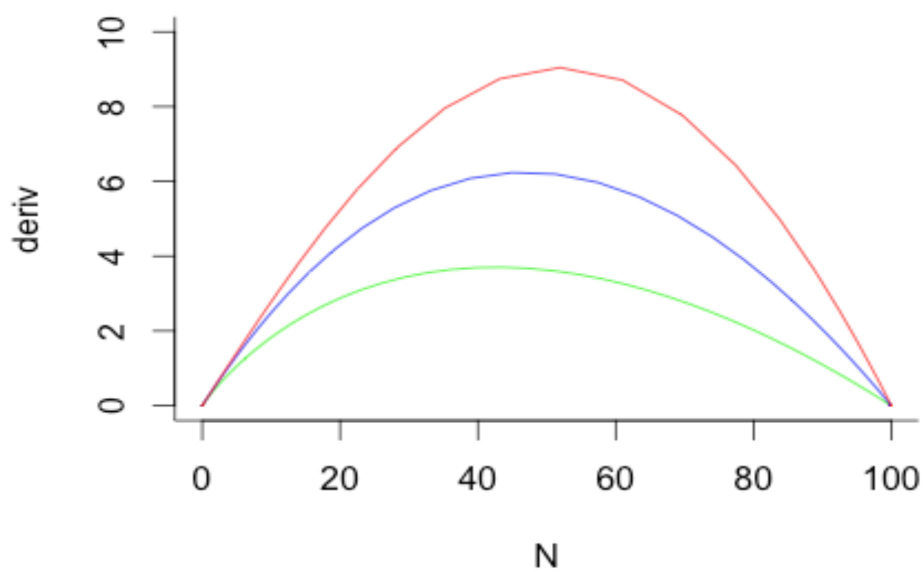
Ty Spangler
4/14/16

4.1 A)

$$rN[1-(N/K)^{\theta}]=0 \qquad N=0$$
$$1-((N/K)^{\theta}=0$$
$$(N/K)^{\theta}=1$$
$$N^{\theta}=K^{\theta}$$
$$N=K$$

The equilibrium point N=0 must be unstable because at a low N value, population would increase, moving away from that equilibrium point. N=K must be stable because as N approaches K, growth will slow down. Also once N is larger than K, population will decline and go towards K.

B)



Values of theta below one will reduce the growth rate, compared to the general logistic growth model. A value of 1 will not change the growth rate. A value greater than 1 would increase the growth rate. Theta has a similar effect on population size at maximum growth rate, where a value less than one would decrease it, a value of one would not change it, and a value greater than one would increase it.

C)

This model may be superior to the logistic growth model because it can be fitted to the dynamics of individual species through the use of theta. If a certain species does not reproduce as often, or with not very much success, their population would grow slower. There could be many factors that affect this such as seasonality, fecundity, resource availability, competition, environmental conditions, etc. So for a slower growing population you can choose a theta value less than one to account for their difference in population growth. Adversely, if a population grows faster, you can adjust theta to account for it. Fast growing and reproducing taxa such as bacteria would likely require a higher theta value. Slower growing and reproducing taxa such as agave plants would likely require a lower theta value.

4.3 A)

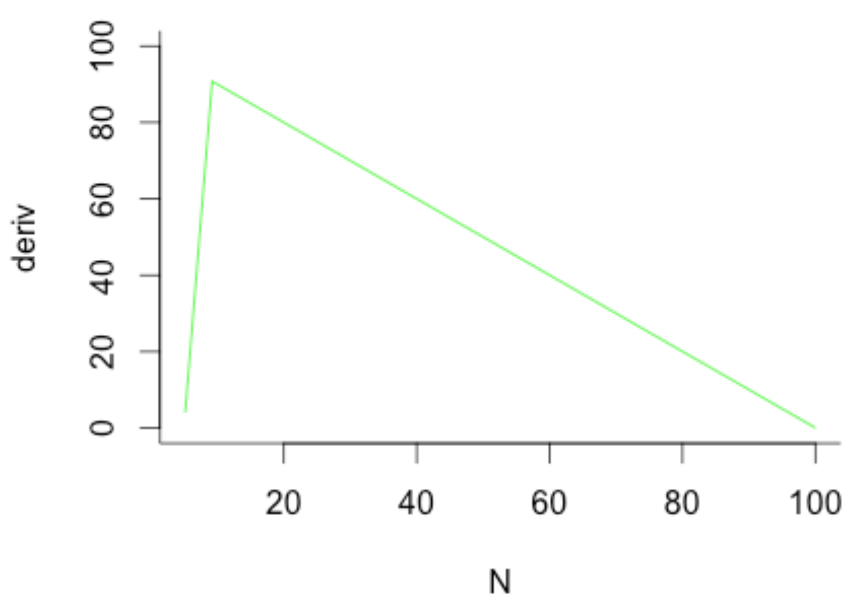
$$\boldsymbol{rN(N-a)\left[1-\left({}^{N}/_{K}\right)\right]}$$

$$\boldsymbol{N=0, \qquad N=a, \qquad N=K}$$

B)

N=0 would be unstable for the same reasons as in 4.1. N=K would be stable for the same reasons as well. N=a would unstable because a value greater than a would cause N to move away from a. A value of N less than a would also cause N to move away because the population would decline.

C) Using K=100, a=4, N_0= 5.14

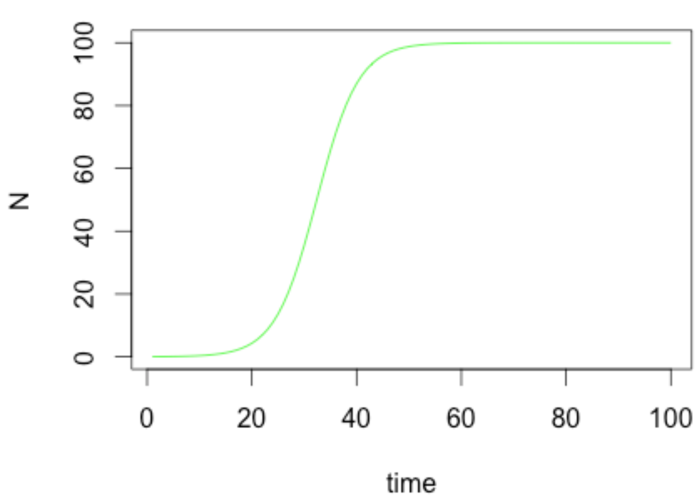


D)

Based on this graph, the model given to incorporate the allee effect causes the population growth rate to sharply rise and then steadily decline as it approaches the carrying capacity. This is very different from the logistic growth model where there is a gradual increase in population growth rate with an almost equally gradual decline in growth rate as the carrying capacity is approached.

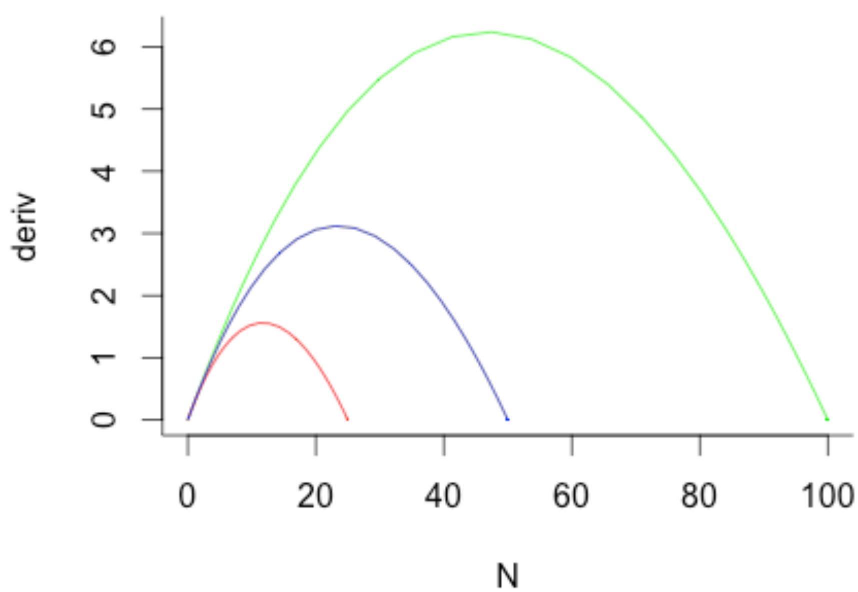
2. A)

```
log.growth <- function(t,y,p){
  N<-y[1]
  with(as.list(p), {
    dN.dt<-r*N*(1-(N/K)^theta)
    return(list(dN.dt))
  })
}


library(deSolve)
p <- c('r' = 0.25, 'K' = 100, 'theta' = 1)
y0 <- c('N' = runif(1, min=0.01, max=0.1))
t <- 1:100

sim <- ode(y = y0, times = t, func = log.growth, parms = p, method = 'lsoda')

sim <- as.data.frame(sim)

plot(N ~ time, data=sim, type='l', col='green')
```



B) ## Redefine parameters with K=50 and K=25

```
p.2 <- c('r' = 0.25, 'K' = 50, 'theta' = 1)

p.3 <- c('r' = 0.25, 'K' = 25, 'theta' = 1)

sim.2 <- ode(y = y0, times = t, func = log.growth, parms = p.2, method = 'lsoda')

sim.2 <- as.data.frame(sim.2)

sim.3 <- ode(y = y0, times = t, func = log.growth, parms = p.3, method = 'lsoda')

sim.3 <- as.data.frame(sim.3)

## Plot the population level growth rate

sim$deriv <- c(diff(sim$N), NA)
plot(deriv ~ N, data = sim, type = 'l', col = 'green', bty = 'l', ylim=c(0,10))

sim.2$deriv <- c(diff(sim.2$N), NA)
points(deriv ~ N, data = sim.2, type = 'l', col = 'blue', bty = 'l')

sim.3$deriv <- c(diff(sim.3$N), NA)
points(deriv ~ N, data = sim.3, type = 'l', col = 'red', bty = 'l')
```



Find the max population growth rates

max(sim$deriv, na.rm = TRUE)
#6.24
which(sim$deriv == max(sim$deriv, na.rm = TRUE))
34
sim$N[which(sim$deriv == max(sim$deriv, na.rm = TRUE))]
47.5 individuals

max(sim.2$deriv, na.rm = TRUE)
#3.12
which(sim.2$deriv == max(sim.2$deriv, na.rm = TRUE))
31
sim.2$N[which(sim.2$deriv == max(sim.2$deriv, na.rm = TRUE))]
23.0 individuals

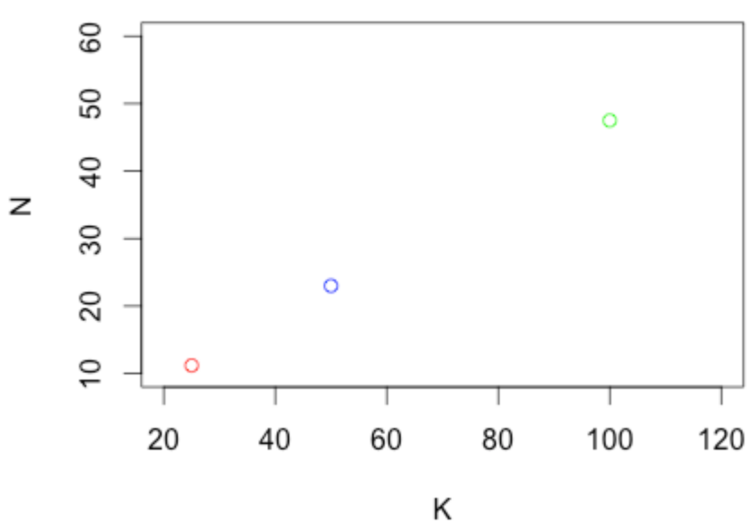
max(sim.3$deriv, na.rm = TRUE)
#1.56
which(sim.3$deriv == max(sim.3$deriv, na.rm = TRUE))
# 28
sim.3$N[which(sim.3$deriv == max(sim.3$deriv, na.rm = TRUE))]
11.2 individuals

Plot the respective carrying capacities against population size at maximum growth rate

plot(100,47.5, col='green', xlab='K', ylab='N', ylim= c(10,60), xlim=c(20,120))

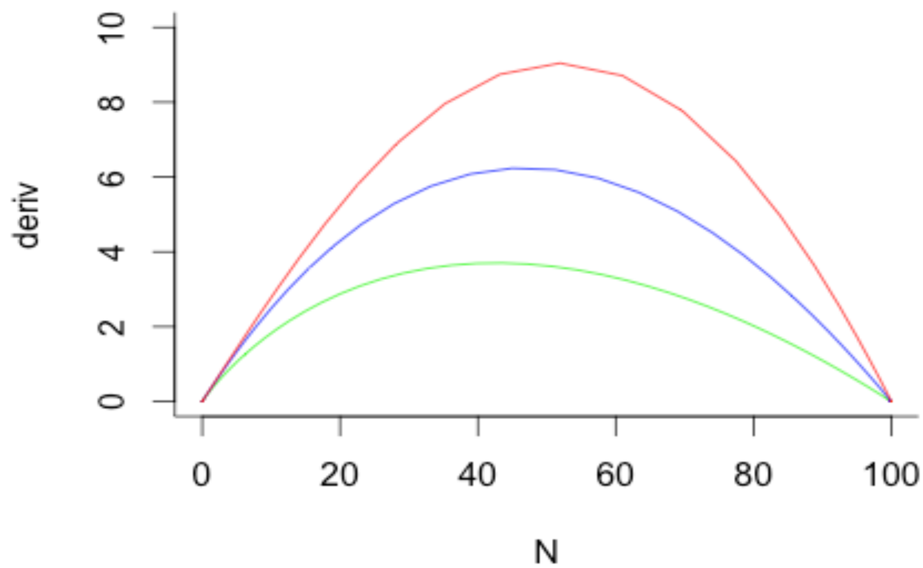
points(50, 23, col='blue')

points(25,11.2, col='red')



C)

```
p <- c('r' = 0.25, 'K' = 100, 'theta' = 0.5)
p.2 <- c('r' = 0.25, 'K' = 100, 'theta' = 1)
p.3 <- c('r' = 0.25, 'K' = 100, 'theta' = 1.8)

y0 <- c('N' = runif(1, min=0.01, max=0.1))

t <- 1:100

log.growth <- function(t,y,p){
  N<-y[1]
  with(as.list(p), {
    dN.dt<-r*N*(1-(N/K)^theta)
    return(list(dN.dt))
  })
}

sim <- ode(y = y0, times = t, func = log.growth, parms = p, method = 'lsoda')

sim <- as.data.frame(sim)

sim.2 <- ode(y = y0, times = t, func = log.growth, parms = p.2, method = 'lsoda')

sim.2 <- as.data.frame(sim.2)

sim.3 <- ode(y = y0, times = t, func = log.growth, parms = p.3, method = 'lsoda')

sim.3 <- as.data.frame(sim.3)

sim$deriv <- c(diff(sim$N), NA)
plot(deriv ~ N, data = sim, type = 'l', col = 'green', bty = 'l', ylim = c(0, 10))

sim.2$deriv <- c(diff(sim.2$N), NA)
points(deriv ~ N, data = sim.2, type = 'l', col = 'blue', bty = 'l')

sim.3$deriv <- c(diff(sim.3$N), NA)
points(deriv ~ N, data = sim.3, type = 'l', col = 'red', bty = 'l')
```




```
## Species A
max(sim$deriv, na.rm = TRUE)
# 3.70
which(sim$deriv == max(sim$deriv, na.rm = TRUE))
# 41
sim$N[which(sim$deriv == max(sim$deriv, na.rm = TRUE))]
#43.53 Individuals

## Species B
max(sim.2$deriv, na.rm = TRUE)
#6.23
which(sim.2$deriv == max(sim.2$deriv, na.rm = TRUE))
#35
sim.2$N[which(sim.2$deriv == max(sim.2$deriv, na.rm = TRUE))]
#44.99 Individuals

## Species 3
max(sim.3$deriv, na.rm = TRUE)
#9.04
which(sim.3$deriv == max(sim.3$deriv, na.rm = TRUE))
#34
sim.3$N[which(sim.3$deriv == max(sim.3$deriv, na.rm = TRUE))]
#51.93

#Species C will be maintained at the highest population abundance
```

4.

$$\frac{dN}{dt} = rN[1-(N/K)]$$

$$\int \frac{dN}{N(1-N/K)} = \int k\,dt$$

$$\frac{1}{N(1-N/K)} = \frac{K}{N(K-N)} = \frac{1}{N} + \frac{1}{K-N}$$

$$\int \frac{dN}{N} + \int \frac{dN}{K-N} = \int k\,dt$$

$$\ln|N| - \ln|\frac{K-N}{N} = kt + C$$

$$\ln\left|\frac{K-N}{N}\right| = -kt - C$$

$$\left|\frac{K-N}{N}\right| = e^{-kt-C}$$

$$\frac{K-N}{N} = Ae^{-kt} \longrightarrow N = \frac{K}{1+Ae^{-kt}}, \quad A = \frac{K-N_0}{N_0}$$