CS 131 – Spring 2019, Discussion Worksheet 1
September 11, 2018

Problem 1.

Prove using truth tables (semantically) that $((A \to B) \wedge A) \to B$ is a tautology (i.e., is always true).

Solution.

A	B	$A \to B$	$(A \to B) \wedge A$	$(A \to B) \wedge A \to B$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Problem 2. Before going out to a restaurant, Jane had told her roommate Katia that she would order at least an appetizer or dessert. Jane came back and told Katia that she had not ordered an appetizer.

a) Define relevant Boolean variables that can be used to formally express the knowledge Katia has about Jane's meal.

Solution. Let p: ordering an appetizer; and q: ordering a dessert.

b) Express the knowledge Katia has about Jane's meal using a single Boolean formula.

Solution. Then $((p \vee q) \wedge \neg p)$ expresses the given conditions.

c) Prove formally (on behalf of Katia, given the Boolean formula you just deduced) that Jane must have had dessert.

Solution. We know that $(p \vee q)$ is true and that p is false. $p \vee q = F \vee q = q$ by identity law, which must also be true.

Problem 3. Write the following sentences as a Boolean formula by defining the right Boolean variables.

a) A sufficient condition for winning a game of chess is having a good endgame technique.

Solution. Let q denote "winning a game of chess", and p denote"having a good endgame technique". Them, the Boolean formula is

$$p \to q$$

b) A dark chocolate is considered bittersweet and high quality whenever it has a minimum of 35 percent chocolate liquor, a cacao content of at least 70 percent, but little or no milk .

Solution. Let b denote "a dark chocolate is considered bittersweet", h denote "a dark chocolate is considered high quality", l denote "a dark chocolate has a minimum of 35 percent chocolate liquor", c denote "a dark chocolate has a cacao content of at least 70 percent", m denote "a dark chocolate has little milk", and n denote "a dark chocolate has no milk". Note that $\neg m \not\equiv n$. Then, the Boolean formula is:

$$(l \wedge c \wedge (m \vee n)) \to (b \wedge h)$$

Problem 4. Prove formally that if p and $(p \to q)$, then q. That is, prove that

$$\Big(p \wedge (p \to q)\Big) \to q$$

is a tautology. This result (often used as a law of logic) is known as the "modus ponens."

Solution. $\Big(p \wedge (p \to q)\Big) \to q$

$$(p \wedge (p \to q)) \to q \tag{1}$$
$$\neg\,(p \wedge ((\neg p) \vee q)) \vee q \quad (\textbf{Conditional identity}) \tag{2}$$
$$((\neg p) \vee (p \wedge (\neg q))) \vee q \quad (\textbf{DeMorgan's law}) \tag{3}$$
$$((\neg p) \vee p) \wedge ((\neg p) \vee (\neg q)) \vee q \quad (\textbf{Distributive law}) \tag{4}$$
$$(\text{True } \wedge ((\neg p) \vee (\neg q))) \vee q \quad (\textbf{Complement law}) \tag{5}$$
$$((\neg p) \vee (\neg q)) \vee q \quad (\textbf{identity law}) \tag{6}$$
$$(\neg p) \vee ((\neg q) \vee q) \quad (\textbf{Associative law}) \tag{7}$$
$$(\neg p) \vee \text{True} \quad (\textbf{Complement law}) \tag{8}$$
$$\text{True} \quad (\textbf{Domination law}) \tag{9}$$