Bank Runs, Sovereign Risk and International Reserves

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Abstract

Policymakers in developing countries often argue that an important reason for having international reserves is to have adequate liquidity in the event of shocks like bank runs. This paper studies the effect of domestic bank run risks on government reserve accumulation in a quantitative model of systemic bank runs and sovereign default with short-term debt and a risk-free asset. When countries' fundamentals are weak, pessimistic expectations of investors can lead to runs on the domestic banking system. A government which commits to aggressively intervene ex-post can help to eliminate the "pessimistic" run equilibrium. Such intervention, by simply borrowing in the face of an impending crisis, is costly because interest rates rise sharply in crisis times. One way to prepare for crises is to build up a stock of international reserves in normal times. A quantitative exercise finds welfare gains of procyclical reserve accumulation policies to be economically substantial, suggesting that precautionary savings in light of domestic financial instability risks is an important channel through which reserve accumulation helps stabilize the economy and improves welfare.

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1 Introduction

Developing countries with open capital accounts are often concerned with volatile capital flows, the extreme of which manifested in sudden-stop episodes that contributed to many emerging market crises in the 80s and 90s. These experiences have prompted policymakers to advocate for accumulating international reserves to protect the sovereign against adverse developments in the financial markets. A recent International Monetary Fund's survey (2013) of member countries' views on the motive and use of reserves finds that a majority of the responses [1] sees external liquidity buffer (on short-term debt, domestic banks, public sector) as a critical reason for building reserves.

A large volumn of the empirical literature has devoted attention to study the precautionary motive lying behind the steady rise in reserve holdings by developing countries.[2] Obstfeld, Shambaugh, and Taylor (2010) develop an empirical model to show that the size of the banking sector liabilities is an important predictor in explaining the accumulation of foreign currency reserves in emerging economies. The authors highlight the precipitous nature of capital flows especially during "double drain" episodes in which a simultaneous withdrawal of bank deposits and domestic capital flight can place an extraordinary demand on banks' dollar liquidity. Foreign reserves then become important in ensuring domestic financial stability.

While this literature is able to represent the empirical facts, the exact mechanism in which rapid adverse developments in the domestic financial market might threaten fiscal solvency of governments needs to be further explored. It can be argued that a sufficiently funded government can intermediate dollar savings on behalf of domestic financial intermediaries and exercise its lender-of-last-resort power to stem the dollar "bleeding" of troubling banks.

1. about three-quarters of 40 responses

2. See Aizenman and Lee (2007), Calvo, Izquierdo, and Loo-Kung (2012), Dominguez, Hashimoto, and Ito (2012), Gourinchas and Obstfeld (2012) for empirical evidence on the precautionary role of reserves.

However, a government with large outstanding debt burdens may find it difficult to obtain funds or to intermediate foreign currency fundings in a crisis situation. Recent experience from the European sovereign debt crisis where sovereign governments were exposed to shocks to their banking sector through both fiscal guarantees and the impact of banking crises on domestic economic activity points to a similar concern.[3] This debate leads to several unanswered questions. Does the risk of domestic financial instability and the associated fiscal costs of promised bailouts necessitate a significant reserve accumulation? What are the welfare implications of reserve policies when sovereign debt sustainability risk and bank run risk are taken into consideration?

To answer these questions, we propose a quantitative model of systemic bank runs and sovereign default with short-term debt and a risk-free asset. The model connects the roles of reserves with private economic activity via the government's financing need for discretionary expenditures that stems from promised bailouts of the banking system in crisis times.

When countries' fundamentals are weak, pessimistic expectations of investors can lead to runs on the domestic banking system. Elevated risk of bank runs induces large increases in the cost of funds which tighten banks' balance sheet, reduce private investment and in turn, future outputs (Gertler and Kiyotaki, 2010, 2015). When runs occur, a government which promises to aggressively intervene ex-post can help to eliminate the "pessimistic" run equilibrium. The interventions, however, need to be fiscally credible to be effective. Dealing with financial instability and the associated recessions is challenging for the governments with significant debt levels. On one hand, financing the interventions by raising taxes would induce severe consumption declines when households' income has already deteriorated. On the other hand, financing the interventions by borrowing would be limited by investors' debt sustainability concerns as the sovereign spreads rise sharply during crisis times. To prepare

3. See Honohan (2010) and Lane (2011) for the background and assessments of the Irish government's management of the crisis, Lane (2012) for an overview of the European sovereign debt crisis.

for crises, the government might adopt rules to accumulate reserves in normal times.

Our model combines ingredients from the macro-banking literature (Gertler and Kiyotaki (2010)) and the quantitative sovereign debt literature (Aguiar and Gopinath (2006) and Arellano (2008)). In the model, there are domestic representative households and banks and risk-neutral foreign investors. Households can save in deposits or directly invest in capital with some efficiency cost. Banks borrow, subject to a collateral constraint, from domestic households using short-term runable deposits to invest in long-term assets. In addition, there is a government who can issue short-term defaultable bond to foreign creditors or decumulate reserves to finance government spending and contingent interventions on the banking system. This framework captures an important fiscal sustainability dimension at the heart of the debate on precautionary role of reserves - without an elastic cost of debt, the government can resort to external borrowing in the international credit markets to sufficiently finance the promised interventions.

In the quantitative analysis, the model is calibrated to a standard set of parameters in the literature. The model is globally solved as a limit of a finite horizon economy when the number of periods is large. We find that a procyclical reserve accumulation policy coupled with a countercyclical bank debt-relief policy produce welfare gains relative to the reference regime without the policy.

To understand this result, consider first the bank intervention policy. When the banking system is in crisis zone, the government commits to provide accumulated reserves to partially relieve banks' debt obligations.[4] On one hand, this policy has an effect of raising the banks' net worth ex-post and the recovery rate on deposits for the households which in turn, lowers the likelihood of runs. Consequently, the policy can reduce the inefficiency that stems from

4. Alternatively, we can think of a discount lending policy where the government extends a credit line to banks at a penalty rate, or an asset purchase policy where the government directly purchases capital from the asset market. See Gertler, Kiyotaki, and Prestipino (2016) for an analysis of these policies in the context of the recent US financial crisis.

the disruption of financial intermediation and prevent severe economic downturns. Since households own the banks, the debt-relief policy effectively acts as a partial bailout on households and raises consumption ex-post. On the other hand, the bailout commitment anticipated by private agents would distort banks' ex-ante borrowing incentives since the transfer each individual bank will receive is increasing in the banks' individual amount of deposits. The net effect of this policy on welfare will depend on particular parameter specifications. In our quantitative exercise, however, we find that the debt-relief policy has a positive welfare effect compared to the reference regime without the policy. This result resonates the findings in Bianchi (2016). Using a quantitative equilibrium model of financial crisis calibrated to the US data, the author finds that the optimal debt-relief policy has little moral hazard effect when it is a systemic bailout policy that depends only on the aggregate states of the economy.

Second, consider a default decision of the government. On one hand, default implies an elimination of servicing cost of debt as outstanding debt is wiped off. This leads to a higher consumption as the government frees up its budget to reduce the tax burden on households. On the other hand, default implies a productivity cost which lowers output and raise the risk of domestic bank runs, and a reduction in the ability to smooth future consumptions as a result of financial exclusion. In the models that incorporate bank run risks, however, periods of low output (that leads to default decisions) tend to coincide with a heightening risk of bank runs. During periods of financial autarky, the lower productivity levels due to the costs of default more likely cause bank runs as an equilibrium outcome. A decumulation of reserves would provide extra source of funds for the government during these exclusion periods.

Moreover, the sovereign bond price schedule depends on both the amount of new debt issuance by the government and the newly-chosen level of deposits by private agents as deposits will affect the government's decision to default in the next period. A higher level of deposits increases the probability of runs and in turn, increases the probability of having lower outputs. Since the government has a larger tendency to default, a higher level of deposits generally reduces the price of government bonds. In the Markov equilibrium, the government chooses the optimal amount of new debt that in turn affects private agents' deposit choice. The dependence of the bond price schedule on deposits introduces additional volatility that is not present in the literature and is crucial for the precautionary saving motive of reserves. In particular, a procyclical reserve policy reduces the borrowing cost via a more stabilized output and a smaller probability of bank runs.

Our model suggests that precautionary saving in light of domestic financial instability risks is an important channel through which reserve accumulation policies help stabilize the economy and improve welfare. The model can be used to quantify the impact of private equilibrium and bank run risks on government borrowing and reserve policies, shedding further light on the importance of financial factors in determining capital account policies in developing countries. Finally, the setup can be adapted to assess the effectiveness of various contingent intervention policies put in place by governments during crises and to conduct counterfactual policy experiments.

Related Literature

This paper relates to the literature that studies the precautionary insurance role of reserves. Jeanne and Ranciere (2011) model reserves as an insurance contract that pays off in sudden stops and characterize a closed-form formula for optimal reserve holdings. Jeanne (2016) analyze the insurance role of reserves and capital controls in a stylized two-period model of capital flows with banking frictions. However, the elastic cost of government's borrowing is not modeled as in our framework.

Regarding the mechanism that connects financial factors to the growth in international reserves, Obstfeld, Shambaugh, and Taylor (2010) develop an empirical model to show that the size of banking sector liabilities is an important predictor in explaining the accumulation of foreign currency reserves in emerging economies. Bocola and Lorenzoni (2017) articulate a theoretical argument for the role of reserves in response to financial panics in a three-period model with financial sector and endogenous currency composition of domestic private savings. Our project shares with these papers the premise that domestic financial instability might prompt a government to pursue precautionary saving policies in the form of reserve accumulation. We contribute to this literature by building a quantitative infinite-horizon model to study the welfare effects of reserve accumulation policies.

In modeling the borrowing decisions of the government, the present paper follows the quantitative sovereign debt literature pioneered by Eaton and Gersovitz (1981) and later advanced by Aguiar and Gopinath (2006) and Arellano (2008).[5] Particularly relevant in this literature are papers by Alfaro and Kanczuk (2009) and Bianchi, Hatchondo and Martinez (2018) who study the insurance role of reserves against the rollover risk of sovereign debt. In the former model, reserves pay off in all states including periods of default, thereby allowing for better consumption smoothing across the repayment and default states. In the latter model, issuing long-term debt to accumulate reserves additionally allows the government to transfer resources across the repayment states, enabling a more significant role of reserves. Similarly, Hernandez (2018) present a model of self-fulfilling debt crises and reserves to quantify the optimal portfolio of reserves and sovereign debt in Mexico. Our model differs from these papers in which we endogenize discretionary fiscal spending arising from government's promised interventions during crisis times. By including a banking sector we can study quantitatively the impact of private equilibrium and bank run risk on government borrowing and reserves policies, shedding further light on the importance of financial factors in determining capital account policies in emerging markets.

5. See also Hatchondo and Martinez (2009), Chatterjee and Eyigungor (2012), Arellano and Ramanarayanan (2012), and Aguiar et. al. (2016).

Our modeling of bank runs follows Gertler and Kiyotaki (2015), extended by Gertler, Kiyotaki, and Prestipino (2017). Since the seminal contribution to the macro-banking literature by Gertler and Kiyotaki (2010), many authors have explicitly included financial intermediaries in their macroeconomic models to study various aspects of unconventional monetary policies (Gertler and Karadi, 2011), macro-prudential policies (Gertler, Kiyotaki, and Queralto, 2012), financial repression policies (Chari, Dovis, and Kehoe, 2016). Apart from working with a small open economy framework with domestic bank runs and sovereign default, we contribute to this literature by studying the fiscal impact of promised interventions intended to reduce the probability of runs in equilibrium. We also take into account the debt sustainability dimension by allowing the government to borrow externally. Regarding concerns on how private sector reacts ex-ante to the contingent intervention policies, our model can be used to study the fiscal costs of distorting ex-ante private incentives and the benefits of eliminating run risks in a coherent manner.

Relatedly, Roch and Uhlig (2016) study bailout guarantees provided by a large, risk-neutral agency in a model of self-fulfilling debt crises in the tradition of Cole and Kehoe (2000) and characterize the minimal actuarially-fair intervention that restores "good" equilibria. In the context of our model, bailout guarantees are fiscally costly to provide and the bailout agency (i.e., the government) needs to consider how to finance those interventions.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 describes the equilibrium concept. Section 4 discusses the main mechanisms in the model. Section 5 presents the quantitative analysis and evaluates the welfare effects of various reserve policies. Section 6 concludes.

2 Model

The model is a blend of the macro-banking model with runs as in Gertler and Kiyotaki (2015) (hereafter, GK15) and a sovereign default model in the tradition of Eaton and Gersovitz (1981) and Arellano (2008). The private sector consists of a continuum of representative households who consume and save in domestic deposits as well as directly invest in capital. Domestic financial intermediaries (or bankers) obtain deposits from households to invest in a production technology that produces final consumption goods, which can then be consumed by the households.

There are two goods, a nondurable good and a durable asset which we call "capital". Capital does not depreciate and is held in fixed supply which we normalize to be unity. Capital can be held by both financial intermediaries and households, so that

$$K^h + K^b = 1$$

where K^h and K^b denote the capital holdings by households and bankers, respectively.

The production technology takes capital as the only input and produces an amount of consumption goods that is equal to the productivity shock, which follows an AR(1) process

$$\ln Z' = \rho_z \ln Z + \epsilon'$$

where $\epsilon' \sim N(0, \sigma_z^2)$ and independently distributed, and $\rho_z \in (0, 1)$.

2.1 Households

The representative household consists of an equal measure of bankers and workers with perfect consumption insurance within the family. Each banker manages a bank and transfers non-negative dividends back to the family. At the end of the period, the household decides how much final goods to consume and the rest of their income can be saved in either deposits or capital.

When directly investing in capital, each household incurs a management cost of capital, $f\left(k^{h\prime}\right) = \alpha^h k^{h\prime}, \quad (\alpha^h > 0)$, where $k^{h\prime}$ denotes individual household's capital holdings. This cost represents the idea that workers are less efficient than bankers in making lending and investment decisions, the extent of which is captured by the parameter α^h. This parameter will be crucial in determining the fire-sale prices of capital in the event of bank runs.

There is a continuum of bankers who use their own net worth and deposits from workers to invest in the production technology. We assume that bankers are more efficient than workers in managing capital and thus do not incur any management cost. It turns out that a bank's assets to net worth ratio, ϕ', is a sufficient statistic to index a bank's identity. From now on, the dependence of bank-specific variables on ϕ' will be made explicit.

Deposit contracts issued by bank ϕ' are one-period risky debt contracts that exchange $Q^d(\phi', S)$ units of consumption good in state S for a promised return of one unit of consumption good in state S' tomorrow when the issuing bank is still in operation. If bank ϕ' defaults in the next period, however, depositors to this bank receive only a fraction $x(\phi', S')$ proportional to their deposits. The fraction $x(\phi', S')$ is specific to both the individual bank's leverage position in the current period as summarized by ϕ' and the aggregate state in the next period, S'. Thus, the effective rate of return on deposits at bank ϕ' is

$$\begin{cases} 1 & \text{if no defaults} \\ x(\phi', S') & \text{if defaults} \end{cases} .$$

A default on bank ϕ' means when there is a run by creditors of every other banks that reduces the prices of capital to the fire-sale levels, it is rational for creditors of this bank to follow the run since its net worth is negative at the fire-sale prices. In the United States, checkable deposits and similar forms of short-term time deposits are guaranteed by the Federal Deposit Insurance Corporation up to 250,000 U.S. dollars per depositor per insured bank. As emphazied in GK15, the systemic bank run in their model is more apt to describe run events on the wholesale banking system that was at the epicenter of the recent U.S. financial crisis. For developing countries, however, deposit insurances on domestic-currency deposits are usually less generous and there is little insurance on dollar- (or foreign-currency) denominated deposits.

We now formulate the representative household's problem recursively. Let S denote the aggregate state variables which will be described later. Also, let (k^h, d) be the individual states consisting of previous direct capital holding and total return on previous deposits at all banks (other than the ones owned by the household's own bankers). Taking as given the deposit price and recovery schedules, $\{Q^d(\phi', S), x(\phi', S')\}$ (one for each bank ϕ'), the price of capital, $Q^k(S)$, the dividend distributions net of bank start-up funds, $\Pi(S)$, and the law of motion for the aggregate state variables $\Gamma(.)$, a representative household decides how much to consume in final consumption goods, how much deposits to put in each bank, and how much direct investment to make. The household's problem can be written recursively as follows

$$V^h(k^h, d, S) = \max_{c, k^{h\prime}, \{d'(\phi')\}_{\phi'}} U(c) + \beta \mathbb{E}_{S'|S} V^h(k^{h\prime}, d', S')$$

$$c + \int_{\phi'} Q^d(\phi', S) d'(\phi') + Q^k(S) k^{h\prime} + \alpha^h k^{h\prime} \leq d + \left[Z + Q^k(S)\right] k^h + \Pi(S)$$

$$d' = \int_{\phi'} d'(\phi') \min\left(1, x(\phi', S')\right)$$

$$S' = \Gamma(S).$$

In the above problem, c is the amount of consumption good, $k^{h\prime}$ is the amount of direct investment, and $\{d'(\phi')\}_{\phi'}$ is the portfolio of deposits (one for each bank ϕ'). The first line of the constraints is the budget constraint, which states that the household's expenditures cannot be higher than its incomes from the total returns on the previous portfolio of deposits, the gross return on capital holdings, and the dividend distributions net of bank start-up transfers. The second line describes how the total return on deposits in state S' in the next period, taking into account whether each bank ϕ' defaults in state S'.

Household's decisions with respect to $k^{h\prime}$and $\{d'(\phi')\}_{\phi'}$ are determined by the Euler equations with respect to $k^{h\prime}$,

$$\mathbb{E}_{S'|S}\left[\Lambda(S,S')\frac{Z'+Q^k(S')}{Q^k(S)+\alpha^h}\right] \leq 1, \text{ and equality when } k^{h\prime} > 0,$$

and $d'(\phi')$,

$$\mathbb{E}_{S'|S}\left[\Lambda(S,S')\frac{\min\left(1, x(\phi', S')\right)}{Q^d(\phi', S)}\right] = 1,$$

where $\Lambda(S,S') = \beta\frac{U'(C(S'))}{U'(C(S))}$ is the representative household's stochastic discount factor between state S and S', and $C(.)$ is the consumption of the representative household in equilibrium.

The modeling choice of the household's linear management cost of capital is done for simplicity but is still relevant to capture the depressed price of capital in a fire-sale event. It turns out that in the no-run equilibrium,[6] banks hold the entire aggregate stock of capital despite their borrowing constraints, while households hold capital only during the run periods.[7] Next, we will turn to describe the representative bank's problem.

6. under a reasonable area of the parameter space

7. Empirical evidence suggests that in many emerging market economies, the majority of private credits are intermediated by the retail banking system.

2.2 Banks

Each banker manages a financial intermediary which we call "bank". Bankers use their own net worth and issue deposits to the workers to fund capital investment. Bankers are constrained in their ability to raise funds due to moral hazard, and they have finite expected lifetime with a probability $(1-\sigma)$ of exiting every period. This device is to motivate dividend payouts from bankers and to guarantee that bankers do not accumulate enough retained earnings to overcome the incentive constraint that is at the heart of the bank problem.

Specifically, each banker manages a bank until retirement and transfers non-negative dividend back to the households. Bankers retire with probability $(1 - \sigma)$ then become workers, or continue until the next period with probability σ. To maintain a fixed mass of bankers and workers, we assume $(1 - \sigma)$ new bankers are born in every period. Each new banker receives a startup capital w^b from his household. A bank ϕ' uses its own net worth (n) and raises deposits (d') at price $Q^d(\phi', S)$ to finance capital investment $Q^k(S)k^{b\prime}$. The balance sheet constraint of bank ϕ' can be written as follows

$$Q^k(S)k^{b\prime} = n + Q^d(\phi', S)d',$$

and its net worth evolves as

$$n' = \left(Z' + Q^k(S')\right) k^{b\prime} - d'$$

where the first term on the right hand side is the gross return on capital in state S' and the second term is the repayment of deposits (recall that $\phi' \equiv \frac{Q^k(S)k^{b\prime}}{n}$).

Following Gertler and Kiyotaki (2010), we introduce a moral hazard friction on part of the bankers. In particular, at the end of every period after the newly issued deposits have been raised and capital investment has been made but before the realization of next period productivity shock, bankers can run away with a fraction θ of their assets. The bankers caught cheating are forced to liquidate their positions and become workers. Anticipating the diversion incentives, households will not lend to banks an amount large enough that renders the market value of diverted assets greater than the franchise value of continuation. In other words, bankers are subjected to an incentive constraint of the form

$$V^b(n, S) \geq \theta Q^k(S) k^{b\prime}$$

in each period of operation. Note that the incentive constraint embeds the constraint that bank's net worth, n, must be positive for the banks to operate. Parameters and shock variables will be chosen such that net worth is non-negative in a no-run equilibrium.

Assume there is no run in state S, taking as given the deposit price schedule, $Q^d(\phi', S)$, each banker chooses new capital and deposit, $k^{b\prime}$ and d', to solve the following recursive problem

$$V^b(n, S) = \max_{k^{b\prime}, d'} \mathbb{E}_{S'|S} \left\{ \Lambda(S, S') \left[(1-\sigma) n' + \sigma V^b(n', S') \right] \right\}$$

$$Q^k(S) k^{b\prime} = n + Q^d(\phi', S) d'$$

$$n' = (Z' + Q^k(S')) k^{b\prime} - d'$$

$$V^b(n, S) \geq \theta Q^k(S) k^{b\prime}$$

$$S' = \Gamma(S),$$

where $\phi' \equiv \frac{Q^k(S) k^{b\prime}}{n}$ is again the bank's leverage ratio, defined to be the ratio of the market value of assets to the beginning-of-period net worth.

Note that bankers use the representative household's stochastic discount factor to value their future streams of net worth since they are a part of the households whom they perfectly share consumption with. Moreover, the bank problem has been written such that there is no dividend payment to households during periods of operation. It can be easily shown that due to the impatience of bankers relative to households and the collateral constraint, bankers have an incentive to retain earnings until retirement.

Thanks to the constant return-to-scale nature of the bank problem,[8] it can be shown that the franchise value of the bank is proportional to its net worth; that is, $V^b(n, S) = \psi(S).n$, where $\psi(S)$ depends only on aggregate variables. The growth rate of net worth can be rewritten as

$$\begin{aligned} \frac{n'}{n} &= \frac{Z' + Q^k(S')}{Q^k(S)} \frac{Q^k(S)k^{b\prime}}{n} - \frac{d'}{n} \\ &= \Big(R^k(S') - R\left(\phi', S\right)\Big) \phi' + R\left(\phi', S\right) \end{aligned}$$

where $R^k(S') \equiv \frac{Z'+Q^k(S')}{Q^k(S)}$ is the realized rate of return on capital between states S and S', and $R\left(\phi', S\right) \equiv \frac{1}{Q^d(\phi', S)}$ is the interest rate on deposits specific to bank ϕ'. We will turn next to describe the systemic runs on banks and characterize bankers' optimal decisions.

2.3 Anticipated Runs on the Banking System

Following Gertler and Kiyotaki (2015), we consider the possibility of self-fulfilling runs on the entire banking system, that is, a symmetric equilibrium in which there exists a positive probability of a run on every bank. Consider now the economy is in state S'. At the beginning of the period after Z' realizes, savers must decide whether to roll over deposits for another period. Each depositor expects that when the others stop rolling over their deposits, the bank has to liquidate its assets and turns over to the workers who are less efficient at managing capital. It is then optimal for the current depositor to stop rolling over his deposit. We focus on the symmetric run equilibrium where all banks liquidate their assets at the fire-

8. If we multiply $n, d', k^{b\prime}$ by a positive constant $\kappa > 0$, the constraint set remains the same and the bank's objective function is also multiplied by κ.

sale price of $Q^{k*}(S')$. This price is determined by the workers who are the marginal investors and hold the entire stock of capital when runs occur. Thus, a run on bank ϕ' in state S' is possible if and only if

$$\left[Z' + Q^{k*}(S')\right] k^{b\prime} < d', \text{ or}$$

$$\left(R^{k*}(S') - R\left(\phi', S\right)\right) \phi' + R\left(\phi', S\right) < 0,$$

where $R^{k*}(S') \equiv \frac{Z'+Q^{k*}(S')}{Q^k(S)}$ denotes the rate of return on capital during the run periods.

To coordinate depositors' beliefs, there is an i.i.d. sunspot random variable $\zeta' \in \{0, 1\}$ such that $\zeta' = 1$ with probability $\bar{\zeta}$ and $\zeta' = 0$ otherwise, where $\bar{\zeta} \in [0, 1]$ is a parameter. When there exists a run equilibrium, all savers stop acquiring new deposits if $\zeta' = 1$, and they all agree to roll over deposits otherwise. Therefore, a bank run arises as an equilibrium outcome if and only if a run equilibrium exists in state S' and $\zeta' = 1$. Denote $\bar{Z}'(\phi', S)$ the *supremum* of productivity shock Z' below which bank ϕ''s net worth is negative under the fire-sale price $Q^{k*}(S')$,

$$\bar{Z}'(\phi', S) = \sup\left\{Z' \,\middle|\, \left(R^{k*}(S') - R\left(\phi', S\right)\right) \phi' + R\left(\phi', S\right) < 0\right\}.[9]$$

Also, let $\iota(\phi', S')$ be the indicator random variable for bank ϕ' in state S' taking value of one if the bank experiences a run and zero otherwise,

$$\iota(\phi', S') = \begin{cases} 1 & \text{if } (Z', \zeta') \in \left\{Z' < \bar{Z}'(\phi', S) \text{ and } \zeta' = 1\right\} \\ 0 & \text{if } (Z', \zeta') \in \left\{Z' < \bar{Z}'(\phi', S) \text{ and } \zeta' = 0\right\} \cup \left\{Z' \geq \bar{Z}'(\phi', S)\right\} \end{cases}.$$

Using the linearity property, the bank's problem can be written as follows

9. Note that the tomorrow run cutoff for bank ϕ' is known today given the rational expectation of what the fire-sale price of capital would be tomorrow, hence the dependency of $\bar{Z}(.)$ only on today's states.

$$\psi(S) = \max_{\phi'} \mathbb{E}_{S'|S} \left[1 - \iota(\phi', S')\right] \{\Lambda(S, S') \left[1 - \sigma + \sigma\psi(S')\right]$$
$$\times \left[\Big(R^k(S') - R(\phi', S)\Big)\phi' + R(\phi', S)\right]\Big\}$$

$$\psi(S) \geq \theta\phi'$$

$$S' = \Gamma(S).$$

When the incentive constraint is binding, banks choose the leverage ratio ϕ' to solve

$$\theta\phi' = \mathbb{E}_{S'|S} \left[1 - \iota(\phi', S')\right] \Big\{\Omega(S, S') \left[\Big(R^k(S') - R(\phi', S)\Big)\phi' + R(\phi', S)\right]\Big\}$$

where $\Omega(S, S') = \Lambda(S, S') \left[1 - \sigma + \sigma\psi(S')\right]$ is the bank's effective stochastic discount factor that takes into the retirement probability and its internal cost of funds, which is higher than the interest rate on deposits when the incentive constraint is binding. This completes the characterization of the bank's problem.

2.4 Aggregations

When there is no run in state S, summing across continuing and newborn banks, we obtain the aggregate net worth at the beginning of the period as

$$N(S) = \sigma\left(\left[Z + Q^k(S)\right] K^b - D\right) + (1 - \sigma)w^b.$$

The absence of individual net worth in the bank problem implies that all banks choose the same leverage ratio, so $\phi' = \Phi'$ in equilibrium, where "script" Φ' denotes the aggregate leverage ratio defined to be the value of aggregate bank assets over the aggregate net worth,$\Phi'(S) \equiv \frac{Q^k(S)K^{b\prime}(S)}{N(S)}$. Summing across banks, we also obtain the aggregate bank balance sheet constraint as

$$Q^k(S)K^{b\prime}(S) = N(S) + Q^d\left(S\right)D'(S),$$

where we have suppressed the dependence of the price of deposits on individual bank's leverage due to the representativeness condition.

Lastly, the evolution of aggregate net worth, taking into account the run possibility in the next period, follows

$$N(S') = \begin{cases} \sigma N(S)\left[\left(R^k(S') - R(S)\right)\Phi'(S) + R(S)\right] + (1-\sigma)w^b \\ \qquad\qquad\qquad\qquad \text{if no run occurs in } S' \\ 0 \qquad\qquad\qquad\qquad\quad \text{if a run occurs in } S' \end{cases}$$

When there is a run in state S, workers hold the entire stock of capital so that $K^{b\prime}(S) = 0$. The aggregate net worth becomes $N(S) = 0$. Following GK15, we assume that when a run occurs, the newborn bankers delay entering until the next period and store their startup capital one-for-one between periods. Therefore, the aggregate net worth in the next period follows

$$N(S') = (1+\sigma)w^b(1-\sigma).$$

2.5 Government

Following the sovereign debt literature, we assume there is a benevolent government who can make external borrowing and saving decisions on behalf of domestic households to smooth out the aggregate income shocks. The government borrows by issuing short-term debt to risk-neutral foreign investors and saves by accumulating reserves in the form of a safe asset. In addition, the government lacks commitment to repay its debt and can default on the outstanding debt obligations. All the proceeds from the government's external operations net of its expenditures are distributed back to domestic households in a lump-sum fashion. This means external capital flows are intermediated by the government instead of banks and take the form of short-term defautable debt.

Each unit of the sovereign bond is sold for a price of q in exchange for a payment of one unit of consumption goods in the next period, conditional on the government not defaulting. In addition, the government accumulates reserves, a', by purchasing one-period safe asset that earns the risk-free foreign interest rate, R^*. The stock of reserves evolves following rules of the form

$$a' = f^a(Z, a, b, D)$$

that potentially depend on both the endogenous and exogenous variables of the economy. In the quantitative exercise, we consider the following reserve accumulation rule

$$\frac{a'}{R^*} = \begin{cases} a + \tau^{res}.Z + T^b & \text{if } \log Z > 0 \\ a - \tau^{res}.Z + T^b & \text{if } \log Z < 0 \end{cases},$$

where τ^{res} is a parameter controlling the magnitude of the changes in reserves with respect to output, and T^b is the intervention spending (described later). In particular, the accumulation of reserves is procyclical when $\tau^{res} > 0$.

Moreover, the government commits to intervene in the domestic banking system during crisis times. We assume the government follows an intervention rule in the form of transfers to the banks when a run possibility opens up. Specifically, the government bank transfers, T^b, are proportional to the outstanding amount of aggregate deposits, D, where the proportion, $t(Z)$, can vary with the aggregate productivity shock; that is,

$$T^b = t(Z).D.$$

We are particularly interested in the case where $t(Z) > 0$ if a run possibility opens up and $t(Z) = 0$ otherwise. When the banking system is in the crisis zone (that is, when the productivity is below the run threshold), this policy can be interpreted as a debt-relief policy for banks. Note that even though the intervention follows an exogenous rule, the total amount of transfers is endogenous since it depends upon the aggregate amount of deposits which is chosen by private agents. It turns out that this form of transfers effectively acts as a bailout on households, and sufficiently large transfers could even eliminate the run possibility and in turn, the run inefficiency stemming from workers' management of the entire stock of capital.[10]

The government budget constraint is

$$T^h + g + \frac{a'}{R^*} + (1-\delta)b = a + (1-\delta)qb'$$

where T^h is the transfers to households, g is a fixed government spending that captures the rigidities in the government budget constraint, and $\delta \in \{0,1\}$ is the government's default decision with one means defaulting and zero otherwise. Whenever the government chooses to repay it can select the level of new debt b'. Default has the benefit that it erases all the debt obligations, but it imposes the cost in terms of productivity reduction and exclusion from international financial markets. If the government defaults, the productivity is reduced to $Z^d \leq Z$. Furthermore, the government is excluded from borrowing in the international markets for a random number of periods and can re-enter with zero debt obligations with probability χ. We assume the government does not lose its reserve balances when defaulting. When the government's revenue exceeds its expenditure, it transfers the difference back to

10. Alternatively, we can think of a discount lending policy of the form $(q_g^D, \bar{d}_g)$ where the government extends a credit line to banks at a penalty rate: banks can borrow from the government upto the amount of $\bar{d}_g$ at price q_g^D. Yet another type of contingent intervention policies is the asset purchase policy $(Q^g, K^{g\prime})$ where the government can purchase capital $K^{g\prime}$ at price Q^g. See Gertler, Kiyotaki, and Prestipino (2016) for an analysis of these policies in the context of the recent U.S. financial crisis.

the households in a lump-sum fashion.

The government's objective is to maximize the expected discounted flow of the representative household's utility derived from the aggregate consumption, where the government's discount factor, β^g, might differ from the household's discount factor, β. We assume the foreign creditors are competitive lenders who discount their future cash flows using the international risk-free rate R^*. Therfore, the sovereign bond price schedule satisfies the break-even condition of foreign creditors

$$q = \frac{\mathbb{E}_{Z'|Z} \text{Prob}\left(\delta' = 1\right)}{R^*}$$

where q is the per-unit price of newly issued bond b', and δ' is the next-period default decision of the government.

3 Equilibrium

We consider a Markov equilibrium where the government makes default and borrowing decisions internalizing the effects of its policies on the allocations and prices in the private equilibrium. At the beginning of the period, the aggregate state of the economy includes the productivity shock, Z, as well as the outstanding amounts of government debt and the aggregate deposits, (b, D). Let $s = (Z, b, D)$ be the state faced by the government. The government then chooses its policies, $\pi = (\delta, b')$, of whether to repay or default and how much to borrow conditional on repaying. Let $S = (s, \pi) = (Z, b, D, \delta, b')$ be the state faced by the private agents.

3.1 Private Equilibrium

Definition: Private Equilibrium.

Given the the aggregate state S, the government policy functions for default $H_D(s') \equiv \delta'(Z', b', D')$, and borrowing $H_B(s') \equiv b''(Z', b', D')$, a symmetric run private equilibrium consists of households' policy functions for consumption and deposits $\{C(S), D'(S)\}$, banks' policy functions for capital holdings, leverage and franchise value $\{K^{b\prime}(S), \Phi'(S), \psi(S)\}$, price functions for capital $\{Q^k(S), Q^{k*}(S)\}$ and for deposits $Q^d(S)$, such that: given the price functions and government policy functions, banks and households choose the allocation rules to solve their respective problems; the government budget constraint holds; and the final goods, deposits, and capital markets clear.

To summarize the set of conditions that a private equilibrium must satisfy, consider two cases. First, there is no run in the current period. If there is a run in the previous period (that is, $D = 0$), the aggregate net worth is

$$N(S) = (1+\sigma)w^b(1-\sigma).$$

Otherwise, taking as given the future allocation and price functions $\{C(S'), Q^k(S'), Q^{k*}(S'), \psi(S')\}$, the private equilibrium allocations and prices can be summarized by the following system of equations

$$C(S) + g \leq (1-\delta)Z + \delta Z^d + a - \frac{a'}{R^*} + (1-\delta)\left\{q(Z, b', D'(S))b' - b\right\} \tag{1}$$

$$\mathbb{E}_{S'|S}\left[\beta \frac{U'\left(C(S')\right)}{U'\left(C(S)\right)} \min\left\{\frac{1}{Q^d(S)}, \frac{\left[\tilde{Z}' + Q^{k*}(S')\right] K^{b\prime}(S) + t(Z')D'(S)}{Q^d(S)D'(S)}\right\}\right] = 1 \tag{2}$$

$$N(S) = \sigma\left[\left(Z + Q^k(S)\right)K^b - D\left(1 - t(Z)\right)\right] + (1-\sigma)w^b \tag{3}$$

$$Q^k(S)K^{b\prime}(S) = N(S) + Q^d\left(S\right)D'(S) \tag{4}$$

$$\psi(S) = \mathbb{E}_{S'|S}\left[1-\iota(S')\right]\left\{\beta\frac{U'\left(C(S')\right)}{U'\left(C(S)\right)}\left[1-\sigma+\sigma\psi(S')\right]\right.$$
$$\left.\times\left[R^k(S')\Phi'(S)+R(S)\left(1-t(Z')\right)\left(1-\Phi'(S)\right)\right]\right\} \tag{5}$$

$$\psi(S) = \theta\Phi'(S) = \frac{Q^k(S)K^{b\prime}(S)}{N(S)} \tag{6}$$

where

$$\iota(S') = \begin{cases} 1 & \text{if } (\tilde{Z}',\zeta') \in \left\{\tilde{Z}' < \bar{Z}'(S) \text{ and } \zeta' = 1\right\} \\ 0 & \text{if } (\tilde{Z}',\zeta') \in \left\{\tilde{Z}' < \bar{Z}'(S) \text{ and } \zeta' = 0\right\} \cup \left\{\tilde{Z}' \geq \bar{Z}'(S)\right\} \end{cases}$$

is the run indicator in state $S' = (s', H_D(s'), H_B(s'))$, and the run cutoff rule follows

$$\bar{Z}'(S) = \sup\left\{Z' \,\middle|\, \frac{\tilde{Z}' + Q^{k*}\left(S'\right)}{Q^k(S)}\Phi'(S) - \left(\Phi'(S)-1\right)\frac{1-t(\tilde{Z}')}{Q^d(S)} < 0\right\}.$$

In the above system of equations

$$\tilde{Z}' = (1-H_D(s'))Z' + H_D(s')Z^{d\prime}$$

is the productivity level tomorrow taking as given the government's policy function for default $H_D(s')$, (1) is the resource constraint, (2) is the households' Euler equation on deposits, (3) describes the aggregate net worth when there is no run in state S, (4) is the aggregate bank balance sheet constraint, (5) describes the bank franchise value in relation to their expected discounted future returns on capital and deposits, and (6) is the bank incentive constraint when binding.

Second, when there is a run in the current state S, taking as given future allocation and price functions $\{C(S'), Q^k(S')\}$, the private equilibrium can be summarized by decision rules for consumption $C(S)$, banks' policies $\{K^{b\prime}(S), D'(S)\}$, and price for capital $Q^{k*}(S)$

that satisfy the system of equations

$$C(S)+g+\alpha^h \leq (1-\delta)Z+\delta Z^d+a-\frac{a'}{R^*}+(1-\delta)\left\{q(Z,b',D'(S))b'-b\right\} \tag{7}$$

$$Z+Q^{k*}(S)<D\left(1-t(Z)\right) \tag{8}$$

$$\mathbb{E}_{S'|S}\left[\beta\frac{U'\left(C(S')\right)}{U'\left(C(S)\right)}\cdot\frac{\tilde{Z}'+Q^k(S')}{Q^{k*}(S)+\alpha^h}\right]=1 \tag{9}$$

$$K^{b\prime}(S)=0, D'(S)=0$$

In the above system of equations, (7) is the resource constraint, (8) is the condition for a run to occur at $Q^{k*}(S)$, and (9) is the households' Euler equation on capital. We are now ready to set up the government recursive problem in the next section.

3.2 Government Problem

Let $V^g(Z,b,D)$ be the option value of default for the government, which can be written as

$$V^g(Z,b,D)=\max_{\delta\in\{0,1\}}\left\{(1-\delta)V^r(Z,b,D)+\delta V^d(Z,D)\right\},$$

where the government can choose whether to repay or default with $\delta=1$ being default and zero otherwise, $V^r(Z,b,D)$ is the value of repayment, and $V^d(Z,D)$ is the value of default. Specifically, the value of repayment is

$$V^r(Z,b,D)=\max_{b'\geq 0}\left\{U(C(S))+\beta^g\mathbb{E}_{Z'|Z}V^g(Z',b',D'(S))\right\}$$

subject to the private equilibrium characterized by conditions (1) - (9) that determines the aggregate consumption and deposits, $C(S)$ and $D'(S)$, and the break-even condition for the sovereign bond price schedule

$$q(Z, b', D') = \frac{1}{R^*} \mathbb{E}_{Z'|Z} \left[1 - H_D(s')\right]$$

where $s' = (Z', b', D')$ is the next-period aggregate state relevant to the government problem, and $H_D(s')$ is the default policy function taken as given by the current government and the foreign creditors.

After default, the government's debt is eliminated, but the government is excluded from borrowing in the international credit markets and can re-enter with probability χ. The value of default is

$$V^d(Z, D) = U(C(S)) + \beta^g \mathbb{E}_{Z'|Z} \left[\chi V^g(Z', 0, D'(S)) + (1 - \chi) V^d(Z', D'(S))\right]$$

subject to the private equilibrium characterized by conditions (1) - (9) with $b' = b = 0$ and $\delta = 1$.

We now define the Markov equilibrium for the economy.

Definition: Markov Equilibrium.

Given the the aggregate state $s = (Z, b, D)$, a Markov equilibrium consists of government policies for default and borrowing $(\delta(s), b'(s))$, government value functions $V^g(Z, b, D)$, $V^r(Z, b, D)$, and $V^d(Z, D)$, and the sovereign bond price schedule $q(Z, b', D')$ such that

- Given the bond price schedule $q(.)$, future policy functions $H_D(Z', b', D')$, $H_B(Z', b', D')$, and value functions $V^g(Z', b', D')$, $V^r(Z', b', D')$, $V^d(Z', D')$, government policies $\delta(s)$, $b'(s)$ solve the government's problem

- Government's policies and value functions are consistent with future policies and value functions
- Bond price schedule is consistent with the pricing equation of the risk-neutral foreign investors.

4 Model Forces

4.1 Bank Intervention Policy

When the banking system is in crisis zone, the government commits to provide accumulated reserves to partially relieve banks' debt obligations. In particular, in every period when the productivity realization is below a run cutoff level that renders runs a positive-probability outcome, the government transfers to banks an amount of funds proportional to the aggregate level of outstanding deposits where the proportions depend on the realizations of the productivity shock, and the government does nothing otherwise. We specify the run cutoff level in the bank transfer rule using the productivity run threshold from the benchmark regime with no intervention and reserve policies, denoted by $\bar{Z}^0$. The amount of bank transfers can be written as

$$T^b = \begin{cases} \tau^{dr}(-\log Z).D & \text{if } Z < \bar{Z}^0 \\ 0 & \text{otherwise} \end{cases}.$$

This intervention resembles a systemic debt-relief policy often implemented by central banks in crisis times. The fact that transfers are proportional to deposits is crucial to maintain the linearity in the bank problem and is useful for aggregation purposes. Parameter $\tau^{dr} > 0$ controls the magnitude of the bailouts where a larger τ^{dr} implies a more complete bailout.

When the amount of aggregate deposits without bailouts falls into the crisis zone, this form of intervention raises the banks' net worth ex-post and the recovery rate on deposits for the households and lowers the likelihood of runs. Consequently, the policy can help to reduce the inefficiency that stems from the disruption of financial intermediation and prevent severe economic downturns. Since households own the banks, the debt-relief policy effectively acts as a partial bailout to households and raises consumption ex-post. On the other hand, the bailout commitment anticipated by private agents would distort banks' ex-ante borrowing incentives since the transfer each individual bank will receive is increasing in the banks' individual amount of deposits. The net effect of this policy on welfare will depend on particular parameter specifications. In the simulation, however, we find that the intervention is welfare improving relative to the reference regime without the policy.

4.2 Reserve Policy

The government accumulates reserves according to

$$\frac{a'}{R^*} = \begin{cases} a + \tau^{res}.Z + T^b & \text{if } \log Z > 0 \\ a - \tau^{res}.Z + T^b & \text{if } \log Z < 0 \end{cases},$$

where τ^{res} is a parameter controlling the magnitude of the changes in reserves with respect to output, and T^b is the intervention spendings. In particular, the accumulation of reserves is procyclical when $\tau^{res} > 0$. This formulation is convenient for computational purposes as the pace of accumulation, $\left(\frac{a'}{R^*} - a\right)$, depends only on the current level of output, avoiding the need to keep track of an additional state variable (that is, the stock of existing reserves). This rule however implies that the changes in reserves do not depend on how large the beginning-of-period reserve holding is, preventing the government from decumulating reserves more aggressively in response to the downturns. We leave this extension for future research.

In normal times when the sovereign borrowing cost is low, the government issues bond and collects taxes from private agents to accumulate reserves. It is important to note that reserves and short-term debt are not perfect substitutes in this model as reserves pay off in all states while debt is defaultable contingent upon states. The government therefore uses reserves to insure against default states and is able to achieve higher welfare as a result of less risky consumption profiles. This hedging mechanism of reserves has been explored in Alfaro and Kanczuk (2009) and Bianchi et. al. (2018). Our model differs, however, in that the default states depend on the amount of deposits selected by private agents which the future government takes as given. The insurance benefit of reserves depends not only on the probability of default in the invariant distribution but also on the marginal utilities in the default states.

In bad times, the government decumulates reserves. Since borrowing becomes more costly when outputs are low, the government can reduce borrowing with less consequence on consumption because of the extra resources coming from the reserve balances. In this sense, a procyclical reserve policy helps stabilize the fluctuation in productivity and achieve better consumption smoothing which results in a higher welfare.

5 Quantitative Analysis

5.1 Solution Method and Calibration

The model is solved on a discretized state space as a limit of a finite horizon economy when the number of periods is large. In particular, starting from the last period we then solve the model backwards taking as given the value functions, bond price functions, policy functions of the government and private agents one period ahead and iterate until convergence.

The technology follows an AR(1) process in logarithm. That is,

$$\log Z_{t+1} = \rho_z \log Z_t + \sigma_z \epsilon_{t+1}$$

Parameter	Description	Value
r	International risk-free rate	0.04
β	Households' time discount factor	0.97
β^g	Government's discount factor	0.92
γ	Coefficient of relative risk aversion	2
ρ_z	Autocorrelation of $\log Z$	0.85
σ_z	Std. dev. of innovation to $\log Z$	0.02
d_0	Default output penalty	- 0.19
d_1	Default output penalty	0.25
χ	Re-entry probability	0.33
σ	Bank survival probability	0.75
θ	Shares of assets divertible	0.39
α^h	HH management cost of capital	0.3
w^b	New bankers endowments	0.49
$\bar{\zeta}$	Sunspot probability	0.1

Table 1: Parameter selection

where $\epsilon_{t+1} \sim N(0,1)$. The autocorrelation $\rho_z = 0.85$ and the standard deviation of the innovation $\sigma_z = 0.02$ are standard in the literature when being calibrated to the GDP data of Mexico or Brazil when each period corresponds to a year. The technology state is then discretized using Tauchen (1986) method.

There are two sets of parameters in the model, one belongs to the sovereign default literature and one is set to the standard value in the macro-banking literature.

The utility function is of a CRRA form

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma}$$

where the coefficient of relative risk aversion parameter is $\gamma = 2$, as is standard in the open-economy real business cyclce literature.

We set the international risk-free rate $r = 0.04$, also standard in the literature in which each period corresponds to one year.

The productivity cost of default is modeled following Chatterjee and Eyigungor (2012).

When government defaults, the productivity is reduced to

$$Z^d = Z - \max\{0, d_0 Z + d_1 Z^2\}, d_1 \geq 0.$$

The re-entry probability is set to $\chi = 0.33$ that implies an average exclusion period of three years. The government is assumed to be more impatient than the private sector, as in Arellano, Bai, and Mihalache (2019). The households' time discount factor is set to $\beta = 0.97$ and the government's discount factor is set to $\beta^g = 0.92$.

The bank survival probability $\sigma = 0.75$ implies an average age of four years for the banks. The share of assets divertible θ and the new bankers' endowment w^b are set to standard values in the literature. The household's management cost of capital $\alpha^h = 0.3$ is chosen to be high enough so that households prefer to save in deposits instead of capital in the steady state.

5.2 Quantitative Performance of the Model

We first consider the performance of the model when there is no bailout or reserve accumulation policies (i.e., the reference regime). Consider first a sovereign's choice to default. On one hand, default implies an elimination of servicing cost of debt as outstanding debt is forgiven. This leads to a higher consumption as the government frees up its budget to reduce tax burden on households. Default is then beneficial in periods with large debt service costs and small sale proceeds from the new debt issuance (due to low realizations of productivity or high levels of deposits chosen by private agents). This is a standard result in the sovereign debt literature (e.g., Eaton and Gersovitz (1981), Grossman and Van Huyck (1988), and Arellano (2008)). In a setup with defaultable bonds, the sovereign government can use default to engineer a de facto contingency on otherwise non state-contingent bond contracts to hedge against periods of unfavorable economic conditions. Effectively, default entails a transfer of resources from foreign creditors to the domestic economy. On the other hand, default implies a productivity cost which lowers output and a reduction in the ability to smooth future consumptions as a result of financial exclusion. In the models that incorporate domestic bank run risks, however, there is an additional novel channel generating endogenous costs of default that is not present in the representative government framework.[11] During periods of financial autarky, the lower productivity levels due to the costs of default render bank runs a more likely private equilibrium outcome. The private equilibrium responses to the government's default decisions endogenously increase the costs of default. The government, when decides whether to default ex-post, internalizes both the exogenous output costs of default and the endogenous costs caused by heightening probabilities of runs and the associated efficiency cost from the disruption of financial intermediation. Ex-ante, foreign creditors take into account the endogenously higher cost of defaults, resulting in a more favorable borrowing terms for the sovereign government. Figure 1 shows, when the output level is high, the government defaults at higher levels of debt than the ones when the productivity level is low.

Second, the sovereign bond price schedule is both sovereign debt and deposit-elastic - that is, it depends on both the amount of new debt issuance by the government and the newly-chosen level of deposits by private agents as this will affect the government's decision to default in the next period. In particular, a higher level of deposits increases the probability of runs in the next period and in turn, increases the probability of having lower outputs. Since the government has a higher tendency to default, a higher level of deposit today generally lowers the price of bonds, as evident in figure 2. The figure depicts the bond price schedule at a high and a low level of new deposits chosen by private agents in the current period, as a function of new amount of debt issuance, keeping output at the mean level. In the Markov equilibrium, the government optimally chooses the new level of

11. Mendoza and Yue (2012), Bocola (2016), and Perez (2015) study different mechanisms to generate endogenous costs of default that are quantitatively relevant in accounting for realistic levels of debt and key business cycle statistics.

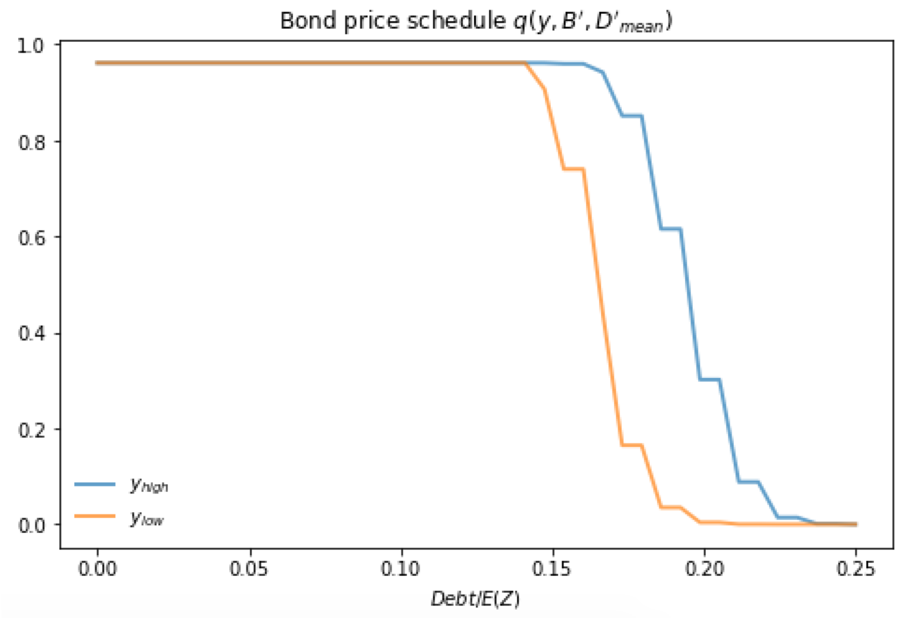

Figure 1: Bond price schedule at different level of outputs

debt that in turn determines the private agents' deposit choice. The dependence of bond price schedule on deposit introduces additional volatility on the sovereign bond spreads that is new to the literature.[12] Moreover, this volatility of borrowing costs that depends on domestic economy's conditions is crucial for the precautionary saving motive of reserves. In particular, a procyclical reserve policy coupled with bank bailout policy reduces the government's borrowing cost via a more stabilized output and smaller probability of bank runs.

Consider the sovereign's choice to borrow. Due to the consumption smoothing motive, the government seeks to borrow more the larger current debt level is. Under a sequence of good shocks, the government keeps borrowing until the service cost of debt is high enough or the output is low enough in which case defaulting is the optimal response. It is also evident that the government borrows more in good times when outputs are high and the

12. Aguiar et al. (2016) in their comprehensive review of the literature emphasize the need to include non-fundamental factors in standard sovereign default frameworks to better account for the volatility of bond spreads.

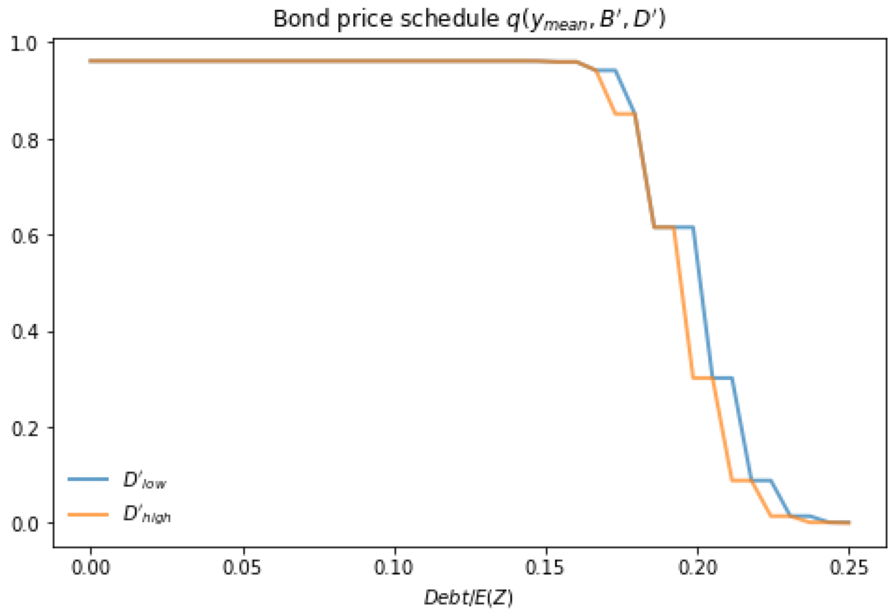

Figure 2: Bond price schedule at different level of deposits

implied spreads are low. This seems to be contradictory with the consumption smoothing motive described above. However, it is well known in the sovereign debt literature that a borrower who is more impatient than the external creditors seeks to front-load consumption and consumption smoothing is mostly achieved via default.

5.3 Evaluation of Reserve Policy

We analyze the nature of procyclical reserve accumulation and countercyclical intervention policies in a welfare comparison exercise. We compare welfare of the government under the benchmark or reference policy regime with $\tau^{dr} = 0$ and $\tau^{res} = 0$, and under various alternative scenarios where we vary the policy parameters τ^{dr} and τ^{res}. Welfare is measured as the unconditional expectation of discounted lifetime utility where the expectation is taken over the invariant distribution of the state. Specifically, for the benchmark case, welfare is given by

Model Specification	Debt if Not Excluded (%GDP)	Welfare (% consumption)
$\tau^{dr} = 0, \tau^{res} = 0$	15.06	0.0
$\tau^{dr} = 1, \tau^{res} = 0$	15.06	0.47
$\tau^{dr} = 1, \tau^{res} = 0.02$	10.504	0.79
$\tau^{dr} = 1, \tau^{res} = 0.05$	3.8	1.92

Table 2: Welfare comparison of reserve policies

$$V^0 = E\sum_{t=0}^{\infty}\beta^t \frac{(C_t^0)^{1-\gamma}}{1-\gamma}.$$

Alternative policy regimes are those of partial bailouts ($\tau^{dr} > 0$) and/or reserve accumulation ($\tau^{res} > 0$), and the welfare is given by

$$V^A = E\sum_{t=0}^{\infty}\beta^t \frac{((1+\lambda^c)C_t^0)^{1-\gamma}}{1-\gamma}.$$

The welfare benefit of an alternative policy is expressed in terms of percentage of consumption equivalence units that households must be given in order to be equally well off under the two specifications. Due to the CRRA preference, the fraction of consumption equivalence units can be written as

$$\lambda^c = \left(\frac{V^A}{V^0}\right)^{\frac{1}{1-\gamma}} - 1,$$

where a positive value of λ^c means that the alternative policy is welfare improving relative to the reference regime.

Starting from the reference regime, with $\tau^{dr} = 0$ and $\tau^{res} = 0$, we explore the impact of alternative specifications by varying the policy parameters τ^{dr} and τ^{res}. Table 2 lists the associated welfare gains (losses, if negative) incurred when departing from the benchmark specification.

Consider going from the benchmark to the policy where $\tau^{dr} = 1$. This means when the banks are in crisis zone, a one percent deviation of output below trend induces the government to relieve one percent of banks' deposit obligations to households. On one hand, this policy has an effect of raising the banks' net worth ex-post and the recovery rate on deposits for the households, and lowering the likelihood of runs. Consequently, the policy can help to reduce the inefficiency that stems from the disruption of financial intermediation and prevent severe economic downturns.

On the other hand, the bailout commitment anticipated by private agents would distort banks' ex-ante borrowing incentives since the transfer each individual bank will receive is increasing in the banks' individual amount of deposits. Even though each bank issue deposit contracts with prices depending on the individual bank's leverage ratio, the interest rate on deposits in equilibrium depends only on the aggregate amount of deposits. When the proportion of deposit relief corresponds one-to-one with respect to the percentage deviation of output from trend, the welfare gain relative to the benchmark regime is 0.47 percent in terms of consumption equivalence units. The increase of deposits due to moral hazard effect is modest. This is because the bailouts are contingent on the productivity run cutoff level for the average banks and not on individual bank's decisions.

We now consider the stabilization role of reserve accumulation policy. Keeping the same debt-relief policy, we set $\tau^{res} = 0.02$. This value means the government accumulates two percent of the output in the form of reserve balances when output is above trend, and decumulate reserves by two percent of the output when the output is below trend. The welfare gain relative to the benchmark regime is 0.79 percent in consumption equivalence units.

To understand this result, first consider a default decision of the government. On one hand, default implies an elimination of servicing cost of debt as outstanding debt is wiped off. This leads to a higher consumption as the government frees up its budget to reduce the tax burden on households. On the other hand, default implies a productivity cost which lowers output and raise the risk of domestic bank runs, and a reduction in the ability to smooth future consumptions as a result of financial exclusion. Moreover, periods of low output (that leads to default decisions) tend to coincide with heightening risks of runs. During periods of financial autarky, the government has to levy extra taxes on households to pay for its spending and to honor its promised bailouts. A decumulation of reserves would provide extra source of funds for the government during these periods of exclusion.

Second, the sovereign bond price schedule depends on both the amount of new debt issuance by the government and the newly-chosen level of deposits by private agents. Since bond price is lower when the output is lower, a higher level of deposits today generally reduces the government bond prices. In equilibrium, the government chooses the optimal amount of new debt that in turn affects private agents' deposit choice. The dependence of bond price schedule on deposit introduces additional volatility that is important for the precautionary saving motive of reserves. In particular, a procyclical reserve policy, coupled with bank bailout policy, reduces the borrowing cost via a more stabilized output and a smaller probability of bank runs.

6 Conclusion

This paper developed a quantitative model of systemic bank runs and sovereign default and international reserves to analyze the welfare consequences of bailout and reserve policies. The model embedded Gertler-Kiyotaki self-fulfilling bank run framework in a standard Eaton-Gersovitz setup. The model connected the roles of reserves with private economic activity via the government's financing need for discretionary expenditures that stem from promised bailouts of the banking system in crisis times. The model highlighted an important fiscal sustainability dimension when addressing government bailout policies.

The precautionary saving role of reserves described here can be interpreted as a stabilization one. In response to adverse developments in the domestic financial market, the government exercises its lender of last resort power to bailout the banks. When the government is highly indebted, these interventions might threaten the government's fiscal solvency. A welfare exercise showed that a procyclical reserve accumulation policy coupled with bank intervention policy is welfare improving.

Future research can extend the developed framework to address various aspects of foreign reserve management policies with proper considerations of financial market factors, including the use of reserve holdings to generate confidence in the market to prevent run-like outcomes.

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A Computational Algorithm

We compute the model as the limit of a finite horizon economy with a large number of periods, T. Starting from the last period T, we then solve the model backwards taking as given the value functions, bond price functions, policy functions of the government and private agents one period ahead and iterate until convergence.

Notation

Aggregate state faced by the government is $s = (Z, b, D)$. We write the aggregate state separately for the repayment and default state:

- Repayment state faced by the government is $s^r = (Z, b, D)$, and the state faced by private agents is $S^r = (Z, b, D, b')$.
- Default state faced by the government is $s^d = (Z, D)$, and the state faced by private agents is $S^d = (Z, D)$.

Note that the run equilibrium outcome in the previous period is encoded in D; that is, $D = 0$ if run occurred in the last period and $D > 0$ otherwise.

Let $V^r(Z, b, D)$ be the government's value of repayment in "good" status (that is, no default in the last period), and $V^d(Z, D)$ be the value of default.

Discretize state space

- Aggregate endogenous state variables: b, D
- Productivity, Z, process is discretized using Tauchen (1986) method

Last period problem: $t = T$

In the last period, there is no new borrowing, $b' = 0$ and no new deposit, $D' = 0$. Assume no run in the last period.

Repayment value

$$V_T^r(s^r) = U(C_T)$$

where

$$C_T^* = C_T = Z - b - g$$

$$N_T = \begin{cases} \sigma\left\{\left[Z + Q_T^k\right] - D\left(1 - t(Z)\right)\right\} + (1-\sigma)w^b & \text{if } D > 0 \\ (1+\sigma)w^b(1-\sigma) & \text{if } D = 0 \end{cases}$$

$$Q_T^{k*} = Q_T^k = N_T$$

Default value

$$V^d(Z, D) = U(C_T)$$

$$C_T^* = C_T = Z^d$$

Value of the government:

$$V^g(Z, b, D) = V^d(Z, D)$$

Assume the probability of default is 1 and when repaying, $b' = b$ with probability one everywhere in the state space.

The bond price that investors offer to the government at $t = T - 1$ is $q_T(Z, b', D') = 0$

Iterate backwards: $t = T - 1$

Repayment

Grid search on b'. Value of repaying old debt and issuing new debt b' is

$$V_t^r(S^r) = U(C(S^r)) + \beta \mathbb{E}_{Z'|Z} \hat{V}_{t+1}^g (Z', b', D'(S^r))$$

where $C(S^r), D'(S^r)$ solve the private equilibrium for given S^r, and $\hat{V}_{t+1}^g (Z', b', D'(S^r))$ is the value V_{t+1}^g linearly interpolated at the point $D' = D'(S^r)$. The equilibrium conditions are summarized as follows:

If no run in state S^r:

- Banks' net worth follows

$$N(S^r) = \begin{cases} \sigma \left\{ \left[Z + Q^k(S^r) \right] - D \left(1 - t(Z)\right) \right\} + (1-\sigma) w^b & \text{if } D > 0 \\ (1+\sigma) w^b (1-\sigma) & \text{if } D = 0 \end{cases}$$

- Solves for $C(S^r)$

$$C(S^r) + g = Z + a - \frac{a'}{R^*} + \hat{q}_{t+1}(Z, b', D'(S^r)) b' - b$$

- From Euler Equation on deposit, solve for $1/Q^d(S^r)$:

$$\mathbb{E}_{S'|S} \left[\beta \left(\frac{\hat{C}_{t+1}(S')}{C(S^r)} \right)^{-\gamma_h} \min \left\{ \frac{1}{Q^d(S^r)}, \frac{\left[Z' + \hat{Q}_{t+1}^{k*}(S') \right] + t(Z') D'(S)}{Q^d(S^r) D'(S^r)} \right\} \right] = 1$$

- From banks' problem,

$$
\begin{aligned}
\theta\frac{Q^k(S^r)}{N(S^r)} =& \mathbb{E}_{S'|S}\left\{\beta\left(\frac{\hat{C}_{t+1}(S')}{C(S^r)}\right)^{-\gamma_h}\left[1-\sigma+\sigma\theta\frac{\hat{Q}^k_{t+1}(S')}{\hat{N}'(S')}\right][1-\iota(S')]\right. \\
&\left.\times\left[\frac{Z'+\hat{Q}^k_{t+1}(S')}{Q^k(S^r)}.\frac{Q^k(S^r)}{N(S^r)}+\frac{1-t(Z')}{Q^d\left(S^r\right)}\left(1-\frac{Q^k(S^r)}{N(S^r)}\right)\right]\right\}
\end{aligned}
$$

where

$$
\iota(S') = \begin{cases} 1 & \text{if } (Z',\zeta')\in\left\{Z'<\bar{Z}'(S) \text{ and } \zeta'=1\right\} \\ 0 & \text{if } (Z',\zeta')\in\left\{Z'<\bar{Z}'(S) \text{ and } \zeta'=0\right\}\cup\left\{Z'\geq\bar{Z}'(S)\right\} \end{cases}
$$

is the run indicator in state $S' = (Z', b', D', H_D(s'), H_B(s'))$.

- In all of the above, the run cutoff rule follows

$$
\bar{Z}'(S^r) = \sup\left\{Z'\left|\frac{Z'+\hat{Q}^{k*}_{t+1}\left(S'\right)}{Q^k(S^r)}.\frac{Q^k(S^r)}{N(S^r)}-\left(\frac{Q^k(S^r)}{N(S^r)}-1\right)\frac{1-t(Z')}{Q^d(S^r)}<0\right.\right\},
$$

where $\hat{C}_{t+1}(S'), \hat{Q}^k_{t+1}(S'), \hat{Q}^{k*}_{t+1}\left(S'\right), \hat{q}_{t+1}(Z,b',D'(S^r))$ are the values $C_{t+1}, Q^k_{t+1}, Q^{k*}_{t+1}, q_{t+1}$ linearly interpolated at the point $D' = D'(S^r)$, and $\hat{N}'(S') = \sigma\left\{\left[Z'+\hat{Q}^k_{t+1}(S')\right]-D'(S^r)\left(1-t(Z')\right)\right\}$ $(1-\sigma)w^b$.

- Update $D'(S^r)$ as

$$
D'(S^r) = \frac{Q^k(S^r)-N(S^r)}{Q^d(S^r)}
$$

- Finally, set $Q^{k*}_t(S^r) = Q^k(S^r)$ when no run in S

If a run occurs in S^r:

- Solves for $C^*(S^r)$

$$
C^*(S^r)+g+\alpha^h = Z+a-\frac{a'}{R^*}+\hat{q}_{t+1}(Z,b',0)b'-b
$$

- From households' Euler equation for capital, solve for $Q^{k*}(S^r)$

$$\mathbb{E}_{S'|S}\left[\beta\left(\frac{\hat{C}_{t+1}(S')}{C^*(S^r)}\right)^{-\gamma_h}\frac{Z'+Q^k(S')}{Q^{k*}(S^r)+\alpha^h}\right]=1$$

  and verify that

$$Z+Q^{k*}(S^r)<D\left(1-t(Z)\right)$$

- Set $D'(S^r)=0$

Default

Value of default in state S is:

$$V_t^d(S^d)=U(C(S^d))+\beta\mathbb{E}_{Z'|Z}\hat{V}_{t+1}^g\left(Z',0,D'(S^d)\right)$$

where $C(S^d),D'(S^d)$ solve the private equilibrium similar to the one in repayment case with $\delta=1$ and $b'=b=0$, and $\hat{V}_{t+1}^g\left(Z',0,D'(S^d)\right)$ is the value V_{t+1}^g linearly interpolated at the point $D'=D'(S^d)$.

Update bond price schedule. Then iterate backwards and check convergence.