Modulation Transfer Function Compensation through a modified Wiener Filter for spatial image quality improvement

JESSICA WONG SOO MEE
Malaysia Space Centre
National Space Agency of Malaysia (ANGKASA)
Lot 2233, Jalan Turi, Sg. Lang, 42700, Banting, Selangor
MALAYSIA
jessica@angkasa.gov.my http://www.angkasa.gov.my

Abstract: - The knowledge of Modulation Transfer Function (MTF) for a degraded image provides a mean to compensate for the image degradation, which improves the image quality in terms of sharpness. MTF compensating is in the image restoration; hence it is fundamentally an ill-posed problem. This paper proposes a stable and flexible filtering technique that executes an optimal tradeoff between sharpness and noise to warrant an acceptable result of image restoration. The MTF compensating is performed through a modified Wiener filter, and can be reduced to a well-posed problem by incorporating the regularization method. The modified Wiener filter employed the L-curve method for selection of optimal regularization parameter, and introduces an inverse control and smooth control parameter that allows freedom of tuning when the importance of image sharpness versus noise is being compromised. The data sets used in the analysis were synthetically blurred remotely-sensed images simulated from Level-2A product of IKONOS. The results by modified Wiener filter were analyzed and presented, they were found to be able to reduce the *Mean Square Error* by more than 50% with respect to the original image, and it results in a significant improvement of spatial image quality.

Key-Words: - Modulation Transfer Function, Spatial image quality, Wiener filter, Image restoration, Ill-posed problem, L-curve

1 Introduction

Spatial image quality is one of the key parameters for characterizing and validating image data. The technical characteristics of image data must be well understood before image analysis can be done, since the quality of the analysis depends on the quality of the data. Image quality can be measured by a vast number of factors, such as contrast, brightness, noise variance, sharpness, radiometric resolution, granularity, modulation, contrast transfer function and many more [1]. Among other measures, image sharpness is vital for characterizing images, for much of the information of an image resides in its edges. Image blurriness, which limits the visibility of details, can be objectively measured by the point spread function (PSF), or its amplitude spectrum, which is the modulation transfer function (MTF). Spatial image degradation can happen in many ways. For satellite imaging, image acquisition occurs while orbiting the earth, and due to the satellite's altitude determination control for maneuvering, the instantaneous field of view (IFOV) of the imaging system can be greater or lesser than the nominal resolution at any point in time during image capturing. This results in MTF degradation proportional to the ratio of IFOV and Ground Sample Distance (GSD) [2]. Moreover, regardless of how well an image system is fabricated, it will inevitably suffer from some degree of blur. These sorts of degradations need to be compensated, and they can be compensated through the knowledge of the degradation function which is the MTF for that image restoration. MTF compensating is in the image restoration; hence it is fundamentally an ill-posed problem. This paper proposes a stable and flexible filtering technique that executes an optimal tradeoff between sharpness and noise to warrant an acceptable result of image restoration.

2 MTF Compensation

MTF Compensation (MTFC) is a restoration technique; Restoration techniques find filters that apply inverse process in order to recover its original image. In image restoration, the main challenge is to prevent noise of input data from being amplified to unacceptable artifacts in the restored image. Image restoration is ill-posed, which means the MTFC is also an ill-posed problem, meaning that it does not have a unique solution, even with the absence of noise [3]. However, scientists and engineers are usually less concerned with existence and uniqueness of the solution to the ill-posed problem and worry more about the stability of the solution. With the concept of regularization, the ill-posed problem of the existing MTFC can be reduced to a well-posed problem by introducing a regularization parameter as *a priori* constraints to produce a stable and flexible deconvolution filter for image restoration. In this case, it is stable in the sense that it will always produce an acceptable image restoration result, while also being flexible by allowing freedom of tuning when the importance of image sharpness versus noise is compromised.

2.1 Deconvolution Filter

As a deconvolution filter, Wiener filtering is an inverse problem, therefore it is likely to be ill-posed. Regularization is necessary while solving inverse problem because of the "naïve" least squares solution by Wiener filter. By using regularization, the error contribution can be damped and the residual norm can be kept in reasonable size. In other words, by incorporating regularization parameter into the Wiener filter, it is possible to control the power of inverse filtering, and subsequently minimizes the amplification of noise in the image. The Wiener filter has been used by other researchers [4] [5] as an MTFC technique to compensate for the degradation of remotely-sensed images. And it also has been used by Shacham *et al*.[6] as blind restoration technique to compensate for the degradation of astronomical data. However, none of them applied the regularization method in their filters, thus a modified Wiener filter based on the idea in the regularization method is employed in this work.

2.1.1 The Modified Wiener Filter

In this paper, the proposed deconvolution filter considers images and noise to be a random process, and is rooted in one of the most widely used deconvolution filters, which is the Wiener filter. In order to regularize the inverse problem of the Wiener filter, the concept of the Tikhonov regularization was adopted. According to Wiener's theory [7], given a degraded image $g(x,y)$, the degraded image can be restored by $W(u,v)$ in a Fourier sense, as shown by the following equations:

$$W(u,v) = \frac{H^*(u,v)S_f(u,v)}{|H(u,v)|^2 S_f(u,v) + S_n(u,v)} \tag{1}$$

$$= \frac{H^*(u,v)}{|H(u,v)|^2 + \frac{S_n(u,v)}{S_f(u,v)}} \tag{2}$$

$$= \frac{1}{H(u,v)} \cdot \frac{|H(u,v)|^2}{|H(u,v)|^2 + \frac{S_n(u,v)}{S_f(u,v)}} \tag{3}$$

where $H(u, v)$ represents the degradation function; $S_n(u,v)$ represents power spectrum of the noise; $S_f(u,v)$ represents power spectrum of the undistorted image and the pair (u,v) represents the location in the spatial frequency domain.

From equation (3), the Wiener filter can be perceived to have two separate parts: an inverse filtering part and a noise smoothing part. It not only performs the deconvolution by inverse filtering but also removes the noise with a compression operation. These properties were advanced to solve the research problem.

In the absence of noise (i.e. $S_n(u,v) = 0$), the Wiener filter reduces to the inverse filter. On the other hand, in the presence of noise (i.e. $\frac{S_n(u,v)}{S_f(u,v)} > 0$), the denominator of (2) is never zero. In the limiting case $S_n(u,v) \to 0$, then

$$W(u,v) = \begin{cases} \frac{1}{H(u,v)} & H(u,v) \neq 0 \\ 0 & H(u,v) = 0 \end{cases} \tag{4}$$

thus no ill-posed problems arise in Wiener filtering. The term $\frac{S_n(u,v)}{S_f(u,v)}$ in (3) can be regarded as a term that smooths the inverse filter $\frac{1}{H(u,v)}$. However, for frequencies at which $S_f(u,v) \geq S_n(u,v)$, estimation of $W(u,v)$ becomes nearly equal to $\frac{1}{H(u,v)}$, which means the Wiener filter behaves as an inverse at the frequencies (u,v). At high spatial frequencies, $S_f(u,v)$ is often much larger than $S_n(u,v)$, thus condition $S_f(u,v) \geq S_n(u,v)$ holds true. For degradation such as blur, $|H(u,v)|$ at high spatial frequencies is relatively small. When (u,v) approaches a region with a large value of $\sqrt{u^2+v^2}$, $|H(u,v)|$ is nearly zero. Therefore, when $S_f(u,v) \geq S_n(u,v)$ and $|H(u,v)| \approx 0$, the Wiener estimate $w(u,v) = \frac{1}{H(u,v)}$ becomes ill-posed. Consequently, this estimate will lead to a poor restoration result.

For the ill-posed inverse problem to be solved, it is necessary to regularize the solution by introducing *a priori* constraints [8]. This can be done by using a numerical regularization parameter to control the power of the inverse filter, thereby penalizing those (u,v) with very high spectral frequencies. The $\frac{S_n(u,v)}{S_f(u,v)}$ in equation (3) is assumed to be constantly proportional to the inverse of the signal-to-noise ratio (SNR) of the image. This leads to the following modification of $W(u, v)$ in equation (3):

$$W(u,v) = \frac{1}{H(u,v)} \cdot \frac{|H(u,v)|^2}{|H(u,v)|^2 + \frac{\lambda}{SNR}} \tag{5}$$

Thus, the frequency domain solution to optimize the degradation problem is expressed as

$$\hat{F}(u,v)=\left[\frac{1}{H(u,v)}\cdot\frac{|H(u,v)|^2}{|H(u,v)|^2+\frac{\lambda}{SNR}}\right]G(u,v) \qquad (6)$$

where $\hat{F}(u,v)$ represents the restored image, $G(u,v)$ represents the degraded image, and λ represents the regularization parameter. The regularization parameter was determined by using the L-curve method. The L-curve is a parametric plot of the size of the regularized solution and its corresponding residual [9]. The underlying idea is that a good method for choosing the regularization parameter for discrete ill-posed problems must incorporate information about the solution size, in addition to using information about the residual size. The L-curve method requires both the regularized solution and its corresponding residual to find the corner of the L-shape curve for the optimal selection of regularization. With the underlying degradation is the linear, position-invariant degradation that is described in a spatial domain by

$$g = Hf + \eta \qquad (7)$$

where g represents the degraded image, f represents the original image, matrix H represents the degradation function, and η represents the additive white noise. In this case, the knowledge of noise variance is not required. Thus the regularization problem can be expressed using the Tikhonov regularization as

$$\left\|g-H\hat{f}_\lambda\right\|^2+\lambda\left\|\hat{f}_\lambda\right\|^2 \qquad (8)$$

Let $\|r\|^2$ be the "residual" vector described by

$$\|r\|^2=\left\|g-H\hat{f}_\lambda\right\|^2 \qquad (9)$$

The $\|r\|^2$ can be calculated using the following equation:

$$\|r\|^2=\sum_{y=0}^{N-1}\sum_{x=0}^{M-1}r^2(x,y) \qquad (10)$$

where $r^2(x,y)=\Im^{-1}\left[G(u,v)-H(u,v)\hat{F}(u,v)\right]$ (11)

substituting the right side of equation (11) for $\hat{F}(u,v)$ in equation (6) yields

$$r^2(x,y)=\Im^{-1}\left[\frac{\left[G(u,v)\frac{\lambda}{SNR}\right]^2}{\left[|H(u,v)|^2+\frac{\lambda}{SNR}\right]^2}\right] \qquad (12)$$

where $\Im^{-1}$ represents inverse Fourier transform. Meanwhile, $\left\|\hat{f}_\lambda\right\|^2$ can be calculated using the following equation:

$$\left\|\hat{f}_\lambda\right\|^2=\sum_{y=0}^{N-1}\sum_{x=0}^{M-1}\hat{f}^2(x,y) \qquad (13)$$

where $\hat{f}^2(x,y)=\Im^{-1}\left[\hat{F}(u,v)\right]$

The procedures for determining the regularization parameter are as follows:

- **Step 1:** Specify an initial value of λ
- **Step 2**: Calculate $\|r\|^2$ and $\left\|\hat{f}_\lambda\right\|^2$
- **Step 3**: Gradually increase λ until $\lambda = 1$;
- **Step 4**: Convert both $\|r\|^2$ and $\left\|\hat{f}_\lambda\right\|^2$ to logarithmic functions.
- **Step 5**: Plot the L-curve in log-log scale (see Fig.1).
- **Step 6:** Trace the transition of the shape of the corner of the L-curve plot, and mark the optimal λ,
- **Step 7:** Use the optimal λ *t*o compute the optimal estimate $\hat{F}(u,v)$

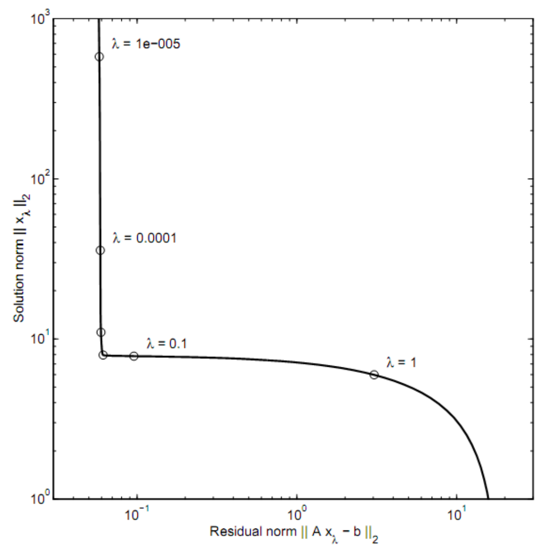


Fig.1. L-curve for Tikhonov regularization [10]

As mentioned earlier, the Wiener filter can be perceived to have two separate parts: an inverse filtering part which contributes to the sharpness for deblurring, and a noise smoothing part for denoising. Therefore it is possible to control the regularization and inverse power independently. Subsequently, it allows more degrees of freedom for image restoration that incorporate both deblurring and denoising techniques in one single filter. Hence the proposed filter is as expressed as:

$$W(u,v)=\left(\frac{1}{H(u,v)}\right)^\beta\cdot\left(\frac{|H(u,v)|^2}{|H(u,v)|^2+\frac{\lambda}{SNR}}\right)^\gamma \qquad (14)$$

where β and γ represent the inverse control parameter and the smooth control parameter respectively.

3 Experimental Results

The original image used in this work is presented in Fig. 2; it is one part of Level 2A product of IKONOS, containing 7800 x 7800 pixels.

Fig. 2. The original image

Fig. 3 presents one of the blurred version of the images obtained by using the inverse Fourier transform, by multiplying the original (in Fourier sense) with a Gaussian filter. The Gaussian filter function, $H_G(u,v)$ which approximates the optical point spread function in the frequency domain is expressed as:

$$H_G(u,v) = e^{\frac{-(u^2+v^2)}{2\sigma^2}} \tag{15}$$

where σ is the standard deviation of Gaussian curve. The standard deviation, σ of Gaussian function for this blurred image was set to 80. The data sets for this work consist of sub-scenes extracted from this synthetically blurred image.

Fig. 3. A synthetically blurred image

The proposed MTFC algorithm was applied to data sets and evaluated quantitatively using two metric, namely *Mean-Squared Error* (MSE) and *Peak Signal-to-Noise Ratio* (PSNR), which are defined as follows:

$$MSE = \frac{1}{MN}\sum_{y=0}^{N-1}\sum_{x=0}^{M-1}[f(x,y) - \hat{f}(x,y)]^2 \tag{16}$$

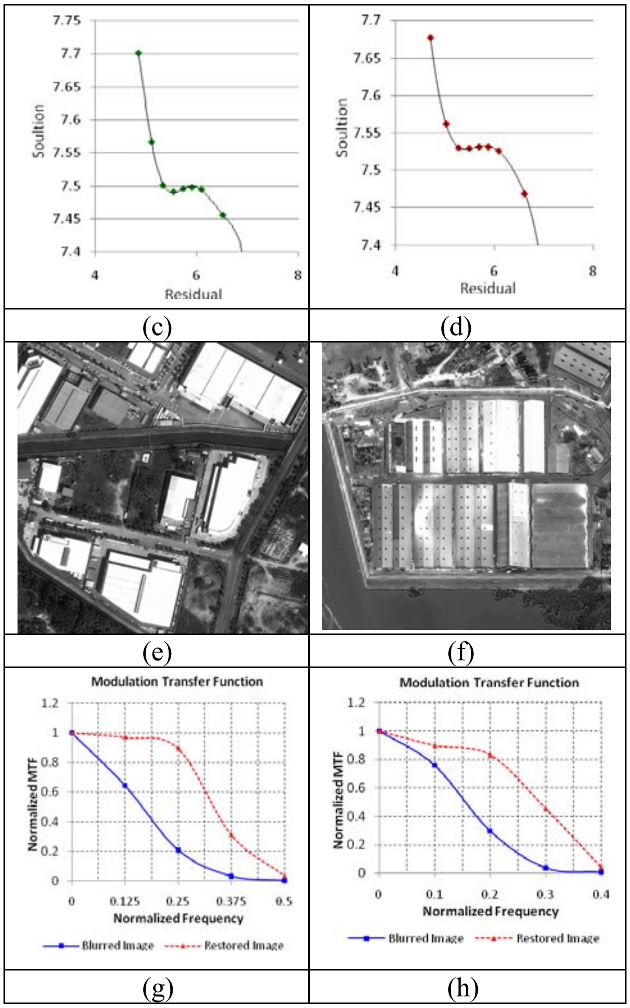
$$PSNR = -10\log_{10}\frac{MSE}{S^2} \tag{17}$$

where f, $\hat{f}$ and S represent the original image, the restored images and the maximum pixel value of the image, respectively.

Two versions of blurred images were used to present the experimental result of MTFC. The standard deviation, σ of Gaussian function for these images is 50 and 80, respectively (see Fig. 4(a) and (b)). The degradation function, $H(u,v)$, which is the MTF of the degraded image, was modeled using the measured MTF by edge input method. The measured (estimated) MTF were compensated using the modified Wiener filter as described in Equation (14) to restore these images. The regularization parameter, λ was tuned between 10^{-6} and 10^2. Experimental result shows that regularization parameter, $\lambda = 10^{-1}$ is relatively good to optimize the regularization of restoration problem. The L-curve plot for regularization process of the blurred image in Fig. 4(a) and (b) is presented in Fig. 4(c) and (d), respectively.

By visual observation, it is clear that the restored images in Fig. 4(e) and (f) are considerably better than their respective blurred image shown in Fig. 4(a) and (b). It can be observed that the blurred images are sharpened without the side effect of noise amplification. Typically, blurred images are assumed to have a lower quality as compared to the restored images. Therefore the MTF for the blurred images are expected to plummet to lower MTF values as compared to the restored images. Under this circumstance, MTF area under the MTF curve after restoration will be larger than those before restoration. This supposition was confirmed by the two MTF plots depicted in Fig. 4.

(a)	(b)



(c) (d) (e) (f) (g) (h)

Fig. 4. Restoration results; (a) Blurred image, σ = 50 with its L-curve plot (c), restored image (e) and MTF plot (g); (b) Blurred image, σ = 80 with its L-curve plot (d), restored image (f) and MTF plot (h);

One criterion for the restoration quality is the ratio between the areas under the MTF curve after and before restoration. The MTF area (MTFA) ratio for the restored image in Figure 4(e) and (f) are 2.08 and 1.84, respectively. Table 1 shows other forms of quantitative results that quantified the image quality improvement.

Table 1: Quantitative measures for the restored images

Performance Measure	Fig. 4(a)	Fig. 4(e)	Fig. 4(b)	Fig. 4(f)
PSNR (dB)	23.17	26.70	25.29	29.07
MSE	313.24	139.09	192.37	80.48

For comparison purpose, the classical Wiener filter was implemented using Equation (3). Both classical and modified Wiener filter were applied to the synthetically blurred image shown in Figure 4(a). The restored images obtained from both classical and modified Wiener filter are presented in Figure 5(a) and 5(b), respectively. Qualitative measures based on visual observation found that 5(b) displays a more enhanced details as compared to 5(a). Fig. 6 shows the comparison of MTF plot between these two filters; MTF area under the MTF curve for the modified Wiener filter are obviously larger than the classical Wiener filter, which means that it produces a better image quality.

(a) Image restoration by classical Wiener Filter

(b) Image restoration by the modified Wiener filter

Fig.5. Comparison of image restoration between the classical Wiener filter and the modified Wiener filter

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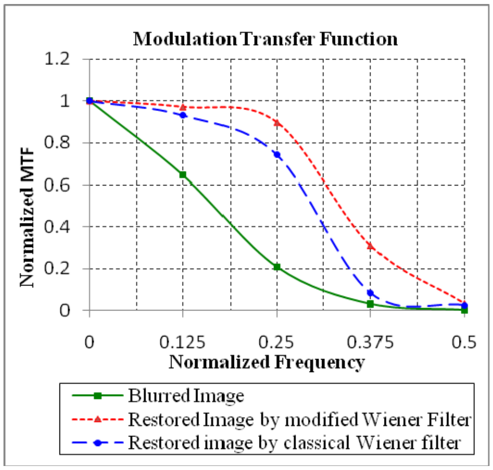

Fig. 6. MTF Comparison between classical and the modified Wiener filter

Table 2 summarizes the performance assessment of these filters; the results demonstrated that the modified Wiener filter was indeed able to produce a better quality of restored image.

Table 2: Performance assessment of classical and the modified Wiener filter

Filter	PSNR (dB)	MSE
Classical Wiener Filter	23.78	283.98
Modified Wiener Filter	26.70	139.09

4 Conclusion

This paper describes a stable and flexible MTFC technique through Wiener filter that aims to execute an optimal tradeoff between sharpness and noise to warrant an acceptable result of image restoration for spatial image quality improvement. The technique is based on derivation of the spatial frequency response to edge target input, which is the MTF measurement. The derived MTF measurement was used to model the degradation function to compensate for image degradation. The main novelty this filter is the incorporation of L-curve regularization method into the classical Wiener filter, which then becomes the modified Wiener filter to overcome the ill-posed problem of image restoration. The algorithm was applied to synthetically blurred images. Restoration results, using the estimated MTF and L-curve regularization show significantly improved of spatial quality image, where the restored image by the modified Wiener filter has the Mean Square Error (MSE) reduced by more than 50% with respect to the original image. The ability of the modified Wiener filter to invert blur and suppress noise amplification at the same time demonstrated the robustness of the proposed MTF compensation technique.

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