CS 131 – Fall 2017, Midterm 1 Study Questions

Students are invited to discuss and post solutions to these problems on Piazza (one solution per person please!), but please wait until after lecture on Tuesday 5PM, so all students have the opportunity to think about them first.

Question 1. *A self-proclaimed "great logician" has invented a new quantifier, on par with $\exists$ and $\forall$. The new quantifier is symbolized by U and is read "there exists a unique". The proposition $Ux.P(x)$ is true iff there is exactly one x for which $P(x)$ is true. The logician has noted, "There used to be two quantifiers, but now there are three! I have extended the field of quantificational logic by 50%!"*

(a) Write a proposition equivalent to $Ux.P(x)$ using only the $\exists$ quantifier, =, and logical connectives.

(b) Write a proposition equivalent to $Ux.P(x)$ using only the $\forall$ quantifier, =, and logical connectives.

Question 2. *Prove this claim two different ways: proof by contrapositive and proof by contradiction.*
If 100 balls are placed in nine boxes, then some box contains 12 or more balls.

Question 3. *Indicate and briefly justify whether these five formulas are true for each of four different domains: $\mathbb{N}$ (natural numbers), $\mathbb{Z}$ (all integers), $\mathbb{Q}$ (rationals), $\mathbb{R}$. (You may want to review posted solutions to Question 3 on Assignment 2 and Question 2 on Lab 2 first).*

$$\begin{array}{rc}
a) & \exists x.x^2 = 2 \\
b) & \forall x.\exists y.x^2 = y \\
c) & \forall y.\exists x.x^2 = y \\
d) & \forall x \neq 0.\exists y.xy = 1 \\
e) & \exists x.\exists y.(x + 2y = 2) \wedge (2x + 4y = 5)
\end{array}$$

Question 4. *(a) Prove there is no positive integer n such that $n^2 + n^3 = 100$.*

(b) Prove or disprove: there is a rational number x and an irrational number y such that x^y is irrational.

(c) Use forward reasoning to show that if x is a non-zero real number, then $x^2 + \frac{1}{x^2} \geq 2$.

(d) Prove that $\min(a, \min(b, c)) = \min(\min(a, b), c)$.

(e) Prove that the product of a non-zero rational number and an irrational number is irrational.

Question 5. *There are 210 students registered in CS131. Prove that among you all there exist two students with exactly the same number of acquaintances in CS131.*

Question 6. *Create a logical circuit that is logically equivalent to XOR using only NAND gates.*

Question 7. *Let $a_n = 2^n + 5 \cdot 3^n$ for $n = 0, 1, 2, \ldots$*

a) Find a_0, a_1, a_2, a_3, a_4.

b) Show that $a_2 = 5a_1 - 6a_0, a_3 = 5a_2 - 6a_1$,and $a_4 = 5a_3 - 6a_2$.

c) Show that $a_n = 5a_{n-1} - 6a_{n-2}$ for all integers n with $n >= 2$.

Question 8. *Translate the following statements in English into logic, and argue whether they are true or false using only rules of inference.*

(a) Every human will die some day. I will never die. Therefore, I am not a human.

(b) All dogs like bones. My pet is a cat. Therefore my pet does not like bones.

(c) When it's warm, we go out. Today is warm. Therefore, today we will go out.