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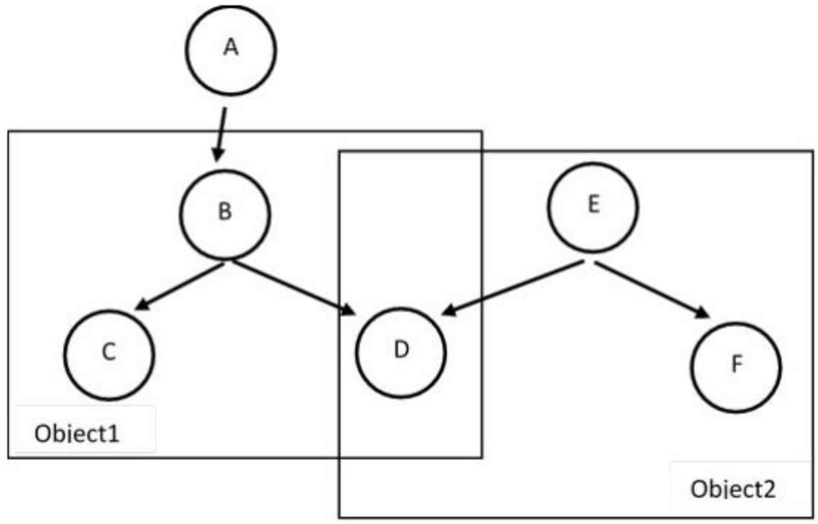
CS 583-01

April 7, 2024

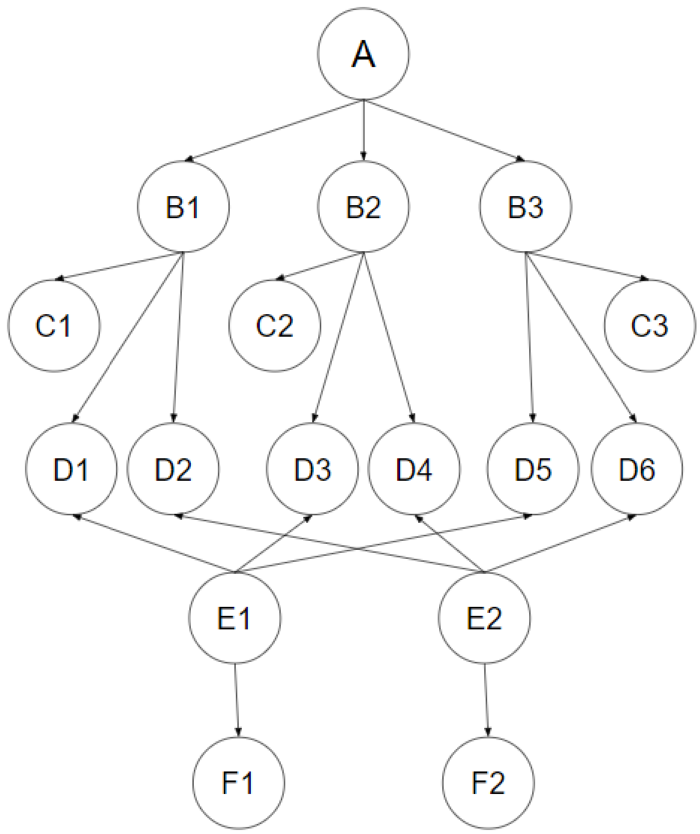
Assignment 3

1. Here is a plate model.




a. What probability distributions do we need to specify for this model?
   P(A)
   P(B | A)
   P (C | B)
   P(E)
   P(F | E)
   P(D | B, E)

b. Draw an unrolled version of the Bayesian network, where there are three items of type Object1 and two items of Object2 type.

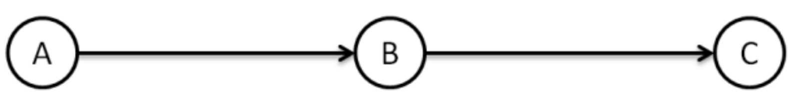



2. For the following linear chain, please calculate the requested probabilities using variable elimination. You can use any order you like. Show your work.

| A | P(A) |
|---|---|
| T | 0.4 |
| F | 0.6 |

| A | B | P(B\|A) |
|---|---|---|
| T | T | 0.3 |
| T | F | 0.7 |
| F | T | 0.8 |
| F | F | 0.2 |

| B | C | P(C\|B) |
|---|---|---|
| T | T | 0.9 |
| T | F | 0.1 |
| F | T | 0.4 |
| F | F | 0.6 |




a. P(C)

$\mathrm{P(C)} = \Sigma_{A,B} P(A)\, P(B|A)\, P(C|B)$

$= \Sigma_B P(C|B)\, T_1(B)$

$= \mathrm{T}(C)$

$\Psi(A,B) = P(A)P(B|A)$

| A | B | P(A)P(B\|A) | Ψ(A, B) |
|---|---|---|---|
| T | T | 0.4 × 0.3 | 0.12 |
| T | F | 0.4 × 0.7 | 0.28 |

| | | | |
|---|---|---|---|
| F | T | 0.6 × 0.8 | 0.48 |
| F | F | 0.6 × 0.2 | 0.12 |

$T_1(B) = \Sigma_A \Psi(A, B)$

| **B** | **T(B)** |
|---|---|
| T | 0.6 |
| F | 0.4 |

$\Psi(B, C) = P(C|B)T(B)$

| **B** | **C** | **P(C\|B)T(B)** | **Ψ(A, B)** |
|---|---|---|---|
| T | T | 0.9 × 0.6 | 0.54 |
| T | F | 0.1 × 0.6 | 0.06 |
| F | T | 0.4 × 0.4 | 0.16 |
| F | F | 0.6 × 0.4 | 0.24 |

$T_2(C) = \Sigma_B \Psi(B, C)$

| **C** | **T(C)** |
|---|---|
| T | 0.7 |
| F | 0.3 |

b. P(C | A=T)

$P(C|A = T) = \Sigma_B P(A = T) \times P(B|A = T) \times P(C|B)$
$= P(A = T)T(C, A = T)$
$\Psi(A, B, C) = P(C|B) \times P(B|A = T)$

| **B** | **C** | **P(C\|B) P(B\|A=T)** | **Ψ(A, B, C)** |
|---|---|---|---|
| T | T | 0.9 × 0.3 | 0.27 |
| T | F | 0.1 × 0.3 | 0.03 |
| F | T | 0.4 × 0.7 | 0.28 |
| F | F | 0.6 × 0.7 | 0.42 |

$T(C, A = T) = \Sigma_B \Psi(A, B, C)$

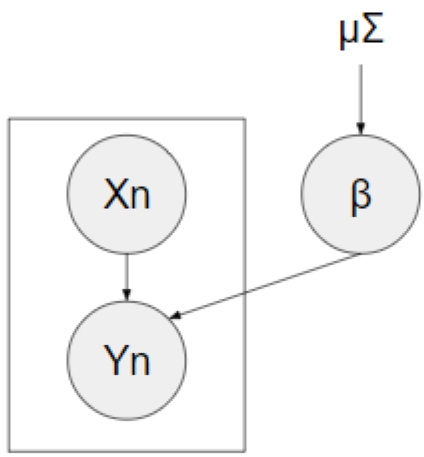
<table>
<tr><th>C</th><th>T(C, A=T)</th></tr>
<tr><td>T</td><td>0.55</td></tr>
<tr><td>F</td><td>0.45</td></tr>
<tr><th>C</th><th>P(A=T) T(C, A=T)</th><th colspan="3">P(C|A=T)</th></tr>
<tr><td>T</td><td>0.4 × 0.55</td><td>0.22</td><td>0.22 / 0.4</td><td>0.55</td></tr>
<tr><td>F</td><td>0.4 × 0.45</td><td>0.18</td><td>0.18 / 0.4</td><td>0.45</td></tr>
</table>

c. P(C | A=T, B=T)

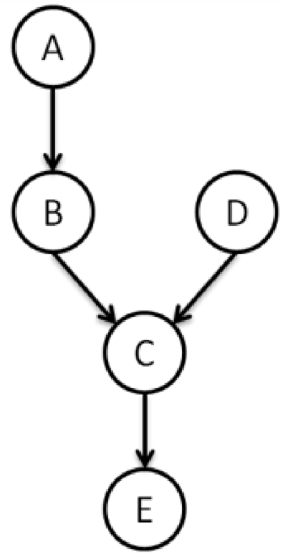
$P(C|A = T, B = T) = P(A = T)P(B = T|A = T)P(C|B = T)$

| C | P(A=T)P(B=T\|A=T)P(C\|B=T) | P(C\|A=T, B=T) |
|---|---|---|
| T | 0.4 × 0.3 × 0.9 | 0.108 |
| F | 0.4 × 0.3 × 0.1 | 0.012 |

3. You are modeling the relationship between a set of N input vectors $X_1, \ldots, X_N$ and a set of N binary outcomes $Y_1, \ldots, Y_N$. We assume there is a single vector of parameters $\beta$ which dictates the relationship between each input vector and its associated output variable. In this model, each output is drawn with $Y_n \sim Bernoulli(invLogit(X_n\beta))$. Additionally, the vector $\beta$ has a prior, given by $\beta \sim Normal(\mu, \Sigma)$. This model is called Bayesian Logistic Regression. Draw its corresponding plate notation.




4. For the following Bayesian network, perform variable elimination to compute P(E). Fill in the table.




| Variable | All Factors | Participates | New Factor After * | New Factor After + |
|---|---|---|---|---|
| **A** | $P(A), P(B\|A), P(D)$<br>$P(C\|B,D), P(E\|C)$ | $P(A), P(B\|A)$ | $\varphi_1(A,B)$ | $T_1(B)$ |
| **B** | $P(D), P(E\|C)$<br>$T_I(B), P(C\|B,D)$ | $P(C\|B,D)$<br>$T_1(B)$ | $\varphi_2(B,C,D)$ | $T_2(C,D)$ |
| **C** | $P(D), P(E\|C)$<br>$T_1(C\|D)$ | $P(D)$<br>$T_2(C,B)$ | $\varphi_3(C,D)$ | $T_3(C)$ |
| **D** | $P(E\|C)$<br>$T_3(C)$ | $P(E\|C)$<br>$T_3(C)$ | $\varphi_4(C,E)$ | $T_4(E)$ |