Tania Soutonglang

CS 583-01

Feb. 5, 2024

Assignment 1

Theoretical Assignments

1. We are given the random variables $X_2, X_3, \dots, X_n$, and $Y_2, Y_3, \dots, Y_m$. Answer the following questions.

    a. Assuming every variable is binary, how many independent parameters are needed to represent $P(X_2, X_3, \dots, X_n, Y_2, Y_3, \dots, Y_m)$?

    $(2^{n-1} * 2^{m-1}) - 1$

    b. Assuming every variable has three possible values, how many independent parameters are needed to represent $P(X_2, X_3, \dots, X_n, Y_2, Y_3, \dots, Y_m)$?

    $(3^{n+1} * 3^{m-1}) - 1$

    c. Assuming each X_i has *i* possible values and similarly, every Y_i has *i* possible values, how many independent parameters are needed to represent $P(X_2, X_3, \dots, X_n, Y_2, Y_3, \dots, Y_m)$?

    $(n!\, m!) - 1$

    d. Assuming every variable is binary, how many independent parameters are needed to represent $P(Y_2, Y_3, \dots, Y_m | X_2, X_3, \dots, X_n)$?

    $(2^{m-1} - 1)(2^{n-1})$

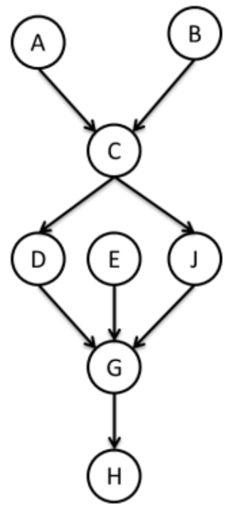
    e. Assuming every variable has three possible values, how many independent parameters are needed to represent $P(Y_2, Y_3, \dots, Y_m | X_2, X_3, \dots, X_n)$?

    $(3^{m-1} - 1)(3^{n-1})$

    f. Assuming each X_i has *i* possible values and similarly every Y_i has *i* possible values, how many independent parameters are needed to represent $P(Y_2, Y_3, \dots, Y_m | X_2, X_3, \dots, X_n)$?

    $(m! - 1)(n!)$

2. We are given the following Bayesian network. Please answer the following questions.



a. Write down the join distribution as a factorization over this Bayesian network.

$$P(A,B,C,D,E,G,H,J) = P(A) * P(B) * P(C|A,B) * P(D|C) * P(E) * P(G|D,E,J) * P(H|G) * P(J|C)$$

b. Assuming each variable is discrete and can take *n* possible values, how many independent parameters are needed for this Bayesian network?

$n^4 + 2n^2 - 3$

c. Are the following independence statements true or false?

i. $A \perp B$ TRUE

ii. $A \perp B \mid C$ FALSE

iii. $A \perp B \mid J$ FALSE

iv. $A \perp B \mid G$ FALSE

v. $A \perp B \mid E$ TRUE

vi. $A \perp B \mid H$ FALSE

vii. $A \perp H$ FALSE

viii. $A \perp H \mid J$ FALSE

ix. $A \perp H \mid D, J$ TRUE

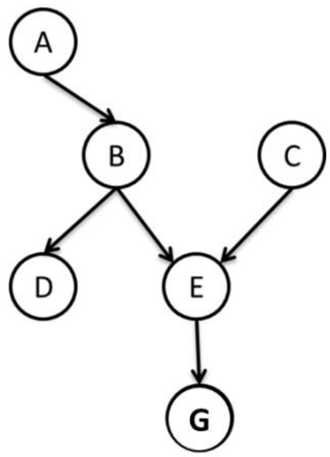
x. $D \perp J$ FALSE

xi. $B \perp E$ TRUE

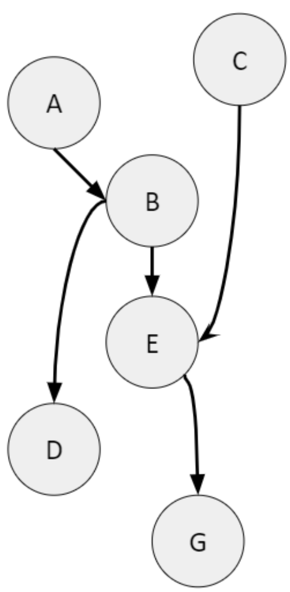
xii. $B \perp E \mid J$ TRUE

xiii. $B \perp E \mid J, H$ FALSE

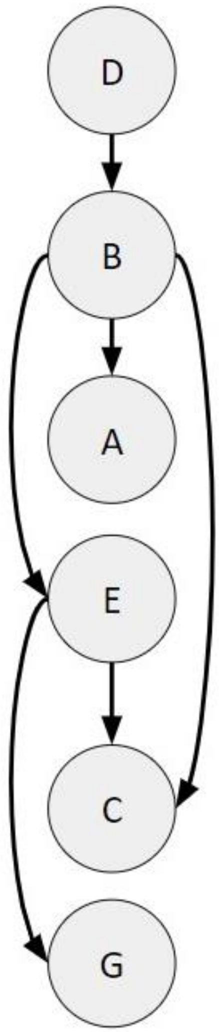
3. We have a distribution P over the variables A, B, C, D, E, and G. We would like to build a Bayesian network that is a minimal I-Map for P. In reality, you have access to P, which you can query for independencies, but for the purposes of this problem, we will assume the following structure is a P-Map for P. Create minimal I-Maps for P, using the following variable orders.



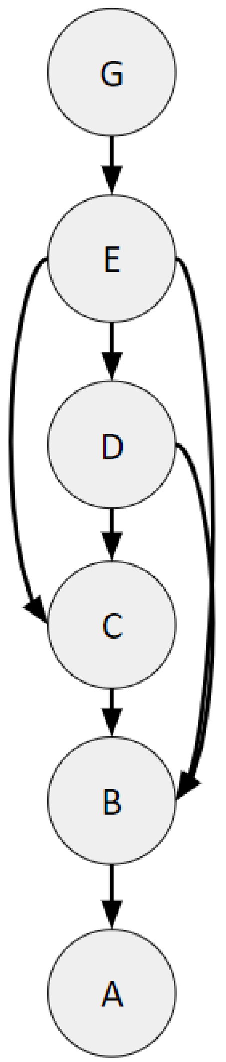
   a. C, A, B, E, D, G



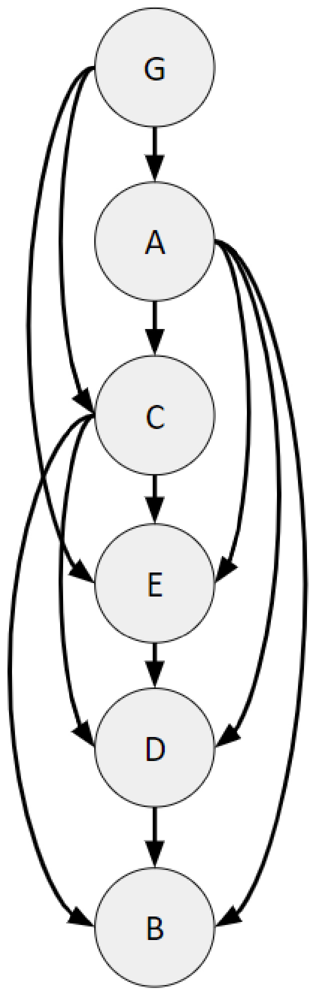
   b. D, B, A, E, C, G



c. G, E, D, C, B, A



d. G, A, C, E, D, B


G
A
C
E
D
B