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CS 583-01

April 20, 2024

CS 583 – Assignment 4

1. We have three variables: X, Y, and Z. X and Z are binary with domain {T, F} and Y has three possible values: {R, G, B}. The Bayesian network has the following structure: $X \rightarrow Y \rightarrow Z$. Here are the counts for a dataset D. If a count is zero, it is not listed.

X	Y	Z	Counts
T	R	T	10
T	R	F	20
T	B	T	30
F	R	F	40
F	B	T	50

Note that we need to estimate $P(X), P(Y|X)$, and $P(Z|Y)$ for this network.

<table>
<tr><th>P(X)</th><th>Counts</th><th colspan="2">P(X) = Counts / 150</th><th colspan="2">P(Xnext | D)</th><th colspan="2">P(Xnext | D)</th></tr>
<tr><td>T</td><td>60</td><td>60/150 = 6/15 = 2/5</td><td>0.4</td><td>61/152</td><td>0.40132</td><td>66/162</td><td>0.40741</td></tr>
<tr><td>F</td><td>90</td><td>90/150 = 9/15 = 3/5</td><td>0.6</td><td>91/152</td><td>0.59868</td><td>96/162</td><td>0.59259</td></tr>
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<table>
<tr><th>P(Y|X = T)</th><th>Counts</th><th colspan="2">P(X) = Counts / 60</th><th colspan="2">P(Xnext | D)</th><th colspan="2">P(Xnext | D)</th></tr>
<tr><td>R</td><td>30</td><td>30/60 = 3/6 = 1/2</td><td>0.5</td><td>31/63</td><td>0.49206</td><td>32/66</td><td>0.48485</td></tr>
<tr><td>G</td><td>0</td><td>0</td><td>0</td><td>1/63</td><td>0.01587</td><td>2/66</td><td>0.03030</td></tr>
<tr><td>B</td><td>30</td><td>30/60 = 3/6 = 1/2</td><td>0.5</td><td>31/63</td><td>0.49206</td><td>32/66</td><td>0.48485</td></tr>
<tr><th>P(Y|X = F)</th><th>Counts</th><th colspan="2">P(X) = Counts / 90</th><th colspan="2">P(Xnext | D)</th><th colspan="2">P(Xnext | D)</th></tr>
<tr><td>R</td><td>40</td><td>40/90 = 4/9</td><td>0.444</td><td>41/93</td><td>0.44086</td><td>42/96</td><td>0.43750</td></tr>
<tr><td>G</td><td>0</td><td>0</td><td>0</td><td>1/93</td><td>0.01075</td><td>2/96</td><td>0.02083</td></tr>
<tr><td>B</td><td>50</td><td>50/90 = 5/9</td><td>0.556</td><td>50/93</td><td>0.53763</td><td>52/96</td><td>0.54167</td></tr>
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<table>
<tr><th>P(Z|Y = R)</th><th>Counts</th><th colspan="2">P(X) = Counts / 70</th><th colspan="2">P(Xnext | D)</th><th colspan="2">P(Xnext | D)</th></tr>
<tr><td>T</td><td>10</td><td>10/70 = 1/7</td><td>0.14286</td><td>11/72</td><td>0.15278</td><td>12/74</td><td>0.16216</td></tr>
<tr><td>F</td><td>60</td><td>60/70 = 6/7</td><td>0.85714</td><td>61/72</td><td>0.84722</td><td>62/74</td><td>0.83784</td></tr>
<tr><th>P(Z|Y = G)</th><th>Counts</th><th colspan="2">P(X) = Counts / 0</th><th colspan="2">P(Xnext | D)</th><th colspan="2">P(Xnext | D)</th></tr>
</table>

T	0	0	0	1/2	0.5	2/4	0.5
F	0	0	0	1/2	0.5	2/4	0.5
P(Z|Y = B)	Counts	P(X) = Counts / 80		P(Xnext | D)		P(Xnext | D)	
T	80	80/80	1.00000	81/82	0.98780	82/84	0.97619
F	0	0	0	1/82	0.01220	2/84	0.02381

a. What are the MLE estimates?

$P(X) = \langle {}^{2}/_{5}, {}^{3}/_{5} \rangle = \langle 0.4, 0.6 \rangle$

$P(Y|X = T) = \langle {}^{1}/_{2}, 0, {}^{1}/_{2} \rangle = \langle 0.5, 0, 0.5 \rangle$

$P(Y|X = F) = \langle {}^{4}/_{9}, 0, {}^{5}/_{9} \rangle = \langle 0.444, 0, 0.556 \rangle$

$P(Z|Y = R) = \langle {}^{1}/_{7}, {}^{6}/_{7} \rangle = \langle 0.14286, 0.85714 \rangle$

$P(Z|Y = G) = \langle 0, 0 \rangle$

$P(Z|Y = R) = \langle 1, 0 \rangle$

b. Assuming a uniform prior and K2 approach to Bayesian estimation, what are the predictive for next X, Y|X, and Z|Y?

$P(X) = \langle {}^{61}/_{152}, {}^{91}/_{152} \rangle = \langle 0.40132, 0.59868 \rangle$

$P(Y|X = T) = \langle {}^{31}/_{63}, {}^{1}/_{63}, {}^{31}/_{63} \rangle = \langle 0.49206, 0.01587, 0.49206 \rangle$

$P(Y|X = F) = \langle {}^{41}/_{93}, {}^{1}/_{93}, {}^{50}/_{93} \rangle = \langle 0.44086, 0.01075, 0.53763 \rangle$

$P(Z|Y = R) = \langle {}^{11}/_{72}, {}^{61}/_{72} \rangle = \langle 0.15278, 0.84722 \rangle$

$P(Z|Y = G) = \langle {}^{1}/_{2}, {}^{1}/_{2} \rangle = \langle 0.5, 0.5 \rangle$

$P(Z|Y = R) = \langle {}^{81}/_{82}, {}^{1}/_{82} \rangle = \langle 0.98780, 0.01220 \rangle$

c. Assuming a |D`| = probabilities 12 and P` is uniform, and a BDE approach to estimation, what are the predictive probabilities for next X, Y|X, and Z|Y?

$P(X) = \langle {}^{66}/_{162}, {}^{96}/_{162} \rangle = \langle 0.40741, 0.59259 \rangle$

$P(Y|X = T) = \langle {}^{32}/_{66}, {}^{2}/_{66}, {}^{32}/_{66} \rangle = \langle 0.48485, 0.03030, 0.48485 \rangle$

$P(Y|X = F) = \langle {}^{42}/_{96}, {}^{2}/_{96}, {}^{52}/_{96} \rangle = \langle 0.43750, 0.02083, 0.54167 \rangle$

$P(Z|Y = R) = \langle {}^{12}/_{74}, {}^{62}/_{74} \rangle = \langle 0.16216, 0.83784 \rangle$

$P(Z|Y = G) = \langle {}^{2}/_{4}, {}^{2}/_{4} \rangle = \langle 0.5, 0.5 \rangle$

$P(Z|Y = R) = \langle {}^{82}/_{84}, {}^{2}/_{84} \rangle = \langle 0.97619, 0.02381 \rangle$