Tania Soutonglang

CS 583

February 27, 2024

Assignment 2

1. We have three random variables, A, B, C, each of which is binary. We have the following factors over these variables:
   1. Draw a Markov network graph over these variables.

A diagram of a triangle with circles and letters

Description automatically generated

* 1. Draw a factor graph over these variables.

A diagram of a triangle with circles and letters

Description automatically generated

* 1. Here are the values of the factors. Compute P(A, B, C).

A black and white lines with letters

Description automatically generated with medium confidence

A close-up of a table

Description automatically generatedA white background with black text

Description automatically generated

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A | B | C | f(A) | f(B) | f(C) | f(A, B) | f(A, C) | f(B, C) | f(D) = f(A) f(B) f(C) f(A,B) f(A,C) f(B,C) | P(A, B, C) = 1/3045 |
| T | T | T | 2 | 1 | 1 | 5 | 6 | 1 | 60 | 0.01970 |
| T | T | F | 2 | 1 | 8 | 5 | 1 | 10 | 800 | 0.26273 |
| T | F | T | 2 | 4 | 1 | 1 | 6 | 10 | 480 | 0.15764 |
| T | F | F | 2 | 4 | 8 | 1 | 1 | 1 | 64 | 0.02102 |
| F | T | T | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0.00033 |
| F | T | F | 1 | 1 | 8 | 1 | 6 | 10 | 480 | 0.15764 |
| F | F | T | 1 | 4 | 1 | 5 | 1 | 10 | 200 | 0.06568 |
| F | F | F | 1 | 4 | 8 | 5 | 6 | 1 | 960 | 0.31527 |
|  |  |  |  |  |  |  |  |  | Z = 3045 |  |

1. For the following Markov network graph, indicate whether the following independence statements are True or False.

A diagram of a network

Description automatically generated

* 1. A Ʇ B FALSE
  2. A Ʇ B | C TRUE
  3. A Ʇ G | D FALSE
  4. A Ʇ G | D, F TRUE
  5. A Ʇ H | G TRUE

1. We have the Markov network over 3 binary variables: . We define a pairwise Markov random field (MRF) over this network. We define the following features.

, 0 otherwise. .

, 0 otherwise. .

, 0 otherwise. .

, 0 otherwise. .

, 0 otherwise. .

Note that to simplify the notation, we simply wrote , though features are defined only over nodes and edges. Assume ignores the variables that it is not defined over. For example, . Populate the following table.

Hints: and .

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A | B | C | f(A) | f(B) | f(C) | f(A,B) | f(B,C) |  | P(A, B, C) |
| T | T | T | 0.69315 | 0.00000 | 1.38629 | -1.60944 | 0 | 0.62500 | 0.03352 |
| T | T | F | 0.69315 | 0.00000 | 0 | -1.60944 | 1.79176 | 0.41667 | 0.02235 |
| T | F | T | 0.69315 | -1.09861 | 1.38629 | 0 | 1.79176 | 0.06250 | 0.00335 |
| T | F | F | 0.69315 | -1.09861 | 0 | 0 | 0 | 1.50000 | 0.08045 |
| F | T | T | 0.00000 | 0.00000 | 1.38629 | 0 | 0 | 0.25000 | 0.01341 |
| F | T | F | 0.00000 | 0.00000 | 0 | 0 | 1.79176 | 0.16667 | 0.00894 |
| F | F | T | 0.00000 | -1.09861 | 1.38629 | -1.60944 | 1.79176 | 0.62500 | 0.03352 |
| F | F | F | 0.00000 | -1.09861 | 0 | -1.60944 | 0 | 15.00000 | 0.80447 |
|  |  |  |  |  |  |  |  | 18.64583 |  |

1. We have a document classification task. We have four documents, D1 and D2. We are interested in classifying these documents into one of two topics: Artificial Intelligence (AI) or Databases (DB). Each document has one or both words: Agent and SQL. We will represent this data as follows: the labels of and are: and . Each can take one of two values: AI or DB. The presence of word Agent in document represented as , and the presence of the word SQL in document represented as . The documents and their contents are as follows:

(i.e, contains only the word Agent)

(i.e, contains both words)

We also have the additional knowledge that cites . We construct the following CRF, with the following feature functions.

Features:

A white sheet with black text and numbers

Description automatically generated

A diagram of a network

Description automatically generated

Calculate , that is . What is the MAP assignment to Y given X? Show your work.

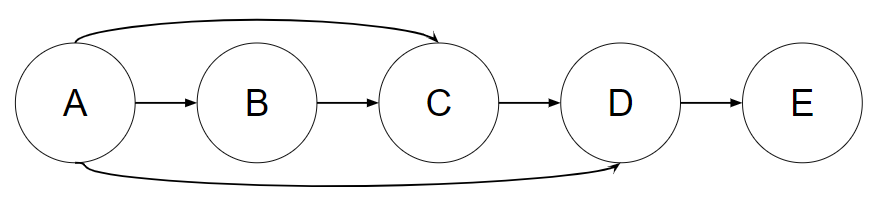
|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Y1 | Y2 | f(Y1, Y2) | f(Y1, X11=T) | f(Y1, X21=F) | f(Y2, X12=T) | f(Y2, X22=T) | f | P(Y|X) |
| AI | AI | 2.71828 | 2.71828 | 1.00000 | 2.71828 | 0.36788 | 7.38906 | 0.77580 |
| AI | DB | 0.36788 | 2.71828 | 1.00000 | 0.36788 | 2.71828 | 1.00000 | 0.10499 |
| DB | AI | 0.36788 | 0.36788 | 1.00000 | 2.71828 | 0.36788 | 0.13534 | 0.01421 |
| DB | DB | 2.71828 | 0.36788 | 1.00000 | 0.36788 | 2.71828 | 1.00000 | 0.10499 |
|  |  |  |  |  |  |  | 9.52439 |  |

1. We are given the following Markov network structure, H.

A diagram of a network

Description automatically generated

* 1. Find a minimal I-Map Bayesian network structure for H. Use the variable order of A, B, C, D, E.



* 1. Find a minimal I-Map Bayesian network structure for H. Use the variable order of E, D, C, B, A.

A diagram of a circle with a letter and arrow

Description automatically generated

* 1. Is a P-Map for H? If not, which independencies are missing?

No it’s not a P-Map for H

A diagram of a network

Description automatically generated

* 1. Is a P-Map for H? If not, which independencies are missing?

No it’s not a P-Map for H because is not true.

A diagram of a network

Description automatically generated

1. We are given the following Bayesian network structure G.

A diagram of a diagram

Description automatically generated

* 1. Find a minimal I-Map Markov network structure H for G. Use any method/variable order you like; make sure H is a *minimal* I-Map.

A diagram of a network

Description automatically generated

* 1. Is H a P-Map for G? If not, which independencies are missing?

No, is not true.