NAME:

CS 583 – Assignment 2

1. We have three random variables, *A*, *B*, *C*, each of which is binary. We have the following factors over these variables: $\phi(A)$, $\phi(B)$, $\phi(C)$, $\phi(A, B)$, $\phi(A, C)$, $\phi(B, C)$.

a. Draw a Markov network graph over these variables.

b. Draw a factor graph over these variables.

c. Here are the values of the factors. Compute $P(A, B, C)$.

A	ϕ(A)
T	2
F	1

B	ϕ(B)
T	1
F	4

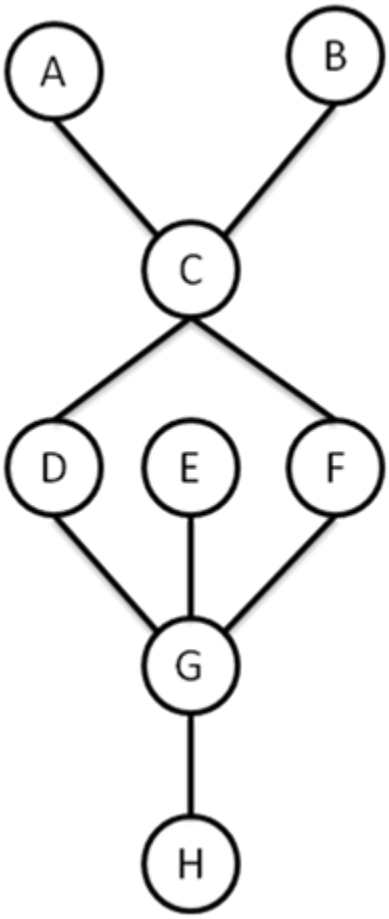
C	ϕ(C)
T	1
F	8

A	B	ϕ(A,B)
T	T	5
T	F	1
F	T	1
F	F	5

A	C	ϕ(A,C)
T	T	6
T	F	1
F	T	1
F	F	6

B	C	ϕ(B,C)
T	T	1
T	F	10
F	T	10
F	F	1

2. For the following Markov network graph, indicate whether the following independence statements are True or False.



a. $A \perp B$

b. $A \perp B \mid C$

c. $A \perp G \mid D$

d. $A \perp G \mid D, F$

e. $A \perp H \mid G$

3. We have the Markov network over 3 binary variables: $A - B - C$. We define a pairwise Markov random field (MRF) over this network. We define the following features.

$f_1(A) = 1\ if\ A = T$, 0 otherwise. $w_1 = \ln(2)$.q

$f_2(B) = 1\ if\ B = F$, 0 otherwise. $w_2 = -\ln(3)$.

$f_3(C) = 1\ if\ C = T$, 0 otherwise. $w_3 = \ln(4)$.

$f_4(A, B) = 1\ if\ A = B$, 0 otherwise. $w_4 = -\ln(5)$.

$f_5(B, C) = 1\ if\ B \neq C$, 0 otherwise. $w_5 = \ln(6)$.

$$P(A, B, C) = \frac{1}{Z} e^{-\sum_{1=1}^{5} w_i \times f_i(A,B,C)}$$

Note that to simplify the notation, we simply wrote $f_i(A, B, C)$, though features are defined only over nodes and edges. Assume f_i ignores the variables that it is not defined over. For example, $f_1(A, B, C) = f_1(A)$. Populate the following table.

A	B	C	$\phi(A) * \phi(B) * \phi(C) * \phi(A, B) * \phi(B, C) = e^{-\sum_{1=1}^{5} w_i \times f_i(A,B,C)}$	$P(A, B, C)$
T	T	T		
T	T	F		
T	F	T		
T	F	F		
F	T	T		
F	T	F		
F	F	T		
F	F	F		

Hints: $e^{\ln(x)} = x$ and $e^{-\ln(x)} = \frac{1}{e^{\ln(x)}} = \frac{1}{x}$.

4. We have a document classification task. We have four documents, D_1 and D_2. We are interested in classifying these documents into one of two topics: Artificial Intelligence (AI) or Databases (DB). Each document has one or both words: Agent and SQL.

We will represent this data as follows: the labels of D_1 and D_2 are: Y_1 and Y_2. Each Y_i can take one of two values: AI or DB. The presence of word Agent in document D_i represented as X_{1i}, and the presence of the word SQL in document D_i represented as X_{2i}.

The documents and their contents are as follows:

D_1: X_{11} = True, X_{21} = False (i.e, D_1 contains only the word Agent)

D_2: X_{12} = True, X_{22} = True (i.e., D_2 contains both words)

We also have the additional knowledge that D_1 cites D_2. We construct the following CRF, with the following feature functions.

Features:

$f_1(X_{1i}, Y_i) = 1$ if X_{1i} = T and Y_i = AI, 0 otherwise; $w_1 = -1$.

$f_2(X_{1i}, Y_i) = 1$ if X_{1i} = T and Y_i = DB, 0 otherwise; $w_2 = +1$.

$f_3(X_{2i}, Y_i) = 1$ if X_{2i} = T and Y_i = AI, 0 otherwise; $w_3 = +1$.

$f_4(X_{2i}, Y_i) = 1$ if X_{2i} = T and Y_i = DB, 0 otherwise; $w_4 = -1$.

$f_5(Y_i, Y_j) = 1$ if $Y_i = Y_j$, 0 otherwise; $w_5 = -1$.

$f_6(Y_i, Y_j) = 1$ if $Y_i \neq Y_j$, 0 otherwise; $w_6 = +1$.

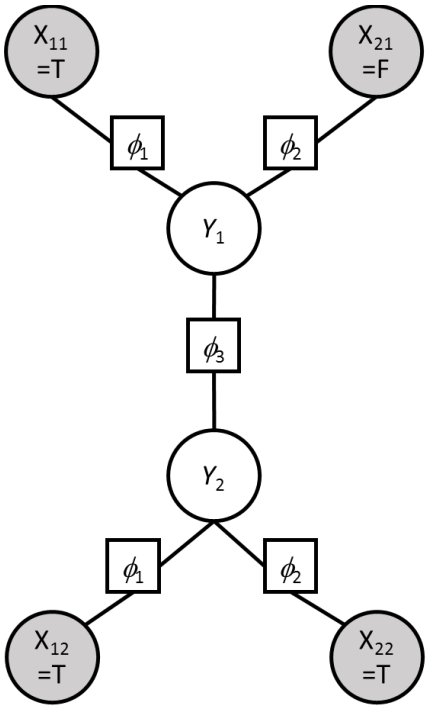
Potentials:

$\phi_1(X_{1i}, Y_i) = e^{-(w1f1+ w2f2)}$

$\phi_2(X_{2i}, Y_i) = e^{-(w3f3+ w4f4)}$

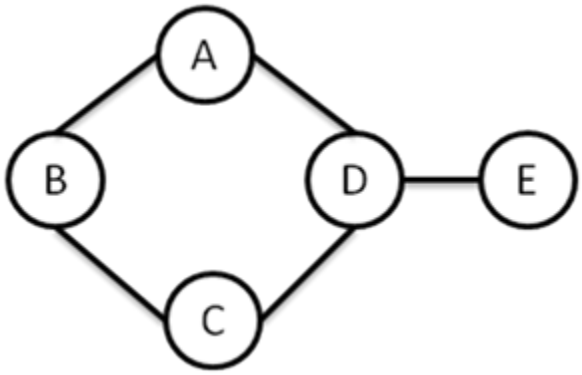
$\phi_3(Y_i, Y_j) = e^{-(w5f5+ w6f6)}$

The factor graph is given as follows:



Calculate P(***Y***|***X***), that is P(Y_1, Y_2 | X_{11}=T, X_{21}=F, X_{12}=T, X_{22}=T). What is the MAP assignment to ***Y*** given ***X***? Show your work.

5. We are given the following Markov network structure, H.



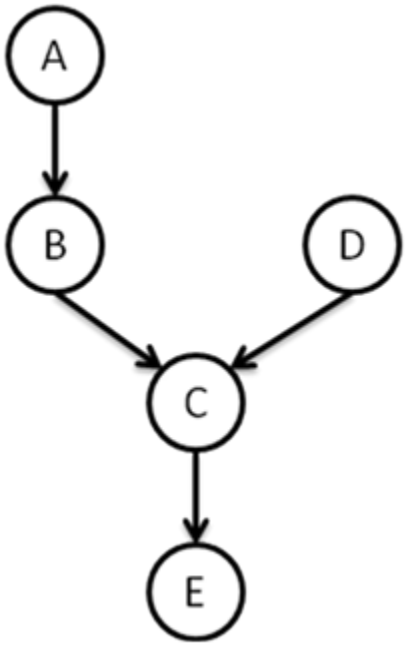
a. Find a minimal I-Map Bayesian network structure G_1 for H. Use the variable order of A, B, C, D, E.

b. Find a minimal I-Map Bayesian network structure G_2 for H. Use the variable order of E, D, C, B, A.

c. Is G_1 a P-Map for H? If not, which independencies are missing?

d. Is G_2 a P-Map for H? If not, which independencies are missing?

6. We are given the following Bayesian network structure G.



a. Find a minimal I-Map Markov network structure H for G. Use any method/variable order you like; make sure H is a *minimal* I-Map.

b. Is H a P-Map for G? If not, which independencies are missing?