Creditor-Sovereign Relationship under Asymmetric Information in a Dynamic Model of Sovereign Debt

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Chapter 1

Introduction

Sovereign debt models traditionally aim to capture how countries borrow in international markets and how creditors respond to the risk of default. In many classic frameworks, researchers consider a single type of bond (often short-term) and focus on understanding the interplay between the sovereign's incentives, income shocks, and enforcement frictions. However, real-world observations suggest that countries often issue multiple classes of debt instruments—short-term and long-term bonds—and may face significant complexities when renegotiating or restructuring defaulted debt. Initially, this study sought to examine a richer environment with long- and short-term bonds, where debt renegotiations would influence default risk. Yet, the complexity of that setup proved challenging: after extensive attempts, a practical solution algorithm for the renegotiation environment remained elusive.

Instead, this paper focuses on two benchmark models of sovereign borrowing and default under a simpler structure of one-period debt: a *full-information* model and an *asymmetric-information* model. The full-information setting corresponds to the canonical view in which international creditors directly observe the sovereign's economic fundamentals, building on the foundational work of Eaton and Gersovitz (1981) and extended by Aguiar and Gopinath (2006). The asymmetric-information variant introduces incomplete observability of the sovereign's income endowment, thereby creating a channel for costly signaling or policy actions to convey partial information. This extension draws on insights from models of incomplete information such as Atkeson (1991) and Sandleris (2008), which argue that limited transparency can amplify borrowing distortions and default risk.

Both models are solved using modern neural-network algorithms that approximate the sovereign's value function, policy decisions, and creditors' default probability in equilibrium. In numerical experiments, each model offers distinct outcomes. Under full information, borrowing decisions respond more flexibly to the debt level, and the sovereign adapts its new bond issuance depending on the gap between current endowment and existing debt. By contrast, under asymmetric information, we find that the sovereign often issues the maximum allowable debt, revealing or signaling its endowment through debt issuance, but at the cost of encountering more frequent defaults.

1.0.1 Future Directions and Extensions

This work has to be continued to the following extensions to provide more valuable results:

- **Alternative Parameter Values:** Our calibration focuses on an "impatient" sovereign with certain benchmark parameters. A broader exploration of discount factors and higher risk-free rates would clarify how patience or global interest conditions affect borrowing decisions and default.
- **Signaling Costs:** Varying the signaling cost could reveal threshold points where the sovereign's signaling strategy changes drastically, potentially leading to multiple equilibria or novel patterns of debt issuance.
- **Debt Renegotiation and Multi-Maturity Bonds:** The original motivation was to consider different maturities (long- and short-term bonds) and a richer renegotiation protocol in the event of default. Although computational challenges arose initially, integrating these features remains a crucial step toward capturing the complexity of real-world debt crises.

Chapter 2

Literature Review

The interaction between sovereign debt, default risk, and information asymmetry has been extensively studied in economic theory. While the full-information model of sovereign borrowing provides a benchmark for understanding debt dynamics, the addition of incomplete information introduces complexities that more closely align with real-world observations. This chapter reviews the foundational works in sovereign debt modeling, emphasizing recent contributions that address issues of asymmetric information, signaling, and the dynamics of contract design.

2.1 Foundational Sovereign Debt Models

The seminal work of Eaton and Gersovitz (1981) introduced the canonical framework for analyzing sovereign borrowing and default. Their model formalized the notion of strategic default in the absence of direct legal enforcement, highlighting the role of reputational penalties in sustaining sovereign credit markets. Extensions of this framework, including those by Aguiar and Gopinath (2006) and Arellano (2008), incorporated income volatility and endogenous default decisions, aligning the theoretical predictions with observed procyclical borrowing patterns in emerging markets.

Aguiar and Gopinath (2006) highlighted the importance of income shocks in driving sovereign defaults, introducing stochastic endowments as a key determinant of debt sustainability. Arellano (2008) extended this framework by modeling long-term debt and introducing heterogeneity in income shocks. These models laid the groundwork for understanding the role of debt maturity, output volatility, and sovereign impatience in default decisions.

2.2 Asymmetric Information in Sovereign Debt

The full-information models of Eaton and Gersovitz (1981) and Aguiar and Gopinath (2006) assume that creditors have complete knowledge of the sovereign's economic fundamentals. However, this assumption often fails in practice, as governments may obscure unfavorable economic indicators. Atkeson (1991) introduced incomplete information into sovereign debt models, demonstrating how private information could lead to adverse selection and distortions in debt pricing. Sandleris (2008) further explored how private information and costly signaling affect borrowing outcomes, illustrating that incomplete transparency raises borrowing costs and amplifies default risk.

Gelos and Wei (2005) provided empirical evidence that investors demand higher risk premia from governments with opaque economic data, validating the theoretical predictions of adverse selection in sovereign debt markets. Similarly, Tomz (2007) emphasized the role of historical reputation in mitigating the impact of incomplete information on debt pricing, highlighting the importance of observable policy signals in reducing creditor uncertainty.

2.3 Dynamic Contracts and Enforcement

The application of contract theory to sovereign debt has advanced our understanding of how creditors and sovereigns design incentive-compatible agreements under uncertainty. The work of Dovis (2019) introduced dynamic incentive-compatible contracts into sovereign debt models, showing how contractual enforcement mechanisms can sustain borrowing in environments with limited commitment. Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012) explored the role of long-term debt in mitigating the time inconsistency problem, emphasizing the trade-offs between debt maturity and default risk.

Recent studies, such as Bianchi et al. (2018), incorporated state-contingent debt instruments into sovereign borrowing models, demonstrating how such contracts can stabilize debt dynamics and improve welfare. Their findings underscore the potential of incorporating state-contingent terms, such as GDP-linked bonds, to address issues of income volatility and asymmetric information.

2.4 Endogenous Information Revelation and Signaling

Endogenous information revelation, whereby the sovereign actively influences creditors' beliefs through observable actions, represents a key mechanism in models of incomplete information. Acharya et al. (2011) examined how signaling and costly disclosure shape borrowing outcomes in environments with private information. Their results suggest that credible signals, such as fiscal reforms, can reduce borrowing costs but may also lead to underinvestment if signaling costs are excessive.

Goldstein and Guembel (2008) explored signaling in financial markets, showing how strategic transparency can mitigate adverse selection and improve market outcomes. This insight extends to sovereign debt, where signaling policies can enhance credibility and reduce default risk. Biais et al. (2010) further emphasized the role of transparency in mitigating moral hazard, arguing that observable policy actions can improve the alignment of incentives between sovereigns and creditors.

2.5 Empirical Evidence and Open Questions

Empirical studies consistently validate the importance of information asymmetry in shaping sovereign borrowing outcomes. Reinhart and Rogoff (2009) documented the historical prevalence of debt crises in low-transparency environments, linking opaque fiscal practices to higher default frequencies. Neumeyer and Perri (2005) provided evidence that sovereign risk premia are sensitive to income volatility, reinforcing the theoretical link between asymmetric information and borrowing costs.

Recent works, such as Collard et al. (2024), have highlighted the limitations of existing models in capturing the dynamics of sovereign default and renegotiation. These studies suggest that integrating dynamic signaling mechanisms into sovereign debt models can better align theoretical predictions with observed outcomes, providing a promising avenue for future research.

2.6 Contribution of This Study

This study builds on the existing literature by incorporating dynamic signaling into a sovereign debt framework. Unlike previous models, which primarily focus on static signaling or exogenous information structures, our model allows the sovereign to actively influence creditor beliefs through costly policy actions. By extending the framework of Atkeson (1991) and Sandleris (2008) with dynamic contracts and endogenous information revelation, this study seeks to provide new insights into the role of transparency and signaling in sovereign debt markets. Furthermore, the integration of long-term and short-term debt instruments adds realism to the model, capturing the trade-offs faced by sovereigns in managing debt maturity and default risk.

2.7 Use of Deep Learning for Dynamic Economic Models

At last we address literature on the chosen model solution method. Recent work has explored machine learning techniques for solving high-dimensional dynamic models and capturing complex agent interactions without relying on discretized grids. Among these, the *Deep Equilibrium Nets* approach proposed by Azinovic et al. (2020) uses neural networks to approximate equilibrium mappings in a style reminiscent of classical fixed-point iterations. Meanwhile, Maliar and Maliar (2021) show that deep learning can handle large state spaces more efficiently than traditional dynamic programming methods. Duarte et al. (2018) and Fernández-Villaverde et al. (2021) argue that separate neural networks for distinct agent objectives—such as sovereign borrowers versus risk-neutral creditors—improves interpretability and flexibility.

Our own solution algorithm in Chapter 3.4 adopts this perspective by pairing one network for the sovereign's Bellman equation residual with another for creditors' default classification under risk neutrality. This method is especially valuable in the asymmetric-information environment, where lenders and borrowers have separate data inputs and objectives, thus enabling a realistic portrayal of how partial transparency or signaling shapes borrowing and default outcomes.

Chapter 3

Model

3.1 Full-Information Model

This section presents the full-information sovereign debt model, which serves as a benchmark for analyzing sovereign borrowing and default decisions when both the sovereign borrower and international creditors have complete information about the economic environment. The model builds on the framework established by Eaton and Gersovitz (1981) and is further developed by Aguiar and Gopinath (2006).

We consider a small open economy that operates in discrete time, indexed by $t = 0, 1, 2, \ldots$. The economy is inhabited by a sovereign government acting as a borrower and a continuum of risk-neutral international creditors. The sovereign government receives a stochastic stream of endowment income y_t, which follows a log-normal (Markov) process with independent increments:

$$\log y_t = \rho \log y_{t-1} + \sigma \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, 1).$$

This endowment represents the total real output of the economy in each period.

3.1.1 Sovereign Problem

A benevolent sovereign seeks to maximize the expected utility of consumption over an infinite horizon, defined as

$$U = \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\},$$

where $\beta \in (0,1)$ is the discount factor, c_t is consumption at time t, and $u(\cdot)$ is a strictly increasing and concave utility function satisfying Inada conditions. The sovereign can borrow from risk-neutral international creditors by issuing one-period, non-contingent bonds with face value b_t at price q_t determined by the risk-neutral creditors. The initial debt level b_{t-1} is given and must be repaid at t. The budget constraint implies that consumption and previous debt servicing are financed by newly-issued debt and the realized stochastic endowment:

$$c_t + b_{t-1} = y_t + q_t b_t,$$

where $c_t \geq 0$ ensures non-negative consumption. Borrowing can be negative, implying that the sovereign can lend at the competitive risk-free rate r. In each period, borrowing (or lending, in absolute value) cannot exceed endowment, $|b_t| \leq y_t$. This constraint is mainly imposed for solution stability and captures lenders' reluctance to issue debt beyond the sovereign's capacity to repay.

Default in this model is involuntary and occurs if the sovereign cannot meet its debt obligations while maintaining non-negative consumption. This design follows Collard et al. (2024), who argue that involuntary defaults are more plausible compared to strategic defaults, as governments often delay default until it becomes unavoidable (Levy Yeyati & Panizza, 2011). Upon default, the sovereign is excluded from international capital markets indefinitely and experiences a proportional reduction $\tau \in (0,1)$ in income. In autarky, consumption is

$$c_t^{\text{default}} = (1-\tau) y_t.$$

The sovereign's optimization problem can be written recursively as

$$V(y_t, b_{t-1}) = \begin{cases} V^{\text{continue}}(y_t, b_{t-1}), & c_{t'} \geq 0 \; \forall t' \leq t, \\ V^{\text{autarky}}(y_t), & \text{otherwise}, \end{cases}$$

with the following values for the continuation and autarky value functions:

$$V^{\text{continue}}(y_t, b_{t-1}) = \max_{\{c_t, b_t\}} \left\{ u(c_t) + \beta \mathbb{E} \left\{ V(y_{t+1}, b_t) \mid y_t, b_t \right\} \right\},$$

$$V^{\text{autarky}}(y_t) = u\big((1-\tau)y_t\big) + \beta \mathbb{E}\big[\{V^{\text{autarky}}(y_{t+1}) \mid y_t\big\}.$$

3.1.2 Creditors' Problem

International creditors are risk-neutral, short-sighted (*e.g.*, each generation of investors cares only about their return and does not remember the history of the sovereign's borrowing), and competitive, requiring an expected return equal to the risk-free rate r. They price sovereign bonds to break even, taking into account the probability of default, but do not account for the previous period payments. The bond price q_t is thus determined by

$$q_t(y_t, b_t) = \frac{1}{1+r} \left(1 - \int_0^{+\infty} \mathbb{P}\{\text{default at } t+1 \mid y_t, b_t\} dP(y_{t+1} \mid y_t) \right) := \frac{1 - \theta_t(y_t, b_t)}{1+r},$$

where $P(\cdot)$ is the conditional law of the exogenous endowment process at $t+1$ given y_t and $\theta_t(y_t)$, which we will use further as θ_t to simplify the notation, is the probability of default of the sovereign given the current state (y_t, b_t) as perceived by the creditors. In the full-information setting, creditors perfectly observe y_t and b_t, enabling them to price each new bond issuance b_t according to the sovereign's repayment capacity.

3.1.3 Timing

If sovereign is not in the state of autarky at the beginning of a period t, the model timing is as follows:

1. The sovereign receives an endowment y_t and chooses the amount b_t of the new debt that must be issued;

2. Observing realized y_t and the newly issued debt b_t, creditors set a risk-neutral price q_t for this new debt;

3. The sovereign observes their income from debt issuance and endowment and decides on consumption:

   (a) If the income from the endowment and the new debt is not sufficient to pay off the old debt b_{t-1} and ensure positive consumption $c_t > 0$, sovereign defaults and stays in autarky for all subsequent periods, consuming $(1-\tau)y_{t'}\ \forall t' \geq t$;

   (b) If the income is sufficient to payoff the old debt with positive consumption, sovereign consumes all that is left after all due payments and the process reinitializes with the start of new period.

3.1.4 Equilibrium

An equilibrium consists of sovereign policy functions $c_t(y_t, b_{t-1})$ and $b_t(y_t, b_{t-1})$, as well as a bond pricing function $q_t(y_t, b_t)$. The sovereign maximizes its value function subject to its budget constraint and default conditions, while creditors break even given their knowledge of the sovereign's income and debt levels. Existence of the equilibrium will be discussed in the appendix.

3.2 Asymmetric-Information Model

In this section, we extend the baseline model from the full-information environment to one characterized by asymmetric information, where international creditors cannot directly observe the sovereign's current endowment or economic fundamentals. Instead, creditors rely on policy actions or signals chosen by the sovereign to update their beliefs about the sovereign's true economic state. This framework captures realistic frictions that arise when official economic data are imperfect or when governments may obscure unfavorable indicators. Models of this nature build on insights from incomplete-information debt theories, such as those proposed by Atkeson (1991) and Sandleris (2008), showing that private information can lead to adverse selection, signaling, and distortions in sovereign borrowing.

3.2.1 Sovereign Problem with Private Information

As in the full-information case, the sovereign government operates in discrete time $t = 0, 1, 2, \ldots$ and receives a random stream of income y_t. However, unlike the previous model, creditors do not directly observe y_t. The sovereign can send a costly signal $a_t \in \{0, 1\}$, intended to convey information about its endowment to lenders. Implementing the signal $a_t = 1$ (such as an expensive reform or transparency initiative) involves a utility cost $\phi(a_t) = \lambda\, a_t$, with $\lambda \geq 0$. The sovereign's goal is still to maximize its expected discounted utility:

$$U = \mathbb{E}_0 \Big\{ \sum_{t=0}^{\infty} \beta^t \big(u(c_t) - \phi(a_t) \big) \Big\},$$

subject to the usual budget constraint and debt dynamics. However, the sovereign now recognizes that its signal a_t affects creditors' beliefs and thereby the price q_t at which it can issue debt b_t.

3.2.2 Creditors and Belief Formation

Creditors remain risk-neutral and seek an expected return equal to the risk-free rate r. Because they cannot observe y_t directly, they form Bayesian beliefs based on the sovereign's observable actions: the newly issued debt b_t and the policy signal a_t. Let

$$\hat{y}_t = \mathbb{E}\{ y_t \mid a_t, b_t \},$$

denote creditors' assessment of the sovereign's income. This belief $\hat{y}_t$ enters their pricing decision. Specifically, the bond price q_t solves:

$$q_t = \frac{1}{1+r} \left(1 - \int_0^{+\infty} \mathbb{P}\{\text{default at } t+1 \mid y_t, b_t\} dP(y_{t+1} \mid a_t, b_t) \right) := \frac{1 - \theta_t(\hat{y}_t, b_t)}{1+r},$$

where $\theta_t(\hat{y}_t, b_t)$ is the probability of default perceived by creditors, conditional on their inferred state $(\hat{y}_t, b_t)$.

If the sovereign defaults, its consumption c_t falls to $(1-\tau)y_t$, as in the full-information environment. In contrast to the full-information setting, though, θ_t depends on the sovereign's signal and debt issuance, forcing creditors to update their beliefs about the underlying y_t.

3.2.3 Signaling Equilibrium and Contract Design

In each period, the sovereign simultaneously chooses: a signal $a_t \in \{0, 1\}$, new debt issuance b_t, and consumption $c_t = y_t + q_t(\hat{y}_t, b_t)\, b_t - b_{t-1}$.

Creditors, in turn, update their beliefs: $\hat{y}_t = \mu(y_t \mid a_t, b_t)$, and set bond prices accordingly. By sending a signal $a_t = 1$ (e.g., implementing a costly reform), the sovereign aims to convince creditors that y_t is higher than what would be inferred from $a_t = 0$. This may reduce the perceived default risk θ_t and yield more favorable debt prices q_t. However, if the cost λ of signaling is too high, or the signal is insufficiently credible, the sovereign may forego signaling, incurring higher borrowing costs.

3.2.4 Equilibrium

An asymmetric-information equilibrium consists of

- Sovereign policy rules for $\{c_t, b_t, a_t\}$ that maximize the sovereign's objective, accounting for creditors' belief-updating mechanism.
- A bond pricing function $q_t(\hat{y}_t, b_t)$ that ensures creditors break even, given their inferred state and the sovereign's default rule.
- A belief function $\mu(y_t \mid a_t, b_t)$ consistent with Bayes' Rule wherever possible (off-equilibrium beliefs may be defined as needed to ensure incentive compatibility).

3.2.5 Empirical and Theoretical Motivation

Empirical work underscores the importance of transparency and credible signals in sovereign debt markets. For instance, Gelos and Wei (2005) find that investors demand higher spreads when governmental data are opaque, consistent with the notion that asymmetric information raises perceived default risk. Additionally, Tomz (2007) documents that governments often adopt policies precisely to distinguish themselves from less creditworthy borrowers.

Theoretically, these results connect to contract theory under asymmetric information, in line with *moral hazard* and *adverse selection* concepts. Similar frameworks have been explored by Cole and Kehoe (1998) in partial reputational settings and by Bi (2012) in dynamic contracting contexts, revealing how private information leads to risk premia and potentially multiple equilibria. By explicitly comparing the full-information benchmark and the asymmetric-information environment, one can quantify the welfare and policy implications of transparency, as well as the role of enforcement mechanisms in limiting opportunistic behavior.

Taken together, these contributions advance the understanding of how incomplete information affects sovereign borrowing choices, contract design, and default outcomes. The following sections attempt at developping the solution methods and illustrate the quantitative effects of signaling, debt pricing, and default risk under asymmetric information.

3.3 Model Calibration

This section describes how we select the parameter values for our model, ensuring they are grounded in both theoretical and empirical considerations. Table 3.1 summarizes the key parameters used in the baseline calibration. The chosen values reflect standard practice in the sovereign debt literature, as well as available data on income processes and policy distortions.

3.3.1 Parameter Choices

Discount Factor, β. We set the discount factor to $\beta = 0.96$, implying that the sovereign has a moderate preference for future consumption over present consumption and is relatively "impatient". This range is common in emerging-market sovereign debt studies, aligning with Aguiar and Gopinath (2006) and Arellano (2008). An alternative specification with $\beta = 0.98$ would correspond to a more "patient" sovereign, potentially resulting in lower default probabilities and more stable debt paths.

Risk Aversion, γ. The risk aversion coefficient $\gamma = 2.0$ follows the constant relative risk aversion (CRRA) utility specification that is standard in international macroeconomics. This choice is supported by macro-finance literature (e.g., Eichengreen et al. (2003)) suggesting moderate levels of risk aversion are sufficient to generate realistic consumption responses to shocks.

Output Loss Due to Default, τ. We set $\tau = 0.2$, implying a 20% loss in income whenever a default occurs. Empirical studies such as Tomz (2007) and Reinhart and Rogoff (2009) document substantial output declines and disruptions following default episodes, making a 20% penalty plausible in this context.

Risk-Free Interest Rate, r. We choose $r = 0.03$ for the baseline specification, which is in line with historical risk-adjusted real returns on safe international assets (see Neumeyer and Perri (2005) for emerging economies). Study on "patient" sovereign will consider higher rate, $r = 0.05$, to capture environments with greater global interest rates or higher opportunity costs of lending.

Income Process: Persistence, ρ, and Volatility, σ. The endowment follows a log AR(1) process:

$$\log y_{t+1} = \rho \log y_t + \epsilon_{t+1}, \quad \epsilon_{t+1} \sim \mathcal{N}(0, \sigma^2).$$

We adopt $\rho = 0.9$, reflecting substantial income persistence often observed in emerging market business cycles (Aguiar and Gopinath (2006)). The shock standard deviation $\sigma = 0.1$ yields moderate volatility in income, consistent with the empirical range estimated for output fluctuations across middle-income countries (Neumeyer and Perri (2005)).

Signaling Cost, λ_{signal}. Finally, $\lambda_{\text{signal}} = 0.05$ is used to represent the sovereign's cost of implementing a policy signal or reform under asymmetric information. Empirical work such as Gelos and Wei (2005) suggests that policy transparency can indeed be costly, in terms of administrative resources or political capital, justifying a positive λ_{signal} that affects the sovereign's decision. Variations in this parameter can significantly alter the model's equilibrium, highlighting the trade-offs between signaling credibility and economic distortions. As an extension of this paper, examination of the signalling cost impact on the optimal model solution is needed.

3.3.2 Summary of Parameter Values

Table 3.1: Baseline Parameter Calibration (Impatient Sovereign)

Parameter	**Value**	**Interpretation**
Discount factor, β	0.96	Sovereign's intertemporal preference
Risk aversion, γ	2.0	CRRA utility curvature
Output loss, τ	0.2	Income penalty in default
Risk-free interest rate, r	0.03	Global safe return
Income persistence, ρ	0.9	AR(1) log-income persistence
Income shock std. dev., σ	0.1	Income volatility
Signaling cost, λ_{signal}	0.05	Resource/political cost of policy signal

3.4 Solution Algorithm Using Neural Networks

Our approach to solving the model relies on recent advances in deep learning methods for dynamic economic models. Following ideas presented in Maliar and Maliar (2021) and Azinovic et al. (2020), we employ two separate neural networks to capture the policy functions of the sovereign and the behavior of competitive creditors, respectively. This methodology falls under the category of *Deep Equilibrium Nets* in the sense that each network converges to a stable solution of the underlying equilibrium conditions, consistent with the use of neural approximations to value functions and default probabilities explored by Duarte et al. (2018) and Fernández-Villaverde et al. (2021).

3.4.1 Overview of the Approach

The model features two key agents: a sovereign that sets borrowing (and potentially signaling) decisions, and risk-neutral creditors that price the sovereign's debt. We implement the solution by training two neural networks in parallel. The first network is assigned to the sovereign, while the second network belongs to the creditors.

Sovereign Neural Network. The sovereign's neural network takes as input the current state (b_{t-1}, y_t). In the case of *asymmetric information*, it additionally manages the policy signal a_t. The network outputs three main components: policy function for new debt b_t; policy decision for signaling a_t (relevant only if the model features private information); and the estimated value function $V(y_t, b_{t-1})$. We train this network by minimizing the residuals of the Bellman equation, comparing predicted continuation values with the target values derived from immediate consumption and future states. In other words, the loss function enforces that

$$V(y_t, b_{t-1}) \approx u(c_t) + \beta \mathbb{E}[V_{t+1} \mid (y_t, b_{t-1})],$$

accounting for the state transitions and the possibility of default with the expectation operator approximated through a sampling technique. This procedure parallels the "Deep Value Function" approaches outlined by Maliar and Maliar (2021) and Azinovic et al. (2020), adapted to the specifics of sovereign debt.

Creditors' Neural Network. A second neural network approximates the probability of default perceived by risk-neutral creditors. In the asymmetric-information model, creditors do not directly observe the sovereign's endowment y_t, but infer part of its economic state through observed debt issuance b_t (and policy signal a_t if included). Thus, the inputs to the creditor network are (b_{t-1}, b_t, a_t) or simply (b_t, a_t) depending on how the competitive market interprets the available information. The network outputs θ_t, the default probability, which then feeds into the bond pricing formula

$$q_t = \frac{1 - \theta_t}{1 + r},$$

reflecting the creditors' risk-neutral behavior. Since this network is trained specifically to match observed default outcomes in simulation data, it does not internalize any benevolent objective for the sovereign. Instead, it tracks realized defaults and risk to update θ_t, ensuring the pricing decision is consistent with historical or simulated default frequencies. This separation of objectives follows the perspective advocated by Duarte et al. (2018) and Fernández-Villaverde et al. (2021), where distinct network architectures capture different agents' problems without conflating their incentives.

3.4.2 Training and Convergence

The core of the algorithm is an iterative procedure:

1. **Simulation or State Generation:** We generate a range of states (y_t, b_{t-1}) (and, if applicable, (b_t, a_t)) that the sovereign and creditors might face.

2. **Sovereign Network Update:** Holding the creditor network fixed, we update the sovereign network weights by minimizing the Bellman residual, comparing the network's predicted value function with the target consumption and future values.

3. **Creditor Network Update:** Holding the sovereign network fixed, we train or update the creditor network to predict default outcomes from the simulated data. The target default indicators are derived from the states in which the sovereign is unable to repay while maintaining non-negative consumption. This step ensures θ_t remains purely risk-neutral and does not inadvertently act in the sovereign's interest.

4. **Policy and Bond Pricing Integration:** We recompute consumption and bond pricing using each updated network, generating new data or checking the existing simulations. Steps 2–3 repeat until both networks converge to a stable solution.

In practical implementations, we often shuffle states in mini-batches and use gradient-based methods such as Adam, as recommended by Maliar and Maliar (2021) and Azinovic et al. (2020). We rely on multi-layer perceptrons with GELU nonlinearities to capture the potentially intricate mapping from states to policy or default probability. The final solution arises when both the Bellman residual (for the sovereign) and the classification error (for default events) no longer significantly improve.

Chapter 4

Simulation

In this section, we present the numerical findings for both the full-information model and the asymmetric-information model. All simulations rely on the calibrated parameters described above, with each model solved under the neural-network-based algorithm previously detailed. We compare the borrowing and consumption policies obtained in these two informational regimes and relate them to empirical observations in the sovereign debt literature.

4.1 Full-Information Model

Under full information (see Figure 4.1 and Figure 4.2), creditors perfectly observe the state (y_t, b_{t-1}), and price new debt competitively based on the sovereign's default boundary. In our baseline runs, we find that:

High Previous Debt. When b_{t-1} is high relative to the realized endowment y_t, the sovereign chooses new debt b_t near the maximum allowed by the constraint $|b_t| \leq y_t$. Because the sovereign must repay a large portion of past debt, it issues as much new debt as possible to meet its obligations and preserve non-negative consumption. Although this policy stabilizes consumption in the short term, it implies larger debt rollovers, consistent with an environment in which any negative shock could push consumption to zero. Nevertheless, it avoids immediate default by maxing out the new borrowing capacity.

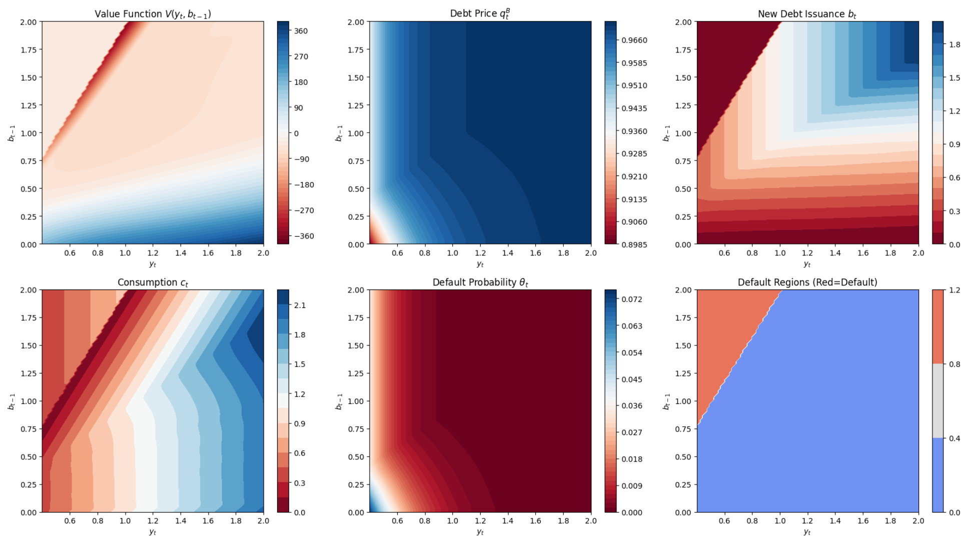

Figure 4.1: **Policy Elasticities under Full Information (Impatient Sovereign)**. This figure illustrates how the sovereign's borrowing choices, consumption, associated value function, and creditor-estimated probability of default change with respect to variations in the state variables (y_t, b_{t-1}). Darker or lighter regions indicate the magnitude of the policy's sensitivity (or elasticity) to the endowment y_t and the previous debt b_{t-1}. In particular, when b_{t-1} is high relative to y_t, the sovereign's borrowing rule approaches the maximum debt limit, reflecting an urgent need to roll over debt and avoid default. Conversely, at lower levels of prior debt, the policy rule indicates more moderate borrowing, highlighting a small precautionary motive in spite of the sovereign's impatience.

Low Previous Debt. Conversely, when b_{t-1} is relatively low compared to y_t, the sovereign refrains from fully utilizing its borrowing capacity. Instead, it chooses $b_t < y_t$, gradually reducing outstanding debt. This pattern suggests some precautionary motive is at play, even though c_t remains far from perfectly smoothed. The result is an overall decline in b_t for these states, effectively indicating that moderate debt levels generate less reliance on external funds.

Consumption Dynamics. Despite the precautionary tendency for lower debt, the simulation indicates limited consumption smoothing. The sovereign's impatience and the calibration lead it to borrow aggressively in states where it sees an immediate benefit. In standard macro models with more patient agents, one often observes greater consumption smoothing, but here consumption is sharply influenced by debt rollovers.[1]

[1] A higher discount factor ($\beta \approx 0.98$ or larger) could bring the model closer to traditional macroeconomic outcomes, yielding better consumption smoothing. Comparison of the results for "patient" and "impatient" sovereigns is nesessary for the extension of this work.

Figure 4.2: **Time-Series Simulation under Full Information (Impatient Sovereign)**. This figure presents a sample simulation path of key variables (endowment y_t, debt b_t, and consumption c_t) in the full-information environment. The sovereign issues near-maximal debt in high-debt states, stabilizing consumption in the short run but raising vulnerability to negative income shocks. When past debt is moderate or low, the sovereign borrows less than its capacity, slowly reducing overall debt levels. Consumption exhibits little smoothing due to the impatience of the sovereign, leading to volatility driven by changes in debt and realized income shocks.

Borrowing Patterns. Overall borrowing remains relatively smooth but not at a constant level. The debt issuance function $b_t(y_t, b_{t-1})$ does not target a fixed steady-state debt level; rather, it adapts to the state, ensuring that default is prevented unless consumption would otherwise be forced below zero.

These findings partially align with empirical observations that emerging-market sovereigns, especially under moderate discount factors, fail to smooth consumption effectively in the face of income volatility (Neumeyer & Perri, 2005). Some evidence also shows that countries with lower debt overhang reduce borrowing to avoid high spreads (Eichengreen et al., 2003), reflecting a pattern similar to our "low previous debt $\implies$ gradually declining new debt" result.

4.2 Asymmetric-Information Model

When creditors cannot directly observe y_t but must infer it from the sovereign's signal (or from debt issuance b_t alone if the signal a_t is absent), our results differ markedly (see Figure 4.3 and 4.4):

Maximal Borrowing. In our baseline asymmetric-information runs, the sovereign systematically chooses $b_t = y_t$, issuing the maximum permissible debt. By doing so, it inadvertently reveals—or at least strongly correlates—new debt issuance with the true income. This helps creditors infer endowment realizations, reducing perceived uncertainty. However, the sovereign's impatience leads it to exploit high income by substantially increasing consumption in good states, which creates heavier debt burdens in future periods.

Default Patterns. Because of the elevated debt rollovers under negative shocks or when y_t drops, the model produces frequent defaults. The sovereign tolerates these defaults due to its high impatience, prioritizing front-loaded consumption. The resulting equilibrium thus shows higher default frequencies than in the full-information model, consistent with the notion that limited transparency can exacerbate borrowing booms and busts.

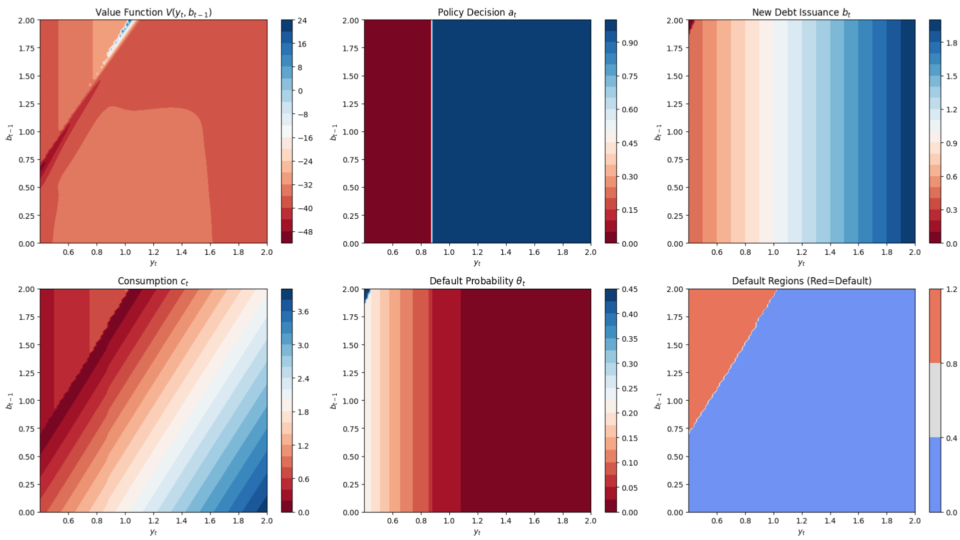

Figure 4.3: **Policy Elasticities under Asymmetric Information (Impatient Sovereign).** This figure illustrates how the sovereign's borrowing policy and associated value function respond to changes in the observed or inferred state variables when creditors have incomplete information about the sovereign's endowment. The color scale reflects the magnitude of policy elasticities with respect to the log-endowment y_t and prior debt b_{t-1}. In contrast to the full-information case, the sovereign tends to issue the maximum allowable new debt $b_t = y_t$, thus revealing (or strongly correlating with) its true income. Because the sovereign is relatively impatient, this policy sacrifices future stability for immediate consumption gains. The result is a pattern of consistently high debt issuance across states where previously one might expect more variation, highlighting the role of information frictions and signaling in shaping borrowing decisions.

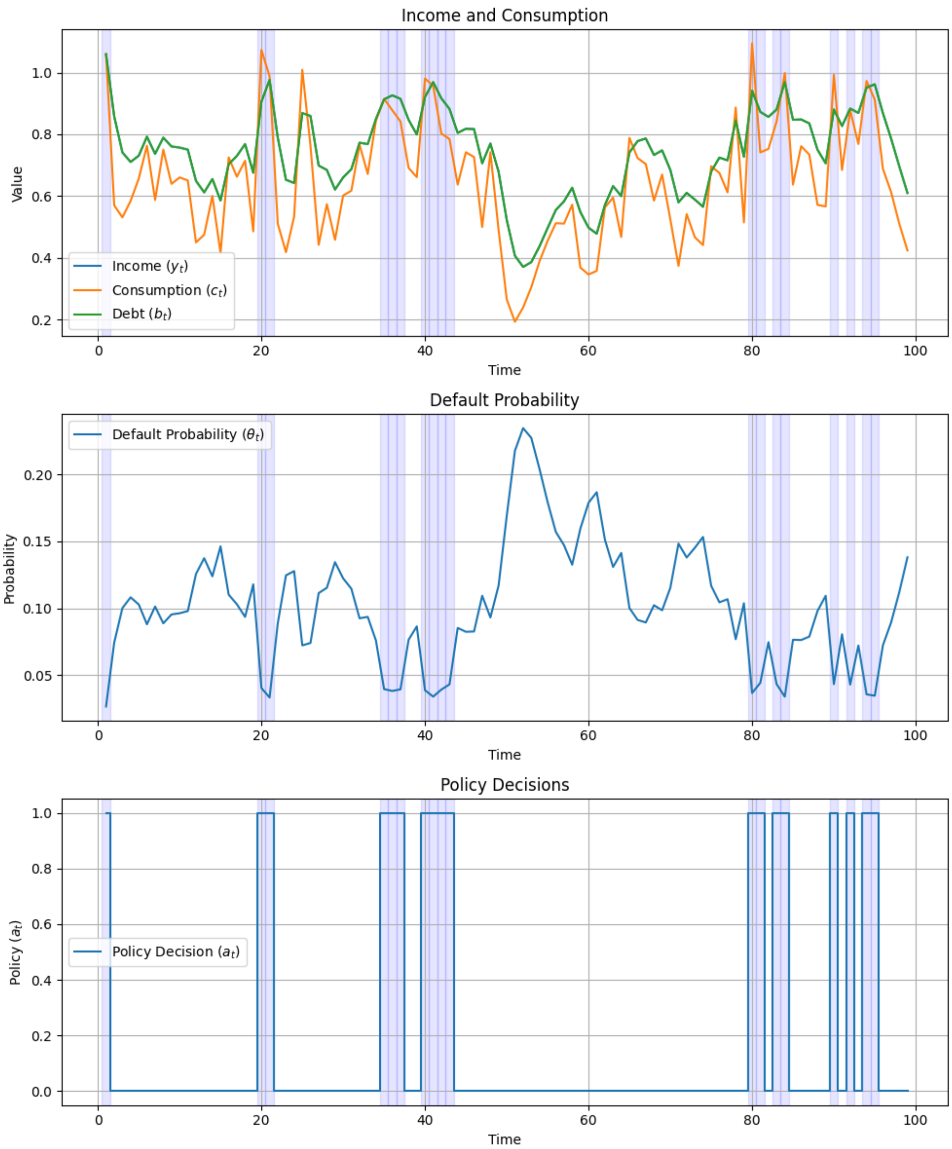

Figure 4.4: **Time-Series Simulation under Asymmetric Information (Impatient Sovereign).** This figure presents a sample simulation path of the model's key variables (endowment y_t, debt b_t, consumption c_t, and any relevant policy signals) when creditors cannot directly observe y_t. Because the sovereign issues near-maximal debt $b_t \approx y_t$, creditors deduce the current endowment from new debt issuance, mitigating some uncertainty. Nevertheless, due to the sovereign's impatience, consumption increases substantially during expansions, leading to a heavier debt burden in downturns and more frequent defaults. This boom-bust cycle, absent in the full-information model, emerges naturally once creditors must infer the sovereign's fundamentals from limited signals.

Threshold Signaling. We also implement a threshold strategy for a_t, defined by $a_t = 1$ if y_t exceeds the 25th percentile of non-defaulting states. This reduces some informational friction by giving creditors a strong signal of "relatively high income," but because of the sovereign's impatience, the fundamental outcome remains: new debt issuance remains at or near $b_t = y_t$, culminating in a cyclical boom-bust cycle. These patterns are in line with certain historical episodes where opaque governments borrowed aggressively in good times, then defaulted when downturns hit (Reinhart & Rogoff, 2009).

Empirical Relevance. Empirical works, such as Tomz (2007) and Gelos and Wei (2005), underline that lack of credibility or transparency can foster higher borrowing in good states, followed by abrupt defaults. This phenomenon emerges naturally in our model, since the sovereign cannot fully commit to lower debt issuance in expansions if the cost of signals is low compared to the short-term gains from borrowing.

4.2.1 Implications for Contracts

From a contractual perspective, these findings suggest that *asymmetric information* can produce contracts with more volatile borrowing and default cycles than under full information. Under incomplete disclosure, the sovereign's best response is to issue the maximum allowed debt to finance consumption, while in high-information regimes it tailors issuance more conservatively—especially after heavy indebtedness. Consequently, one sees that contract design under incomplete information might need additional enforcement or conditionality provisions to limit excessive borrowing, or else it must tolerate higher default risk.

In practice, these results highlight the importance of transparency and credible signals: if the sovereign's signal is costless (or the sovereign is sufficiently impatient), the equilibrium can shift to extreme borrowing strategies. By contrast, a more patient sovereign or stronger transparency constraints can reduce such boom-bust cycles, mitigating default frequencies.

Chapter 5

Conclusion

This study set out to explore how the introduction of incomplete information about a sovereign's income stream affects borrowing decisions, default risk, and contract design. We began with a *full-information* model, where international creditors directly observe the country's endowment and price debt accordingly. Although the sovereign faces unavoidable constraints on borrowing (including a non-negativity condition on consumption), we found that when the sovereign is relatively impatient, its policy often involves near-maximal borrowing under high debt states, thereby limiting consumption smoothing and heightening the risk of default under negative income shocks.

We then extended the analysis to an *asymmetric-information* environment, where creditors cannot observe the sovereign's endowment. In this setting, the sovereign can use debt issuance (and potentially costly policy signals) to reveal or partially mask its economic fundamentals. Paradoxically, our results show that an impatient sovereign might consistently issue *maximum* debt when it is free to do so, because revealing a high endowment accelerates immediate consumption, even if it intensifies default episodes in downturns. This "boom-bust" dynamic underscores the potential for incomplete information to amplify borrowing volatility and default risk.

From a contractual standpoint, our findings suggest that policy interventions focused on transparency—either by compelling the sovereign to disclose reliable data or by requiring more credible signals—could help mitigate these borrowing cycles. Indeed, high-frequency defaults may be stabilized if reforms convincingly indicate robust fundamentals, though the cost of signaling can itself reduce net welfare if not aligned with actual productivity gains. Hence, whether policy actions truly foster credibility or prove too expensive to adopt remains a critical question in practice.

Although the paper relies on a relatively simple one-period debt structure, it highlights the tractability of using deep learning tools to approximate policies and default probabilities. By training separate neural networks for the sovereign and for creditors, we ensure each agent pursues its own objective without artificially intertwining incentives. This setup can be generalized to richer environments, including multi-maturity debt structures, more patient (or impatient) sovereigns, and time-varying interest rates.

Future Work. First, calibrating the model to alternative discount rates and risk-free rates can elucidate the boundary between "patient" and "impatient" regimes, clarifying when consumption smoothing is feasible. Second, extending the signaling component to allow for a continuum of policy intensities (rather than binary signals) might more closely align with real-world structural reforms. Finally, reintroducing long- and short-term debt under incomplete information, along with the possibility of renegotiation, would capture a deeper range of empirical sovereign debt patterns—albeit at the cost of more complex solution algorithms.

By focusing on these narrower yet tractable models, we have demonstrated how modern computational methods yield valuable insights into the interplay of information asymmetry, debt issuance, and default.

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