 0." I will be inverting all of the clauses (effectively repeating step 2 until there is no more satisfiability).

B.

SAT -1 -2 3 -4 -5 -6 -7 -8 -9 10 11 -12 -13 -14 -15 -16 -17 -18 19 -20 -21 22 -23 -24 -
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47 48 -49 -50 51 -52 -53 -54 -55 -56 57 -58 -59 -60 -61 -62 -63 64 -65 -66 -67 68 -69 -
-70 -71 -72 -73 -74 75 -76 77 -78 -79 -80 81 -82 -83 -84 -85 -86 -87 88 -89 -90 -91 -
92 -93 94 -95 -96 -97 -98 -99 100 -101 -102 -103 -104 105 106 -107 -108 -109 -110
-111 -112 -113 114 -115 -116 -117 118 -119 -120 -121 122 -123 -124 -125 -126 -
127 128 -129 -130 -131 -132 -133 -134 -135 -136 137 -138 139 -140 -141 142 -143
-144 -145 -146 -147 148 149 -150 151 -152 -153 154 -155

SAT -1 -2 3 -4 -5 -6 -7 -8 -9 10 11 -12 -13 -14 -15 -16 -17 -18 19 -20 -21 22 -23 -24 -
25 -26 -27 -28 29 -30 -31 -32 -33 -34 35 36 -37 -38 -39 -40 -41 42 -43 -44 -45 -46 -
47 48 -49 -50 51 -52 -53 -54 -55 -56 57 -58 -59 -60 -61 -62 -63 64 -65 -66 -67 68 -69 -
-70 -71 -72 -73 -74 75 -76 77 -78 -79 -80 81 -82 -83 -84 -85 -86 -87 88 -89 -90 -91 -
92 -93 94 -95 -96 -97 -98 -99 100 -101 -102 -103 -104 105 106 -107 -108 -109 -110
-111 -112 -113 114 -115 -116 -117 118 -119 -120 -121 122 -123 -124 -125 -126 -
127 128 -129 -130 -131 -132 -133 -134 -135 -136 137 -138 139 -140 -141 142 -143
-144 -145 -146 -147 148 149 -150 151 -152 -153 154 -155

SAT -1 -2 -3 -4 5 -6 -7 8 -9 -10 11 -12 -13 -14 -15 -16 -17 -18 19 -20 -21 22 -23 -24 -
25 -26 -27 -28 29 -30 -31 -32 -33 -34 35 36 -37 -38 -39 -40 -41 42 -43 -44 -45 -46 -
47 48 -49 -50 51 -52 -53 -54 -55 -56 -57 58 -59 -60 -61 -62 -63 64 -65 -66 -67 -68 -69 -
-70 -71 -72 -73 -74 75 -76 -77 -78 79 -80 81 -82 -83 -84 -85 -86 -87 88 -89 -90 -91 -

92 -93 -94 95 -96 97 -98 -99 -100 -101 -102 103 -104 -105 -106 -107 -108 -109 110
-111 -112 -113 114 -115 116 -117 -118 -119 -120 -121 122 -123 -124 -125 -126 -
127 -128 -129 -130 -131 132 -133 -134 -135 -136 137 138 -139 -140 -141 -142 143
-144 -145 -146 -147 148 -149 -150 151 152 -153 154 -155

SAT -1 -2 -3 -4 5 -6 -7 -8 9 -10 11 -12 -13 -14 -15 -16 -17 18 -19 -20 -21 22 -23 -24 -
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47 48 -49 -50 51 -52 -53 -54 -55 -56 57 -58 -59 -60 -61 -62 -63 64 -65 -66 -67 68 -69
-70 -71 -72 -73 -74 75 -76 -77 -78 79 -80 81 -82 -83 -84 -85 -86 -87 88 -89 -90 -91
92 -93 -94 -95 -96 -97 -98 -99 100 -101 -102 103 -104 -105 106 -107 -108 -109 -
110 -111 -112 -113 114 -115 -116 -117 -118 -119 120 -121 122 -123 -124 -125 -
126 -127 -128 -129 130 -131 -132 -133 -134 -135 -136 137 -138 139 -140 -141 -
142 -143 -144 145 146 -147 -148 -149 -150 151 -152 -153 154 155

Englishman can own: Zebra and Fox

Japanese can own: Snail

Norwegian can own: Zebra and Fox

Spaniard can own: Dog

Ukranian can own: Horse

Problem 5

$$(\forall X)(\exists Y)(\text{loves}(X, Y))$$

Domain: All persons living on earth

a) True: Everyone in this world loves somebody

False: There exists one person that loves no one

$$b) p(X) \rightarrow \neg p(X) \quad (\neg p(X) \vee \neg p(X))$$

Domain: All X

True: $p(X)$ is false

False: $p(X)$ is true

$$c) (\exists X)p(X) \rightarrow \forall X p(X)$$

Domain: All players on soccer team

True: If one team member got cake, everyone gets cake

False: someone ^{got} cake, but someone else on the team did not get cake

$$d) (p(X, Y) \wedge p(Y, Z)) \rightarrow p(X, Z)$$

Domain: All integers

True: $p(A, B)$ is true when $A < B$

False: X is less than Y and Y is less than Z , so X is less than Z

False: $Z - X = 1$

There is no integer between X and Z such that it is

greater than X but less than Z

Problem 6

$$a) \neg ((\forall X)(\exists Y)p(X, Y))$$

$$(\exists X)(\forall Y) \neg p(X, Y)$$

$$b) \neg ((\forall X)p(X) \vee (\exists Y)q(X, Y))$$

$$(\exists X)\neg p(X) \wedge (\forall Y)\neg q(X, Y)$$

Problem 7

Is E a tautology when $(\exists x)E$ is a tautology?

Yes. Consider when expression E is accounting for multiple variables such as x, y, z . Then, regardless of y and z , if E is a tautology, this is still a tautology for $(\exists x)E$. That then means that it must be a tautology even without the specific $\exists x$, and we can then assume a different x . This then continues to hold true until we reach that $\forall x E$ is a tautology.

Problems

1 Actor X was in movie 1 with actor Y

2 Actor Y was in movie 2 with actor Z

$$\forall x \exists_{M_1, M_2} [C(x, M_1) \wedge C(y, M_1) \wedge C(y, M_2) \wedge C(z, M_2)]$$

The above is also true if it is in less than

2 hops, as then $x = y$