Analyzing Helicopter Emergency Transport in Upstate New York

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I. Executive Summary

Our team was tasked to determine where helicopters and their crews should be based around the upstate New York region to best respond to emergency calls. We conducted a simulation analysis using a historical dataset from the existing operation to make this determination. We found that the optimal locations for helicopters in the upstate area were in Albany, Rochester, Syracuse, Buffalo, and Binghamton, and Watertown, with 11 helicopters spread across those cities.

Optimal Placement of Helicopters Around Upstate New York		
Helicopter Number	Base Location	Latitude Longitude Coordinates
1	Albany	42.65 N, 73.76 W
2	Albany	42.65 N, 73.76 W
3	Rochester	43.16 N, 77.61 W
4	Syracuse	43.05 N, 76.15 W
5	Syracuse	43.05 N, 76.15 W
6	Syracuse	43.05 N, 76.15 W
7	Syracuse	43.05 N, 76.15 W
8	Buffalo	42.89 N, 78.88 W
9	Buffalo	42.89 N, 78.88 W
10	Binghamton	42.10 N, 75.92 W
11	Watertown	43.97 N, 75.91 W

These locations and helicopter assignments are the most optimal combination in terms of average response time and cancellation rate, taking into account the cost of deploying a helicopter and its crew at a location. We concentrated our helicopter bases around the cities with hospitals that historically have tended to most calls (Albany, Rochester, Syracuse, Buffalo, Binghamton), as well as Watertown to ensure enough coverage in the upstate area. We emphasized the importance of a low average response time of less than one hour, so that the emergency calls would be attended to in a timely manner. We also wanted to ensure that the majority of calls across the upstate area would be attended to and that those outside of major cities would not have to cancel their emergency call. In our distribution of cities and helicopters, the average cancellation rate is only 8%. Our distribution of cities and helicopters also emphasize that the majority of the time, helicopters are being utilized, with an average utilization of around 2 calls per helicopter per day.

However, we wanted to make sure that helicopters were not being placed in excess, which would incur additional implementation costs. We at first limited our optimal solution to 10 helicopters and five base locations, excluding the Watertown base. However, an additional simulation that we conducted, to ensure a greater coverage of the upstate area by adding a 6th base in Watertown, yielded a faster response time by 2 minutes. Although an additional base would yield more implementation costs, the 6 base locations resulted in an improvement of our metrics. Thus, we ultimately recommend the 6th base to be included at Watertown, with a total of 11 helicopters across the region.

This combination was generated from a simulation model based on historical data. We analyzed the data to understand the distribution of data within the simulation, and developed a model to simulate calls occurring across a years worth of time.

Going forward, a further analysis should be conducted that implements more details surrounding helicopter operations as well as the feasibility of locations and developing hospital bases. This initial model relies on assumptions about constant helicopter base and crew operations, which can be modelled in further detail. Similarly, we considered all locations in upstate New York to be able to feasibly handle any number of helicopters, which realistically may be more constrained. Ultimately our solution is an optimal solution when looking at average response time and cancellation time, while prioritizing high-call-attending hospitals and coverage of the upstate New York area, utilizing a simplified model of helicopter operations.

II. Problem Description

Our team was tasked with re-designing and optimizing helicopter transport operations in the upstate New York region, by completing a simulation analysis of the situation. The current operation follows a set procedure in which a first responder arrives on the scene and determines if helicopter transport is needed. If deemed necessary, a call is placed to helicopter dispatch (HD) and they determine if it's safe to fly and if a helicopter is available within range. Afterwards, a helicopter is dispatched to the scene, but some flights are cancelled en-route, such as if a patient dies, and upon cancellation they return to their base. If the helicopter makes it to the scene, a determination is made for which hospital to fly to, and the receiving facility is not always the closest trauma center, since some injuries require specialized care. After transporting the patient to the hospital, the helicopter then returns to its base.

In order to complete this re-design, we determined the optimal number of helicopters that need to be deployed and their locations to ultimately minimize the average response time while emphasizing cancellation rate, utilization, and cost. We limited the scope of our simulation model to 12 helicopters and only 5 base locations, to minimize the scope as well as the implementation costs of our solution. However, we experimented with the number of helicopters and increased the number of base locations to 6, to ensure that our solution was optimal. To assess the optimality of our solution, we examined the following performance measures.

The percentage of calls dispatched measures the percentage of calls in which a helicopter is actually sent to a location. A call is not always dispatched because the flying conditions could be unsafe or no helicopters are within range of the call. The average response time is the time interval between the moment a call is received at the helicopter dispatch to the moment the helicopter arrives on scene, and is only computed for calls in which a helicopter actually arrives on scene. The response fraction is the fraction of calls received at dispatch where a helicopter actually arrives at the scene. Finally, the utilization of helicopters is a metric for measuring the cost/profitability of this service, and is computed by finding the average number of calls satisfactorily transported per day divided by the number of helicopters.

The goal of this study was to analyze the given performance metrics to then find the optimal number of helicopters to deploy, as well as the right locations to station our helicopters.

III. Modeling Approach and Assumptions

Our understanding of helicopter transport operations in the NY State area was based on a simulation model. This model factored in trends and patterns of real-time helicopter operations data to simulate how helicopter operations would handle incoming emergency calls over a set time-period. By generating numerous instances of emergency calls and helicopter operations over a set time-period, we were able to determine what the average helicopter utilization and metrics on emergency call handling would be, based on where the helicopter's bases were. Running this model multiple times with different base locations for the helicopters allowed us to determine the best placement of helicopters throughout the NY State area. A more detailed breakdown of the model can be found in Appendix 1 and Appendix 2.

In our model, we generated set "events" that would occur along a time horizon, corresponding to how helicopter dispatch and the individual helicopters handle emergency calls. In broad terms, as each call came in, we assigned the call to a helicopter (if it was safe to fly), the helicopter flew from its base to the call location, then to the medical facility to drop off the patient, and then back to its base location. However, to capture the nuances of these operations, we made specific decisions at each time step, which were broken up into these events. We defined them as:

- Helicopter dispatch receives a call
- Dispatch assigns a helicopter to an emergency call
- Helicopter finishes its flight checks and safety preparations
- Helicopter reaches the call location
- Helicopter finishes tending to the patient on the scene
- Helicopter reaches the medical facility
- Helicopter comes back to its base location

To accurately simulate these operations, we analyzed the real-time operational data to determine the probability of these events occurring and the distributions of how long these events would take in time (i.e. how long it would take a helicopter to go from base A to call location B). We also assessed the probability that dispatch assesses it is not safe to fly, the probability that there are no helicopters available, as well as a likely times after which each call would cancel.

After determining the likelihood of specific events happening and their probable time duration, the model made specific determinations about the helicopter and call status. After helicopter dispatch received a call, our model determined whether it was safe to fly to that call and whether there was an available helicopter available. The helicopter would determine if it was safe to fly based on the likelihood in the real-life data of flights that were labelled as safe or unsafe to fly. In reality, this determination would most likely be made based on base or call location, but for the simplicity of the model, we ignore this case. A helicopter would only be assigned if it was at its base location, and did not have a call assigned to it. We also assumed that a helicopter would only be assigned if its base location was less than 180 km away from the call location. The nearest helicopter to the call location, given that it fit all of the criteria, would be assigned. If it was not safe to fly or there were no helicopters available, the call was determined 'Not Serviced.' We also assume that these calls only require the transport of a single patient to a single destination. If a helicopter was safely assigned, the helicopter would take the time to go through flight checks and preparation, before departing the base to fly to the call location scene. We estimated a helicopter would fly at a speed of 160km per hour. A call could be cancelled at any time until the helicopter reached the scene. If the call was cancelled, the helicopter would take the time to fly back to the base, from the location it was at when the cancellation time was reached.

After the helicopter reached the scene, it would spend a simulated amount of time at the scene. Then, it would transport the patient to the receiving facility. From the data, we found that if the trauma center was closest, the patient would go there. If not, the patient would go to either the nearest facility, which would be a hospital, or the nearest trauma center. We analyzed the data to determine the likelihood of a patient going to a trauma center versus a hospital, when a trauma center was not the closest to the scene. After the helicopter flew to the hospital and spent some time unloading the patient, it would fly back to the base location. Once it flew back to the base location, the helicopter would become immediately available to be assigned to a new patient. For the constraints of this model, we assumed that helicopters would not require time to refuel and be serviced between calls, helicopter crews would not be restricted in how many hours they can fly in a shift, and that shift handovers between helicopter crews take a negligible amount of time. We also assumed that a helicopter can only be assigned to a new call once it has landed back at its base station. During this process, multiple calls could come into helicopter dispatch, and other helicopters could be assigned to helicopters while other helicopters were out in the field.

We started off the model by generating a single call, and then its subsequent events. As we ran the model, we kept track of the total number of calls received, the number of times a helicopter was dispatched, the number of times a helicopter reached the scene, the number of cancelled calls, the average response time, and the utilization of helicopters. From this model simulation, we were able to understand the helicopter operations and emergency call response metrics of the upstate NY area.
To ensure that our model accurately reflected the status of helicopter operations, our model utilized the method of batch means. This is a method that allows the simulation model to have a warmup period, in order to allow the simulation to start off at a randomly generated state, instead of at a "cold" state. As an

example, a “cold” state would be one where there are no calls in the system and no helicopters have been dispatched. A warmup period guarantees that the model will start off where there may be some calls already in the system, and some helicopters may be out on the field. When determining the warm-up period, we found the length at which point the metrics did not fluctuate greatly. (Appendix 3) For purposes of this simulation, we simulated helicopter operations for a year, and split the year up into 26 batches of 2 weeks, to track the performance metrics. We determined the batch size to be 2 weeks, as 2 weeks would generate enough events while ensuring that the batch means were approximately normally distributed. (Appendix 4)

IV. Data Analysis

In order to drive our model, we needed to define several data distributions in order to accurately model how helicopter operations worked. For example, we wanted to understand how often a call was placed, how long a helicopter usually took at the scene of a call, and how long it would take for a helicopter to be dispatched to a call. By analyzing historical data, we were able to identify the distributions of our data in order to model the correct input parameters. A description of the technical details can be found in the appendix of this report, but the general parameters are as follows.

First, we found that the probability that a helicopter is unable to fly due to unsafe weather conditions is 10.1% This implies that the probability of times we have safe conditions and can actually send out a helicopter is 89.9%. Once helicopters are sent out, we found that the cancellation times for a call are Exponentially distributed with a mean of 4.97 hours. (Appendix 5) We also found that the scene time of a helicopter is Gamma distributed with a mean of 0.354 hours. (Appendix 6) After treatment at the scene, the helicopter will go to a trauma center if it is closer than a hospital. However, it only goes to the closest hospital 80.7% of the time. The other 19.3% of the time it will go to the nearest trauma center. (Appendix 7) The time at the hospital follows a Gamma distribution with a mean of 0.495 hours. (Appendix 8) In terms of call, we found that the interarrival times between them are Exponentially distributed with specific rates based on the hour. (Appendix 9)

There are also some inputs that we had little data on. In those cases, a Triangular distribution best represents the data. For the dispatch delay time, we found that it is commonly 7 mins, but has a min of 5 mins and a max of 10 mins. For the flight delay time, we found that it is most commonly 7.5 mins, and a min of 5 mins and a max of 10 mins.

Lastly, we resampled the call locations given the call data provided. This allowed us to accurately represent the data, while quantifying the possible uncertainty. All of these inputs fitted with the distributions discovered have allowed for us to accurately simulate the helicopter's responses and discover the best placement of bases and the optimal number of helicopters to have.

V. Model Verification

To verify our model, we tested the edge cases of our possible scenarios. We then hypothesized what we would expect using our background knowledge and compared that with what our actual results from the model were. For details regarding each of the edge case results, please refer to Appendix 10. To start, we simulated one run of a normal scenario of 100 hours as well as a constant set of helicopter locations to establish a base case.

Below are the results:

```
[('Dispatch Rate', [0.8262988180952956, 0.8554042784438664]),
 ('Cancel Rate', [0.04981148160404921, 0.06667883136167359]),
 ('Response Rate', [0.7659004063330245, 0.7927003553824913]),
 ('Utilization of Helicopters', [7.877963296112004, 8.602036703887993]),
 ('Average Response Time', [0.5616609062274653, 0.5849097378102153])]
```

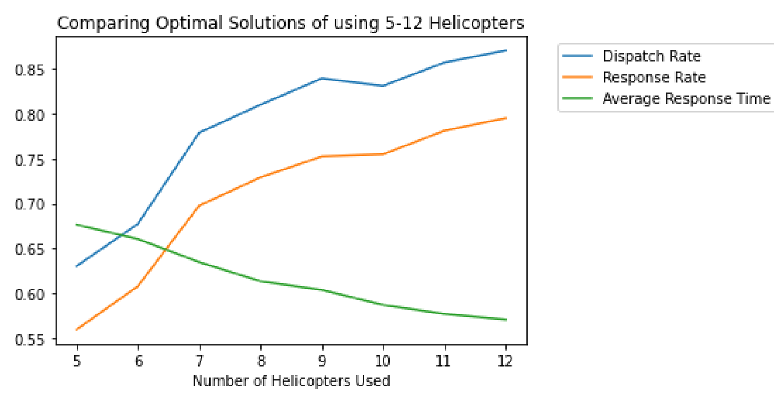
Next, we looked at our extreme cases to insure the model would be valid despite odd scenarios. First, we set all the call locations to a single spot and we put one helicopter at that location. We expected the utilization of the helicopter to be very high and the response fraction low as we only have one helicopter. We expected the average response time to be the mean of our dispatch and flight delay times (about 0.242 hours). This scenario was validated by our model results. From there, we simulated the same scenario, except increasing the number of helicopters at the location each time. We expected the utilization to decrease and response fraction to increase with each incremental change in helicopters. This was all validated by our model results.

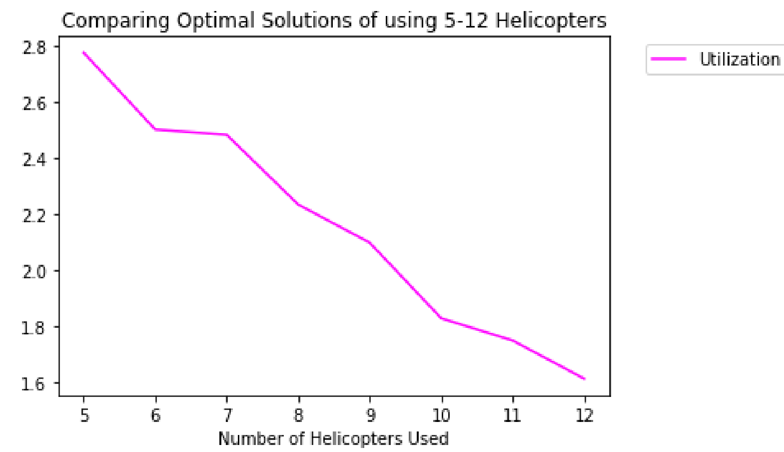
Another edge case tested was when all of the calls were outside of a 180km radius from the base location. In this case, there would be no helicopters assigned and all our metric outputs should be zero, which we found to be true of our model. Then, we looked at when the calls were right at the edge of how far helicopters could travel to, which was 179.56km away from the 12 helicopters. Here we would expect a high chance of cancellation as the average response times should be very long. A direct result of these high cancellation rates would be a low utilization rate and low response fraction. We were able to confirm these ideas with our model, indicating validity.

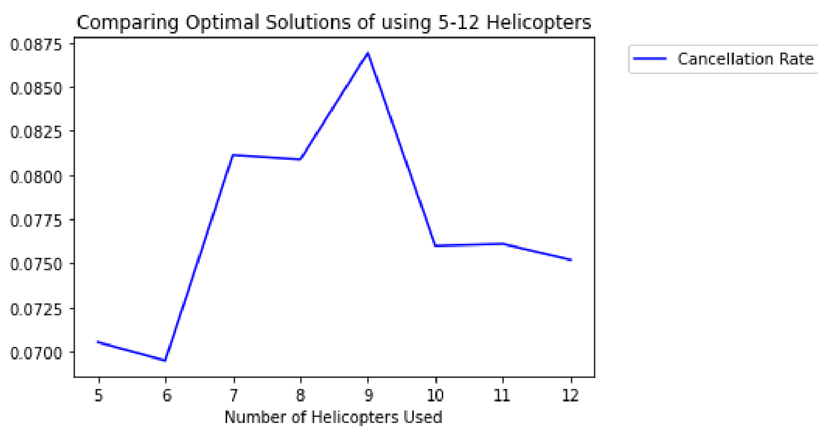
We also analyzed call samples to make sure that the times were recorded in chronological order. We wanted the sequence to go: call time, then the time the helicopter left the base, then the time the helicopter arrived at the scene, and finally when the helicopter arrived at the hospital. This ensured we were handling the correct calls for each event. (Appendix 11) Finally, we looked at the helicopter assignments, to make sure two calls were not assigned to a helicopter at the same time. For example, after the 4th hour, three helicopters #8, #7, and #2 were in use. These findings from our model match what one would expect to see as the results, indicating that our model is correct and valid for simulating helicopter transport.

VI. Model Analysis

To find our optimal solution, we began by running simulations for 5 through 12 helicopters. We ran every possible combination of these helicopters spread across the 5 most commonly visited bases. After that, we selected, for each number of helicopters, the top five solutions with the lowest average response time. We then, of those five helicopter placements, selected the one with the lowest cancel rate. This method allowed us to most heavily weight the average response time, while still paying attention to maintaining a low cancellation rate. After this was done, we were left with eight solutions of helicopter assignments, one for each number of helicopters between 5 and 12. We then plotted graphs of the dispatch rate, cancel rate, response rate, utilization, and average response time against the number of helicopters. Plotting these graphs allowed us to visualize the change in our metrics as we change the number of helicopters we use. Each of the graphs can be seen in Appendix 12 along with the mapped locations of the helicopters in Appendix 13. Below are the three graphs created showing the changes:







As shown above, we noticed that there was not a significant difference between 10-12 helicopters for dispatch, response rate, and average response time. Additionally, the cancellation rate had a local minimum at 10 helicopters. We were not concerned with having a lower utilization as this could indicate that the helicopters could handle a higher amount of calls if there is variability. We then investigated adding one more helicopter to this 10-helicopter solution, considering helicopter bases that were not in the top five for call demand. We analyzed the heat map of calls and noticed that there was a dense amount of calls around Watertown, NY. (Appendix 14) Watertown, NY is also not close to any of the five highly-demanded hospital locations, meaning that calls from this area had a higher probability of being cancelled. We then compared our 10-helicopter solution to our new 11-helicopter solution that uses the same 10 base assignments with one more helicopter assigned to Watertown. This 11-helicopter solution has a lower average response time than the 10-helicopter solution and we chose this as our optimal solution for helicopter number and base assignments. Below is a table comparing the simulations between the 10-helicopter solution and our optimal 11-helicopter solution:

		Optimal Helicopters	Optimal Helicopters Plus Additional Base
0	Number of Helicopter	10.000000	11.000000
1	Dispatch Rate	0.830742	0.839286
2	Cancelation Rate	0.075988	0.077818
3	Response Rate	0.754656	0.761459
4	Utilization	1.828219	1.654795
5	Average Response Time	0.587161	0.568516

VII. Sensitivity Analysis

To understand the uncertainty in our model, we conducted a sensitivity analysis on the call rates as well as the distribution for the hospital and scene times. First, investigating the call rates, we ran a simulation with call rates increased by 20% and another one with call rates decreased by 20%. We then compared the outputs between original, increased, and decreased call rates. This was done through updating the call_rates dictionary, running the simulations, and then resetting the values back to ensure that no other aspects of our model were accidentally altered. The following is the output for our optimal solution and 5-helicopter solution after increasing and decreasing call rates:

<u>Using Optimal Solution (11 Helicopters)</u>

No Change in Call Rates

```
[('Dispatch Rate', [0.8544704590694735, 0.8585193622186452]),
 ('Cancel Rate', [0.0757923522126493, 0.07737272932596548]),
 ('Response Rate', [0.7779465163382413, 0.78221686894884]),
 ('Utilization of Helicopters', [1.7408067505933886, 1.7690313565049929]),
 ('Average Response Time', [0.5689976199833628, 0.5717254981504474])]
```

Increase Call Rates by 20%

```
[('Dispatch Rate', [0.8365750608005669, 0.8390705477789095]),
 ('Cancel Rate', [0.0753520927655983, 0.07712461800162539]),
 ('Response Rate', [0.759719523231184, 0.7634821618517049]),
 ('Utilization of Helicopters', [2.2325772668639527, 2.2565883620277014]),
 ('Average Response Time', [0.5891871582860956, 0.5914874206338158])]
```

Decrease Call Rates by 20%

```
[('Dispatch Rate', [0.8606989657769198, 0.8632957977029169]),
 ('Cancel Rate', [0.0696222670688757, 0.07185614597752504]),
 ('Response Rate', [0.7896903127972178, 0.792873720341769]),
 ('Utilization of Helicopters', [1.414675263022556, 1.4311528814357253]),
 ('Average Response Time', [0.5618066645445955, 0.5645414575306434])]
```

<u>Using 5 Helicopters</u>

No Change in Call Rates

```
[('Dispatch Rate', [0.6674729845176479, 0.6714721538676007]),
 ('Cancel Rate', [0.07082691435402189, 0.07305931580258668]),
 ('Response Rate', [0.5953908674023741, 0.5999161585251643]),
 ('Utilization of Helicopters', [2.8725734043536635, 2.9104402942764738]),
 ('Average Response Time', [0.6779705018728434, 0.6821605716844591])]
```

Increase Call Rates by 20%

```
[('Dispatch Rate', [0.6274199659090585, 0.6318211614316269]),
 ('Cancel Rate', [0.07013360762113448, 0.07195750119982962]),
 ('Response Rate', [0.5564639547602879, 0.5606535837716282]),
 ('Utilization of Helicopters', [3.5299377183461087, 3.564856802201836]),
 ('Average Response Time', [0.6864377683244333, 0.6895816363490305])]
```

Decrease Call Rates by 20%

```
[('Dispatch Rate', [0.8738299815243262, 0.8763893443826734]),
 ('Cancel Rate', [0.07848685557089098, 0.08106036008159843]),
 ('Response Rate', [0.7939084550755392, 0.7968003963631946]),
 ('Utilization of Helicopters', [1.3344796461309665, 1.3504518607183487]),
 ('Average Response Time', [0.5711426020934655, 0.5736021968949937])]
```

From the analysis above, there is significant sensitivity with respect to the utilization of the helicopters. There is a 25% increase in utilization for the optimal solution by increasing the call rates, and an 18% decrease by decreasing the call rates. The cancel rate and average response time are not significantly affected by the change in call rates. Additionally, by looking at the 5-helicopter solution, the values are more sensitive to a change in call rates than the 11-helicopter optimal solution. This is seen through the almost 60% decrease in utilization after decreasing the call rates by 20%. Though the dispatch rate and response rate are not significantly affected in the optimal solution, it is clear that these values become much more sensitive as the number of helicopters goes down. For example, the dispatch rate increases by 28% and the response rate increases by 32% after decreasing the call rates by 20%. Though the average response time, our highest-valued metric, has little sensitivity in our optimal solution, it is important to note the sensitivity when the number of helicopters used is decreased. We also investigated the sensitivity with our 12-helicopter optimal solution, but given that 12 helicopters is very close to our 11-helicopter optimal solution, the sensitivity was very similar between the two.

After testing the sensitivity on our simulation from a change in call rates, we investigated the sensitivity in our simulation from a change in the distribution for hospital and scene times. Though both were originally Gamma distributed, we re-ran our simulation using Beta random variables with the same mean and variance as before. The equations for alpha and beta (parameters in the Beta distribution) in terms of mean and variance are as follows:

$$\alpha = [((1 - \mu)/\sigma^2) + 1/\mu)] * \mu^2$$
$$\beta = \alpha * (1/\mu - 1)$$

After simulating our optimal solution with Beta-distributed hospital times, Beta-distributed scene times, and Beta-distributed hospital and scene times, we analyzed the following output:

<u>Using Optimal Solution (11 Helicopters)</u>

No Change (Gamma-Distributed Hospital and Scene Times)

```
[('Dispatch Rate', [0.8544704590694735, 0.8585193622186452]),
 ('Cancel Rate', [0.0757923522126493, 0.07737272932596548]),
 ('Response Rate', [0.7779465163382413, 0.7822168689488 4]),
 ('Utilization of Helicopters', [1.7408067505933886, 1.7690313565049929]),
 ('Average Response Time', [0.5689976199833628, 0.5717254981504474])]
```

Beta-Distributed Hospital Times

```
[('Dispatch Rate', [0.8511858513436702, 0.8539834884998881]),
 ('Cancel Rate', [0.07719457013735691, 0.07962614658730278]),
 ('Response Rate', [0.7724326584480843, 0.7758342557357953]),
 ('Utilization of Helicopters', [1.6863046694614292, 1.7099593405012108]),
 ('Average Response Time', [0.5695315610429252, 0.5720663017459082])]
```

Beta-Distributed Scene Times

```
[('Dispatch Rate', [0.85642452449967, 0.8597772764734626]),
 ('Cancel Rate', [0.0741733766486581, 0.07610620660671867]),
 ('Response Rate', [0.7810973929686741, 0.7848052785741545]),
 ('Utilization of Helicopters', [1.688675287972651, 1.7115737780298392]),
 ('Average Response Time', [0.574257437450843, 0.5769365416749835])]
```

Beta-Distributed Hospital and Scene Times

```
[('Dispatch Rate', [0.8591308445207747, 0.8621451876540905]),
 ('Cancel Rate', [0.07342642584246034, 0.07594912138207147]),
 ('Response Rate', [0.7839506339678038, 0.7879712515488894]),
 ('Utilization of Helicopters', [1.752310137001422, 1.7724719302463978]),
 ('Average Response Time', [0.573886794249847, 0.5762171172189184])]
```

After running the sensitivity analysis on Gamma versus Beta distributions for scene and hospital times, there is some sensitivity with the cancel rate, but the other metrics have minimal change. The cancellation rate is sensitive to a change to a beta-distributed hospital time, decreasing by about 11%. The dispatch rate, response rate, utilization, and average response time see little change when switching to beta distributions in hospital and scene times. Overall, the sensitivity is low when changing to a Beta distribution, likely due to the fact that the two distributions have the same mean and variance given their parameters.

VIII. Conclusions

Utilizing our knowledge of simulation and understanding of statistics, we were able to optimize the way upstate New York handles emergency helicopter transport. By analyzing historical data and simulating existing helicopter operations on different scenarios, we were able to generate model results over the course of a year. We verified that our model was correct, by testing edge cases and comparing against expected results. We tested a solution of our model that would utilize 6 helicopter bases, rather than 5, to increase our coverage of the upstate region, which yielded better response times. Although 6 bases would yield a higher implementation cost, this improvement in the performance metrics as well as the increased coverage of the region indicated that the 6 bases would be the more optimal solution.

With our solution of utilizing 2 helicopters in Albany, 1 in Rochester, 4 in Syracuse, 2 in Buffalo, 1 in Binghamton, and 1 in Watertown, we were able to maintain an average response time below an hour with an average cancellation rate of 8%. Similarly, we ensured that the helicopters would be utilized the majority of time, with an average of around 2 calls per helicopter per day. From our solution, we found that the results were fairly sensitive to the call rates, indicating the rate at which calls arrive into helicopter dispatch. Further analysis could be done to investigate the details and sensitivity of the incoming calls.

Our final recommendation is an optimal solution when looking at average response time and cancellation time, while prioritizing high-call-attending hospitals and coverage of the upstate New York area, utilizing a simplified model of helicopter operations.

IX. Appendices

Appendix 1: Simulation Overview

1. Start off simulation with a call event.
   - Generate a dispatch event for that call and put in event list.
   - Generate a next call event and put in event list.
   - Throughout the code, sort the event list with the smallest time value at the start.
2. Iterate through the event list and handle each event according to what it is
   - In the event list, events have one of the following labels.
     - "Call Recieved"
     - "Dispatch Finished"
     - "Flight Check Finished"
     - "Arrived At Scene"
     - "Finished at Scene"
     - "Arrived at Hospital"
     - "Finished at Hospital"
     - "Arrived Back at Base"
     - "Call Cancelled"
   - Each event is formatted as a tuple (event time, event label, associated helicopter number)
   - The helicopter number will be used to reference the specific call that it is assigned to. The helicopter number also corresponds to the index that the helicopter is in, in the list of helicopters X.
     - This works, because a call will always contain the helicopter number that it is assigned to, and only one helicopter will be in operation at all times, so these pairings are unique at each point in time.
3. After each event, delete that event from the event list

Appendix 2: Simulation Event Handling

If the event is "Call Received"

1. Set current time to time of the event
2. Generate a new call instance at current time
3. Add the new call instance to a list of ongoing calls
4. Generate a new dispatch event "Dispatch Finished"
5. Generate a new call event "Call Received"
6. Increase num_calls by 1
7. Sort events again, so they are now in increasing order by time

If the event is "Dispatch Finished"

1. Set current time to time of the event
2. Find the specific call that the event is for and then delete the event from the event list
    - This will be the only call in the list of ongoing calls that is not yet assigned to a helicopter yet
3. Check to see if the call has been determined safe to fly, the cancellation time has not been exceeded yet, and that there are available helicopters in the area
    - If all these conditions hold
        1. Assign the nearest available helicopter to the call
        2. Generate a cancellation event using the cancellation time of the call and include the new assigned helicopter number to the event
        3. Generate a new flight check event
    - If all these conditions do not hold
        1. Delete the call from the list
4. Sort events again, so they are now in increasing order by time

If the event is "Flight Check Finished"

1. Set current time to time of the event
2. Find the specific call that the event is for and then delete the event from the event list
    - This will be the the call in the ongoing calls list with the helicopter number given within the event
3. Update the 'Process' and 'LeftBase' attributes of the helicopter
4. Calculate the travel time from the helicopter base to the call location by finding the travel distance and the known travel speed of 160km per hour
5. Generate a new travel to scene event "Arrived at Scene" using the calculated travel time
6. Sort events again, so they are now in increasing order by time

If the event is "Arrived At Scene"

1. Set current time to time of the event
2. Find the specific call that the event is for and then delete the event from the event list
    - This will be the the call in the ongoing calls list with the helicopter number given within the event
3. Update the 'Process' and 'Location' attributes of the helicopter

4. Calculate the response time of the call by subtracting the current time from the time that the call was received 'CallTime'
5. Update the 'ArrivedAtScene' time of the call to the current time
6. Delete the 'Call Cancelled' event from the event list, since it is no longer needed
7. Generate a new scene time event 'Finished at Scene'
8. Sort events again, so they are now in increasing order by time

If the event is "Finished At Scene"

1. Set current time to time of the event
2. Find the specific call that the event is for and then delete the event from the event list
    - This will be the the call in the ongoing calls list with the helicopter number given within the event
3. Update the 'Process' and 'Location' attributes of the helicopter
4. Calculate the travel time from the call location to the hospital destination by finding the travel distance and the known travel speed of 160km per hour
5. Generate a new travel to hospital event "Arrived at Hospital" using the calculated travel time
6. Sort events again, so they are now in increasing order by time

If the event is "Arrived At Hospital"

1. Set current time to time of the event
2. Find the specific call that the event is for and then delete the event from the event list
    - This will be the the call in the ongoing calls list with the helicopter number given within the event
3. Update the 'Process' and 'Location' attributes of the helicopter
4. Update the 'ArrivedAtHosp' time of the call to the current time
5. Generate a new hospital time event 'Finished at Hospital'
6. Sort events again, so they are now in increasing order by time

If the event is "Finished At Hospital"

1. Set current time to time of the event
2. Find the specific call that the event is for and then delete the event from the event list
    - This will be the call in the ongoing calls list with the helicopter number given within the event
3. Update the 'Process' attribute of the helicopter
4. Delete the call from the list of ongoing calls
5. Calculate the travel time from the hospital back to the helicopter base by finding the travel distance and the known travel speed of 160km per hour
6. Generate a new travel to base event "Arrived Back at Base" using the calculated travel time
7. Sort events again, so they are now in increasing order by time

If the event is "Arrived Back at Base"

1. Set current time to time of the event
2. Find the specific call that the event is for and then delete the event from the event list

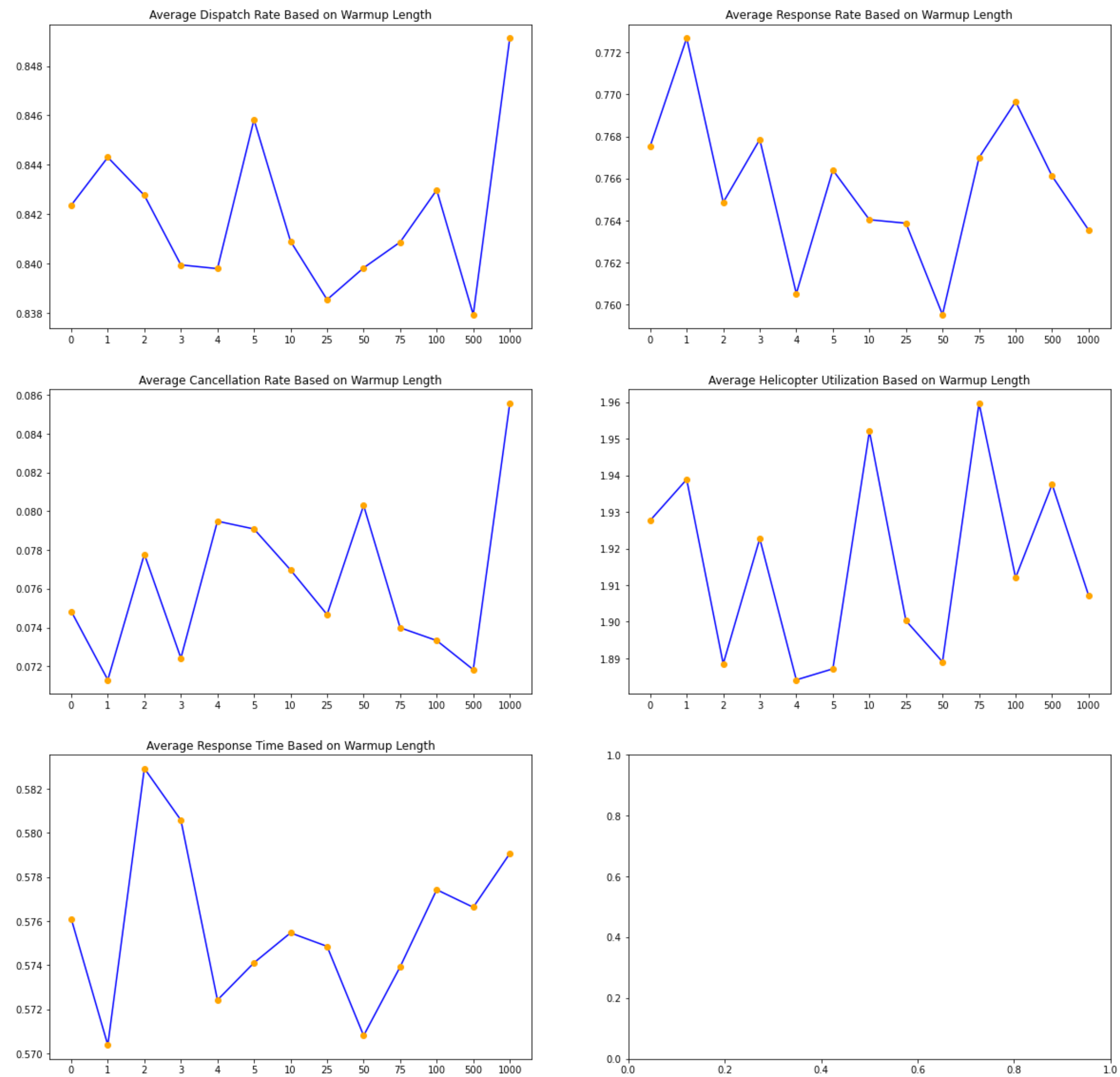
    - This will be the the call in the ongoing calls list with the helicopter number given within the event
3. Update the 'Process', 'LeftBase', and 'Location' attributes of the helicopter
4. Update the 'HeliAvail' attribute of the helicopter to 1, to indicate that the helicopter is ready for transport again
5. Sort events again, so they are now in increasing order by time

If the event is "Call Cancelled"

1. Set current time to time of the event
2. Find the specific call that the event is for and then delete the event from the event list
    a. This will be the call in the ongoing calls list with the helicopter number given within the event
3. Increase the number of cancelled calls by 1
4. Identify which process the call is in
5. If the helicopter has already left the base yet and is on its way to the scene
    a. Calculate the time the helicopter has been flying for by subtracting the time it left the base from the current time
    b. Calculate the distance already travelled by the helicopter by multiplying the travel time by the travel speed
    c. Find the total distance from the helicopter base to the call location
    d. Calculate the percentage of the total distance already travelled
    e. Assuming that the helicopter travels along a straight line from the base to the call location, find the straight line vector (change in latitude, change in longitude) from the base to the call location
    f. Multiply the vector by the percentage, to find how much the helicopter has travelled in vector form, to find change in latitude and change in longitude
    g. Add that distance to the starting point of the helicopter base, to find the current location
    h. Calculate the travel time from the current location back to the helicopter base by finding the travel distance and the known travel speed of 160km per hour
        - Note: you could also simply use the time travelled.
    i. Generate a new travel to base event "Arrived Back at Base" using the calculated travel time
    j. Delete the 'Arrived at Scene' event from the event list for that helicopter since it won't be arriving at the scene anymore.
    k. Delete the call from the ongoing call list.
6. If the helicopter has not left the base yet
    a. Update the 'Process', 'LeftBase', and 'Location' attributes of the helicopter
    b. Update the 'HeliAvail' attribute of the helicopter to 1, to indicate that the helicopter is ready for transport again
    c. Delete the 'Flight Check Finished' event from the event list for that helicopter, since the call is cancelled.
    d. Delete the call from the ongoing call list.
    e. Sort events again, so they are now in increasing order by time

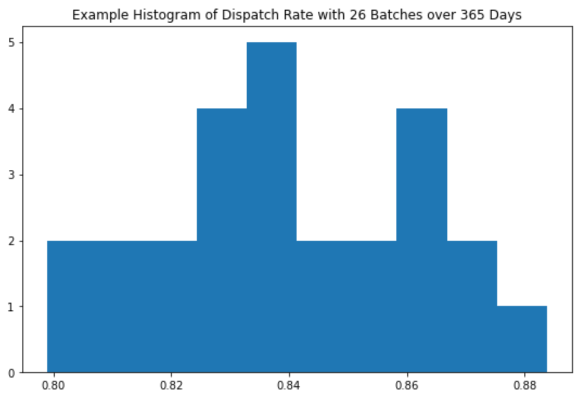
Appendix 3: Warmup Period

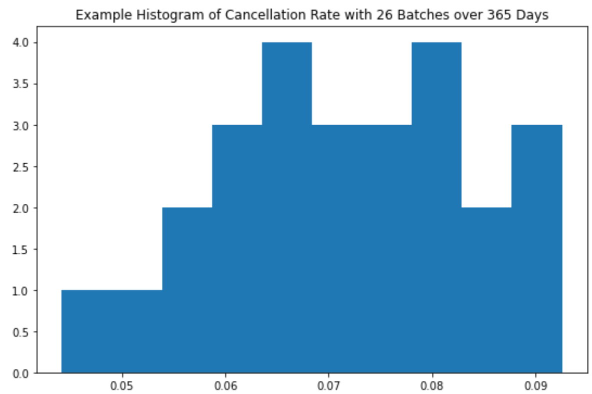
Although most of the graphs display fluctuation, the Average Response Rate and Average Cancellation Time graphs below show that after hour 10, the metrics do not fluctuate as greatly.

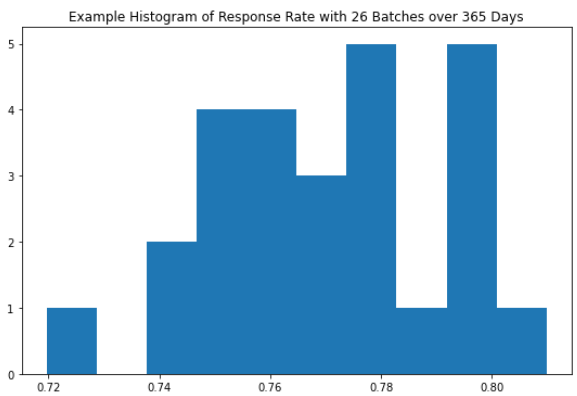


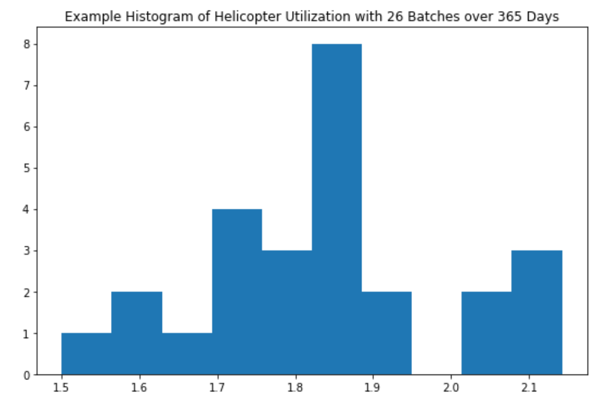
Appendix 4: Normally Distributed Batch Means

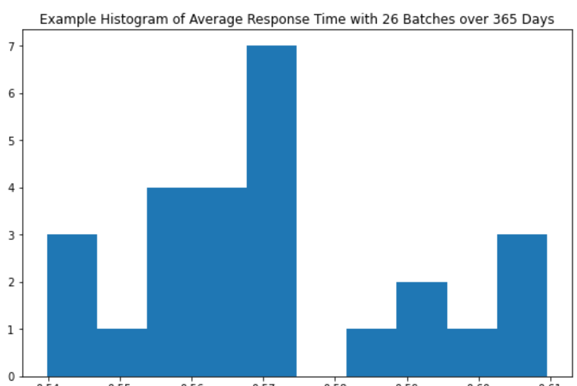
The graphs below show that 2 week batches result in approximately normally distributed batch means, across the 5 metrics of dispatch rate, cancellation rate, response rate, utilization, and average response time.

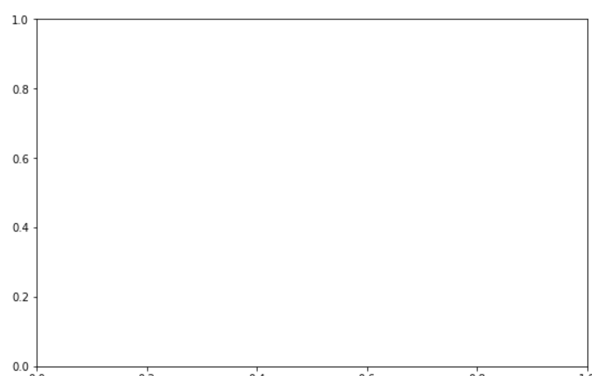












Appendix 5: Call Cancellation Time

We used the Maximum Likelihood Estimation method to generate values for the parameters of an Exponential distribution. By taking into account the calls that were censored at 30 minutes, we estimated a mean cancellation time of 4.87, using the estimated rate parameter $\hat{\lambda} = r / \sum_{i=1}^{r} (x_i + (n - r)/2)$ where $r$was the number of observed/uncensored data and $n - r$ was the number of unobserved/censored data. Ultimately, we found the cancellation time to be $Exp(\hat{\lambda} = 0.205)$

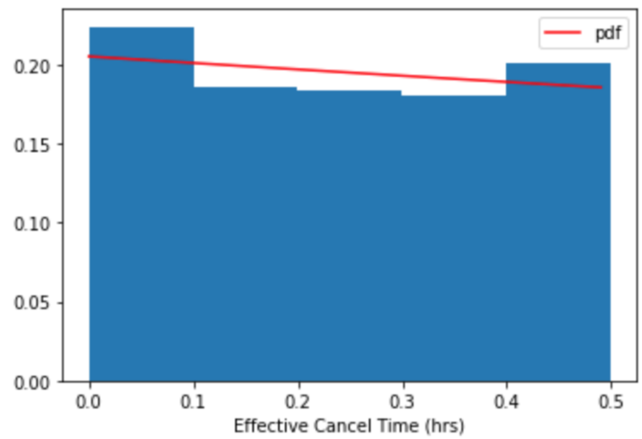


Figure Source: Project HW Solutions

Appendix 6: Scene Time

To determine the distribution of scene times, we fitted and plotted multiple distributions to the given historical data, to determine which would be a good fit. We plotted Gamma, Beta, Lognormal, Exponential, Pareto, and Weibull distributions.

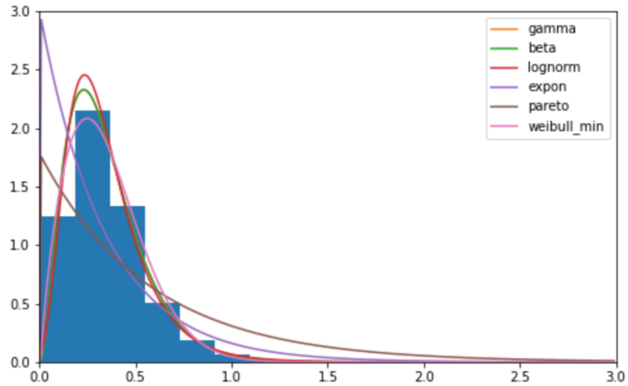


Figure Source: Project HW Solutions

From the plotted distributions, we immediately assumed that the Gamma and Beta distributions were best fit to the data. However, we analyzed Q-Q plots to verify our assumptions.

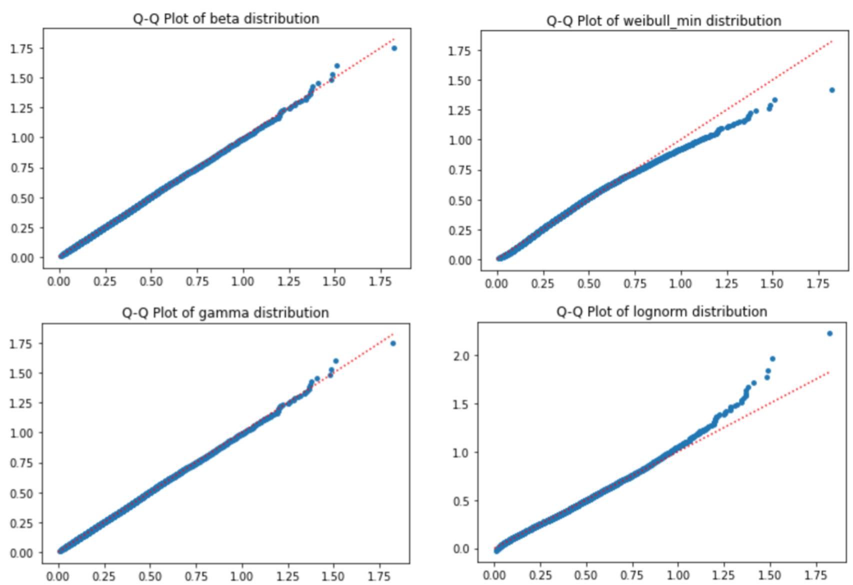


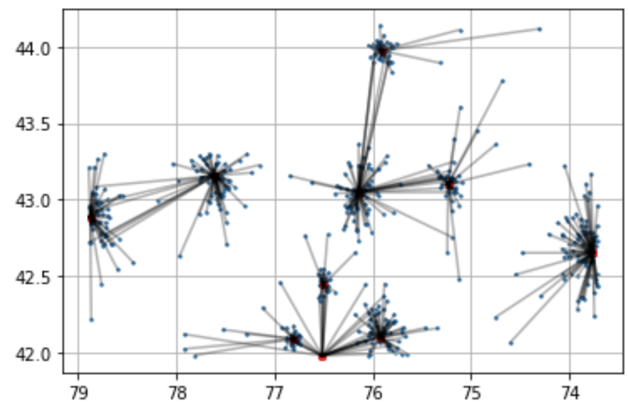
Figure Source: Project HW Solutions

From the Q-Q plots, we could distinguish the risk of the distribution, to determine how good the fit of the cdf was. We knew that values at the extremes have a higher variance than those in the middle, so greater departures from linearity at the ends of the plot are acceptable. We found that the Gamma and Beta distributions both fit the data well, and thus used $Gamma(a = 2.95, scale = 0.12)$ as our distribution. However, we also conducted a sensitivity analysis with the Beta distribution to verify our assumptions.

Appendix 7: Hospital or Trauma Center Probability

In historical helicopter operations, the helicopter chooses the destination, depending on where the call arises. A helicopter will always fly to the nearest facility if it is a trauma center. However, if the nearest facility is a hospital, it will sometimes go to the hospital and other times will go to the trauma center.

This figure below shows the route that helicopters transfer patients from scenes to hospitals, where the calls are the blue dots and the lines indicate the routes taken by the helicopter to transfer the patients.



Using the call and facility locations, we found that the probability of a patient being transferred to the nearby hospital if the nearest hospital was not a trauma center was 0.807, while the probability of a patient being transferred to a trauma center was 0.193.

Appendix 8: Hospital Time

To determine the distribution of hospital times, we fitted and plotted multiple distributions to the given historical data, to determine which would be a good fit. We plotted Gamma, Beta, Lognormal, Exponential, Pareto, and Weibull distributions.

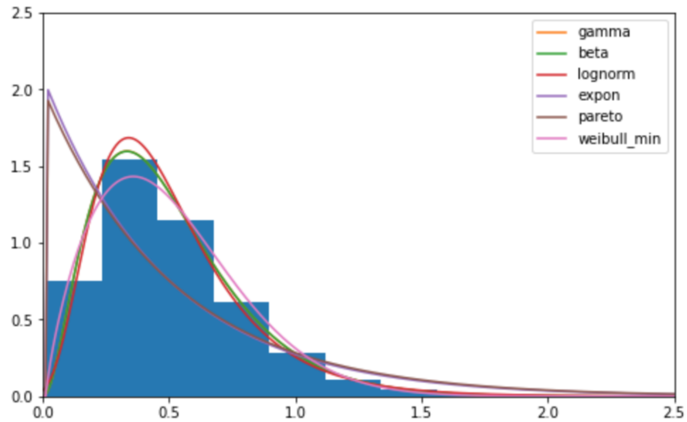


Figure Source: Project HW Solutions

From the plotted distributions, we immediately assumed that the Gamma and Beta distributions were best fit to the data. However, we analyzed Q-Q plots to verify our assumptions.

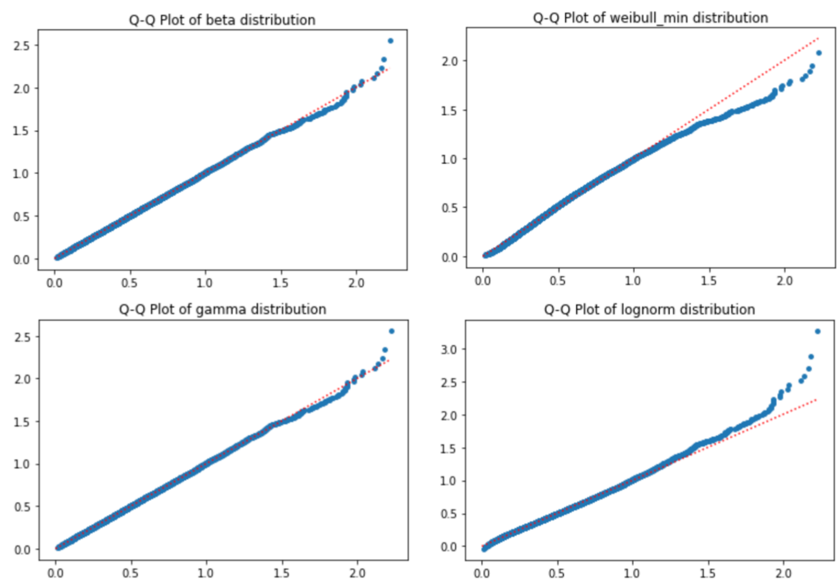
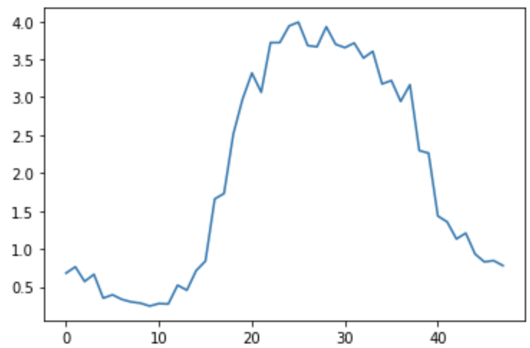


Figure Source: Project HW Solutions

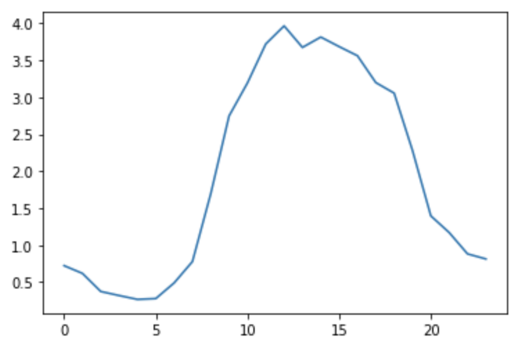
From the Q-Q plots, we could distinguish the risk of the distribution, to determine how good the fit of the cdf was. We knew that values at the extremes have a higher variance than those in the middle, so greater departures from linearity at the ends of the plot are acceptable. We found that the Gamma and Beta distributions both fit the data well, and thus used $Gamma(a = 2.91,\ scale = 0.17)$ as our distribution. However, we also conducted a sensitivity analysis with the Beta distribution to verify our assumptions.

Appendix 9: Call Rates

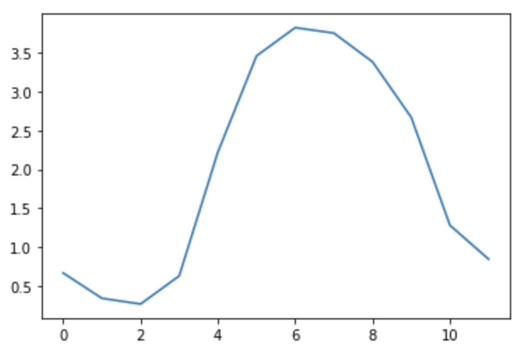
To determine the call rates and call arrival times at helicopter dispatch, we fit a nonstationary Poisson process to the customer request arrival data. We assumed only time-of-day effects in the data, ignoring day-of-week and seasonality effects. We plotted arrival rates by the half-hour, hour, and every two-hours, to determine the arrival rates that best fit the data.



Arrival Rate by Half-Hour

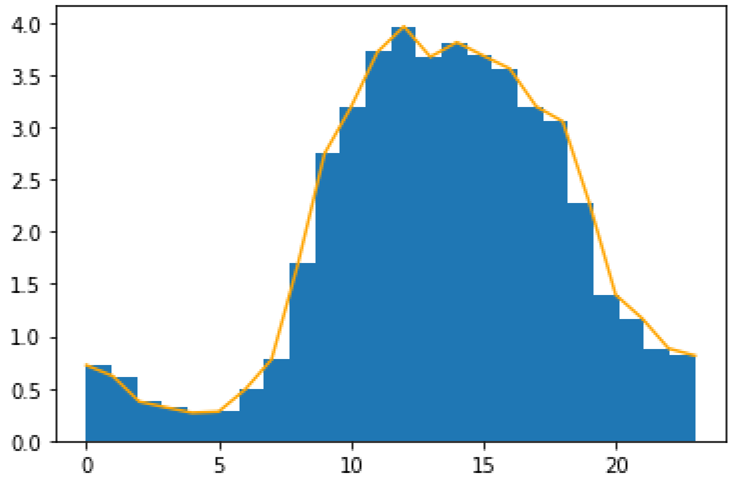


Arrival Rate by Hour



Arrival Rate by Two Hours

Ultimately, we determined that the arrival rate by every hour best-fit the data, after overlaying a histogram over the data.



The resulting arrival rates in terms of calls per hour looked like the following.

Hour	Rate	Hour	Rate	Hour	Rate
1	0.723	9	1.70	17	3.56
2	0.618	10	2.75	18	3.20
3	0.374	11	3.19	19	3.05
4	0.319	12	3.72	20	2.28
5	0.266	13	3.96	21	1.40
6	0.277	14	3.67	22	1.17
7	0.489	15	3.81	23	0.881
8	0.777	16	3.68	24	0.813

Figure Source: Project HW Solutions

Appendix 10: Model Verification Results

Base results:

```
[('Dispatch Rate', [0.8262988180952956, 0.8554042784438664]),
 ('Cancel Rate', [0.04981148160404921, 0.06667883136167359]),
 ('Response Rate', [0.7659004063330245, 0.7927003553824913]),
 ('Utilization of Helicopters', [7.877963296112004, 8.602036703887993]),
 ('Average Response Time', [0.5616609062274653, 0.5849097378102153])]
```

	Scenario	Expected	Actual
1	Call locations are all at one location One helicopter located at same location	Utilization of helicopters is high Average response time is the mean of the dispatch delay and flight delay times (~ 0.2417) Response fraction is low because there's only one helicopter operating	[('Dispatch Rate', [0.19541989758964196, 0.2248179873541254]), ('Cancel Rate', [0.0021328545879455265, 0.007344269595061009]), ('Response Rate', [0.19104020838914626, 0.21972055237161442]), ('Utilization of Helicopters', [17.996653821894565, 19.603346178105436]), ('Average Response Time', [0.2451646321812733, 0.2504750555980543])]
2	Call locations are all at one location 50 helicopters located at same location	Utilization of helicopters is lower than Scenario 1 and lower than normal, because there are more helicopters Average response time is the mean of the dispatch delay and flight delay times (~ 0.2417) Response fraction is higher than Scenario 1	[('Dispatch Rate', [0.84333134722337, 0.8749280195796859]), ('Cancel Rate', [0.019208100545380656, 0.02734118852818026]), ('Response Rate', [0.8155286786422608, 0.852577795483631]), ('Utilization of Helicopters', [1.5852027154400312, 1.670797284559969]), ('Average Response Time', [0.24677235434980488, 0.24870248554049318])]

3	Call locations are all at one location 5,000 helicopters located at same location	Utilization of helicopters is closer to 0 Dispatch is close to one minus the probability of a flight being unsafe to fly (~ 0.899) Average response time is the mean of the dispatch delay and flight delay times (~ 0.2417)	`[('Dispatch Rate', [0.8815630001705933, 0.9047354971976714]), ('Cancel Rate', [0.007886045147749536, 0.013078598355881654]), ('Response Rate', [0.8659466283959423, 0.8857668716622908]), ('Utilization of Helicopters', [0.01820036643541734, 0.019879633564582663]), ('Average Response Time', [0.2465426895379322, 0.2488412615663285 5])]`
4	Call locations are all at one location outside of the 180km radius from a base location 50 helicopters located at one location 180km away from call location	No helicopters are assigned and everything is zero.	`[('Dispatch Rate', [0.0, 0.0]), ('Cancel Rate', [0.0, 0.0]), ('Response Rate', [0.0, 0.0]), ('Utilization of Helicopters', [0.0, 0.0]), ('Average Response Time', [0.0, 0.0])]`
5	Call locations are all at one location exactly `179.56` km away from a base location 12 helicopters located at one location `179.56` km away from call location	Percent cancelled is greater than normal Response fraction is lower than normal because of a high chance of cancellation Utilization of helicopters is lower than normal because of high cancellation probability Average response time is very long	`[('Dispatch Rate', [0.8488683706289015, 0.8890006617530852]), ('Cancel Rate', [0.19343844343703484, 0.21701152544242244]), ('Response Rate', [0.6154820695486758, 0.6642217958628265]), ('Utilization of Helicopters', [1.2585405117594408, 1.3574594882405593]), ('Average Response Time', [1.3692188680273878, 1.369824875381354])]`

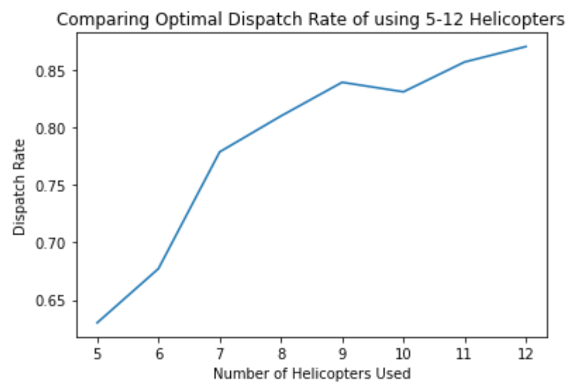
Appendix 11: Chronological Order Validation

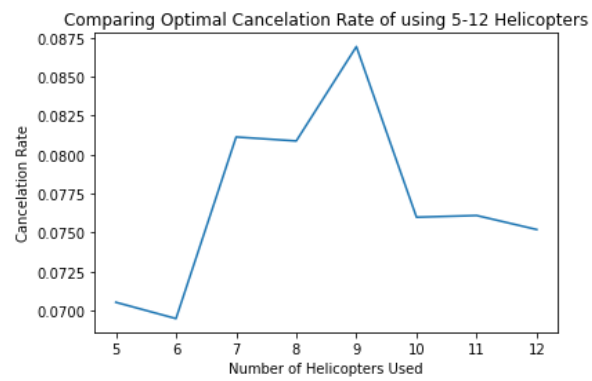
Below are a set of example outputs validating the chronological order:

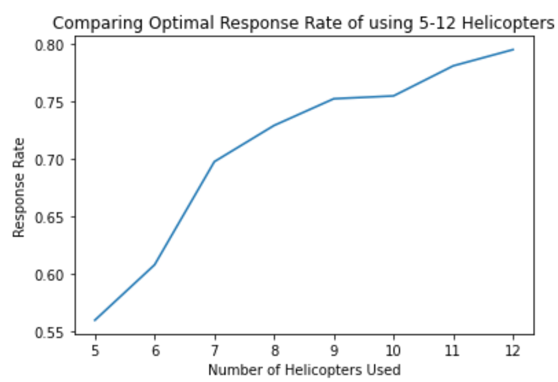

```
{'CallTime': 0.03110334184113149,
'LeftBase': 0.2568258941876057,
'ArrivedAtScene': 0.5339735260966023,
'ArrivedAtHosp': 1.1197836454360623}

{'CallTime': 1.851381309153024,
'LeftBase': 2.062719009847849,
'ArrivedAtScene': 2.2668401708315122,
'ArrivedAtHosp': 3.2239504421010303}

{'CallTime': 0.9992231065648307,
'LeftBase': 1.2518563556221665,
'ArrivedAtScene': 2.1121462131010738,
'ArrivedAtHosp': 3.769291328735129}
```

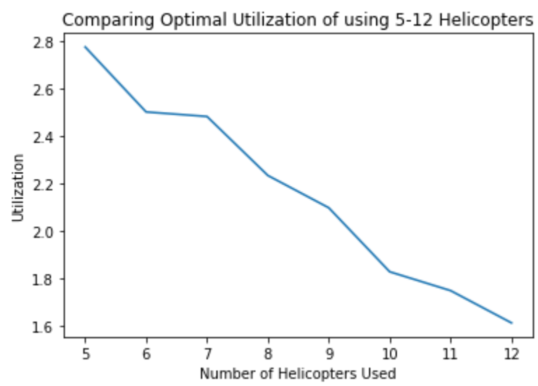
Appendix 12: Comparing Results from 5-12 Helicopters

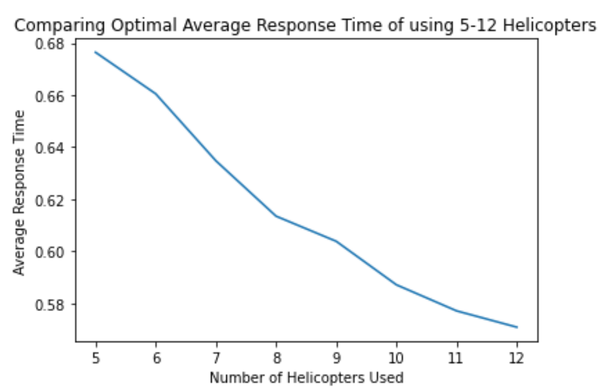
We plotted the performance metrics of model results from 5 to 12 helicopters.





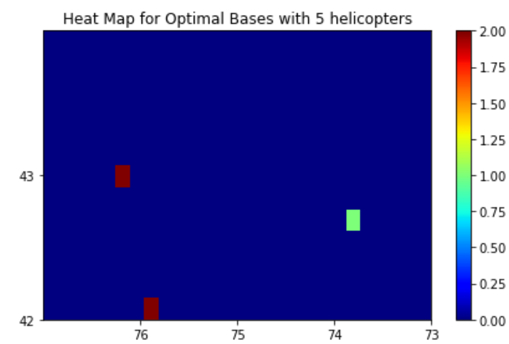


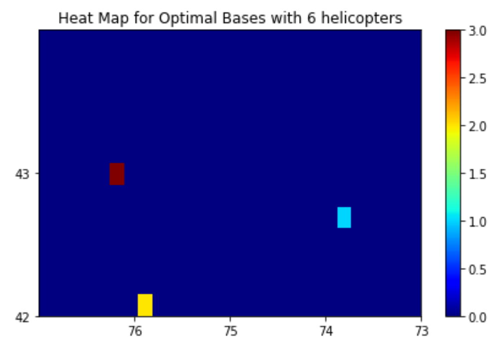


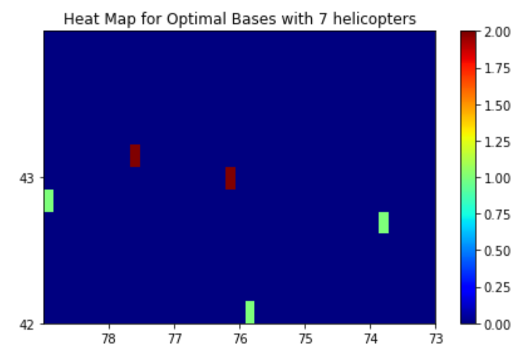


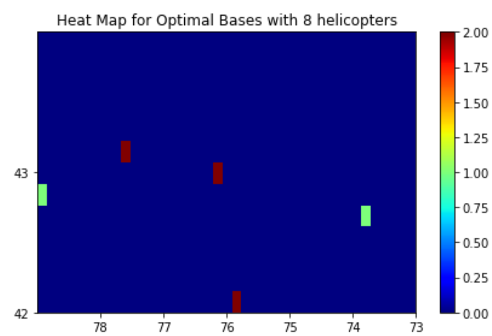
Appendix 13: Map of 5-12 Helicopters

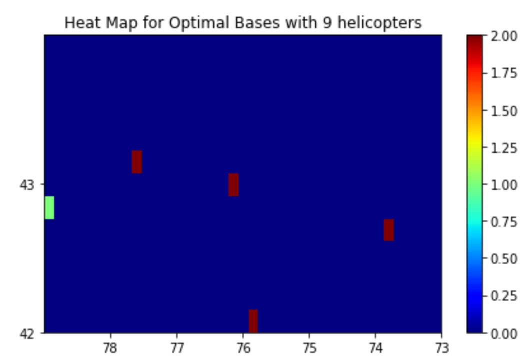
We mapped the locations on a plot with the latitude and longitude on the x and y axis. The color scale indicates the number of helicopters at each location.

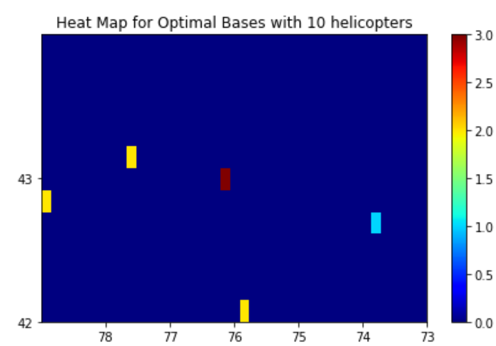


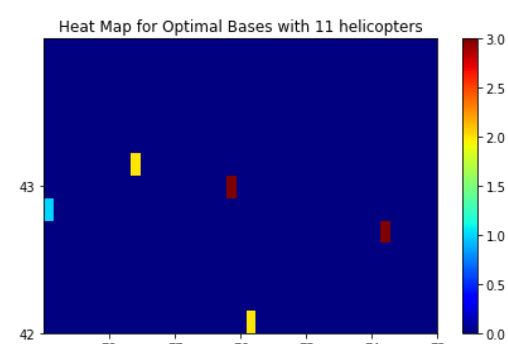


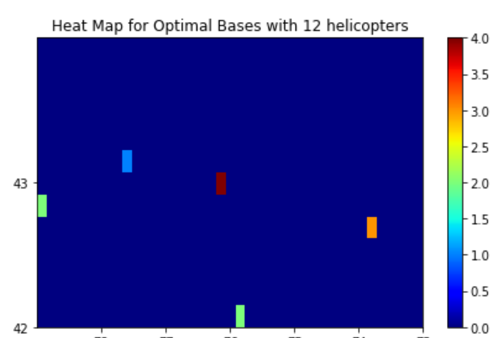












Appendix 14: Call Probabilities in Upstate New York

From the historical data, we plotted the density of calls in the upstate region, using a heat map probability distribution.

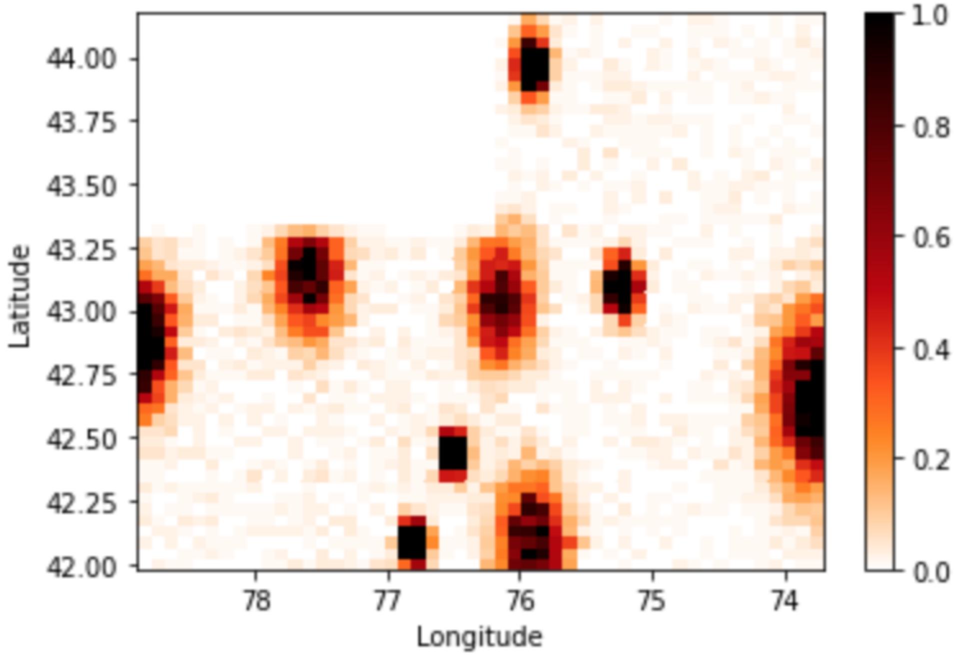


Figure Source: Project HW Solutions