ML Series - Linear Regression

Linear regression is a supervised machine learning algorithm used to model the linear relationship between a target variable (or dependent variable) and one or more independent variables. The goal of linear regression is to find the best-fit line that minimizes the sum of the squared differences between the predicted and actual values of the dependent variable.

In Statistical terms, regression models help us make predictions about the population based on sample data.

`Variables`

`Target(or dependent): Y`

`Independent:` $x1$, $x2$, $x3 \ldots xk$

Simple linear regression model (population):

Y = $\beta 0 + \beta 1 x1 + \varepsilon$

1. Y – dependent variable.
2. $x1$ - independent variable.
3. $\beta 0$ (constant/intercept) - It is the constant term in the regression equation. It represents the predicted value of Y when the independent variable $x1$ is 0.
4. $\beta 1$ (coefficient) – quantifies the effect of $x1$ on y,it shows how much Y is expected to change when $x1$ increases by one unit, holding all other variables constant.
5. ε (error of estimation) - Also known as the residual or disturbance term, represents the difference between the observed value of Y and the value predicted by the model. It accounts for the variation in Y that cannot be explained by the independent variable $x1$.

Simple linear regression equation (sample):

$\hat{y} = b0 + b1x1$

1. $\hat{y}$ – target value
2. $b0$ – coefficient – estimate of $\beta 0$
3. $b1$ – coefficient – estimate of $\beta 1$
4. $x1$ – sample data for the independent variable
5. ε – error of estimation

Geometrical representation of Linear regression model

Election

Regression line or the best fitting line through data points.

Why Linear regression:

1. **Simplicity and interpretability**: It's a relatively easy concept to understand and apply. The resulting simple linear regression model is a straightforward equation that shows how one variable affects another. This makes it easier to explain and trust the results compared to more complex models.
2. **Prediction**: Linear regression allows you to predict future values based on existing data. For instance, you can use it to predict sales based on marketing spend or house prices based on square footage.
3. **Foundation for other techniques**: It serves as a building block for many other data science and machine learning methods. Even complex algorithms often rely on linear regression as a starting point or for comparison purposes.

Difference between regression and correlation

1. Correlation - measures the degree of relationship between two variables
2. Regression - shows how one variable affects another or what changes it causes to the other

Muliple Linear Regression in python

Implenting basic libraries

In [1]:
```
# basic python libraries
import pandas as pd
import numpy as np
```

```
import matplotlib.pyplot as plt
import seaborn as sns
%matplotlib inline

# advance libaries
from sklearn.model_selection import train_test_split
from sklearn.metrics import mean_squared_error
from sklearn.metrics import r2_score
```

In [3]:
```
import warnings
warnings.filterwarnings('ignore')
```

Loading and describing data

Dataset Link - https://www.kaggle.com/code/ashydv/sales-prediction-simple-linear-regression/notebook

In [113…
```
df = pd.read_csv("advertising.csv")
```

In [7]:
```
df.shape
```

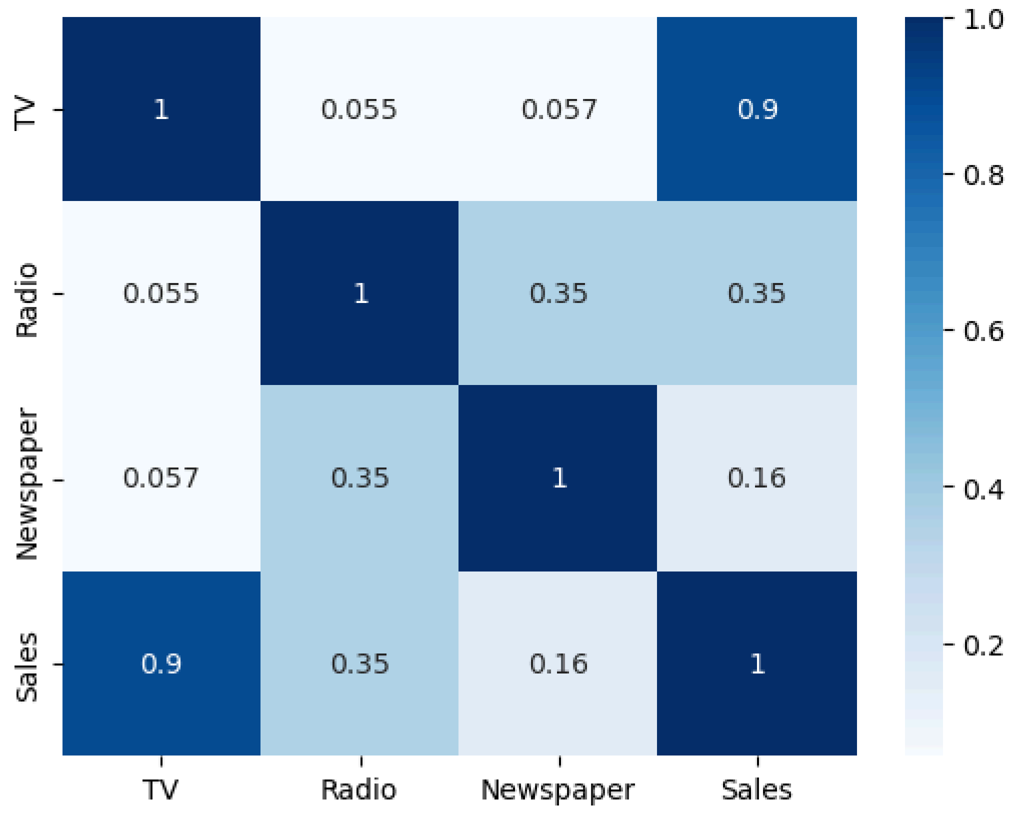
Out[7]:
```
(200, 4)
```

In [9]:
```
df.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 200 entries, 0 to 199
Data columns (total 4 columns):
 #   Column     Non-Null Count  Dtype  
---  ------     --------------  -----  
 0   TV         200 non-null    float64
 1   Radio      200 non-null    float64
 2   Newspaper  200 non-null    float64
 3   Sales      200 non-null    float64
dtypes: float64(4)
memory usage: 6.4 KB
```

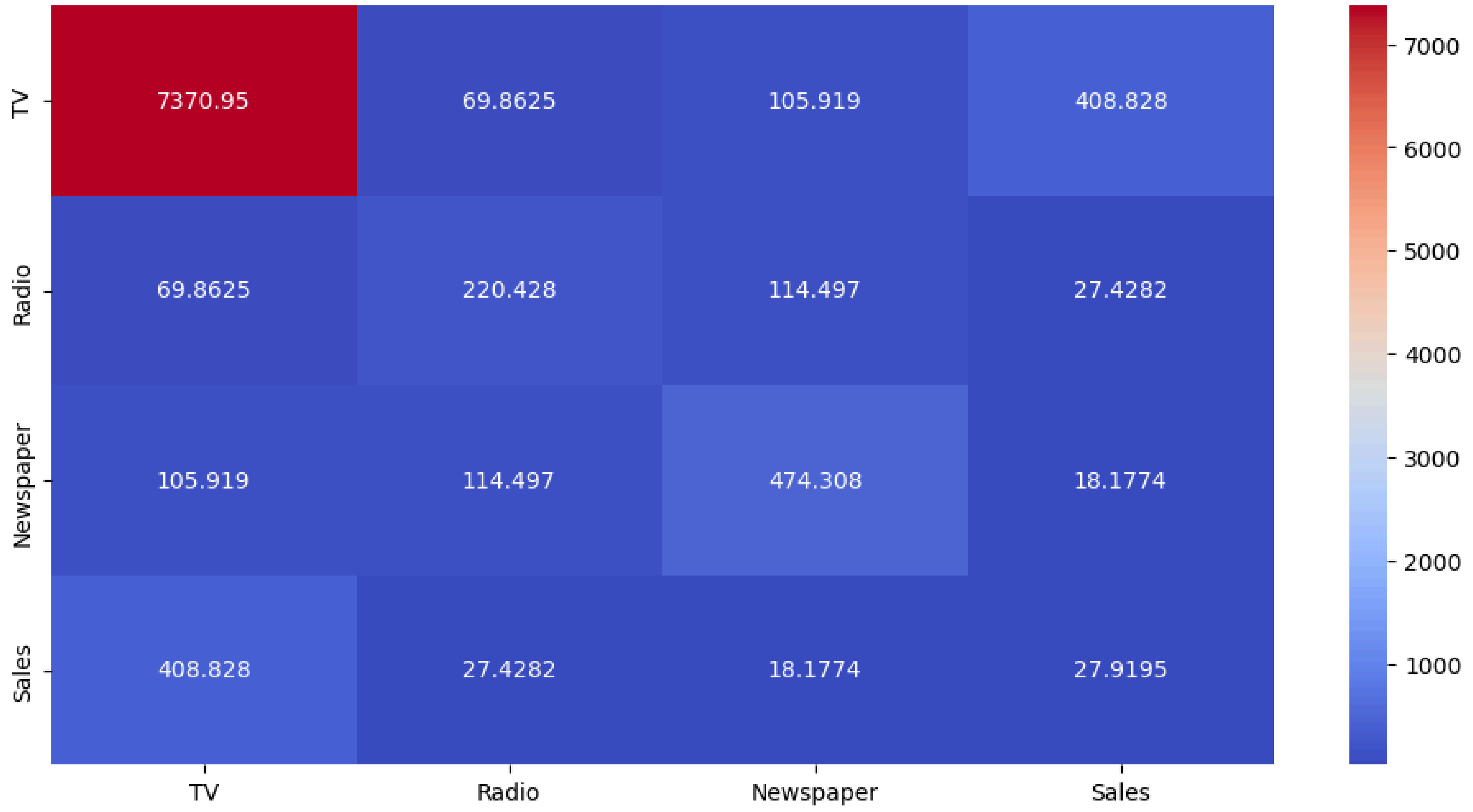
In [13]:
```
sns.heatmap(df.corr(),annot = True, cmap = "Blues", fmt= ".2g")
```

Out[13]:
```
<Axes: >
```



In [17]:
```
plt.figure(figsize=(12,6))
sns.heatmap(df.cov(), annot = True, cmap ='coolwarm',fmt=".6g")
```

Out[17]:
```
<Axes: >
```



In [19]:
```
df.describe()
```

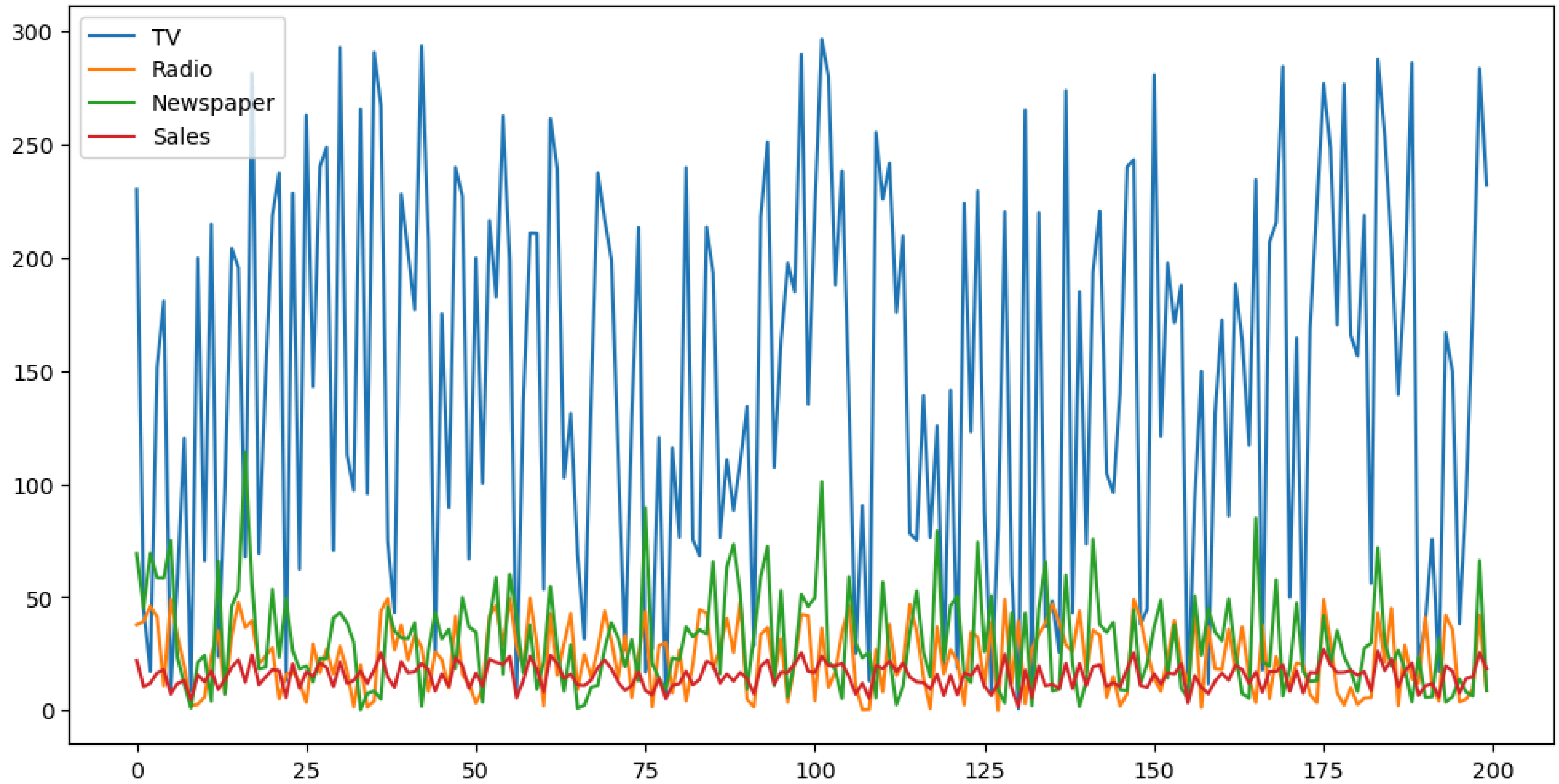
Out[19]:

	TV	Radio	Newspaper	Sales
count	200.000000	200.000000	200.000000	200.000000
mean	147.042500	23.264000	30.554000	15.130500
std	85.854236	14.846809	21.778621	5.283892
min	0.700000	0.000000	0.300000	1.600000
25%	74.375000	9.975000	12.750000	11.000000
50%	149.750000	22.900000	25.750000	16.000000
75%	218.825000	36.525000	45.100000	19.050000
max	296.400000	49.600000	114.000000	27.000000

Exploring the dataset

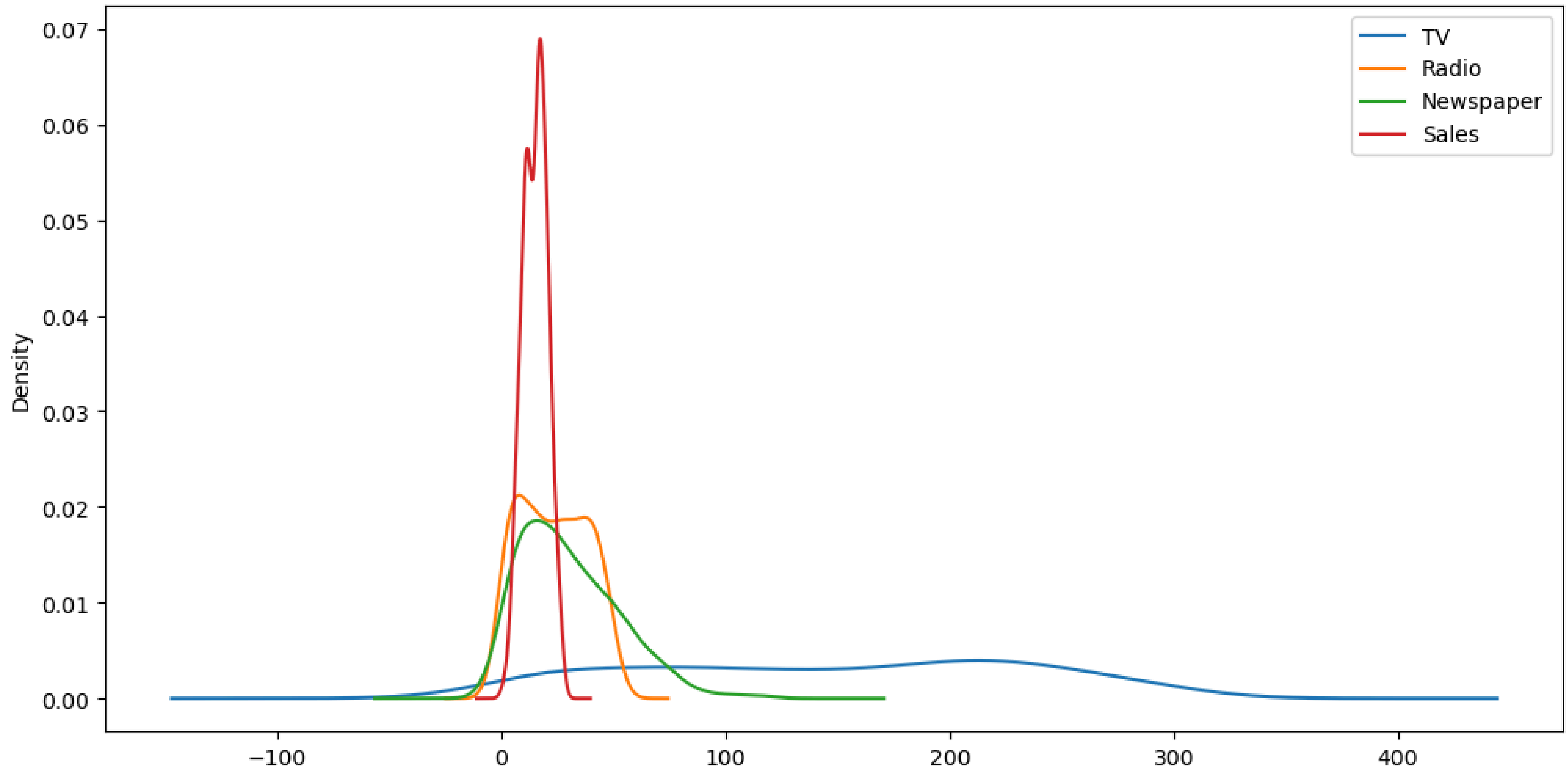
In [22]:
```
df.plot(figsize=(12, 6))
```

Out[22]: <Axes: >

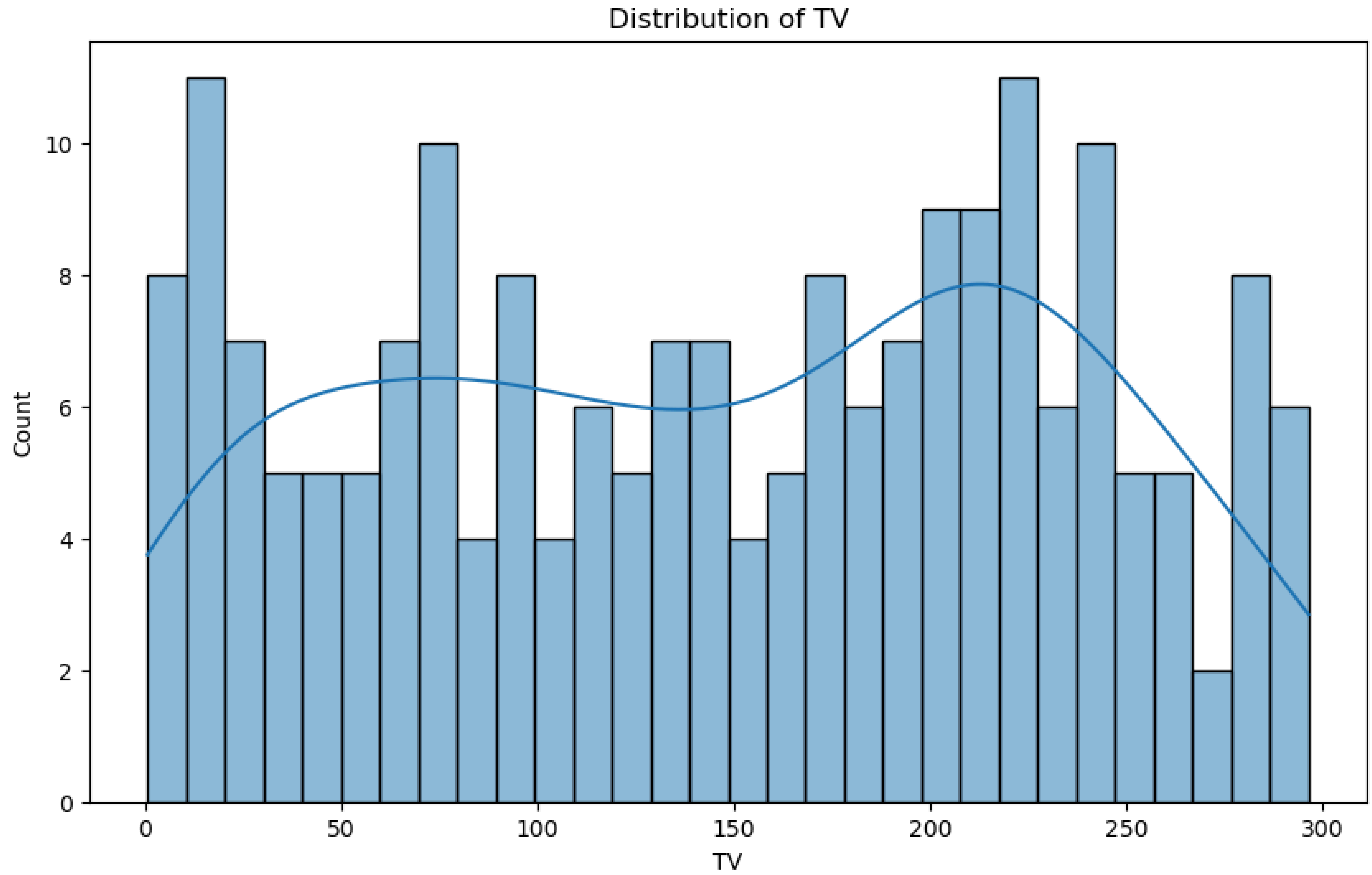


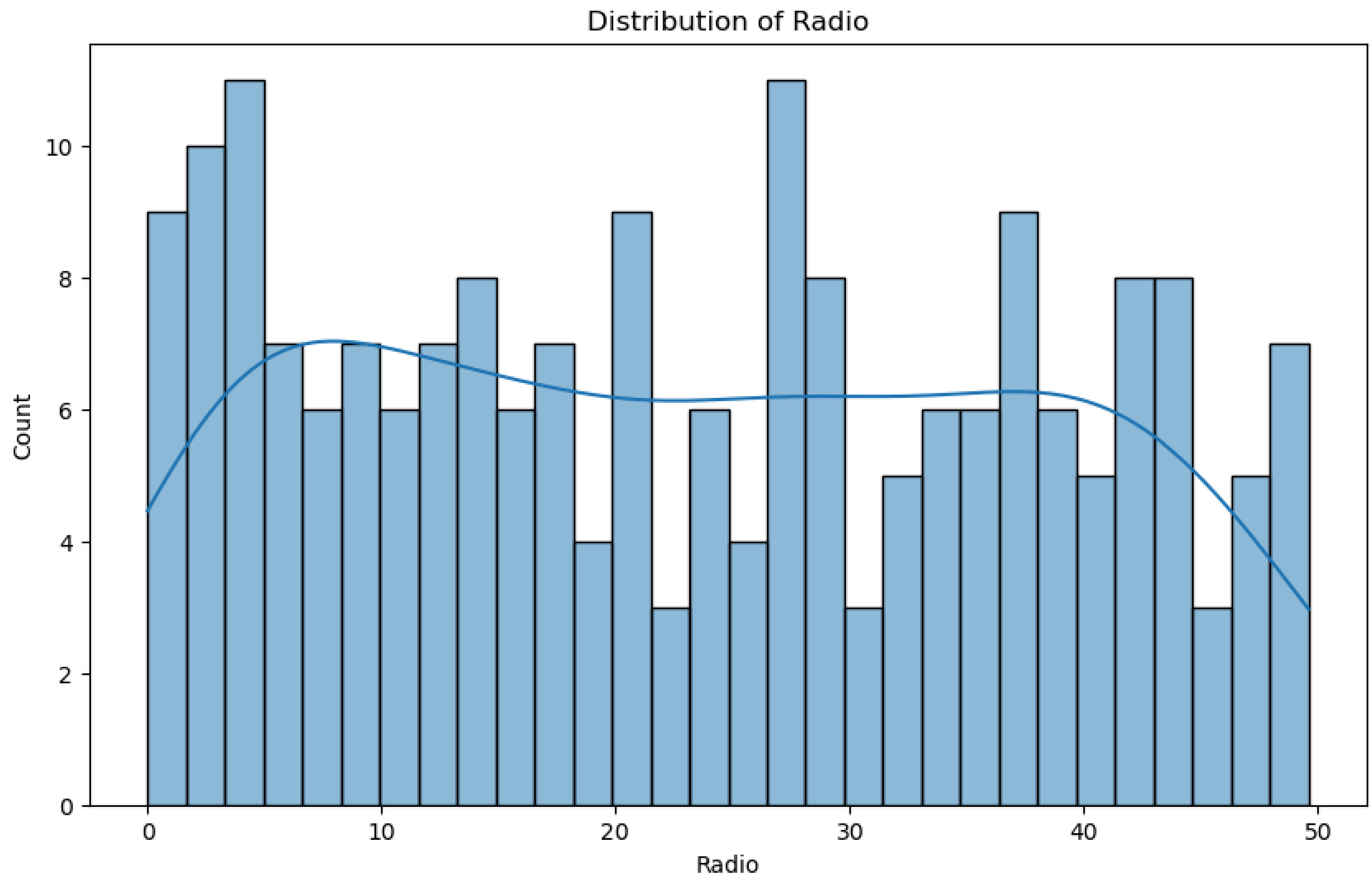
In [254…
```
for i in df.columns:
    df[i].plot.kde(figsize=(12,6))
```

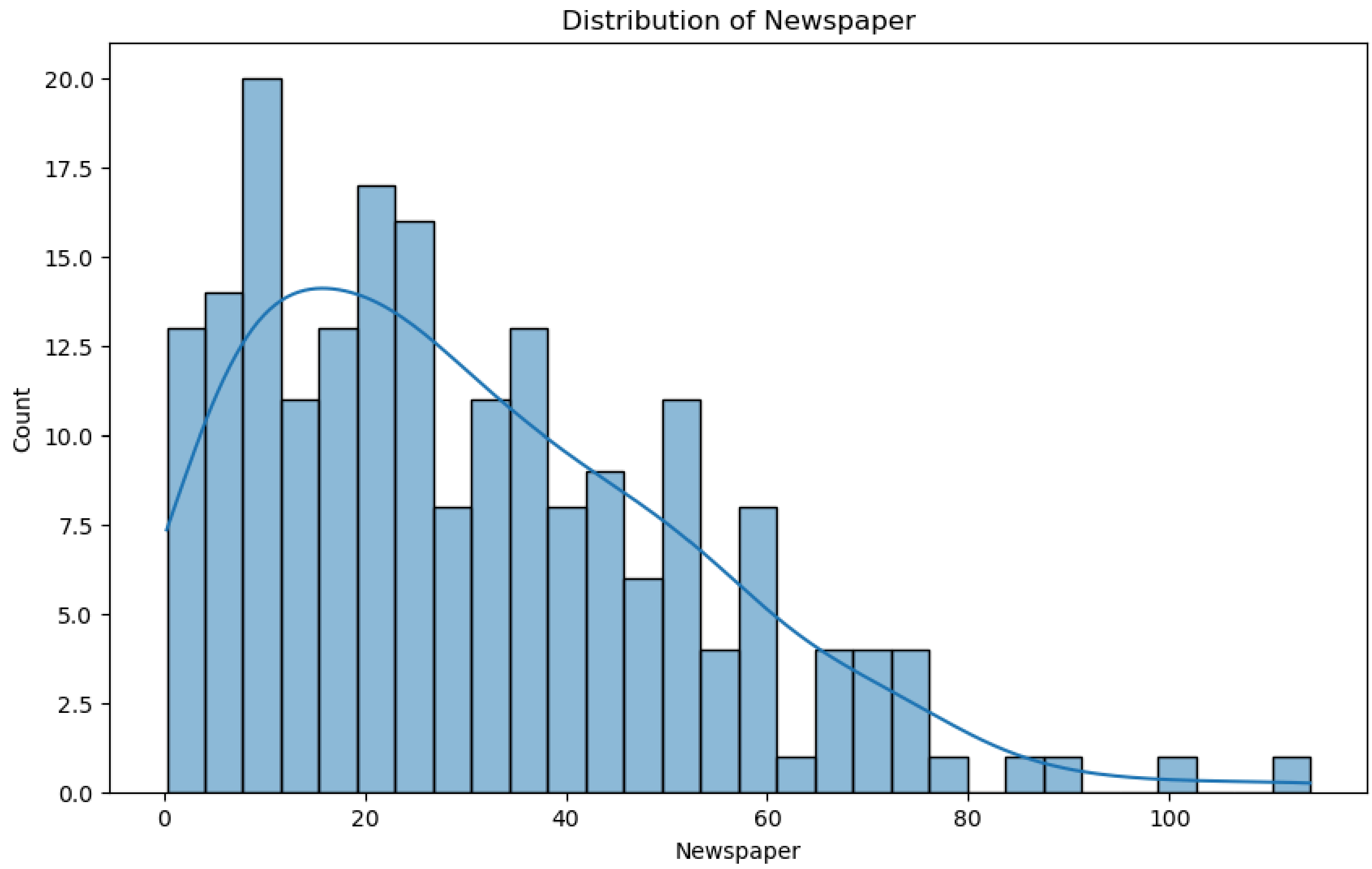
```
    plt.legend()
```

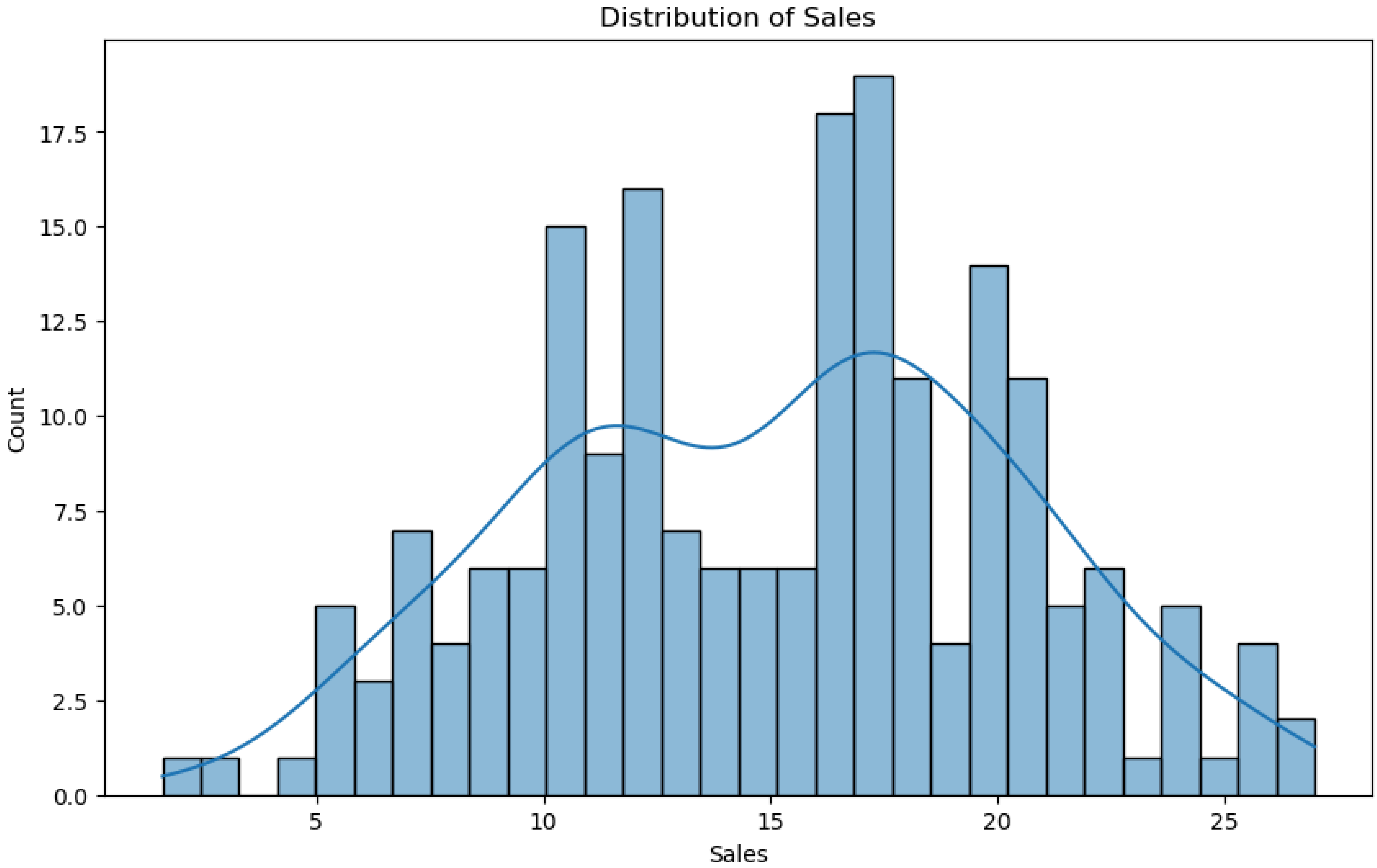


In [28]:
```
for i in df.columns:
    plt.figure(figsize=(10,6))
    sns.histplot(df, x=i, kde=True, bins=30)
    plt.title(f'Distribution of {i}')
    plt.show()
```



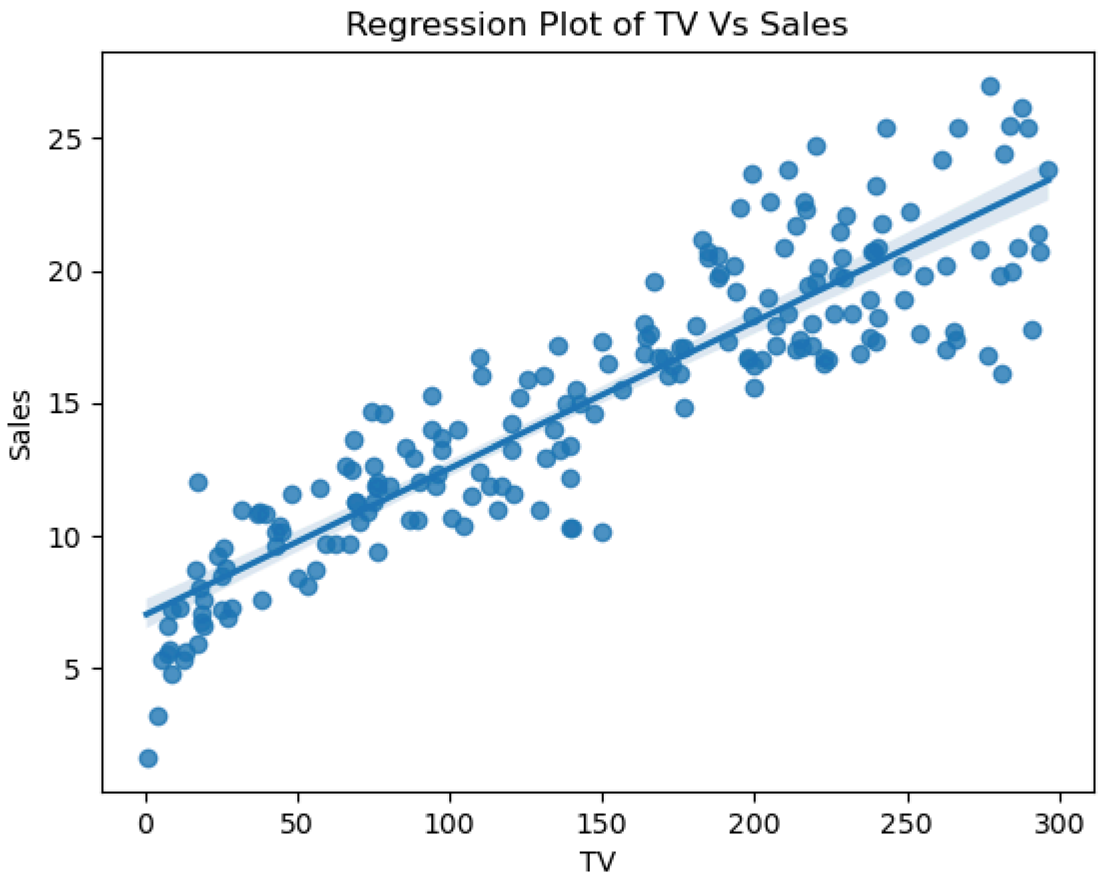

Distribution of Radio
Count
Radio
0
2
4
6
8
10
0
10
20
30
40
50

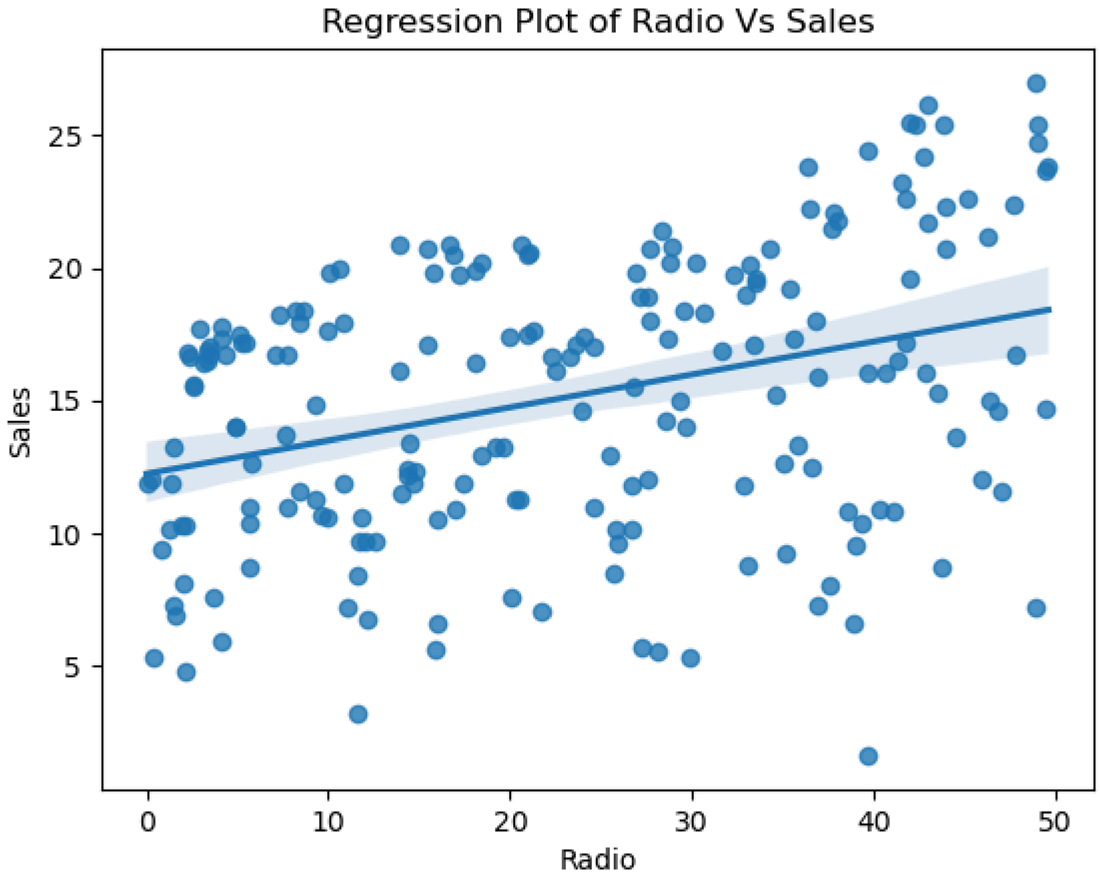

Distribution of Newspaper
Count
Newspaper
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
0
20
40
60
80
100

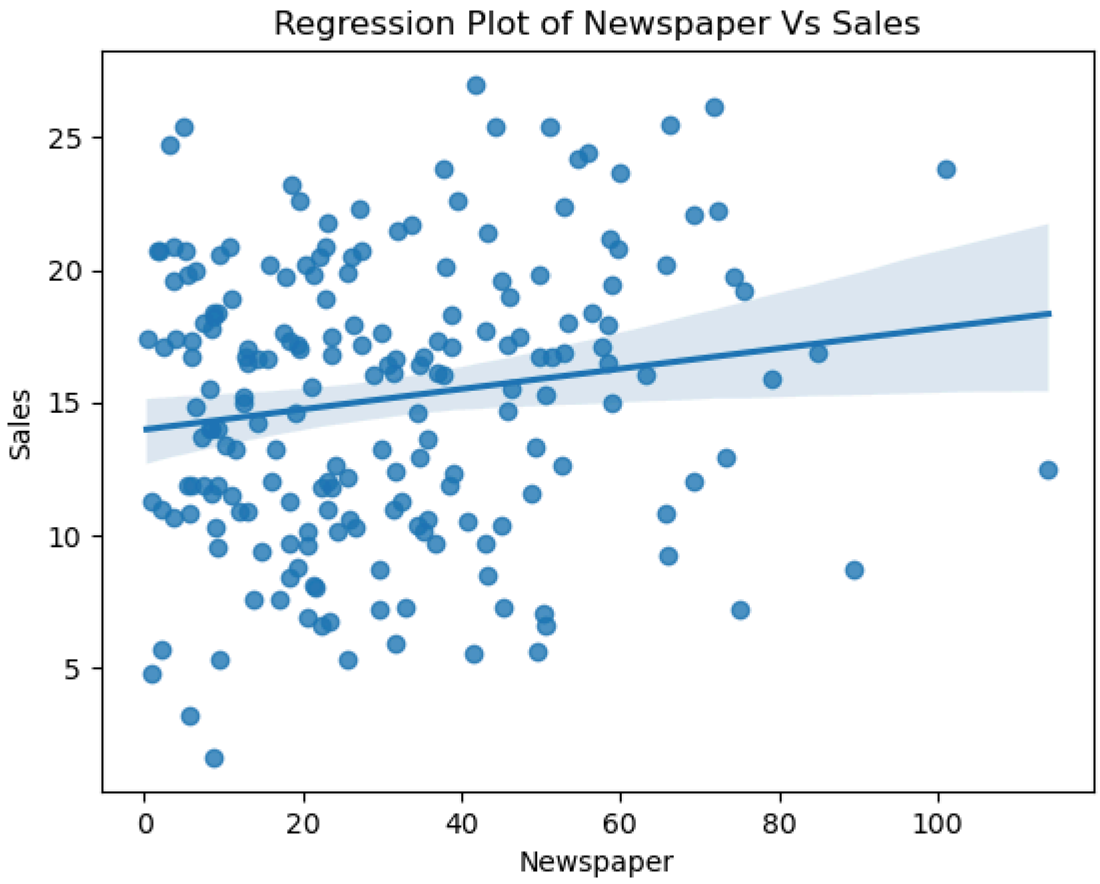


In [109…

```
for i in df.columns:
    if i != 'Sales':
        sns.regplot(data = df, x=i, y ='Sales')
        plt.title(f'Regression Plot of {i} Vs Sales')
        plt.show()
```

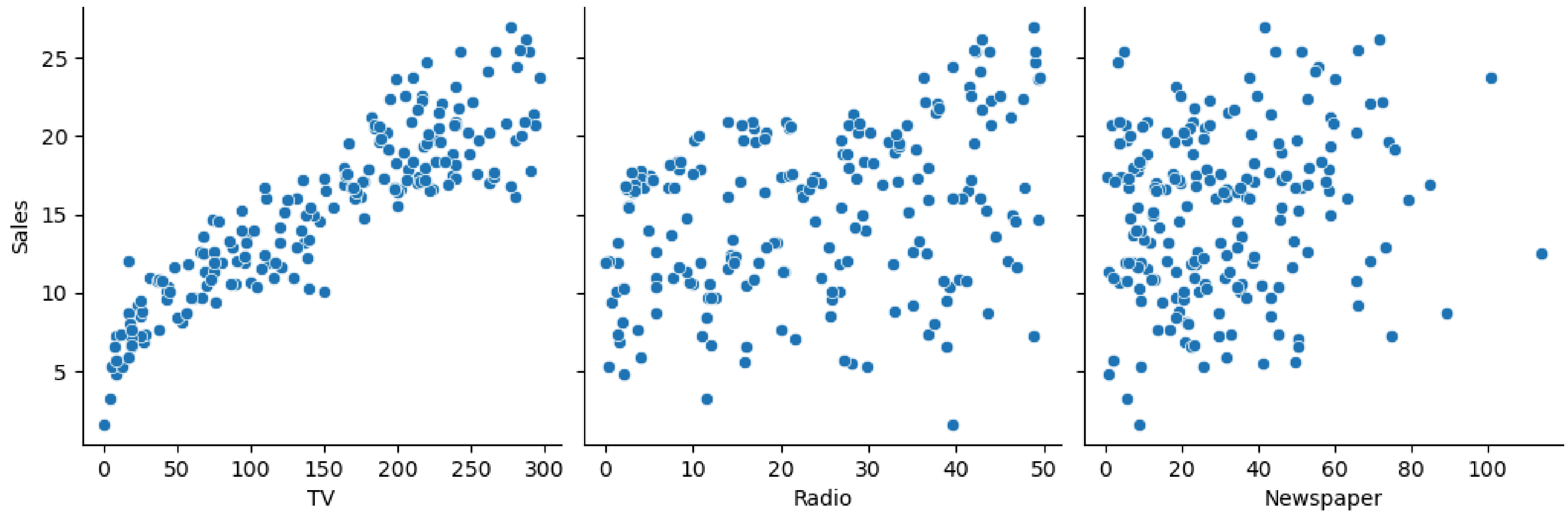






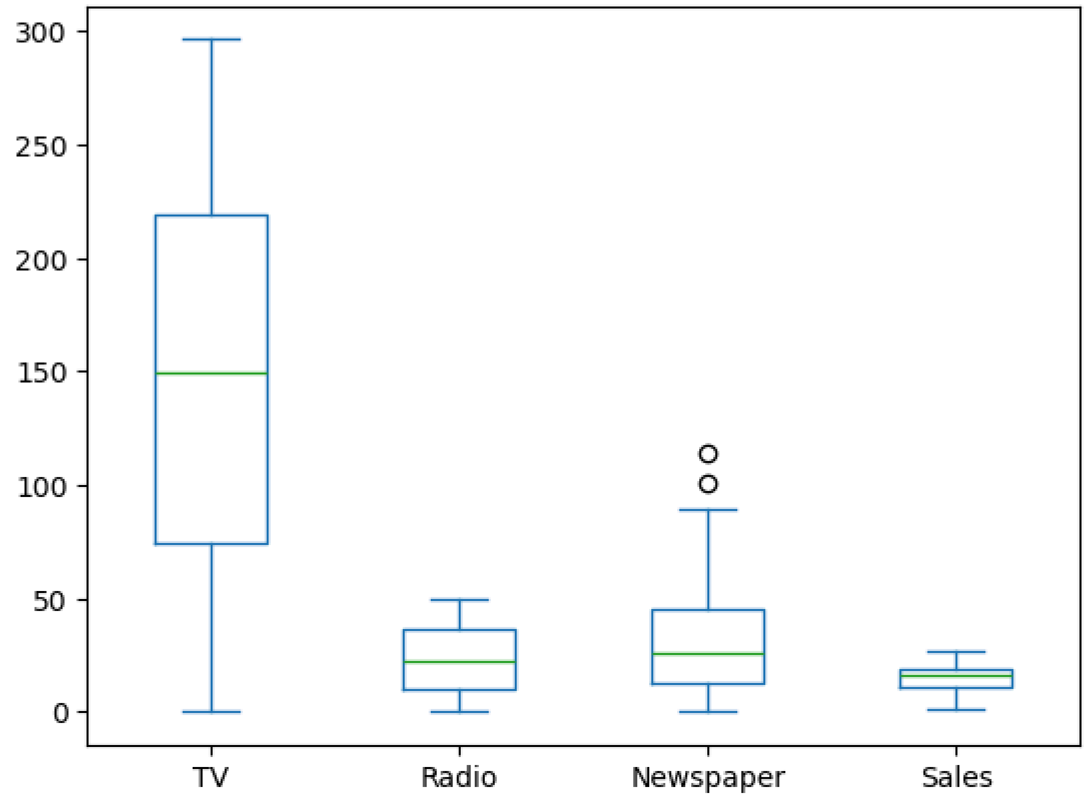

```
In [30]: sns.pairplot(df, x_vars = ['TV','Radio','Newspaper'], y_vars = ['Sales'], kind = 'scatter',aspect=1, size = 3.5)
```

```
Out[30]: <seaborn.axisgrid.PairGrid at 0x168bf75f0>
```




```
In [24]: df.plot.box()
```

```
Out[24]: <Axes: >
```

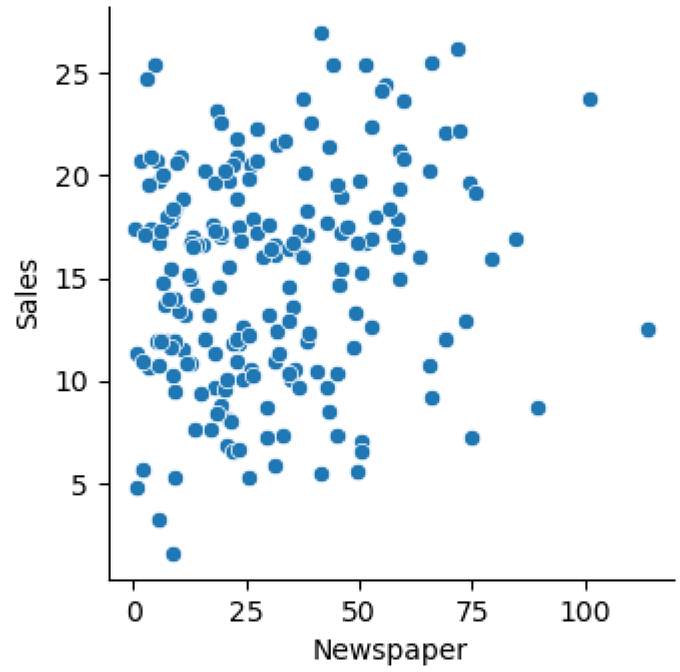


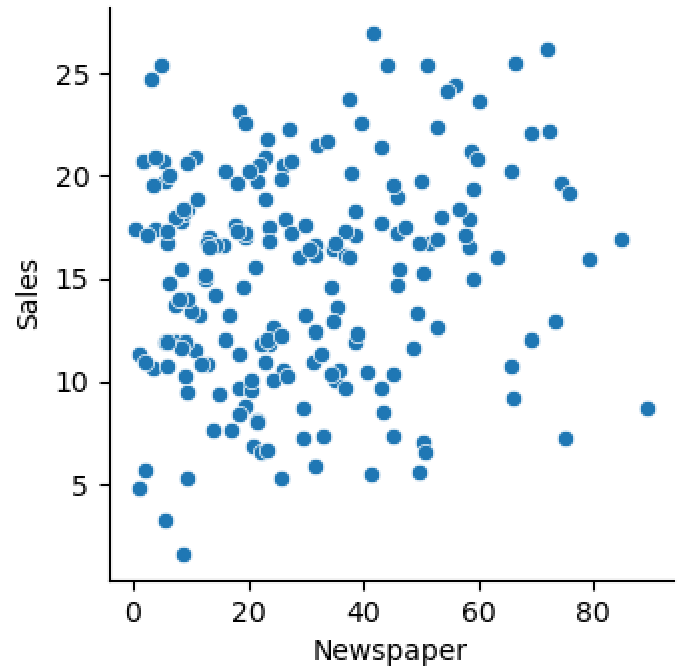
As we can see, there are outliers in the 'Newspaper' column. Let's remove these outliers from the dataset to make it more accurate and reliable

In [416…
```
Q1 = df.quantile(0.25) # define first quartile
Q3 = df.quantile(0.75) # define third quartile
IQR = Q3 - Q1 #calculating IQR
df1 = df[~((df < (Q1 - 1.5 * IQR)) |(df > (Q3 + 1.5 * IQR)))] # defining df1 with removing outlier from df
```

In [34]:
```
# Compairison of Newspaper column with with outlier and witout outlier
sns.pairplot(df, x_vars = ['Newspaper'], y_vars = ['Sales'], kind = 'scatter',aspect=1, size = 3.5)
sns.pairplot(df1, x_vars = ['Newspaper'], y_vars = ['Sales'], kind = 'scatter',aspect=1, size = 3.5)
```

Out[34]: <seaborn.axisgrid.PairGrid at 0x1690cb050>





In [36]:
```
df1['Newspaper'].equals(df['Newspaper']) #checking weather df1(with no outlier) is equal is df(with outlier)
```

Out[36]: False

Customizing preprocessing

In [39]:
```
df1 = df1.dropna()
```

In [41]:
```
df1
```

Out[41]:

	TV	Radio	Newspaper	Sales
0	230.1	37.8	69.2	22.1
1	44.5	39.3	45.1	10.4
2	17.2	45.9	69.3	12.0
3	151.5	41.3	58.5	16.5
4	180.8	10.8	58.4	17.9
...	...	...	...	...
195	38.2	3.7	13.8	7.6
196	94.2	4.9	8.1	14.0
197	177.0	9.3	6.4	14.8
198	283.6	42.0	66.2	25.5
199	232.1	8.6	8.7	18.4

198 rows × 4 columns

Deifining X and y varaible as independent and dependent variable.

In [117…
```
X = df1.drop(['Sales'],axis=1) # independent variable
y = df1['Sales'] # dependent variable
```

Deviding data into train_test_split

In [47]:
```
X_train, X_test, y_train, y_test = train_test_split(X,y, test_size=0.2, random_state = 42) # with test_size = 20 %
```

Building the linear regression model from sklearn

In [121…
```
from sklearn.linear_model import LinearRegression
lr_model = LinearRegression(fit_intercept=True, positive = True)
lr_model.fit(X_train, y_train)
```

Out[121…
```
▾ LinearRegression
LinearRegression(positive=True)
```

Predicting dependent value in terms of testing value

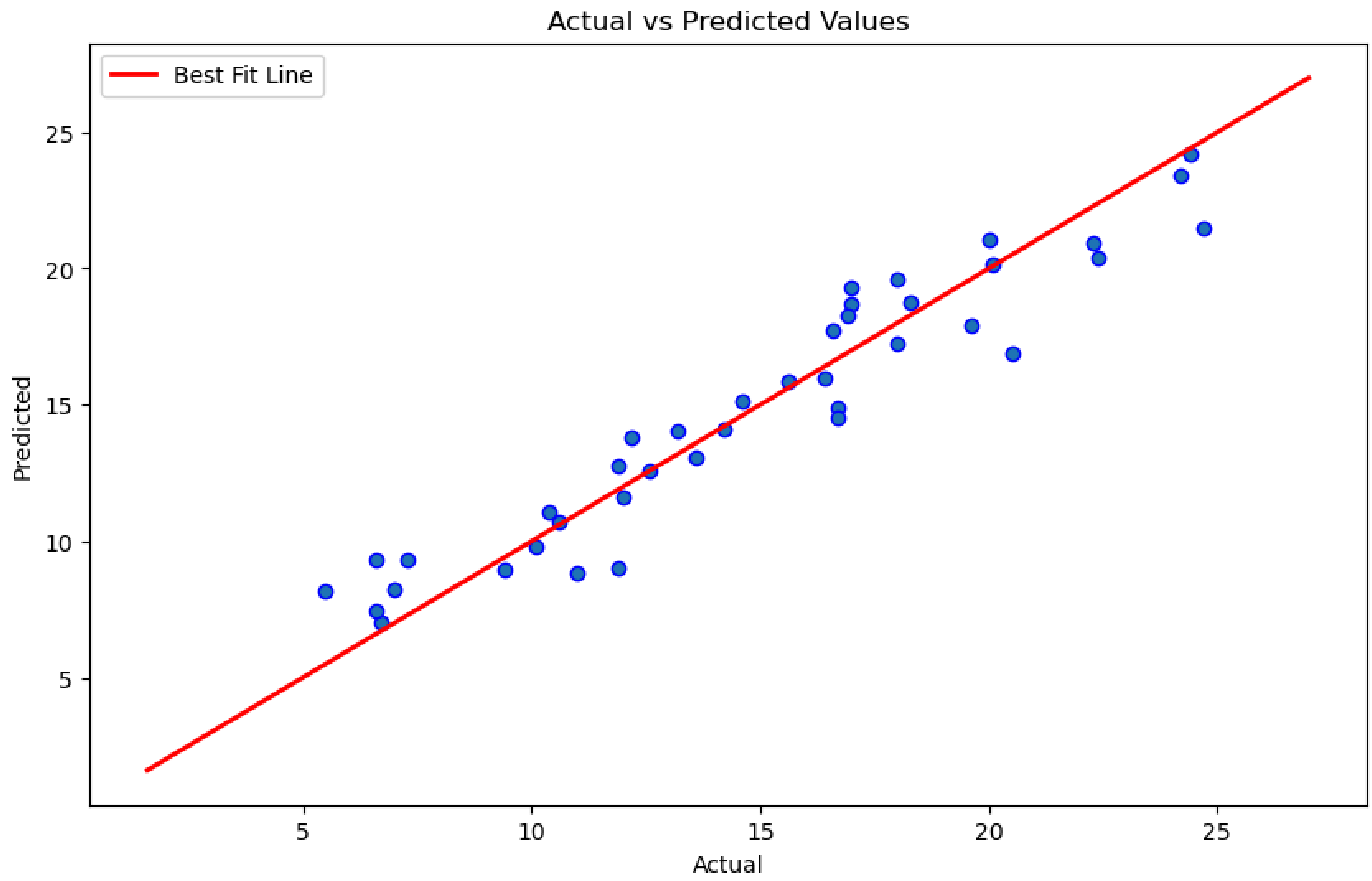
In [125…
```
y_pred = lr_model.predict(X_test)
y_pred
```

Out[125…
```
array([ 8.82481623, 13.77055468, 24.22481723, 11.10056512,  9.32009201,
       21.48456053, 20.15850056, 12.76167515, 15.12881886, 21.08287367,
       15.98934888, 20.38088326,  8.16535464, 14.03642672,  8.9375406 ,
       10.73054602,  9.35201578, 15.82574293,  9.79573284, 11.63821168,
       12.60519985, 17.21866213, 23.44516693,  7.02979947, 18.7214195 ,
       20.9597909 ,  8.20661353, 18.25399281,  7.45331361, 14.85839901,
       18.74710884, 14.11078068,  9.01994517, 13.07559033, 19.30878714,
       17.89362267, 16.88640389, 14.52324791, 19.60537819, 17.74576003])
```

Best fit line for acrtual test value vs predicted value

Best Fit Line - In simple terms, the best-fit line is a line that best fits the given scatter plot(i.e. y_test). Mathematically, you obtain the best-fit line by minimizing the Residual Sum of Squares (RSS).

In [293…
```
plt.figure(figsize=(10, 6))
plt.scatter(y_test, y_pred, edgecolors='blue')
plt.plot([y.min(), y.max()], [y.min(), y.max()], 'r', lw=2, label = "Best Fit Line") #xlim, ylim, color, linewidth,
plt.legend(loc=2)
plt.xlabel('Actual')
plt.ylabel('Predicted')
plt.title('Actual vs Predicted Values')
plt.show()
```



Checking the coefficients of independent varibale with dependent varaible by keeping all variables constant

In [244…

```
for i, col in enumerate(X.columns):
    if i != 'Sales':
        print(f'The coefficient of {col} is: {lr_model.coef_[i+1]:.16f}')
```

```
The coefficient of TV is: 0.0539592475959363
The coefficient of Radio is: 0.1010151220694665
The coefficient of Newspaper is: 0.0074693692578945
```

Gettting the intercept - constant term which indicates the value of dependent varibale when independent variable is at 0.

In [242…

```
print("The y intercept is : ", lr_model.intercept_)
```

```
The y intercept is :  4.623695319484105
```

Getting various evaluation metrics (we did not calculate MAE, which is robust to outliers, because the outliers were already removed from the dataset)."

In [250…

```
print('MSE of LR_model:', mean_squared_error(y_test, y_pred))
print("RMSE for LR_model: ",np.sqrt( mean_squared_error(y_test, y_pred)))
print("R-squared for LR_model: ",r2_score( y_test, y_pred ))
```

```
MSE of LR_model: 2.415728945593508
RMSE for LR_model:  1.5542615434969458
R-squared for LR_model:  0.9130003152798273
```

Predicting the value for new unseen data

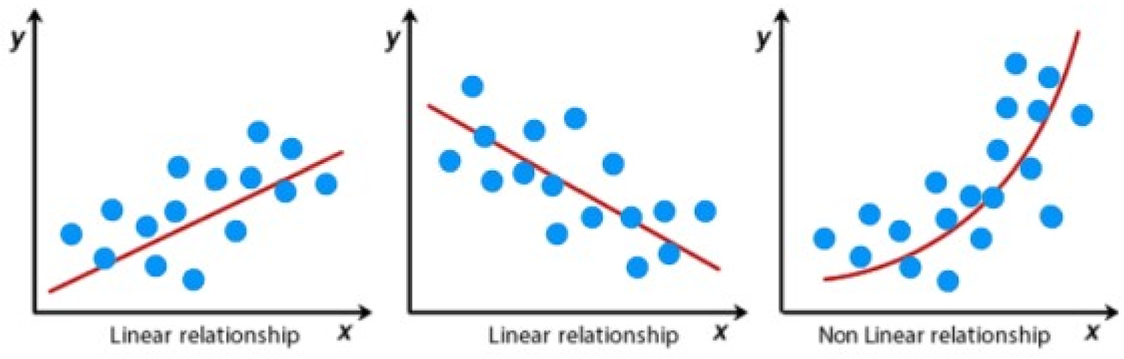
In [418…

```
user_input = {'const': 1,
              'TV': float(input("Enter value for TV: ")),
              'Radio': float(input("Enter value for Radio: ")),
              'Newspaper': float(input("Enter value for Newspaper: "))}

user_data = pd.DataFrame([user_input])
print(user_data)
new_prediction = lr_model.predict(user_data)
print("Predicted response value:", new_prediction[0])
```

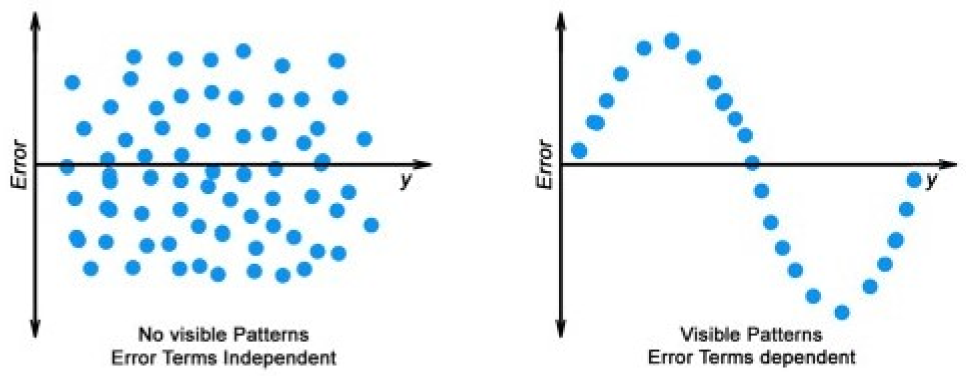
```
   const     TV  Radio  Newspaper
0      1  178.9   68.7       59.4
Predicted response value: 21.660424134488395
```

Assumption in Linear Regression

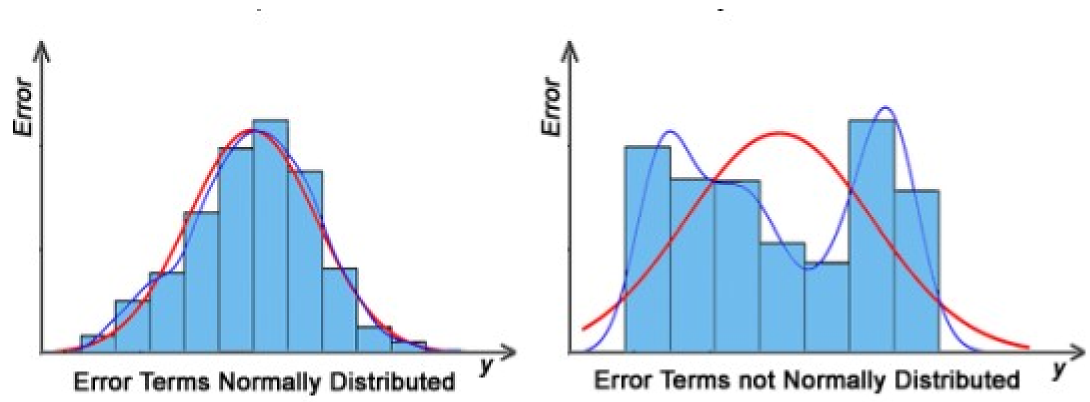
1. **Linearity of residuals**: There needs to be a linear relationship between the dependent variable and independent variable(s).



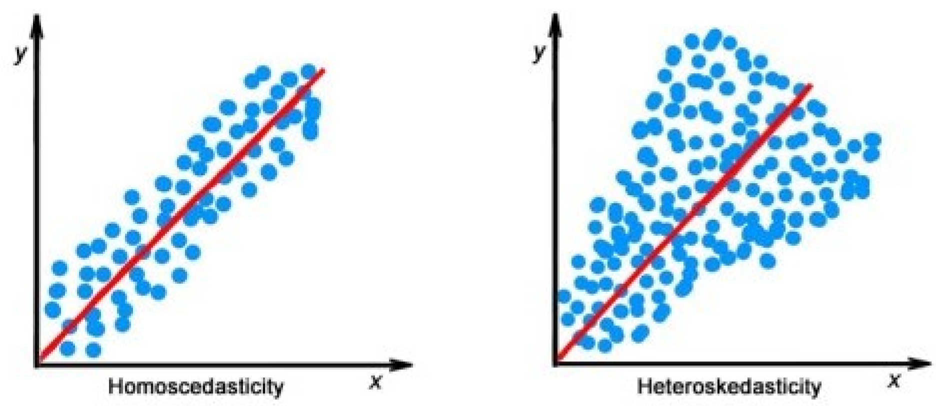
2. **Independence of residuals**: The error terms should not be dependent on one another (like in time-series data wherein the next value is dependent on the previous one). There should be no correlation between the residual terms. The absence of this phenomenon is known as Autocorrelation.



3. **Normal distribution of residuals**: The mean of residuals should follow a normal distribution with a mean equal to zero or close to zero. This is done to check whether the selected line is the line of best fit or not. If the error terms are non-normally distributed, suggests that there are a few unusual data points that must be studied closely to make a better model.



4. **The equal variance of residuals**: The error terms must have constant variance. This phenomenon is known as Homoscedasticity. The presence of non-constant variance in the error terms is referred to as Heteroscedasticity. Generally, non-constant variance arises in the presence of outliers or extreme leverage values.



5. **Overfitting**: When more and more variables are added to a model, the model may become far too complex and usually ends up memorizing all the data points in the training set. This phenomenon is known as the overfitting of a model. This usually leads to high training accuracy and very low test accuracy.
6. **Multicollinearity**: It is the phenomenon where a model with several independent variables, may have some variables interrelated.
7. **Feature Selection**: With more variables present, selecting the optimal set of predictors from the pool of given features (many of which might be redundant) becomes an important task for building a relevant and better model.

Reference Link - https://www.analyticsvidhya.com/blog/2021/10/everything-you-need-to-know-about-linear-regression/

Building Linear regression using OLS Model

OLS or the ordinary least squares is the most common method to do estimate of the linear regression equation. “Least squares” stands for the minimum squares error, or SSE. This method aims to find the line, which minimizes the sum of the squared errors.

In [326…
```
import statsmodels.api as sm
X_train = sm.add_constant(X_train) # adding constant or intercept to X_train
sm_model = sm.OLS(y_train, X_train).fit() # building model by fitting y_train and X_train
```

Getting parameters of OLS Model

In [330…
```
sm_model.params
```

Out[330…
```
const        4.623695
TV           0.053959
Radio        0.101015
Newspaper    0.007469
dtype: float64
```

In [332…
```
w0 = sm_model.params[0]  # Intercept
w1, w2, w3 = sm_model.params[1], sm_model.params[2], sm_model.params[3] # Coefficients
```

In [334…
```
print(f'Intecept of statsmodel : {w0}')
print(f'Coefficients of Features of statsmodel : {w1, w2, w3}')
```

```
Intecept of statsmodel : 4.6236953194840655
Coefficients of Features of statsmodel : (0.053959247595936476, 0.10101512206946696, 0.007469369257894682)
```

In [336…
```
print(sm_model.summary())
```

```
                            OLS Regression Results                            
==============================================================================
Dep. Variable:                  Sales   R-squared:                       0.897
Model:                            OLS   Adj. R-squared:                  0.895
Method:                 Least Squares   F-statistic:                     448.5
Date:                Fri, 09 Aug 2024   Prob (F-statistic):           7.30e-76
Time:                        23:14:40   Log-Likelihood:                -306.46
No. Observations:                 158   AIC:                             620.9
Df Residuals:                     154   BIC:                             633.2
Df Model:                           3                                         
Covariance Type:            nonrobust                                         
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          4.6237      0.354     13.070      0.000       3.925       5.323
TV             0.0540      0.002     34.088      0.000       0.051       0.057
Radio          0.1010      0.010     10.303      0.000       0.082       0.120
Newspaper      0.0075      0.007      1.060      0.291      -0.006       0.021
==============================================================================
Omnibus:                       15.937   Durbin-Watson:                   2.278
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               25.727
Skew:                          -0.525   Prob(JB):                     2.59e-06
Kurtosis:                       4.676   Cond. No.                         455.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

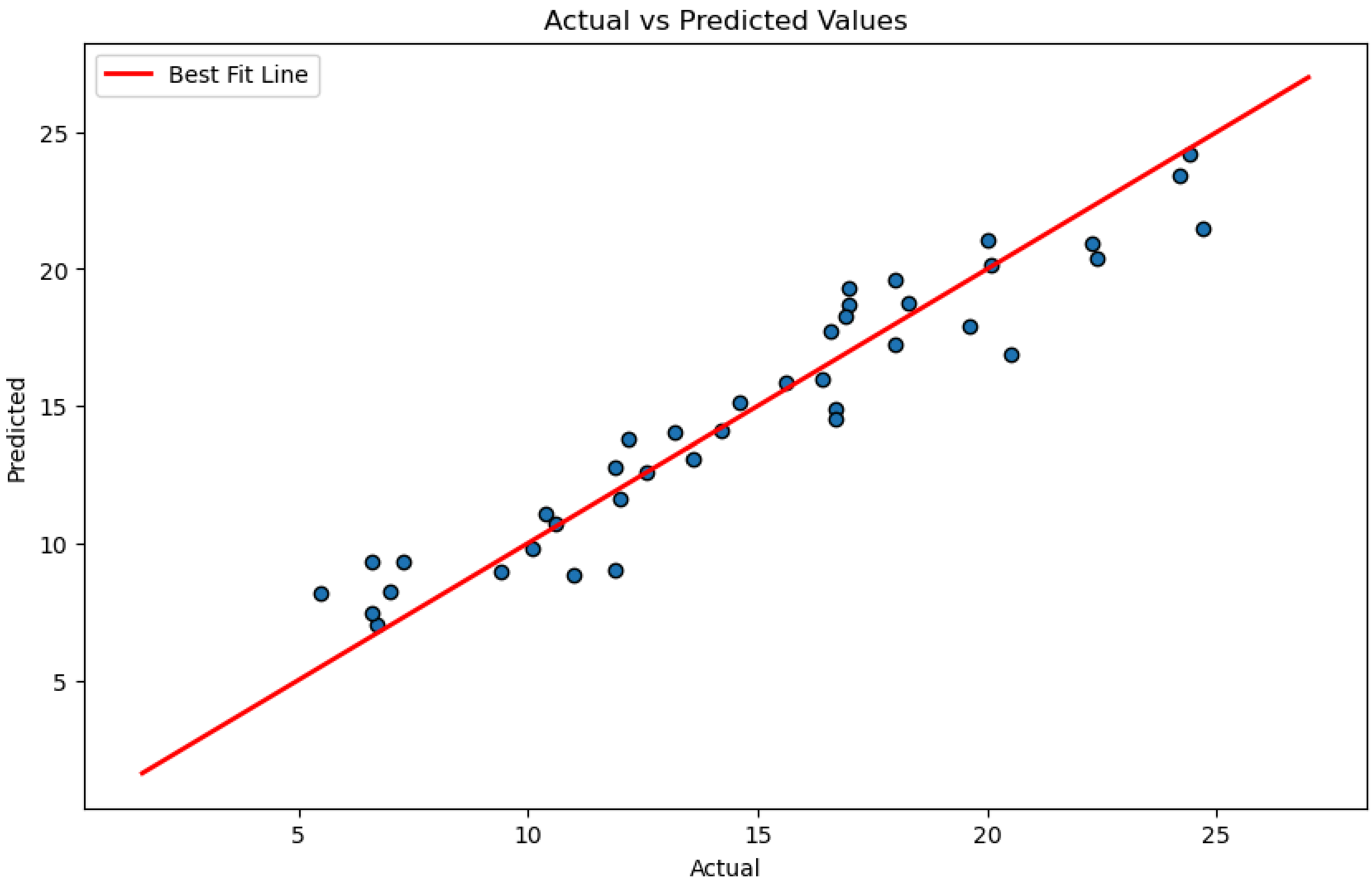
In [338…
```
X_test = sm.add_constant(X_test)
y_pred = sm_model.predict(X_test)
```

In [340…
```
y_pred
```

```
Out[340…  66      8.824816
116     13.770555
17      24.224817
143     11.100565
158      9.320092
128     21.484561
142     20.158501
31      12.761675
19      15.128819
169     21.082874
160     15.989349
15      20.380883
56       8.165355
57      14.036427
117      8.937541
46      10.730546
126      9.352016
9       15.825743
149      9.795733
86      11.638212
115     12.605200
163     17.218662
61      23.445167
189      7.029799
74      18.721419
69      20.959791
121      8.206614
165     18.253993
119      7.453314
177     14.858399
70      18.747109
77      14.110781
127      9.019945
83      13.075590
25      19.308787
193     17.893623
97      16.886404
173     14.523248
20      19.605378
152     17.745760
dtype: float64
```

Getting Best Fit Line for OLS based Linear regression

In [350…
```
plt.figure(figsize=(10, 6))
plt.scatter(y_test, y_pred, edgecolors=(0, 0, 0))
plt.plot([y.min(), y.max()], [y.min(), y.max()], 'r', lw=2, label = "Best Fit Line")
plt.xlabel('Actual')
plt.ylabel('Predicted')
plt.title('Actual vs Predicted Values')
plt.legend(loc=2)
plt.show()
```



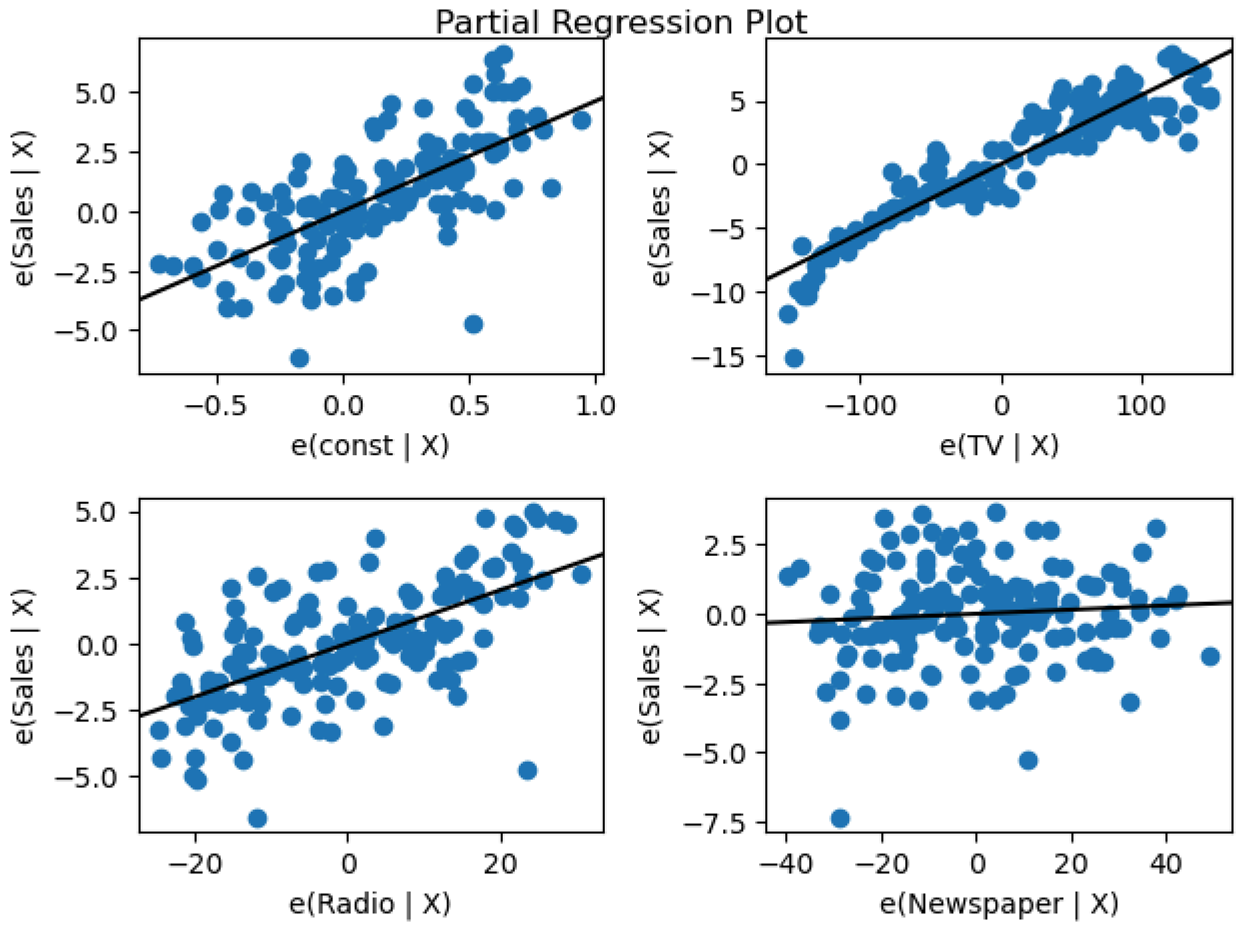
Getting Partial Regression Plot- Which is useful for understanding the relationships between individual predictor variables and the target variable in the context of a multiple regression model.

Partial regression plots are useful for:

1. **Understanding Relationships**: They help visualize the unique contribution of each predictor variable to the target variable, independent of other predictors.
2. **Detecting Multicollinearity**: They can reveal if any predictor variables are collinear with others, which might indicate multicollinearity issues.
3. **Diagnosing Model Issues**: These plots can help in diagnosing issues with the regression model by showing how well each predictor is explaining the variation in the target variable.

In [354…

```
import statsmodels.graphics.api as smg
smg.plot_partregress_grid(sm_model)
plt.figure(figsize=(10,8))
plt.show()
```

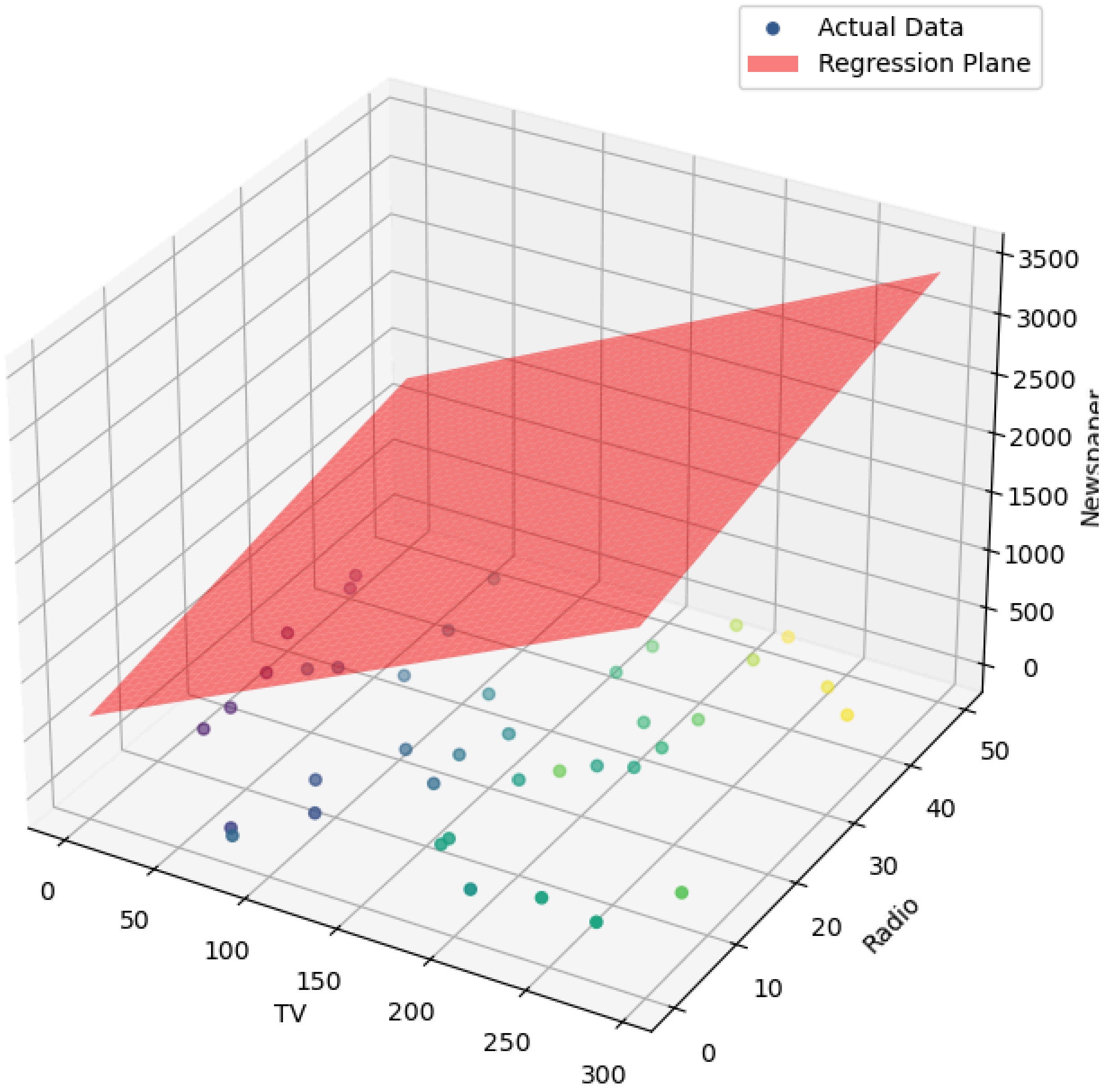



```
<Figure size 1000x800 with 0 Axes>
```

Visualizaing the regressiong in 3d plane

In [408…

```
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Create a 3D plot
fig = plt.figure(figsize=(15, 8))
ax = fig.add_subplot(111, projection='3d')

# Scatter plot for the actual data
ax.scatter(X_test['TV'], X_test['Radio'], X_test['Newspaper'], c=y_test, marker='o', label='Actual Data')

# Create a grid for the surface plot
x1_range = np.linspace(X_train['TV'].min(), X_train['TV'].max(), 100)
x2_range = np.linspace(X_train['Radio'].min(), X_train['Radio'].max(), 100)
x1, x2 = np.meshgrid(x1_range, x2_range)
x3 = (sm_model.params[0] + sm_model.params[1] * x1 + sm_model.params[2] * x2) / sm_model.params[3]  # Solve for x3

# Plot the surface
ax.plot_surface(x1, x2, x3, alpha=0.5, color='r', label='Regression Plane')

ax.set_xlabel('TV')
ax.set_ylabel('Radio')
ax.set_zlabel('Newspaper')

cbar = plt.colorbar(sc)
cbar.set_label('Sales')


plt.legend()
plt.show()
```



Predicting the sales price in unseen data

In [420…
```
user_input = {'const': 1,  # Add constant term manually
              'TV': float(input("Enter value for TV: ")),
              'Radio': float(input("Enter value for Radio: ")),
              'Newspaper': float(input("Enter value for Newspaper: "))}

user_df = pd.DataFrame([user_input])
print(user_df)
predicted_y = sm_model.predict(user_df)

print(f"Predicted value of Sales: {predicted_y[0]}")
```

```
   const     TV  Radio  Newspaper
0      1  178.9   67.9       56.4
Predicted value of Sales: 21.557203929059167
```

In [ ]: