CS 131 – Fall 2019, Prof. Tsourakakis
Assignment 2 must be submitted by Friday September 20, on Gradescope.

Problem 1. (12 Points) Write logical expressions in first-order logic for the following sentence:

a) (4 Points) Every human has a stomach.

b) (4 Points) Everyone is a friend of someone.

c) (4 Points) Nobody likes everybody.

Problem 2. (12 Points) Negate the following logical statements

a) (4 Points) $\forall x \exists y P(x, y)$ (Assume that x and y belong to the same domain, and this domain is arbitrary).

b) (8 Points) $\exists x \Big(F(x) \rightarrow \forall y \big(\neg P(y) \wedge \forall z Q(z) \big) \Big)$. (Simplify this expression until you have no negation operator).

Problem 3. (13 Points) State whether the following sentences are true or false. Justify your answer or try to give a counter example when necessary.

a) (6 Points) Let x be a real number. Then $\forall x \; 0 \leq \frac{1}{x}$ is always true.

b) (7 Points) $\forall x \; x^2 + 4x + 5 > 0$ where x is in the set of real numbers

Problem 4. (15 Points) Prove by using the laws of deduction that the following logical statement is unsatisfiable:
$$\varphi := \neg(A \rightarrow C) \wedge \Big(\big((A \wedge B) \rightarrow C\big) \wedge (A \rightarrow B) \Big)$$

Problem 5.

a) (18 Points) Write a first-order logical expression for the following sentences by using the predicate $Cup(x)$ to mean x is a cup.

i) (9 Points) There are at most two cups.

ii) (9 Points) There are exactly two cups.

b) (30 Points) Let the predicate $P(x, y)$ mean x is a parent of y. You are allowed to use the quantifiers, the predicates P, $=$, and $\neq$, logical operators and variables, and the predicates from the previous subproblems. For the purposes of the problem below, the notion of "sibling" includes half-siblings. For definitions of "first cousin" or "first cousin once removed," please take a look here
`https://en.wikipedia.org/wiki/Cousin#Non-blood_relations`.

i) (5 Points) Define a predicate $G(a, b)$ which is true if and only if a is a grandparent of b.

ii) (5 Points) Define a predicate $S(a, b)$ which is true if and only if a is a sibling of b (recall that a person is not his/her own sibling).

iii) (5 Points) Define a predicate $A(a, b)$ which is true if and only if a is an aunt or uncle of b.

iv) (5 Points) Define a predicate $C(a, b)$ which is true if and only if a is a first cousin of b (think about the warning in part (b) and how it generalizes to this case).

v) (5 Points) Define a predicate $R(a, b)$ which is true if and only if a is a first cousin once removed of b.

vi) (5 Points) Let a be a man and b be a woman. Massachusetts law prohibits marriage between them if they are related in certain ways. Define a predicate $I(a, b)$ which is true if and only if the marriage between a and b is prohibited according to `https://malegislature.gov/Laws/GeneralLaws/PartII/TitleIII/Chapter207/Section1` and/or `https://malegislature.gov/Laws/GeneralLaws/PartII/TitleIII/Chapter207/Section2`. For the purposes of this problem, ignore prohibitions that involve the word "husband," "wife," or the prefix "step" (because we have no language to express them). To get full credit, avoid making your statement too repetitive – strive for simplicity.