Manual Compute Exercise 1

Exercise 1.1

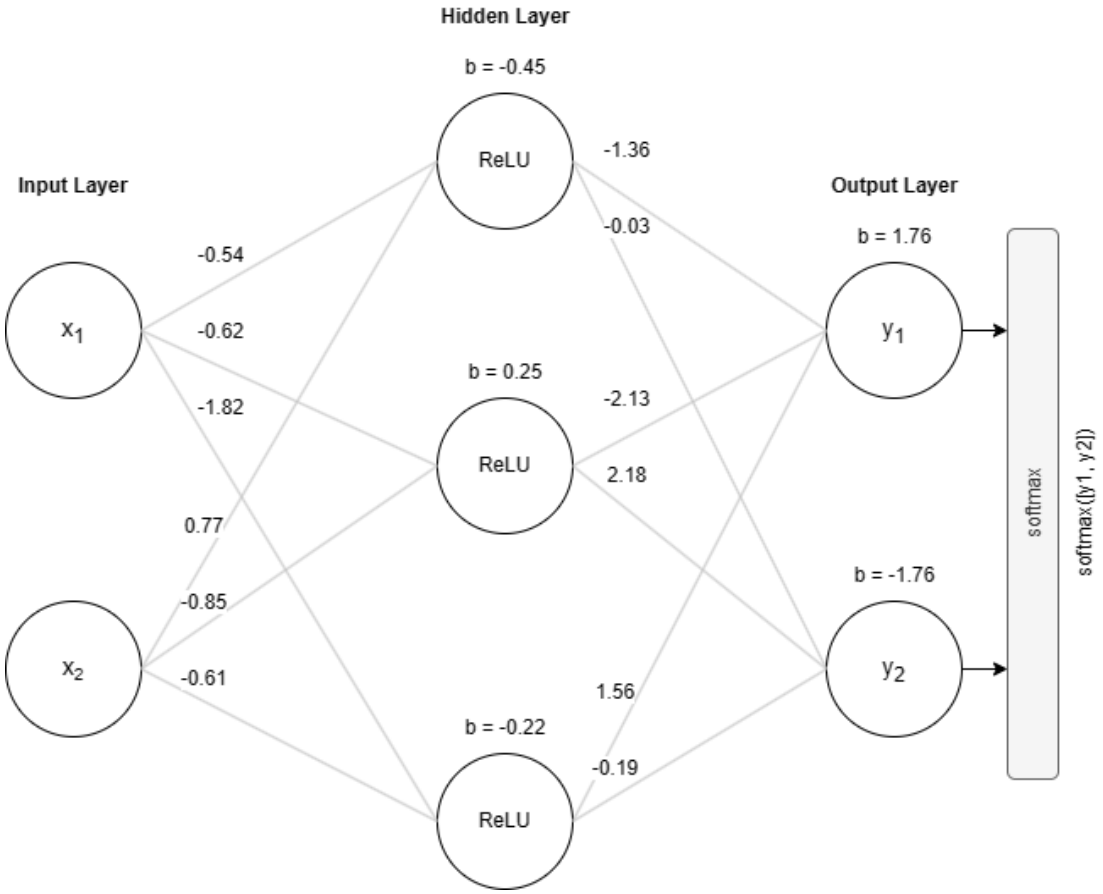

Figure 1: Simple Artificial Neural Network Layers

Exercise 1.2

Assume there are 2 new inputs:

$x_{new\_1} = [0, 5]$
$x_{new\_2} = [-1, -1]$

According to Figure 1, the formula can be defined as:

$$z = \left(\sum_{i=1}^{n} w_i * x_i\right) + b$$

The activation function used in the hidden layer is the ReLU function.

$$\hat{y} = ReLU(z)$$

The ReLU function is defined as:

$$f(x) = \max(0, x) = \begin{cases} 0, & x \leq 0 \\ x, & x > 0 \end{cases}$$

For the output layer, softmax is used to convert a vector of real numbers into a probability distribution.

$$softmax(\boldsymbol{x})_i = \frac{e^{x_i}}{\sum_{j=1}^{n} e^{x_j}}$$

In case of $\boldsymbol{x}_{new\_1}$:

$z_1 = (-0.54 * 0) + (0.77 * 5) - 0.45 = 3.4$
$z_2 = (-0.62 * 0) + (-0.85 * 5) + 0.25 = -4$
$z_3 = (-1.82 * 0) + (-0.61 * 5) - 0.22 = -3.27$

The value in the layer 1 will be:

$hidden_1 = ReLU(z_1) = 1$
$hidden_2 = ReLU(z_2) = 0$
$hidden_3 = ReLU(z_3) = 0$

After that, calculate the value in the output layer.

$output_1 = (-1.36 * 1) + (-2.13 * 0) + (1.56 * 0) + 1.76 = 0.4$
$output_2 = (-0.03 * 1) + (2.18 * 0) + (-0.19 * 0) - 1.76 = -1.79$

Finally, the probability distribution will be:

$y_{prob} = softmax([0.4, -1.79]) = [0.9, 0.1]$

The result is **Class 0**

In case of x_{new_2}:

$$z_1 = (-0.54 * -1) + (0.77 * -1) - 0.45 = -0.68$$
$$z_2 = (-0.62 * -1) + (-0.85 * -1) + 0.25 = 1.72$$
$$z_3 = (-1.82 * -1) + (-0.61 * -1) - 0.22 = 2.21$$

The value in the layer 1 will be:

$$hidden_1 = ReLU(z_1) = 0$$
$$hidden_2 = ReLU(z_2) = 1$$
$$hidden_3 = ReLU(z_3) = 1$$

After that, calculate the value in the output layer.

$$output_1 = (-1.36 * 0) + (-2.13 * 1) + (1.56 * 1) + 1.76 = 1.19$$
$$output_2 = (-0.03 * 0) + (2.18 * 1) + (-0.19 * 1) - 1.76 = 0.23$$

Finally, the probability distribution will be:

$$y_{prob} = softmax([1.19, 0.23]) = [0.72, 0.28]$$

The result is **Class 0**