Problem 1. Suppose P is a predicate on one argument and Q is a predicate on two arguments. Evaluate if statements above are equivalent.

a) $\forall x \forall y\, Q(x, y)$ and $\forall y \forall x\, Q(x, y)$

b) $\forall x \exists y\, Q(x, y)$ and $\exists y \forall x\, Q(x, y)$

c) $\forall x(P(x) \vee (\exists y Q(x, y)))$ and $\forall x \exists y(P(x) \vee (Q(x, y)))$

Problem 2.

Let $T(x, y)$ means that student x likes cuisine y, where the domain of x consists of all students at BU and the domain of y consists of all cuisines. Express each of these statements by a simple English sentence.

a) $\forall x \forall z \exists y((x \neq z) \rightarrow \neg(T(x, y) \wedge T(z, y)))$

b) $\exists x \exists z \forall y(T(x, y) \leftrightarrow T(z, y))$

c) $\forall x \forall z \exists y(T(x, y) \leftrightarrow T(z, y))$

Let $S(x)$ be a predicate "x is a student", $F(x)$ be a predicate "x is a faculty member", and $A(x, y)$ the predicate "x has asked y a question", where the domain consists of all people associated with BU. Use quantifiers to express each of these statements.

d) Some student asked every faculty member a question.

e) There is a faculty member who has asked every other faculty member a question.

f) some student has never been asked a question by a faculty member

Problem 3.

a) Express the following sums and products by using $\sum$ and/or $\prod$ notations:

i) $\frac{1}{4} + 1 + \frac{9}{4} + 4 + \frac{25}{4} + ...$

ii) $-8 + 13 - 18 + 23 - 28$

iii) $1 \cdot \frac{1}{3} \cdot \frac{1}{9} \cdot \frac{1}{27} \cdot \frac{1}{81} \cdots$

b) Evaluate/Compute the following sums and product sums:

i) $\sum_{i=1}^{3} \sum_{j=1}^{3} \frac{i}{j}$

ii) $\prod_{i=0}^{3} \frac{2^{2i}}{3^i}$

iii) $\prod_{i=1}^{3} \prod_{j=1}^{2} ij^2$

Problem 4. Prove that if $x = 1$ or $x = 2$, then $x^2 - 3x + 2 = 0$.

Problem 5. Prove that there is an integer N such that for all integers $n \geq N$, $2^n < n!$.